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Actuator saturation control of uncertain structures with input time delay

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ABSTRACT

This paper presents a robust saturation control approach for active vibration attenuation of building structures involving parameter uncertainties and input time delay. The parameter uncertainties are described in both polytopic and norm-bounded forms and represent the variations of floor masses, stiffnesses and damping coefficients. The input time delay can be time-varying within a known bound. In terms of the feasibility of certain delay-dependent linear matrix inequalities (LMIs), a state feedback controller can be designed to guarantee the robust stability and performance of the closed-loop system in the presence of parameter uncertainties, actuator saturation, and input time delay. The effectiveness of the proposed approach is investigated by numerical simulations on the vibration control of a three-storey building structure subject to seismic excitation. It is validated that the designed robust saturation controller can effectively suppress the structural vibration and keep the system stability when there are parameter uncertainties and input time delay.

1. Introduction

Vibration control of building structures subject to seismic excitation has been an active topic for decades, and a lot of research effort has been devoted to the development of advanced control algorithms and devices. When designing a controller for active vibration control of structures, besides the control performance that must be considered, some practical issues should be considered in the controller design process as well.

One of the important issues is the actuator saturation problem because any actuation mechanisms are subject to inherent physical limitations. The saturation on actuator capacity takes on added importance in structural applications, and in earthquake design in particular [1]. Vibration control of nominal structures subject to actuator saturation has been studied by some researchers, see for example [1–3], and the references therein. Another one of the most critical issues is the parameter uncertainty problem as it can affect both the performance and the stability of the control system. Parameter uncertainties may come from modelling errors, variations in material properties, and changing load environments, which make system description for the structural models inevitably containing uncertainties [4]. For uncertain structural systems, robust controller design considering both parameter uncertainties and actuator saturation was recently studied by, for example, [5–7]. However, in these studies, only the variations of floor stiffnesses and damping coefficients can be dealt with due to the use of affine quadratic stability. The changes on floor masses is not able to be analytically addressed

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using the proposed approach. The third important issue is time delay. As indicated in [5], time delay is one of the inevitable problems in actual engineering applications. Considering time delay in the controller design process will be very important to the system stability and performance. For structural control systems, particularly with the digital controllers, the sum of input time delay is due to online data acquisition from sensors at different points of the structure, filtering, processing of data, calculating control forces and transmitting the control force signals from computer to the actuator. When using actuators, like electrohydraulic actuators, as the control force devices, the time delays will be generally taken by the actuators to build up the required control forces. In recent years, the efforts on the stability criterion analysis of the mechanical and structural systems with time delays have been made through characteristic equation analysis [8–10]; however, the controller synthesis problems have not been fully addressed in these works. On the other hand, several compensation strategies to deal with the time delay effect in vibration control of civil engineering structures have been presented in [11], where the compensator is designed to compensate a specified delay and the system will be stable in a region where the actual delay is varied around this specified delay. An optimal control method was studied in [12] to deal with the time delay effect on linear time delay systems, and an energy-to-peak control of a building structure with an input delay was studied in [13]. It has been shown from the above-mentioned studies that controller design for vibration control of building structures with considering more practical issues, such as parameter uncertainties, actuator saturation, and control input time delay, etc., is becoming more and more important.

In recent years, stability analysis, stabilisation, and control synthesis for linear time-delay systems subject to actuator saturation have been addressed by many authors (see, for example, [14–23] and the references therein). Cao et al. [15] and Zuo et al. [17] studied the stability analysis problem of linear time delays subject to actuator saturation, in which they considered the state time delay and used the approach proposed by [24] to deal with the actuator saturation problem. The stabilisation problem of time-delay systems with saturating actuators was studied by Su et al. [14], Trabouriech et al. [19], Zhou et al. [18], Zhou et al. [21] and Liu [22], where the input delay was considered by [14,18,21] and the external disturbance was considered by [19]. Oucheriah [16], Zhang et al. [20], and Zhao et al. [23] studied the controller synthesis problem for time-delay systems with actuator saturation. The state time delay was considered by [16,20] and the input delay was considered for seat suspensions in [23]. An auxiliary feedback matrix method [25] was used by [20,23] to handle the actuator saturation problem. The auxiliary feedback matrix method is shown less conservative in estimating the domain of attraction than other existing methods which are based on circle criterion or the vertex analysis [25]. However, this method may lead to a low-gain controller design, which, on the contrary, is not suitable to structural vibration control because a high-gain controller is able to take advantage of the available control and utilise the capacity of an actuator sufficiently [2,26].

In this paper, the robust controller design approach for the uncertain structural systems considering parameter uncertainties, actuator saturation, and input time delay will be presented. The main objective is to design a state feedback controller such that the closed-loop system is asymptotically stable with the optimal energy-to-peak disturbance attenuation performance subject to parameter uncertainties (variations of floor masses, stiffnesses, and damping coefficients), actuator saturation, and input time delay. Sufficient conditions for designing such a controller are given in terms of delay-dependent linear matrix inequalities (LMIs), which can be efficiently solved using available software Matlab LMI Toobox. To validate the effectiveness of the approach, the designed controller is applied to reduce the vibration of a seismic-excited building structure. Simulation results show that the designed controller can achieve good vibration attenuation performance and keep the system robust stability in spite of the presence of parameter uncertainties, actuator saturation, and input time delay.

The rest of this paper is organised as follows. Section 2 presents the description of saturation control of uncertain structure with input time delay. Section 3 gives the controller design approach. Section 4 provides an application example to validate the effectiveness of the approach developed in this paper. Finally, we conclude our findings in Section 5.

Notation: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ the set of all $n\times m$ real matrices. For a real symmetric matrix W, the notation of $W>0$ ($W<0$) is used to denote its positive- (negative-) definiteness. I is used to denote the identity matrix of appropriate dimension. When a matrix is equal to 0, in such case, 0 is used to denote the zero matrix of appropriate dimension. To simplify notation, $*$ is used to represent a block matrix which is readily inferred by symmetry.

2. Saturation control of uncertain structure with input delay

The first-order model of uncertain structure with actuator saturation constraint and input delay can be expressed as

$$x(t) = A_\xi x(t) + B_w w(t) + (B+\Delta B)\mathrm{SAT}(u(t-\tau(t))),$$

$$x(t) = \phi(t), \quad t \in [-\overline{\tau},0], \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state vector, $w(t)\in\mathbb{R}$ and $u(t)\in\mathbb{R}^r$ are the external disturbance and the control input, respectively. $\tau(t)$ is the time-varying input delay satisfying $0<\tau(t)\le\overline{\tau}$, $\overline{\tau}$ is the maximum time delay. $\phi(t)$ is a continuous vector-valued initial function on $[-\overline{\tau},0]$. Matrix $A_\xi\in\mathbb{R}^{n\times n}$ represents uncertain system matrix belonging to a given convex polytope Θ

described by κ vertices in the following form:

$$A_\xi \in \Theta \triangleq \left\{ A_\xi | A_\xi = \sum_{i=1}^{\kappa} \xi_i (A_i + \Delta A_i) = \sum_{i=1}^{\kappa} \xi_i \overline{A}_i; \xi_i \geq 0; \sum_{i=1}^{\kappa} \xi_i = 1 \right\}, \tag{2}$$

where ξ is used to characterise the parameter uncertainty and is assumed to be varied in a polytope of vertices $\xi_1, \xi_2, \ldots, \xi_\kappa$, i.e., $\xi \in \Theta \triangleq Co\{\xi_1, \xi_2, \ldots, \xi_\kappa\}$, where the symbol Co denotes the convex hull and Θ denotes a given convex bounded polyhedral domain. ΔA_i and ΔB are uncertainties in system and control matrices and they are in the norm-bounded form of

$$[\Delta A_i, \Delta B] = MF(t)[E_i, E_b], \tag{3}$$

where M, E_i, and E_b are known constant matrices, and $F(t)$ is an unknown matrix function with the property $F^{\mathrm{T}}(t)F(t) \leq I$. It is noted from (2) and (3) that the parameter uncertainties are described in both polytopic and norm-bounded forms. These two uncertainty forms can fully represent the parameter uncertainties that are induced by the variations of floor masses, stiffnesses, and damping coefficients. The actuator saturation expression $\mathrm{SAT}(u)$ is in the decentralised saturation form, that is, $[\mathrm{SAT}(u)]_i = \mathrm{sat}(u_i)$, where $i = 1, 2, \ldots, r$, and $\mathrm{sat}(u_i)$ is the standard saturation function with the limit of $u_{\lim_i}$ for the ith actuator, that is,

$$\mathrm{sat}(u_i) = \begin{cases} u_i, & |u_i| \leq u_{\lim_i}, \\ \mathrm{sign}(u_i) u_{\lim_i}, & |u_i| > u_{\lim_i} \end{cases} \tag{4}$$

Using the following transform [26–28]:

$$\mathrm{SAT}(u) = \Psi_\eta u, \tag{5}$$

where $\Psi_\eta = \mathrm{diag}\{\eta_1, \ldots, \eta_i, \ldots, \eta_r\}$, $\eta_i \triangleq \mathrm{sat}(u_i)/u_i$ with $\eta_i = 1$ if $u_i = 0$, Eq. (1) can now be written as

$$\dot{x}(t) = A_\xi x(t) + B_w w(t) + \overline{B} \Psi_\eta u(t - \tau(t)), \tag{6}$$

where $\overline{B} = B + \Delta B$.

To obtain a high-gain controller as that done in [26], the command to the ith actuator is allowed to be $\delta_i u_{\lim_i}$ for an arbitrary scalar $\delta_i > 1$. Therefore, the resulting η_i will be bounded by 1 and $1/\delta_i$, that is,

$$\eta \in \mathcal{P} \triangleq \left\{ \eta : \frac{1}{\delta_i} \leq \eta_i \leq 1,\ i = 1, 2, \ldots, r \right\}. \tag{7}$$

Accordingly, the vertex set associated with (7) is denoted as

$$\mathcal{P}_{vex} \triangleq \left\{ \eta : \eta_i = \frac{1}{\delta_i} \text{ or } \eta_i = 1,\ i = 1, 2, \ldots, r \right\}, \tag{8}$$

and Ψ_η can be expressed as $\Psi_\eta = \sum_{i=1}^{2r} \zeta_i \Psi_{\eta_i} = \sum_{i=1}^{2r} \zeta_i \Psi_i$, where $\zeta_i \geq 0$ and $\sum_{i=1}^{2r} \zeta_i = 1$.

In this paper, the disturbance signal $w(t)$ is assumed to be bounded and with finite energy, that is,

$$\|w\|_2 \triangleq \sqrt{\int_0^\infty w^{\mathrm{T}}(t) w(t)\, \mathrm{d}t} < \infty, \tag{9}$$

i.e., $w(t) \in L_2[0, \infty)$. This is one possible specification for a class of design loads that the engineering structures are designed to resist, for example, a class of design earthquakes whose total energy is specified as Richter scale [29]. And for system (6), we define the control output as

$$z(t) = Cx(t), \tag{10}$$

where C is constant matrix. Then, for the uncertain system (6), we are interested in designing a state feedback control law

$$u(t) = K_c x(t), \tag{11}$$

where $K_c \in \mathbb{R}^{r \times n}$ is the control gain matrix to be designed, such that the closed-loop system given by

$$\dot{x}(t) = A_\xi x(t) + B_w w(t) + \overline{B} \Psi_\eta K_c x(t - \tau(t)) \tag{12}$$

is asymptotically stable for all admissible parameter uncertainties, and the closed-loop system guarantees, under zero initial condition, $\|z\|_\infty < \gamma \|w\|_2$, i.e., energy-to-peak performance, where $\gamma > 0$ is a prescribed constant, for any non-zero $w \in L_2[0, \infty)$.

3. Controller design

The following lemma will be used to derive the main results.

Lemma 1 (*Zhou and Khargonekar [30]*). *For real matrices of appropriate dimensions, M, E, and $F(t)$, and $F(t)$ satisfying $F^{\mathrm{T}}(t)F(t) \le I$, the following inequality holds for any scalar $\varepsilon > 0$*

$$MF(t)E + E^{\mathrm{T}}F^{\mathrm{T}}(t)M^{\mathrm{T}} \le \varepsilon MM^{\mathrm{T}} + \varepsilon^{-1}E^{\mathrm{T}}E.$$

The following theorem will be used to design the controller (11).

Theorem 1. *For given scalars $\rho > 0$, $\gamma > 0$, $\delta > 0$, and $\overline{\tau} > 0$, if there exist matrices $L > 0$, $R > 0$, $\overline{X}_i$, $\overline{Y}_i$, $i = 1,2,\ldots,\kappa$, and $\overline{K}_c$, scalars $\varepsilon_{ij} > 0$, $i = 1,2,\ldots,\kappa$, and $j = 1,2,\ldots,2r$, satisfying LMIs* (13)–(15), *then the closed-loop system* (12) *is asymptotically stable and $\|z\|_\infty < \gamma\|w\|_2$.*

$$\begin{bmatrix} \begin{matrix} A_iL + LA_i^{\mathrm{T}} \\ +\overline{X}_i + \overline{X}_i^{\mathrm{T}} + \varepsilon_{ij}MM^{\mathrm{T}} \end{matrix} & \begin{matrix} B\Psi_j\overline{K}_c \\ -\overline{X}_i + \overline{Y}_i^{\mathrm{T}} \end{matrix} & -\overline{X}_i & B_w & \begin{matrix} LA_i^{\mathrm{T}} \\ +\varepsilon_{ij}MM^{\mathrm{T}} \end{matrix} & (E_iL)^{\mathrm{T}} \\ * & -\overline{Y}_i^{\mathrm{T}} - \overline{Y}_i & -\overline{Y}_i & 0 & \overline{K}_c^{\mathrm{T}}\Psi_j^{\mathrm{T}}B^{\mathrm{T}} & (E_b\Psi_j\overline{K}_c)^{\mathrm{T}} \\ * & * & -\overline{\tau}^{-1}(R-2L) & 0 & 0 & 0 \\ * & * & * & -I & B_w^{\mathrm{T}} & 0 \\ * & * & * & * & -\overline{\tau}^{-1}R + \varepsilon_{ij}MM^{\mathrm{T}} & 0 \\ * & * & * & * & * & -\varepsilon_{ij}I \end{bmatrix}$$
$$< 0,\ i \in [1,\kappa],\ j \in [1,2r]. \tag{13}$$

$$\begin{bmatrix} L & LC^{\mathrm{T}} \\ CL & \gamma^2 I \end{bmatrix} > 0, \tag{14}$$

$$\begin{bmatrix} (\delta_i u_{\mathrm{lim}i})^2 I & \overline{K}_{c_i} \\ \overline{K}_{c_i}^{\mathrm{T}} & \frac{L}{\rho} \end{bmatrix} \ge 0. \tag{15}$$

Furthermore, the controller gain matrix is obtained as $K_c = \overline{K}_cL^{-1}$. If the performance index γ is minimised subject to the LMIs (13)–(15), *the optimal controller will be obtained.*

Proof. Choose a Lyapunov–Krasovskii functional candidate for system (12) as

$$V(t) = x^{\mathrm{T}}(t)Px(t) + \int_{-\overline{\tau}}^{0}\int_{t+\beta}^{t} \dot{x}^{\mathrm{T}}(\alpha)Q\dot{x}(\alpha)\,\mathrm{d}\alpha\,\mathrm{d}\beta, \tag{16}$$

where $P = P^{\mathrm{T}}$, $P > 0$, $Q = Q^{\mathrm{T}}$, $Q > 0$. Then, the time derivative of $V(t)$ along the solution of system (12) gives

$$\begin{aligned} \dot{V}(t) &= \dot{x}^{\mathrm{T}}(t)Px(t) + x^{\mathrm{T}}(t)P\dot{x}(t) + \overline{\tau}\dot{x}^{\mathrm{T}}(t)Q\dot{x}(t) - \int_{t-\overline{\tau}}^{t} \dot{x}^{\mathrm{T}}(\alpha)Q\dot{x}(\alpha)\,\mathrm{d}\alpha \\ &\le \dot{x}^{\mathrm{T}}(t)Px(t) + x^{\mathrm{T}}(t)P\dot{x}(t) + \overline{\tau}\dot{x}^{\mathrm{T}}(t)Q\dot{x}(t) - \int_{t-\tau(t)}^{t} \dot{x}^{\mathrm{T}}(\alpha)Q\dot{x}(\alpha)\,\mathrm{d}\alpha = \frac{1}{\tau(t)}\int_{t-\tau}^{t} \Sigma(t,\alpha)\,\mathrm{d}\alpha, \end{aligned} \tag{17}$$

where

$$\begin{aligned} \Sigma(t,\alpha) &= \dot{x}^{\mathrm{T}}(t)Px(t) + x^{\mathrm{T}}(t)P\dot{x}(t) + \overline{\tau}\dot{x}^{\mathrm{T}}(t)Q\dot{x}(t) - \tau(t)\dot{x}^{\mathrm{T}}(\alpha)Q\dot{x}(\alpha) = 2x^{\mathrm{T}}(t)P(A_\xi x(t) + B_w w(t) + \overline{B}\Psi_\eta K_c x(t-\tau(t)) \\ &\quad + (A_\xi x(t) + B_w w(t) + \overline{B}\Psi_\eta K_c x(t-\tau(t))^{\mathrm{T}}\overline{\tau}Q(A_\xi x(t) + B_w w(t) + \overline{B}\Psi_\eta K_c x(t-\tau(t)) - \tau(t)\dot{x}^{\mathrm{T}}(\alpha)Q\dot{x}(\alpha). \end{aligned}$$

By the Newton–Leibniz formula, we have

$$\int_{t-\tau(t)}^{t} \dot{x}(\alpha)\,\mathrm{d}\alpha = x(t) - x(t-\tau(t)). \tag{18}$$

Then, for any appropriately dimensioned matrices

$$X_\xi = \sum_{i=1}^{\kappa} \xi_i X_i, \quad Y_\xi = \sum_{i=1}^{\kappa} \xi_i Y_i, \tag{19}$$

we have

$$\Lambda = \frac{1}{\tau(t)}\int_{t-\tau(t)}^{t} \begin{bmatrix} x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-\tau(t)) \end{bmatrix} \begin{bmatrix} X_\xi \\ Y_\xi \end{bmatrix} [x(t) - x(t-\tau(t)) - \tau(t)\dot{x}(\alpha)]\,\mathrm{d}\alpha = 0. \tag{20}$$

Adding 2Λ to the right hand of (17), we have

$$\dot{V}(t) - w^{\mathrm{T}}(t)w(t) \le \frac{1}{\tau(t)}\int_{t-\tau(t)}^{t} \vartheta^{\mathrm{T}}(t,\alpha)\Pi\vartheta(t,\alpha)\,\mathrm{d}\alpha, \tag{21}$$

where $\vartheta^{\mathrm{T}}(t,\alpha)=[x^{\mathrm{T}}(t)\ x^{\mathrm{T}}(t-\tau(t))\ \dot{x}^{\mathrm{T}}(\alpha)\ w^{\mathrm{T}}(t)]$ and

$$\Pi=\begin{bmatrix} \begin{matrix} PA_\xi+A_\xi^{\mathrm{T}}P \\ +\overline{\tau}A_\xi^{\mathrm{T}}QA_\xi+X_\xi+X_\xi^{\mathrm{T}} \end{matrix} & \begin{matrix} P\overline{B}\Psi_\eta K_c \\ +\overline{\tau}A_\xi^{\mathrm{T}}Q\overline{B}\Psi_\eta K_c \\ -X_\xi+Y_\xi^{\mathrm{T}} \end{matrix} & -\tau(t)X_\xi & PB_w+\overline{\tau}A_\xi^{\mathrm{T}}QB_w \\ * & \begin{matrix} \overline{\tau}K_c^{\mathrm{T}}\Psi_\eta^{\mathrm{T}}\overline{B}^{\mathrm{T}}Q\overline{B}\Psi_\eta K_c \\ -Y_\xi^{\mathrm{T}}-Y_\xi \end{matrix} & -\tau(t)Y_\xi & \overline{\tau}K_c^{\mathrm{T}}\Psi_\eta^{\mathrm{T}}\overline{B}^{\mathrm{T}}QB_w \\ * & * & -\tau(t)Q & 0 \\ * & * & * & \overline{\tau}B_w^{\mathrm{T}}QB_w-I \end{bmatrix}. \tag{22}$$

When assuming the zero-disturbance input, i.e., $w(t)\equiv 0$, if $\Pi<0$, then from (21), $\dot{V}(t)<0$ is established and the asymptotic stability of the closed-loop system (12) is guaranteed.

Assume zero initial condition, i.e., $x(t)=\phi(t)=0,\ \forall t\in[-\overline{\tau},0]$, then, we have $V(t)|_{t=0}=0$. And for any non-zero disturbance $w\in L_2[0,\infty)$ and $t\geq 0$, if $\Pi<0$, there holds,

$$V(t)-V(t)|_{t=0}-\int_0^t w^{\mathrm{T}}(s)w(s)\,\mathrm{d}s<0, \tag{23}$$

and $V(t)<\int_0^t w^{\mathrm{T}}(s)w(s)\,\mathrm{d}s$.

By the Schur complement, $\Pi<0$ is equivalent to

$$\begin{bmatrix} PA_\xi+A_\xi^{\mathrm{T}}P+X_\xi+X_\xi^{\mathrm{T}} & P\overline{B}\Psi_\eta K_c-X_\xi+Y_\xi^{\mathrm{T}} & -X_\xi & PB_w & A_\xi^{\mathrm{T}} \\ * & -Y_\xi^{\mathrm{T}}-Y_\xi & -Y_\xi & 0 & K_c^{\mathrm{T}}\Psi_\eta^{\mathrm{T}}\overline{B}^{\mathrm{T}} \\ * & * & -\tau^{-1}(t)Q & 0 & 0 \\ * & * & * & -I & B_w^{\mathrm{T}} \\ * & * & * & * & -\overline{\tau}^{-1}Q^{-1} \end{bmatrix}<0. \tag{24}$$

Define $L\triangleq P^{-1}$, and pre- and post-multiplying (24) by $\mathrm{diag}(L\ L\ L\ I\ I)^{\mathrm{T}}$ and its transpose, respectively, we obtain

$$\begin{bmatrix} \begin{matrix} A_\xi L+LA_\xi^{\mathrm{T}} \\ +LX\xi L+LX_\xi^{\mathrm{T}}L \end{matrix} & \overline{B}\Psi_\eta K_c L-LX_\xi L+LY_\xi^{\mathrm{T}}L & -LX_\xi L & B_w & LA_\xi^{\mathrm{T}} \\ * & -LY_\xi^{\mathrm{T}}L-LY_\xi L & -LY_\xi L & 0 & LK_c^{\mathrm{T}}\Psi^{\mathrm{T}}(\eta)\overline{B}^{\mathrm{T}} \\ * & * & -\tau^{-1}(t)LQL & 0 & 0 \\ * & * & * & -I & B_w^{\mathrm{T}} \\ * & * & * & * & -\overline{\tau}^{-1}Q^{-1} \end{bmatrix}<0. \tag{25}$$

By defining $\overline{X}_\xi=LX_\xi L$, $\overline{Y}_\xi=LY_\xi L$, $\overline{K}_c=K_cL$, and $R=Q^{-1}$ in (25), we obtain

$$\begin{bmatrix} A_\xi L+LA_\xi^{\mathrm{T}}+\overline{X}_\xi+\overline{X}_\xi^{\mathrm{T}} & \overline{B}\Psi_\eta\overline{K}_c-\overline{X}_\xi+\overline{Y}_\xi^{\mathrm{T}} & -\overline{X}_\xi & B_w & LA_\xi^{\mathrm{T}} \\ * & -\overline{Y}_\xi^{\mathrm{T}}-\overline{Y}_\xi & -\overline{Y}_\xi & 0 & \overline{K}_c^{\mathrm{T}}\Psi_\eta^{\mathrm{T}}\overline{B}^{\mathrm{T}} \\ * & * & -\tau^{-1}(t)LR^{-1}L & 0 & 0 \\ * & * & * & -I & B_w^{\mathrm{T}} \\ * & * & * & * & -\overline{\tau}^{-1}R \end{bmatrix}<0. \tag{26}$$

It is noticed that $(R-L)R^{-1}(R-L)\geq 0$ when $R>0$, which is equivalent to

$$LR^{-1}L\leq R-2L. \tag{27}$$

Therefore, from (27) and $\tau(t)\leq\overline{\tau}$, if

$$\begin{bmatrix} A_\xi L+LA_\xi^{\mathrm{T}}+\overline{X}_\xi+\overline{X}_\xi^{\mathrm{T}} & \overline{B}\Psi_\eta\overline{K}_c-\overline{X}_\xi+\overline{Y}_\xi^{\mathrm{T}} & -\overline{X}_\xi & B_w & LA_\xi^{\mathrm{T}} \\ * & -\overline{Y}_\xi^{\mathrm{T}}-\overline{Y}_\xi & -\overline{Y}_\xi & 0 & \overline{K}_c^{\mathrm{T}}\Psi_\eta^{\mathrm{T}}\overline{B}^{\mathrm{T}} \\ * & * & -\overline{\tau}^{-1}(R-2L) & 0 & 0 \\ * & * & * & -I & B_w^{\mathrm{T}} \\ * & * & * & * & -\overline{\tau}^{-1}R \end{bmatrix}<0, \tag{28}$$

then, inequality (26) can be established. Substituting $A_\xi=\sum_{i=1}^{\kappa}\xi_i\overline{A}_i$, $\overline{X}_\xi=\sum_{i=1}^{\kappa}\xi_i\overline{X}_i$, $\overline{Y}_\xi=\sum_{i=1}^{\kappa}\xi_i\overline{Y}_i$, and $\Psi_\eta=\sum_{i=1}^{2r}\Psi_i$ into (28), we readily obtain the equivalent condition for inequality (28) as

$$\Sigma=\begin{bmatrix}\overline{A}_iL+L\overline{A}_i^{\mathrm{T}}+\overline{X}_i+\overline{X}_i^{\mathrm{T}} & \overline{B}\Psi_j\overline{K}_c-\overline{X}_i+\overline{Y}_i^{\mathrm{T}} & -\overline{X}_i & B_w & L\overline{A}_i^{\mathrm{T}}\\ * & -\overline{Y}_i^{\mathrm{T}}-\overline{Y}_i & -\overline{Y}_i & 0 & \overline{K}_c^{\mathrm{T}}\Psi_j^{\mathrm{T}}\overline{B}^{\mathrm{T}}\\ * & * & -\overline{\tau}^{-1}(R-2L) & 0 & 0\\ * & * & * & -I & B_w^{\mathrm{T}}\\ * & * & * & * & -\overline{\tau}^{-1}R\end{bmatrix}<0,$$

$$i\in[1,\kappa],\quad j\in[1,2r]. \tag{29}$$

By the definition of (3), it is noticed that

$$\Sigma=\Sigma_0+\Delta\Sigma,$$

where

$$\Sigma_0=\begin{bmatrix}A_iL+LA_i^{\mathrm{T}}+\overline{X}_i+\overline{X}_i^{\mathrm{T}} & B\Psi_j\overline{K}_c-\overline{X}_i+\overline{Y}_i^{\mathrm{T}} & -\overline{X}_i & B_w & LA_i^{\mathrm{T}}\\ * & -\overline{Y}_i^{\mathrm{T}}-\overline{Y}_i & -\overline{Y}_i & 0 & \overline{K}_c^{\mathrm{T}}\Psi_j^{\mathrm{T}}B^{\mathrm{T}}\\ * & * & -\overline{\tau}^{-1}(R-2L) & 0 & 0\\ * & * & * & -I & B_w^{\mathrm{T}}\\ * & * & * & * & -\overline{\tau}^{-1}R\end{bmatrix}$$

and by Lemma 1,

$$\Delta\Sigma=\begin{bmatrix}MF(t)E_iL+L(MF(t)E_i)^{\mathrm{T}} & MF(t)E_b\Psi_j\overline{K}_c & 0 & 0 & L(MF(t)E_i)^{\mathrm{T}}\\ * & 0 & 0 & 0 & \overline{K}_c^{\mathrm{T}}\Psi_j^{\mathrm{T}}(MF(t)E_b)^{\mathrm{T}}\\ * & * & 0 & 0 & 0\\ * & * & * & -I & 0\\ * & * & * & * & 0\end{bmatrix}$$

$$=\begin{bmatrix}M\\0\\0\\0\\M\end{bmatrix}F(t)[E_iL\;\;E_b\Psi_j\overline{K}_c\;\;0\;\;0\;\;0]+[E_iL\;\;E_b\Psi_j\overline{K}_c\;\;0\;\;0\;\;0]^{\mathrm{T}}F^{\mathrm{T}}(t)\begin{bmatrix}M\\0\\0\\0\\M\end{bmatrix}^{\mathrm{T}}$$

$$\le\varepsilon_{ij}\begin{bmatrix}MM^{\mathrm{T}} & 0 & 0 & 0 & MM^{\mathrm{T}}\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ MM^{\mathrm{T}} & 0 & 0 & 0 & MM^{\mathrm{T}}\end{bmatrix}+\varepsilon_{ij}^{-1}\begin{bmatrix}(E_iL)^{\mathrm{T}}\\(E_b\Psi_j\overline{K}_c)^{\mathrm{T}}\\0\\0\\0\end{bmatrix}[E_iL\;\;E_b\Psi_j\overline{K}_c\;\;0\;\;0\;\;0].$$

Therefore, it can be inferred that if the inequality (13) holds, then $\Sigma<0$ can be established.

Furthermore, using the Schur complement, the feasibility of the following inequality:

$$\begin{bmatrix}P & C^{\mathrm{T}}\\ C & \gamma^2 I\end{bmatrix}>0 \tag{30}$$

guarantees $C^{\mathrm{T}}C<\gamma^2P$. At the same time, it can be derived from (23) that $x^{\mathrm{T}}(t)Px(t)<\gamma^2\int_0^t w^{\mathrm{T}}(s)w(s)\,\mathrm{d}s$ if $\Pi<0$ is guaranteed. Then, it can be easily established from (10) that for all $t\ge 0$,

$$z^{\mathrm{T}}(t)z(t)=x^{\mathrm{T}}(t)C^{\mathrm{T}}Cx(t)<\gamma^2x^{\mathrm{T}}(t)Px(t)<\gamma^2\int_0^t w^{\mathrm{T}}(s)w(s)\,\mathrm{d}s\le\gamma^2\int_0^\infty w^{\mathrm{T}}(s)w(s)\,\mathrm{d}s. \tag{31}$$

Taking the supremum over $t\ge 0$ yields $\|z\|_\infty<\gamma\|w\|_2$ for all $w\in L_2[0,\infty)$, that is, the energy-to-peak performance is established. Similarly, condition (30) can be transformed to (14) by pre- and post-multiplying (30) by $\mathrm{diag}(L\;I)^{\mathrm{T}}$ and its transpose, respectively.

On the other hand, from (11), the constraint $|u_i|\le\delta_i u_{\mathrm{lim}i}$ can be expressed as

$$|K_{ci}x(t)|\le\delta_i u_{\mathrm{lim}i}, \tag{32}$$

where K_{ci} is the ith row of K_c. Let $\Omega(K_c)=\{x(t)||x^{\mathrm{T}}(t)K_{ci}^{\mathrm{T}}K_{ci}x(t)|\le(\delta_i u_{\mathrm{lim}i})^2\}$, the equivalent condition for an ellipsoid $\Omega(P,\rho)=\{x(t)|x^{\mathrm{T}}(t)Px(t)\le\rho\}$ being a subset of $\Omega(K_c)$ is [31]

$$K_{ci}\left(\frac{P}{\rho}\right)^{-1}K_{ci}^{\mathrm{T}}\le(\delta_i u_{\mathrm{lim}i})^2. \tag{33}$$

By the Schur complement, inequality (33) can be written as

$$\begin{bmatrix}(\delta_i u_{\mathrm{lim}i})^2 I & K_{c_i}\\ K_{c_i}^{\mathrm{T}} & \frac{P}{\rho}\end{bmatrix}\ge 0. \tag{34}$$

Using the definitions $L=P^{-1}$ and $\overline{K}_c=K_cP^{-1}$, and pre- and post-multiplying (34) by diag$(I\ L)^{\mathrm{T}}$ and its transpose, respectively, inequality (34) can be transformed to inequality (15). This completes the proof. □

4. Numerical example

In this section, a practical numerical example is presented to verify the effectiveness and applicability of the proposed robust saturation controller to a seismic-excited building with parameter uncertainties and input time delay.

A three-storey shear-beam building model is considered [32], where the active bracing system (ABS) is installed at the first floor to control the vibration of the structure as shown in Fig. 1. It is assumed that all the masses, stiffnesses, and damping coefficients for each floor are identical, and the nominal structural parameters are given as m_i=1000 kg, c_i=1.407 kN s/m, and k_i=980 kN/m, where i=1,2,3, respectively. The state space equation of the three-storey shear-beam building model is obtained similar to Eq. (1), in which

$$A_\xi=\begin{bmatrix}0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\-(k_1+k_2)/m_1&k_2/m_1&0&-(c_1+c_2)/m_1&c_2/m_1&0\\k_2/m_2&-(k_2+k_3)/m_2&k_3/m_2&c_2/m_2&-(c_2+c_3)/m_2&c_3/m_2\\0&k3/m_3&-k_3/m_3&0&c_3/m_3&-c_3/m_3\end{bmatrix},$$

$$B=[0\ \ 0\ \ 0\ \ 1/m_1\ \ 0\ \ 0]^{\mathrm{T}},$$

and

$$B_w=[0\ \ 0\ \ 0\ \ -1\ \ -1\ \ -1]^{\mathrm{T}}.$$

Consider the uncertainties of stiffnesses and damping coefficients are 40 percent of their nominal values, respectively, we can obtain the matrices A_i as the following. Define the maximum and minimum values for the uncertain parameters as $k_{\min}$=0.6 × 980 kN/m, $k_{\max}$=1.4 × 980 kN/m, $c_{\min}$=0.6 × 1.407, and $c_{\max}$=1.4 × 1.407 kN s/m, then the vertices $\theta_i(i=1,\ldots,4)$ of the polynomial set for matrix A_ξ can be defined as

$$\theta_1=[k_{\min}\ \ c_{\min}],\quad \theta_2=[k_{\max}\ \ c_{\min}],\quad \theta_3=[k_{\min}\ \ c_{\max}],\quad \theta_4=[k_{\max}\ \ c_{\max}].$$

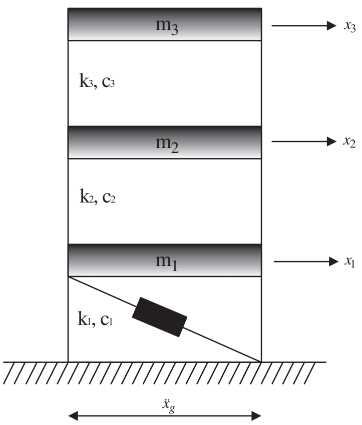

Fig. 1. Three DOF building model with ABS.

After substituting $\theta_i (i=1,\ldots,4)$ into A_ξ, respectively, the matrices A_i $(i=1,\ldots,4)$ can be obtained. In fact, by defining the coefficients as

$$\xi_1=\mu v,\quad \xi_2=(1-\mu)v,\quad \xi_3=\mu(1-v),\quad \xi_4=(1-\mu)(1-v),$$

where $\mu=(k_{\max}-k_i)/(k_{\max}-k_{\min})$, $v=(c_{\max}-c_i)/(c_{\max}-c_{\min})$, the uncertain matrix A_ξ can be expressed as $A_\xi=\sum_{i=1}^4 \xi_i(A_i+MF(t)E_i)$, where $\xi_i \geq 0$ and $\sum_{i=1}^4 \xi_i=1$. In addition, consider the variation of the first floor mass, the uncertain matrices can be defined as

$$M=[0\ \ 0\ \ 0\ \ 0.2/m_1\ \ 0\ \ 0]^{\mathrm{T}},$$

$$E_1=[-(k_{\min}+k_{\min})\ \ k_{\min}\ \ 0\ \ -(c_{\min}+c_{\min})\ \ c_{\min}\ \ 0],$$

$$E_2=[-(k_{\max}+k_{\max})\ \ k_{\max}\ \ 0\ \ -(c_{\min}+c_{\min})\ \ c_{\min}\ \ 0],$$

$$E_3=[-(k_{\min}+k_{\min})\ \ k_{\min}\ \ 0\ \ -(c_{\max}+c_{\max})\ \ c_{\max}\ \ 0],$$

$$E_4=[-(k_{\max}+k_{\max})\ \ k_{\max}\ \ 0\ \ -(c_{\max}+c_{\max})\ \ c_{\max}\ \ 0],$$

$$E_b=1.$$

In this study, the control output, $z(t)$, is defined as the relative displacement of the first floor, that is,

$$z(t)=[1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]x(t).$$

In order to study the structural responses under seismic excitations, the El Centro 1940 earthquake excitation of which peak ground acceleration is scaled to 0.112 g is used in this study. Assume all the relative displacements and the relative velocities of the three floors can be measured for feedback, the state feedback control can be realised. Since there are few cases where the time delay is larger than the one sample rate if the time delay is mainly from the computation of control laws [33] and the sampling frequencies of most vibration control systems are on the order of 100–500 Hz [34], the maximum time delay $\overline{\tau}$ allowed is selected as 20 ms. Consider the maximum actuator output force limit $u_{\lim}$ as 700 N (about 2.3 percent building weight), and define $\delta=10$, $\rho=0.01$, and $\overline{\tau}=20$ ms, then use the approach presented in Section 3, we obtain the controller gain as

$$K_c=10^5\times[0.7829\ \ -3.6796\ \ 2.4813\ \ -0.5351\ \ -0.0472\ \ 0.0427].$$

To evaluate the control system performance, three evaluation criteria are used. The first evaluation criterion is a measure of the normalised maximum floor displacement relative to the ground, given as

$$J_1=\max_{t,i}\left(\frac{|x_i(t)|}{x_{\max}}\right), \tag{35}$$

where $x_i(t)$ is the relative displacement of the ith floor over entire response, $x_{\max}$ denotes the uncontrolled maximum displacement. The second evaluation criterion is a measure of the reduction in the interstorey drift. The maximum of the normalised interstorey drift is

$$J_2=\max_{t,i}\left(\frac{|d_i(t)|}{d_{\max}}\right), \tag{36}$$

where $d_i(t)$ is the interstorey drift of the above ground floors over response history, and $d_{\max}$ denotes the peak interstorey drift in the uncontrolled response. The third evaluation criterion is a measure of the peak floor accelerations, given by

$$J_3=\max_{t,i}\left(\frac{|\ddot{x}_{ai}(t)|}{\ddot{x}_{a\max}}\right), \tag{37}$$

where $\ddot{x}_{ai}(t)$ is the absolute acceleration of the ith floor, and $\ddot{x}_{a\max}$ is the peak uncontrolled absolute acceleration.

Now, the control performance of the proposed controller for the nominal system will be checked. When there is no time delay on input, i.e., $\tau=0$, the responses of the open-loop system ($u(t)=0$) and the closed-loop system are compared in Fig. 2, where only the interstorey drift of the first floor and the absolute acceleration of the third floor, and control force are shown for clarity. It can be seen from Fig. 2 that better responses are obtained for the closed-loop system when $\tau=0$. It is also verified that using energy-to-peak performance guarantees good response quantities.

When the time delay $\tau=20$ ms is introduced at the control input, under the same earthquake excitation, the responses of the first floor interstorey drift, the third floor absolute acceleration, and control force for the nominal system are plotted in Fig. 3. It is noted that using our algorithm presented in Section 3, the controller is feasible for the given maximum time delay $\overline{\tau}=20$ ms. This means that the controller can stabilise the system (6) with good energy-to-peak performance under parameter uncertainties and actuator saturation described above for any time delay satisfying $0\leq\tau\leq 20$ ms. It can be seen from Fig. 3 that the closed-loop system is stable and the closed-loop performance is similar to that of no time delay in input case as shown in Fig. 2.

For detailed comparison, the values for J_2 and J_3 are summarised in Table 1, where the results obtained by some other methods are also listed for comparison (note that J_1 is not listed as it cannot be obtained from [5] for the other methods to

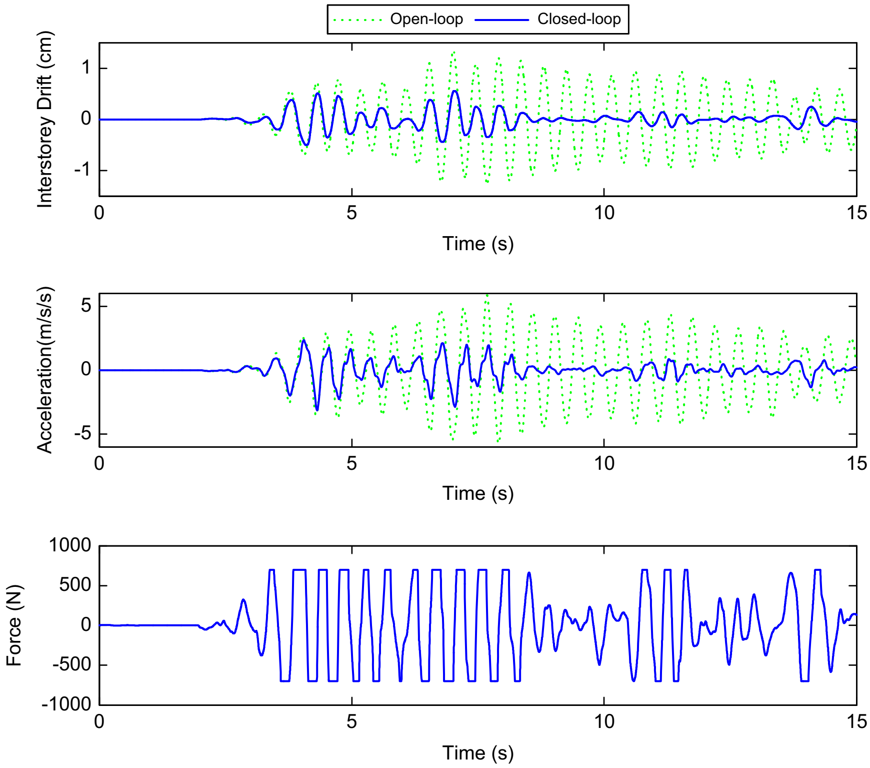

Fig. 2. Responses of interstorey drift of the first floor and absolute acceleration of the third floor, and control force for the nominal system when $\tau = 0$ ms.

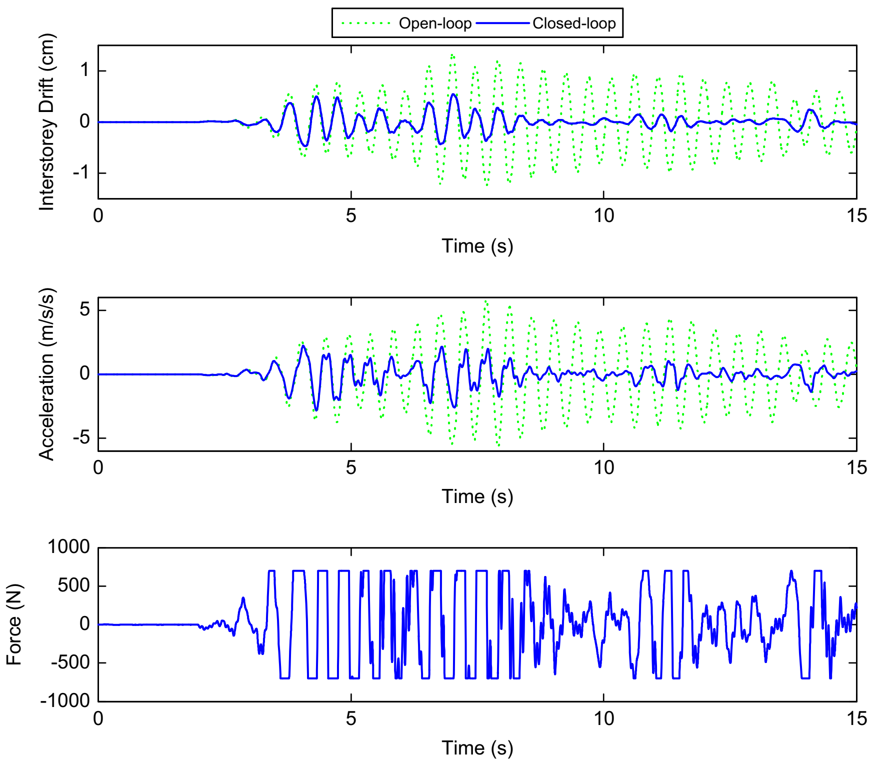

Fig. 3. Responses of interstorey drift of the first floor and absolute acceleration of the third floor, and control force for the nominal system when $\tau = 20$ ms.

Table 1
Normalised maximum response values for nominal system.

Control strategy	LQR [5]	MBBC [5]	SSMC [5]	Lim et al. [5]	Proposed	
Time delay τ (ms)	0	0	0	0	0	20
J_2	0.66	0.38	0.39	0.40	0.41	0.40
J_3	0.58	0.55	0.56	0.54	0.53	0.48

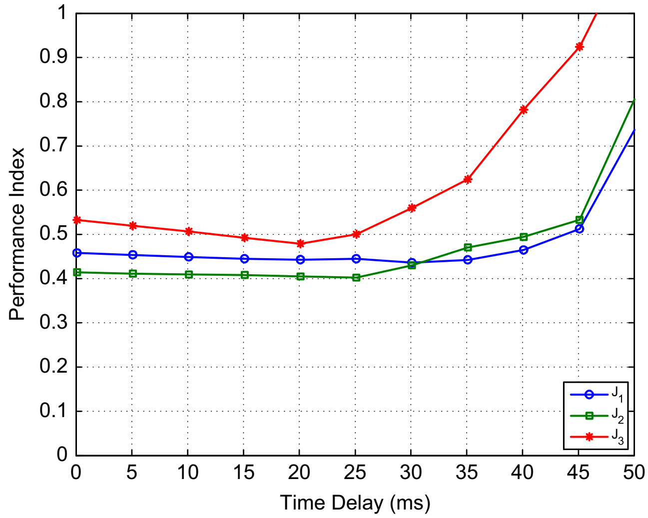

Fig. 4. Performance index versus time delay for nominal system.

compare the results). It can be seen that the proposed controller produces better performance than all of the other methods in terms of maximum absolute acceleration reduction under the same maximum control force and the good peak response quantities are not affected even when time delay $\tau = 20$ ms exists in the control input.

To further validate the effectiveness of the designed controller in dealing with the time delay problem, the effect of time delay on the responses of the structure is studied by calculating the values of J_1, J_2, and J_3 versus the time delay τ, as shown in Fig. 4. It can be seen from this figure that the closed-loop performances are all better than the corresponding open-loop system performances and these good performances can be kept up to the maximum time delay (20 ms) with no more degradation in peak response quantities.

For uncertain system, the proposed controller guarantees robust stability and performance within all the ranges of parameter uncertainties considered in controller design. Robust stability and performance of the proposed controller are verified through numerical simulations for the cases with various parameter uncertainties. For brevity, the responses of the interstorey drift and the absolute acceleration of the uncertain system considering six cases when time delay $\tau = 0$ and 20, respectively, are studied. Firstly, we only consider four-vertex cases where the system stiffnesses and damping coefficients are given as their vertex values, respectively. In the following, Case 1 corresponds to $k_i = 1.4 \times 980$ kN/m and $c_i = 1.4 \times 1.407$ kN s/m, Case 2 corresponds to $k_i = 0.6 \times 980$ kN/m and $c_i = 0.6 \times 1.407$ kN s/m, Case 3 corresponds to $k_i = 1.4 \times 980$ kN/m and $c_i = 0.6 \times 1.407$ kN s/m, and Case 4 corresponds to $k_i = 0.6 \times 980$ kN/m and $c_i = 1.4 \times 1.407$ kN s/m. Then, we consider another two cases where the system stiffnesses and damping coefficients are kept as their nominal values but the first floor mass changes. In such cases, Case 5 corresponds to $m_1 = 1200$ kg, Case 6 corresponds to $m_1 = 850$ kg.

Among them, the interstorey drift of the first floor and the absolute acceleration of the third floor, and control force for Case 3 are shown in Fig. 5 when $\tau = 0$ ms and plotted in Fig. 6 when $\tau = 20$ ms. It can be seen from Fig. 5 that in spite of the changes on floor stiffness and damping coefficients, the closed-loop systems can achieve the good peak responses. Compared with Fig. 5, it can be seen from Fig. 6 that the peak responses of the closed-loop system are similar in spite of the presence of input time delay. It verifies that the designed controller can robustly stabilise the system no matter of the parameter uncertainties and input time delay. The values of J_1, J_2, and J_3 versus time delay τ for Case 3 are shown in Fig. 7. It is seen from this figure that the closed-loop performances are all better than the corresponding open-loop performances and these good performances can be kept up to the maximum time delay (20 ms) with a few degradation in peak response quantities. Similarly, for Case 5, the interstorey drift of the first floor and the absolute acceleration of the third floor, and control force are plotted in Fig. 8 when $\tau = 0$ ms and Fig. 9 when $\tau = 20$ ms. From Figs. 8 and 9, it is observed that the

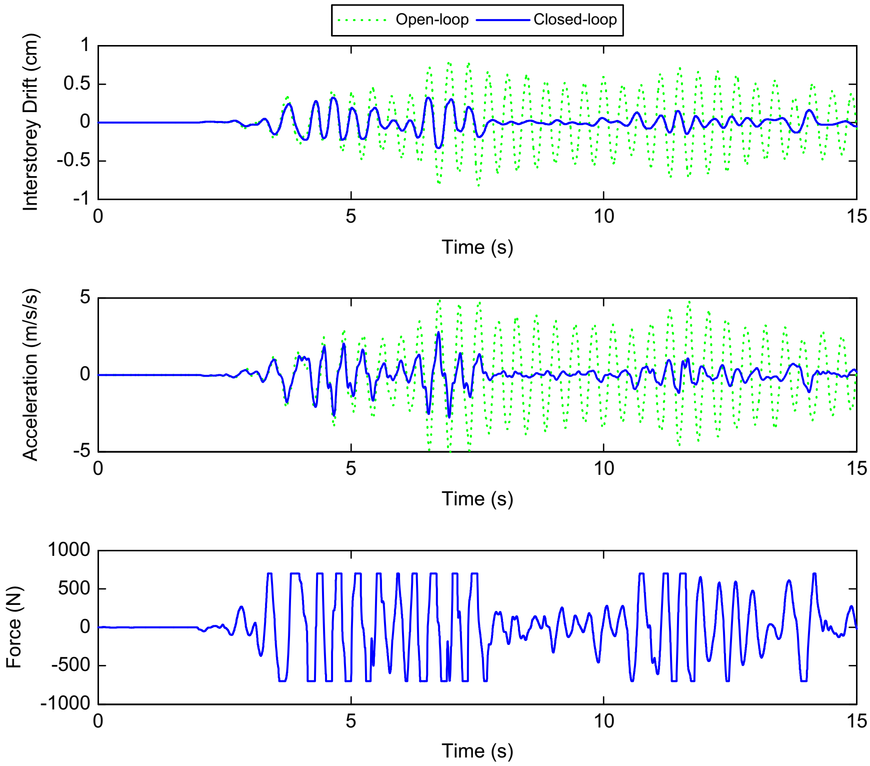

Fig. 5. Responses of interstorey drift of the first floor and absolute acceleration of the third floor, and control force for Case 3 when $\tau = 0$ ms.

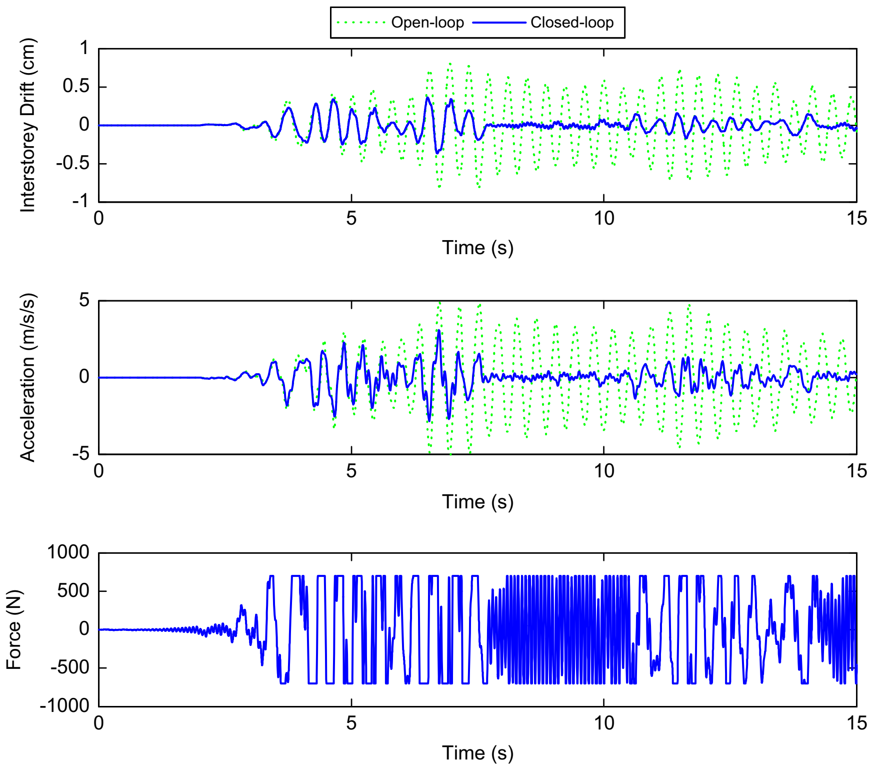

Fig. 6. Responses of interstorey drift of the first floor and absolute acceleration of the third floor, and control force for Case 3 when $\tau = 20$ ms.

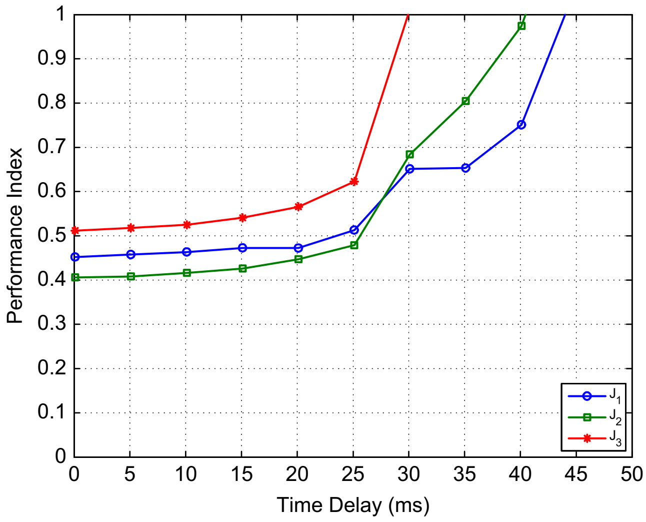

Fig. 7. Performance index versus time delay for Case 3.

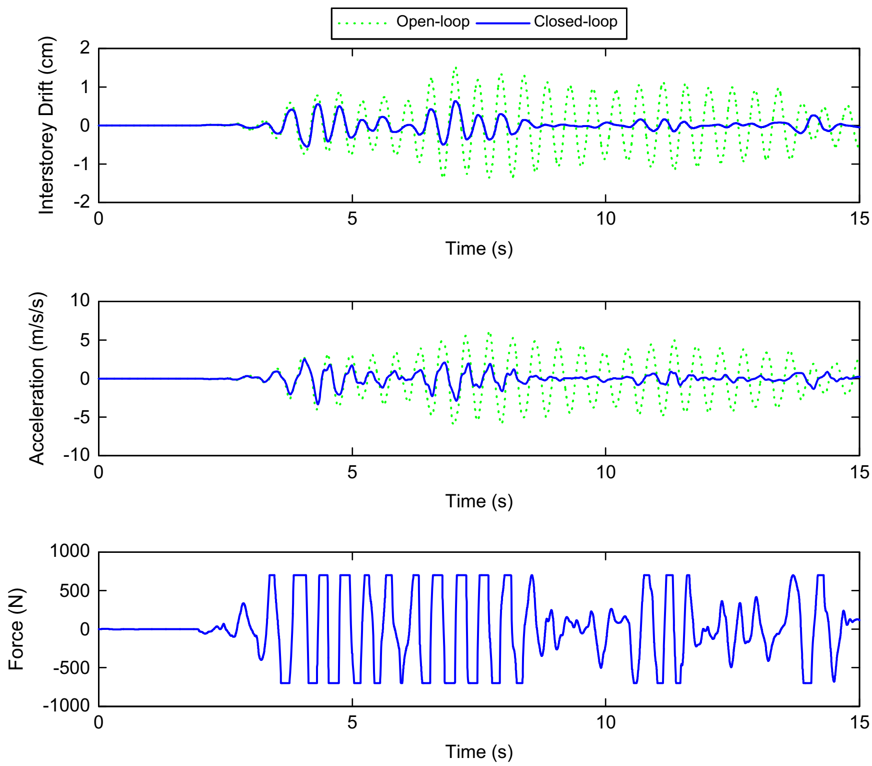

Fig. 8. Responses of interstorey drift of the first floor and absolute acceleration of the third floor, and control force for Case 5 when $\tau = 0$ ms.

closed-loop system keeps the good peak responses regardless of the floor mass variation and the presence of input time delay. The values of J_1, J_2, and J_3 versus time delay τ are shown in Fig. 10, where good closed-loop performances are achieved and kept even after the input time delay exceed 20 ms. Through extensive numerical simulations, it is checked within considered bounds of parameter uncertainties that the proposed controller achieves almost the same effectiveness in maximum responses reduction in comparison with some methods presented in [5]. For brevity, the detailed structural responses for all these cases are not shown here. But for the above-mentioned six cases, the values for J_2 and J_3 are summarised in Table 2. Besides the robust stability, it is observed that the proposed controller can achieve good performance no matter the presence of input time delay or not.

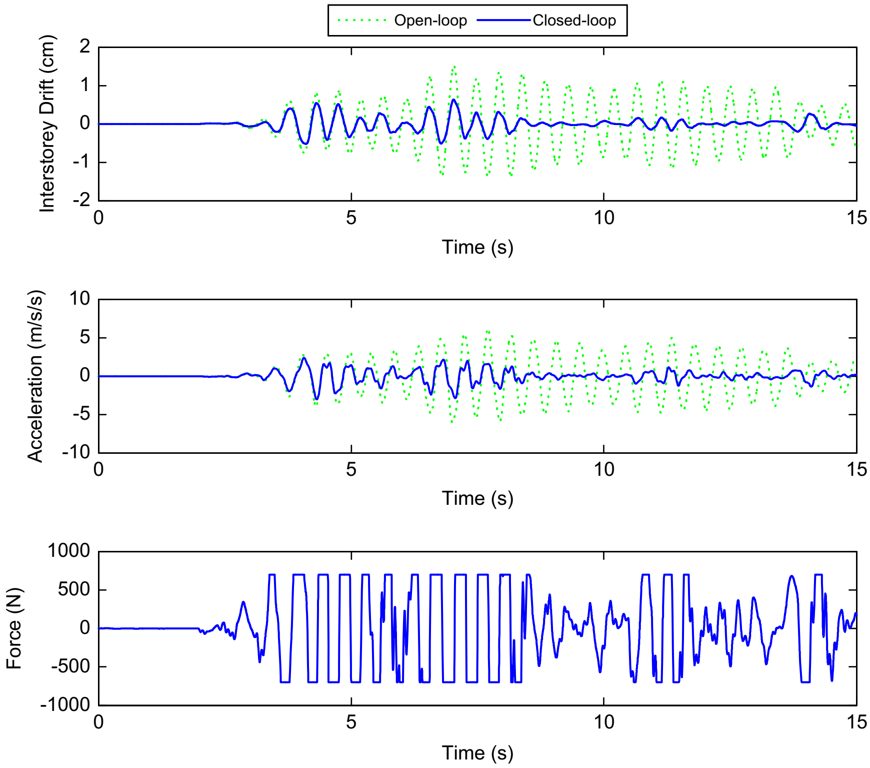

Fig. 9. Responses of interstorey drift of the first floor and absolute acceleration of the third floor, and control force for Case 5 when $\tau = 20$ ms.

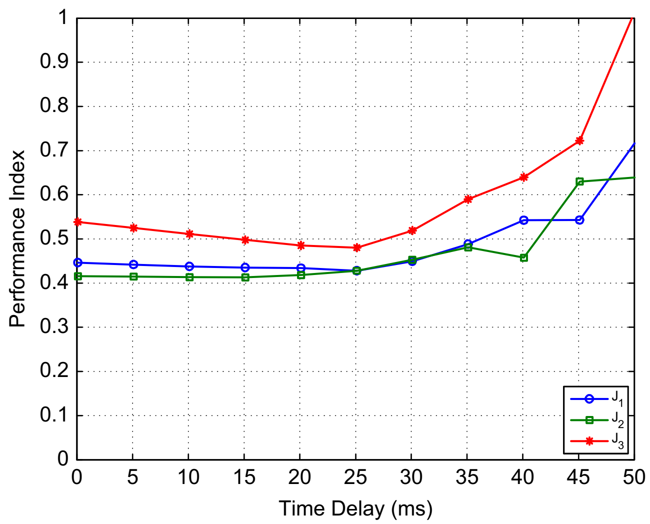

Fig. 10. Performance index versus time delay for Case 5.

Table 2
Normalised maximum response values for uncertain system.

	Case 1				Case 2				Case 3		Case 4		Case 5		Case 6	
	SSMC	[5]	Proposed		SSMC	[5]	Proposed		Proposed		Proposed		Proposed		Proposed	
τ	0	0	0	20	0	0	0	20	0	20	0	20	0	20	0	20
J_2	0.44	0.44	0.44	0.47	0.64	0.65	0.68	0.71	0.41	0.45	0.71	0.73	0.42	0.42	0.43	0.42
J_3	0.58	0.57	0.55	0.61	0.78	0.78	0.78	0.80	0.51	0.57	0.86	0.88	0.54	0.49	0.54	0.48

5. Conclusions

This paper presents an approach for designing robust saturation controller to attenuate the vibration occurred in uncertain structures with input time delay. The required feedback control gain matrix can be determined by solving a set of LMIs. Simulation example shows that the saturation controller designed using the presented approach can effectively achieve the attenuation objective even when the system has parameter uncertainties and input time delay. Considering parameter uncertainties, actuator saturation, and input time delay into the controller design process provides more realistic implementation for the vibration control of building structures.

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