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Design of Four-wave Oscillating Cellular-Neural-Network

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Abstract: This paper proposes a method to analyze the behavior of the oscillating CNN (Cellular-Neural-Networks), and develops a way to design this system of Four-waves. After investigating the behaviors of the CNN cells, it provides a simplified model named One-cell model to calculate the frequency and the amplitude of the oscillating wave. And it finds a way to make CNN cells synchronized with fewer subharmonics. Then a method is presented to design the four-wave CNN array. Some corresponding simulation results are present to demonstrate the consistence between the results calculated mathematically and those simulated with SPICE. Simulation results on the designed 32×32 CNN array demonstrates that the designed system can synchronize well with four waves.

Keywords: Cellular-Neural-Networks, Four-wave, synchronization

Classification: Science and engineering for electronics

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1 Introduction

With a regular locally interconnected array, CNN (cellular neural networks) can be easily implemented by VLSI technology. The behavior of this nonlinear system provides a versatile of models for varieties phenomena of real systems. The study and realization of CNN has been received considerable attention. Many researches have studied the dynamical behavior of the CNN related to the values of network parameters, such as coupling coefficients between neurons. Some of CNN applications in image processing, pattern recognition, biology and combinatorial optimization problems have been reported in [1, 2, 3]. Among various behaviors of CNN, synchronization is a key feature. Thus, the stability and synchronization phenomena of the oscillating CNN becomes an important study topic [4]. The complete stability of planar piecewise linear dynamical systems consisting of two cells was studied in [5].

The CNN array can be described and analyzed by several methods. It can be described mathematically as a set of equations, and some software like MATLAB, C or other special programs can be used for analysis. Methods presented up to now have been mostly with these methods and the analysis is time consuming and with less electrical meaning. We simulate and analyze the CNN array both mathematically and at circuit level with SPICE. Based on the corresponding simulation results, a simplified model is built to investigate the multi-phase synchronization of oscillating CNN. The behaviors of verities CNN cells are investigated firstly. Then a simplified model is presented to analyze the oscillating frequency and amplitude. Based on the investigations, we develop a way to design a four-wave CNN array, and give the simulation results.

2 Description of the CNN Array of 32×32 Cells

The proposed CNN is a spatial array of 32×32 cells, whose configuration is shown in Fig. 1 (a). Each cell, as shown in Fig. 1 (b), consists of a linear inductor L, a linear capacitor C, an nonlinear conductance G and four controlling current sources whose expressions are

$$G = f(V_k, \sin) = \frac{\alpha V_k}{\beta - \sin(\gamma V_k)} \quad (k \in (1, 2, 3, \cdots, 1024)) \tag{1}$$

or

$$G = f(V_k, \cos) = \frac{\alpha V_k}{\beta - \cos(\gamma V_k)} \quad (k \in (1, 2, 3, \cdots, 1024)) \tag{2}$$

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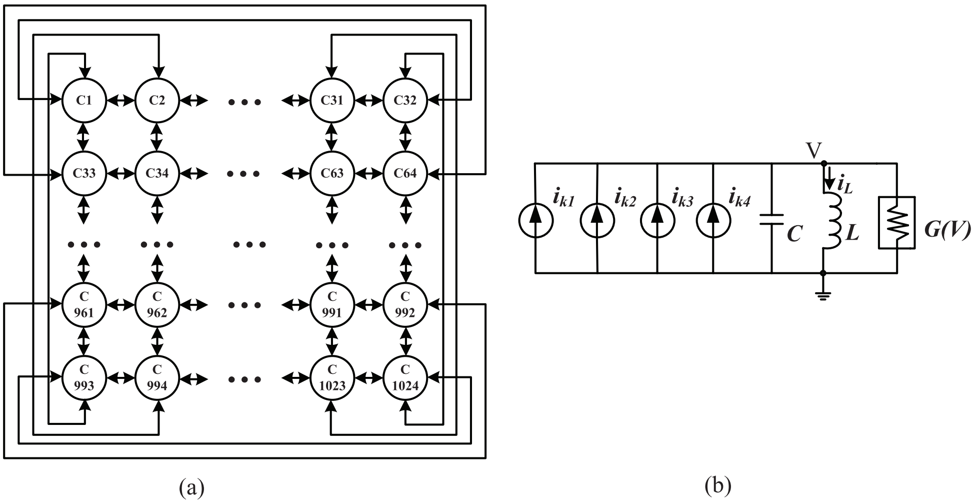

Fig. 1. (a) The proposed CNN array of 32x32 cells. (b) One cell for the CNN array

where k is the serial number of a cell, V is the voltage across the capacitor C at node k, and α, β and γ are real parameters. For simplicity, either (i, j) or (k) ($k = (i-1)\times(maximum\ of\ a\ row)+j$) can be alternatively used to denote a cell at i-th row and j-th column. The symbol ki denotes i-th neighborhood cell of the cell k. The coupling function, which defines the coupling of cell k with the voltage V_{ki} of its neighbor cells ki ($ki \in (1,2,3,4)$), can be expressed as

$$I_{(k,ki)} = \frac{1}{2}A\cdot(|V_{ki}+1| - |V_{ki}-1|) \quad (ki \in (1,2,3,4)) \tag{3}$$

Each cell of the CNN array is a dynamical system, and satisfies the following set of state equations

$$C\frac{dV_k}{dt} + i_L + f(V_k) = \sum_{ki=1}^{ki=4} I_{(k,ki)} \tag{4}$$

$$I_L = \frac{1}{L}\int_{0+}^{t} V_k dt \tag{5}$$

$$V_k(0-) = \text{constant} \tag{6}$$

where I_L denotes the current through the inductor L at node k, and V_k (0-) denotes its initial voltage.

2.1 Behavior Analysis

There are many factors contribute to the behaviors of the CNN cells. These factors can be summarized into four types: 1) Parameters of the cell: L, R, C, A, α, β, γ, 2) Controlling current sources of a cell, 3) Initial states of a cell, and 4) Arrangements of cells. Factors of types 1) and type 2) dominate a cell's intrinsic feature, such as oscillating frequency and amplitude. For a given circuit, the parameters in (1) to (5) are given, so the initial values and cells locations become controlling factors affecting the behavior of the CNN. Combination of mathematical expression and SPICE description provides a better solution to the convergence in the calculation and the computational

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complexity. We built model for (1)-(6) mathematically, and investigate its behaviors. For the whole array we describe and simulate it at circuit level with HSPICE.

From above discusses it can conclude that to make the cell of the CNN at node k synchronized with its four neighbors, the requirements are as follows:

1) the four waves of its neighbors should be synchronization first.

2) the phase of the wave at node k should be opposite to the phases of its neighbors.

3) the initial states of all cells can not be the same.

Combing above analysis results and (4), (5) and (6), we develop a simplified model to describe the synchronizing cells of the CNN array

$$\begin{cases} C\frac{dV_k}{dt}+\frac{1}{L}\int_{0+}^{t} dV_k dt+f(V_k)=4A\cdot V_{ki}\approx -4A\cdot V_k(\,|V_{ki}|<1)\,|V_k(0-)|=V_m \\ C\frac{dV_k}{dt}+\frac{1}{L}\int_{0+}^{t} dV_k dt+f(V_k)=4A(\,|V_{ki}|\geq 1)\,|V_k(0-)|=1(\mathrm{V}) \end{cases} \tag{7}$$

This model is named one-cell model in this paper. Without losing generation, assuming the stable solution for (7) is

$$V_k(t)=V_m e^{j\omega t} \tag{8}$$

where V_m is the amplitude of the voltage at node k, and ω is its oscillating angular frequency.

Also because there is constrain for the first equation in (7), we only use the second equation in (7) to analyze oscillating frequency and amplitude. Substituting (8) into (7), we can get

$$j\omega C V_m e^{j\omega t} + \frac{1}{j\omega L} V_m e^{j\omega t} + \frac{\alpha V_m e^{j\omega t+\pi}}{\beta - \sin(\gamma V_m e^{j\omega t+\pi})} = 4\cdot A \tag{9}$$

Let $\omega t = \pi/2$. For the real part and image part of (12), we get respectively

$$\omega C V_m - \frac{1}{\omega L} V_m = 0 \tag{10}$$

$$\alpha V_m - 4A\beta + 4A\sin(\gamma V_m) = 0 \tag{11}$$

The simulation data of the frequency and the amplitude is shown in Table I. It can be seen that there are little differences between the frequencies calculated with MATLAB and simulated with SPICE. Also, the calculated frequency varies little when the values of parameters change greatly. As for the amplitude, the differences between the values calculated with MATLAB and simulated with SPICE are less than 10% except all γ, A=0.03, 0.12 and $\alpha = 0.03$. That is to say, simulation results of the CNN with MATLAB and SPICE are coincidence. It can also be seen from Table I that the amplitude of the calculated waves varies nearly proportional to A, whereas inversely proportional to α. When γ changes from 0.2 to 5, the amplitude simulated with SPICE varies not so greatly, only from 3.93 to 4.56, and the results calculated with MATLAB change a little, form 2.17 to 4.99. When β changes from 1.7 to 6.8, the amplitude simulated both with MATLAB and SPICE enlarges more than three times. It can be concluded that α, A, and β affect the amplitudes of the CNN waves greatly, whereas contribute a little to the amplitude. L and C dominate the oscillating frequency.

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Table I. Simulation results with MATLAB and SPICE

Parameter		$Vm(V)$		f (KHz)	
		MAT	SPICE	MAT	SPICE
α	0.03	9.69	12.57	2.52	2.41
	0.06	9.12	8.42		2.41
	0.12	3.19	3.59		2.41
γ	0.2	3.1	4.46		2.41
	0.5	2.17	3.93		2.4
	5	4.99	4.56		2.41
A	0.03	2.9	2.19		2.41
	0.08	5.77	5.89		2.41
	0.12	6.53	8.67		2.38
β	1.1	3.11	3.23		2.21
	1.7	3.37	3.89		2.38
	3.4	9.37	9.98		2.39
	6.8	16.13	20.81		2.41

Default values are α=0.09, β=1.7, A=0.06, γ=2, L=200 μH and C=20 μF.

3 Designing Oscillating CNN of Four-Waves

Based on above research results, this paper presents a way to design the synchronization CNN array of 32×32 cells that can generate four different waves, which is named four-wave CNN. The requirements for four-wave CNN are 1) Four types of cells; 2) Appropriate arrangements of cells; 3) Appropriate initial state assignment to all cells.

The basic requirement to generate oscillating CNN of four-wave is to design four different types of cells. The configuration of the 32×32 cells is the same as shown in Fig. 1 (a), and the construction of each cell is similar to that in Fig. 1 (b). However, there are four types of nonlinear conductance, whose expressions are

$$\text{TypeA}: \begin{cases} f(V) = \frac{\alpha V}{\beta - \sin(\gamma V)} \\ I = A \cdot \frac{|V_{ki}+1| - |V_{ki}-1|}{2} \end{cases} \qquad \text{TypeB}: \begin{cases} f(V) = \frac{\alpha V}{\beta - \cos(\gamma V)} \\ I = A \cdot \frac{|V_{ki}+1| - |V_{ki}-1|}{2} \end{cases}$$

$$\text{TypeC}: \begin{cases} f(V) = \frac{\alpha V}{\beta - \sin(\gamma V)} \\ I = A \cdot \frac{|V_{ki}-1| - |V_{ki}+1|}{2} \end{cases} \qquad \text{TypeD}: \begin{cases} f(V) = \frac{\alpha V}{\beta - \cos(\gamma V)} \\ I = A \cdot \frac{|V_{ki}-1| - |V_{ki}+1|}{2} \end{cases}$$

In all types, $ki \in (1, 2, 3, 4)$.

Four-wave CNN also need appropriate arrangements for cells. A careful examination reveals that the four types of cells should be arranged in the following way: the same type of cells should be arranged along the same diagonal line, and for different diagonal lines the type of cells should be in orders as Type A, type B, Type C, Type D, Type D, Type C, Type B, Type A, Type A, ..., as shown in Fig. 2 (a). These symmetries are very similar to the Four-Element Tours that Professor Chua summarized in [6].

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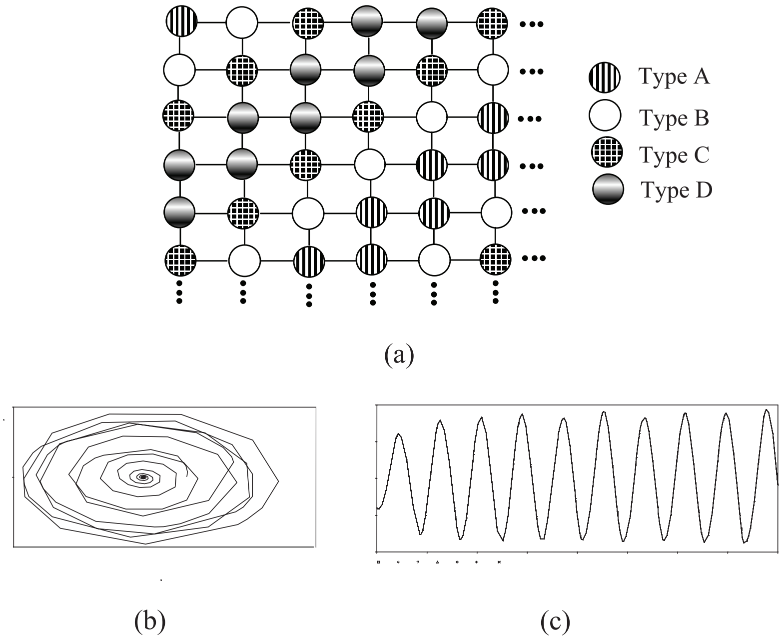

Fig. 2. (a) Arrangement for cells of Four-wave oscillating CNN, (b) I-V curve of a cell, (c) Voltage curves of Type A cells in row 1

Initial states contribute greatly to the behavior of the CNN. For Four-wave CNN, more carefully calculations are needed to select initial states. In simulation, we only simply assign V(1) =4 V, and assign other initial states of cells to 0 V. Fig. 2 (b) shows the wave of the current-voltage curve of a cell. It can be seen that the cells in the designed array are stable. Fig. 2 (c) shows the waves of Type A cell in the first row. It can be seen that the voltages in all cells become stable in little time and demonstrate perfect synchronization; even no high orders of waves can be seen.

4 Conclusion

This paper investigates the behavior of the oscillating CNN comprehensively, and develops a method to design the complex Four-wave array. Instead of simulating and analyzing the CNN array only mathematically, it also builds a mode at circuit level in SPICE. It finds that the requirements to make a cell synchronize with its neighbors, and develops a simplified model named one-cell model. With this simplified model, some important properties of this nonlinear array can be easily drawn. It found that initial states of CNN cells have great influence on the stabilization and synchronization of the CNN array, and can even ease the synchronization phenomena. Based on the research results, the Four-wave CNN is designed, and simulation results are provided to demonstrate the correctness of the proposed methods.

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