

# A Study on Control of Heater Power and Heating Time for Thermoforming

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*Obtaining a uniform thickness of the final product using thermoforming is difficult, and the thickness distribution depends strongly on the distribution of the sheet temperature. In this paper, the time-dependent temperature distribution of the total sheets in the storing stage was studied because the temperature after the storing stage is the initial temperature of the heating process. An analytic solution for simulating the storing stage was derived. Using the solved analytic solution, the time-dependent temperature distribution of the total sheets was found out under the condition of assuming that the temperature-dependent specific heat of the ABS sheets was a certain constant value. Finally, the control method for a successful thermoforming using the heater power or heating time was researched in order to improve the quality of the final products. The results show that the satisfied temperature distribution can be obtained by adjusting the heater power or heating time. The method for analysis in this study will be used to improve the quality of the final products.*

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## NOMENCLATURE

$C_p$ =specific heat of ABS sheet, J/kg.K  
 $H$ =thickness of total sheets, m  
 $h$ =heat transfer coefficient, W/m<sup>2</sup>.K  
 $k$ =thermal conductivity of ABS, 0.174 W/m.K  
 $k_{eq}$ =thermal conductance, W/m.K  
 $L$ =half of the total thickness, m  
 $N$ =total number of sheets  
 $q_{in}$ =inputted heat flux, W/m<sup>2</sup>  
 $R_c$ =contact resistance, m. K/W  
 $T_a$ =average temperature, K  
 $T_c$ =center temperature, K  
 $T_s$ =surface temperature, K  
 $T_0$ =initial temperature, K  
 $T_\infty$ =environmental temperature, K  
 $t$ =time, s  
 $t_h$ =heating time, s  
 $\rho$ =density of ABS sheet, 1050 kg m<sup>-3</sup>  
 $\alpha$ =thermal diffusivity of ABS, m<sup>2</sup>/s  
 $\eta_n$ =eigen value

## 1. Introduction

Thermoforming is a method of manufacturing plastic parts by preheating a flat sheet of plastic to its forming temperature, then bringing it into contact with a mold whose shape it takes. The sheet is held against the mold surface unit until cooled. The formed part is then trimmed from the sheet.<sup>1,2</sup>

Thermoforming is one of the most versatile and economical processes available for manufacturing returnable packaging and many other products. However, obtaining a uniform thickness of the final product with thermoforming is difficult. The thickness distribution strongly depends on the distribution of the sheet temperature. Table 1 shows the temperature of the forming window, which is the marginal temperature of the ABS sheet.<sup>3-5</sup> The uneven temperature of the upper and lower surfaces, and the thickness direction may cause defects such as cracks and wrinkles.

In this paper, the time-dependent temperature distribution of the total sheets in the storing stage was studied for investigating real status because the temperature distribution of the total sheets after the storing stage is the initial temperature of the heating process. Figure 1 shows the schematic of the storing stage.

Table 1 Temperature of forming window

Forming Window	Lower Forming Temperature(°C)	Normal Forming Temperature(°C)	Upper Forming Temperature(°C)
Objective	140	145	155

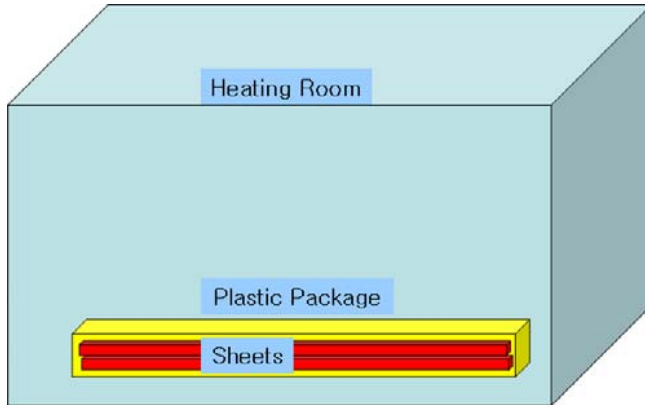


Fig. 1 Schematic of storing stage

Also, a new method for control using the heating time and heater power was introduced for a successful thermoforming.

## 2. Governing Equation and Method for Analysis

Before the heating process of thermoforming, the ABS sheet was stored in a room which has a constant temperature and humidity under the condition of packaging with plastic. The constant temperature of the room makes the sheets keep a suitable temperature for thermoforming. The constant humidity and plastic package defend the sheets from water or the other things. The initial temperature of the ABS sheets which has been pressed out is 60°C~65°C, and the temperature of the room is 50°C. In order to simulate the storing stage, natural convection and heat conduction must be considered. Also, the heat transfer caused by contact resistance between the sheets should be included.

### 2.1 Convective Heat Transfer

Natural convection must be considered in analyzing the storing stage before the heating process of thermoforming.<sup>6,7</sup> The heat transfer coefficient can be calculated using Goldstein, Lloyd, and Moran's correlation.<sup>6-8</sup> The convective heat transfer from the sheets to the environment can be obtained by using the calculated heat transfer coefficient.

### 2.2 Conductive Heat Transfer

#### 2.2.1 Analytic Solution

Conductive heat transfer through the thickness direction was considered to simulate the temperature distribution. When assuming that the heat transfer coefficients of the upper and lower surfaces are the same, the symmetry condition can be used. The governing equation, boundary conditions and initial condition are as follow.

$$\rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}, \quad 0 \leq x \leq L, \quad (1)$$

$$x = 0 : k \frac{\partial T}{\partial x} = 0 \quad (2)$$

$$x = L : -k \frac{\partial T}{\partial x} = h(T_s - T_\infty), \quad 0 \leq t$$

$$t = 0 \quad T = T_0 \quad (3)$$

Where  $\rho$  is the density of the ABS sheets ( $\text{kg/m}^3$ ),  $L$  is the half of the total thickness (m),  $k$  is thermal conductivity of the ABS sheets ( $\text{Wm}^{-1}\text{K}^{-1}$ ), and  $c$  is the specific heat of the ABS sheets ( $\text{Jkg}^{-1}\text{K}^{-1}$ ),  $T_0$  is the initial temperature of the ABS sheets (K),  $T_\infty$  is the environmental temperature (K), and  $T_s$  is the surface temperature of the total sheets.

In order to calculate an analytic solution, the specific heat of the ABS sheets were assumed as a constant value(mean value). The analytic solution can be obtained using Eq. (5).

$$\theta = \frac{T - T_\infty}{T_s - T_\infty}, \quad x' = \frac{x}{L}, \quad t' = \frac{\alpha t}{L^2} = F_0 \quad (4)$$

$$\theta = \sum_{n=1}^{\infty} C_n \exp(-\eta_n^2 F_0) \cos(\eta_n x') \quad (5)$$

$$C_n = \frac{4 \sin(\eta_n)}{2\eta_n + \sin(2\eta_n)}, \quad B_i = \frac{hL}{k}$$

Where  $\eta_n$  are the eigen values which are the solutions of  $\eta_n \tan(\eta_n) = B_i$ , and  $\alpha$  is the thermal diffusivity defined as  $\alpha = k/\rho c$ .

#### 2.2.2 Consideration of Contact Resistance

The thermal conductance caused by contact resistance between the sheets ( $k_{eq}$ ) must be considered in the conductive heat transfer.  $k_{eq}$  can be obtained using Eq. (6).

$$\frac{1}{k_{eq}} = N \frac{H}{k} + (N-1)R_c \quad (6)$$

Where  $N$  is the total number of the sheets, and  $H$  is the total thickness of the sheets. The value of  $k_{eq}$  was 0.174W/mK under the condition of assuming that the total number of the sheets and contact resistance ( $R_c$ ) were 200 and  $0.3 \times 10^{-4} \text{mK/W}$ , respectively.<sup>3</sup>

## 3. Time-Dependent Temperature Distribution

In order to obtain the analytic solution, the heat transfer coefficients of the upper and lower surfaces were assumed as a constant value. The constant value was the bigger one of the upper and lower surfaces at the initial temperature of the sheets which were calculated using Goldstein, Lloyd, and Moran's correlation.<sup>6-8</sup> The heat transfer coefficient of the case which has a plastic package was assumed as the half of the constant value. Also, Biot number can be obtained using the calculated heat transfer coefficient, half of the total thickness of ABS sheets and thermal conductivity. Table 2 shows the heat transfer coefficient, Biot number of each different analysis condition.

The heat transfer coefficient was larger under the condition of having a high temperature difference between the surface and center of the total sheets as shown in Table 2.

Table 2 Heat transfer coefficient, Biot number of each different condition

Item		Environmental Temp(°C)	$h(W/m^2k)$	Biot Number
With Plastic Package	Room	50	2.044	3.524
	Outside	25	2.9106	5.0183
Without Plastic Package	Room	50	4.088	7.0483
	Outside	25	5.821	10.036

Table 3 Time-dependent temperature distribution of total sheets

CASE		Environmental Temp(°C)	24h		48h		72h	
			Surface (°C)	Center (°C)	Surface (°C)	Center (°C)	Surface (°C)	Center (°C)
A	Room, With Plastic Package	50	56	64.5	54.5	63	53.5	62
B	Outside, With Plastic Package	25	37	64	34	59.5	32.5	54
C	Room, Without Plastic Package	50	53	64.5	52.5	62	52	60.5
D	Outside, Without Plastic Package	25	32	63.5	29	57.5	28	52.5

The analytic solution solved in the previous chapter was used to simulate the time-dependent temperature distribution of the total sheets under the condition of using a constant value of specific heat of the ABS sheets(mean value). In this analysis, the thermal conductivity, density and specific heat of the ABS sheets were 0.174W/mK, 1050kg/m<sup>3</sup> and 2590J/kg·K, respectively. The heat transfer coefficients were from Table 2, and the total number of the sheets was 200.

Table 3 shows the time-dependent temperature distribution of the total ABS sheets. The temperature of surface and center means the temperature of the 1<sup>st</sup> (or 200<sup>th</sup>) and 100<sup>th</sup> sheet surfaces respectively. The surface temperature was changed much faster, but the temperature of the center was changed a little.

From the analysis results, the temperature of the total sheets cannot be equal to the temperature of the storing room. So, the temperature distribution must be considered in the heating process of thermoforming for improving the quality of the final outputs.

#### 4. Control of Heater Power and Heating Time

In this study, it was assumed that the machine used to thermoforming was suitable for the sheet whose initial temperature was 58°C under the condition of considering the value in mass production. Using the optimization method developed in a previous study, the optimal heater power distribution can be obtained as shown in Table 4.<sup>8</sup> The values mean the percentage to the maximum usable heater power. Table 5 shows the temperature distribution of the sheet after 100s using the heater power distribution shown in Table 4.

Temperature difference is more than 10°C between the sheets after the storing stage as discussed in previous chapter. So, the 2 cases which having initial temperature of 52°C and 64°C have been investigated because 52°C and 64°C are the representative values in real production. Using the optimal heater power distribution shown in Table 4, the 2 cases were simulated. The analysis results show

Table 4 Optimal heater power distribution

%	1	2	3	4	5	6	7	8
1	100.0	80.7	88.5	86.4	86.3	88.6	80.7	100.0
2	49.8	0.0	0.0	0.0	0.0	0.0	0.0	49.8
3	8.7	11.5	28.0	22.9	23.0	27.8	11.6	8.6
4	70.8	34.5	29.8	31.2	30.8	30.1	34.3	70.9
5	30.7	12.7	22.3	19.1	19.6	21.9	13.0	30.5
6	30.5	13.0	21.9	19.6	19.1	22.4	12.7	30.6
7	70.9	34.3	30.1	30.8	31.2	29.8	34.5	70.8
8	8.6	11.6	27.8	23.0	22.9	28.0	11.5	8.6
9	49.8	0.0	0.0	0.0	0.0	0.0	0.0	49.8
10	100.0	80.6	88.6	86.3	86.4	88.5	80.7	100.0

Table 5 Temperature distribution using optimal heater power distribution(initial temperature: 58°C)

°C	1	2	3	4	5	6	7	8
1	140.32	142.98	142.82	142.97	142.97	142.82	142.98	140.32
2	145.71	145.13	144.92	145.01	145.01	144.92	145.13	145.71
3	141.98	142.85	143.56	143.59	143.59	143.56	142.85	141.98
4	144.31	145.04	145.02	145.12	145.12	145.02	145.04	144.31
5	143.45	144.21	144.56	144.63	144.63	144.56	144.21	143.45
6	143.45	144.21	144.56	144.63	144.63	144.56	144.21	143.45
7	144.31	145.04	145.02	145.12	145.12	145.02	145.04	144.31
8	141.98	142.85	143.56	143.59	143.59	143.57	142.85	141.98
9	145.71	145.13	144.92	145.01	145.01	144.92	145.13	145.71
10	140.32	142.98	142.82	142.97	142.97	142.82	142.98	140.32

Table 6 Temperature difference between the 3 cases

	Initial Temperature: 52°C	Initial Temperature: 58°C	Initial Temperature: 64°C
Mean Temperature(°C)	139	145	148
Temperature Difference(°C)	-6	0	3

Table 7 Temperature difference between the case having initial temperature of 52°C and the baseline case

°C	1	2	3	4	5	6	7	8
1	-4.58	-4.69	-4.72	-4.73	-4.73	-4.72	-4.69	-4.58
2	-4.68	-4.81	-4.87	-4.88	-4.88	-4.87	-4.81	-4.68
3	-4.73	-4.9	-4.96	-4.98	-4.98	-4.96	-4.9	-4.73
4	-4.77	-4.96	-5.02	-5.04	-5.04	-5.02	-4.96	-4.77
5	-4.79	-4.98	-5.05	-5.07	-5.07	-5.04	-4.97	-4.78
6	-4.79	-4.98	-5.05	-5.07	-5.07	-5.05	-4.98	-4.78
7	-4.77	-4.95	-5.02	-5.04	-5.04	-5.02	-4.95	-4.77
8	-4.73	-4.9	-4.96	-4.97	-4.98	-4.96	-4.9	-4.73
9	-4.68	-4.81	-4.87	-4.88	-4.88	-4.87	-4.81	-4.68
10	-4.58	-4.69	-4.72	-4.73	-4.73	-4.72	-4.68	-4.57

that the temperature difference is 9°C between the 2 cases as shown in Table 6. The effect of the 2 cases on the temperature difference was different(6°C and 3°C) as shown in Table 6 although the difference magnitude of the initial temperature was the same. Table 7 and Table 8 show the temperature difference between the proposed case(initial temperature: 52°C or 64°C) and baseline case which having initial temperature of 58°C, respectively.

Table 8 Temperature difference between the case having initial temperature of 64 °C and the baseline case

°C	1	2	3	4	5	6	7	8
1	3.41	3.49	3.53	3.54	3.54	3.53	3.49	3.41
2	3.45	3.6	3.65	3.67	3.67	3.65	3.6	3.45
3	3.54	3.71	3.76	3.78	3.78	3.76	3.71	3.55
4	3.57	3.74	3.8	3.82	3.83	3.8	3.74	3.56
5	3.58	3.77	3.83	3.86	3.86	3.84	3.77	3.59
6	3.58	3.77	3.83	3.86	3.86	3.83	3.77	3.58
7	3.56	3.74	3.8	3.83	3.83	3.8	3.74	3.56
8	3.54	3.71	3.76	3.79	3.78	3.76	3.71	3.54
9	3.45	3.6	3.65	3.66	3.67	3.65	3.6	3.45
10	3.41	3.49	3.53	3.54	3.54	3.53	3.49	3.42

#### 4.1 Control of Heating Time

The required heating time suitable for a certain heater power can be easily found out using the analytic solution which has been solved under the condition that the specific heat of the ABS sheet and the heat flux in the heating process are constant values. The analytic solution can be expressed as Eq. (7) under the condition that symmetry condition was used. Where  $L$  is the half of the sheet thickness (m), and  $q_{in}$  is the constant heat flux ( $3.6 \text{ kW/m}^2$ ).

$$\frac{v}{q_{in} \cdot L / k} = F_0 + \frac{3x^2 - L^2}{6L^2} - \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} e^{-n^2 \pi^2 F_0} \cos \frac{n\pi x}{L} \quad (7)$$

$$F_0 = \frac{\alpha t}{L^2}, \quad \alpha = \frac{k}{\rho c} \quad (8)$$

The center temperature ( $T_c$ ,  $x=0$ ), surface temperature ( $T_s$ ,  $x=L$ ) and average temperature ( $T_a$ ) can be calculated using Eqs. (9)-(11). Where  $v$  is equal to  $(T - T_{\infty})$ , and  $T_{\infty}$  is RT (room temperature).

$$\frac{v_c}{q_{in} \cdot L / k} = F_0 - \frac{1}{6} \quad (9)$$

$$\frac{v_s}{q_{in} \cdot L / k} = F_0 + \frac{1}{3} \quad (10)$$

$$\frac{v_a}{q_{in} \cdot L / k} = F_0 \quad (11)$$

Eqs. (9)-(11) can be changed as follow.

$$T_a = T_{\infty} + \frac{q_{in} \cdot t}{\rho c L} \quad (12)$$

$$T_c = T_a - \frac{1}{6} \frac{q_{in} \cdot L}{k} \quad (13)$$

$$T_s = T_a + \frac{1}{3} \frac{q_{in} \cdot L}{k} \quad (14)$$

The heating time ( $t_h$ ) can be expressed as Eq. (15) which means the time that  $T_a$  is equal to NFT (Normal Forming Temperature) shown in Table 1.

$$t_h = \frac{NFT - T_{\infty}}{q_{in} / \rho c L} \quad (15)$$

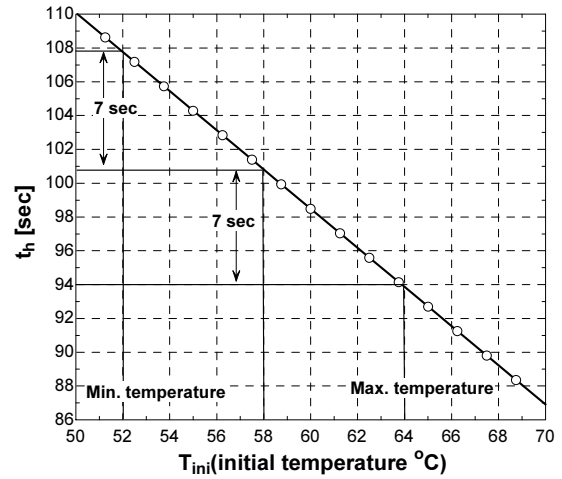


Fig. 2 Heating time under the condition of considering initial temperature of sheet using analytic solution

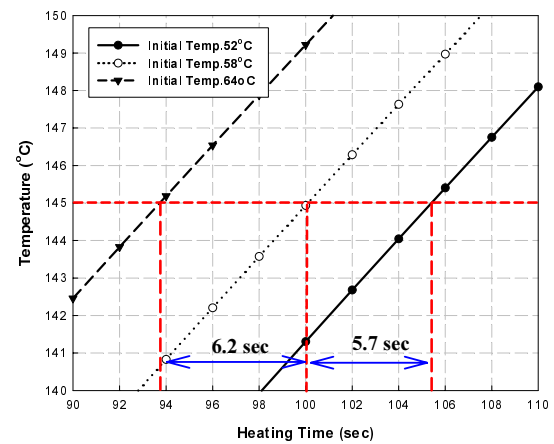


Fig. 3 Heating time under the condition of considering initial temperature of sheet using heating process analysis code

Table 9 Heater power with each different initial temperature

Initial Temperature(°C)	40	50	60	70
Total Heater Power(kW)	13.24	12.36	11.45	10.52
Mean Working Rate(%)	41.4	38.6	35.8	32.9
Difference between each case and baseline(W)	-1.6	-0.72	0.19	1.12

The heating time of the 2 cases(initial temperature: 52 °C and 64 °C) for obtaining NFT (Normal Forming Temperature) are calculated using the analytic solution, and the heating time of the 2 cases should be adjusted 7s and -7s respectively for getting NFT (Normal Forming Temperature) as shown in Fig. 2.

For improving the accuracy, the heating process analysis code developed in the previous study was used to obtain the real required heating time. The heating time of the 2 cases(initial temperature: 52 °C and 64 °C) for obtaining NFT (Normal Forming Temperature) should be adjusted 6.2s and -5.7s respectively as shown in Fig. 3.

#### 4.2 Control of Heater Power

A method for control using heater power was carried out for a successful thermoforming. Table 9 and Fig. 4 show optimal working rate of each case having a different initial temperature. The

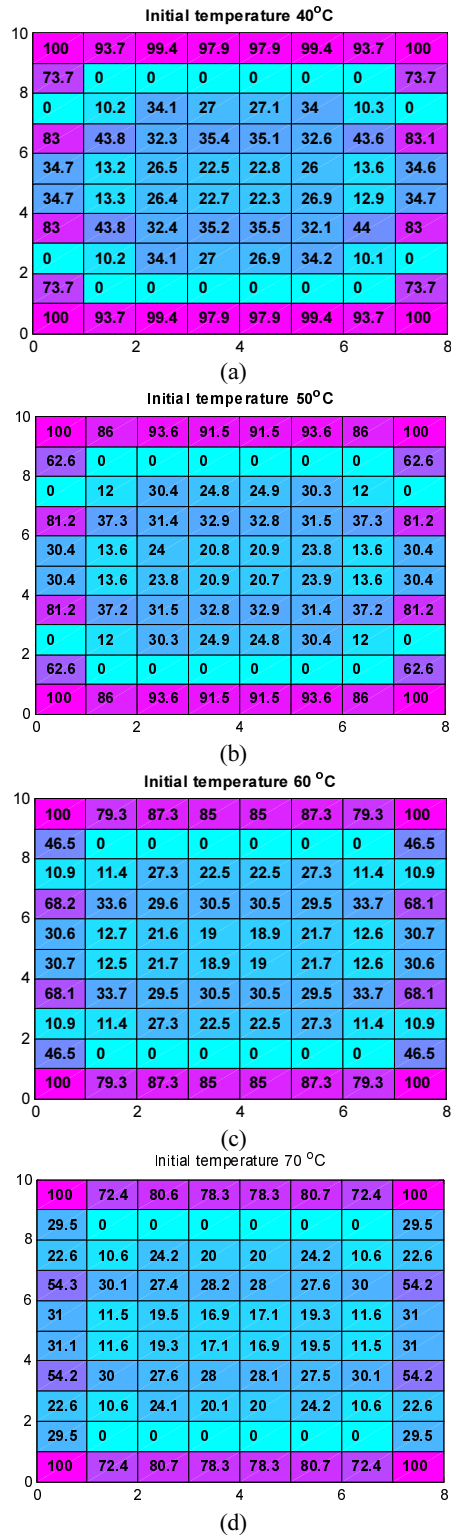


Fig. 4 (a) Optimal heater working rate of the case having initial temperature of 40°C(%) (b) Optimal heater working rate of the case having initial temperature of 50°C(%) (c) Optimal heater working rate of the case having initial temperature of 60°C(%) (d) Optimal heater working rate of the case having initial temperature of 70°C(%)

working rates mean the percentage to the maximum usable heater power. The mean working rate and total heater power of the baseline case(initial temperature: 58°C) were 36.4% and 11.64kW.

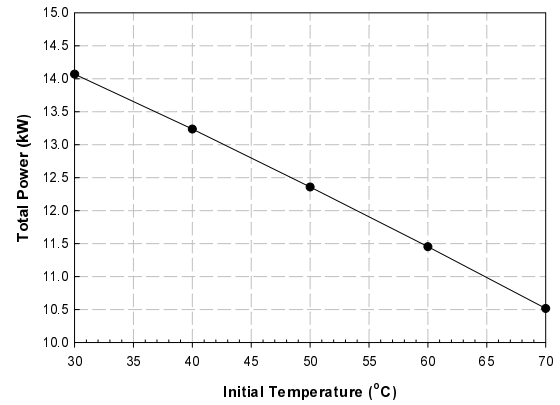


Fig. 5 Total heater power of each initial temperature

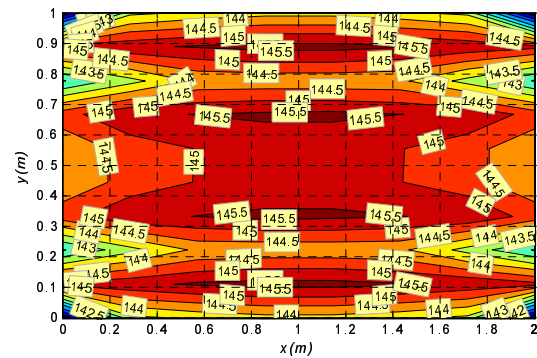


Fig. 6 Temperature distribution of the case which having initial temperature of 52°C

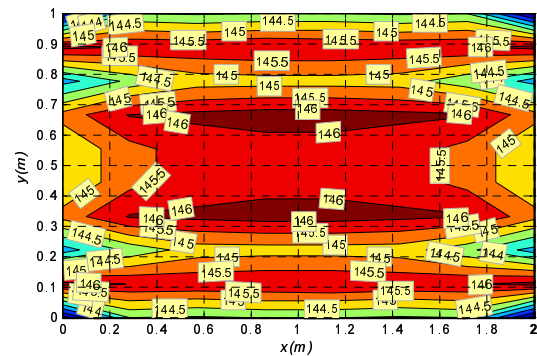


Fig. 7 Temperature distribution of the case which having initial temperature of 64°C

Fig. 5 shows the total heater power of each case which has the gradient of 90W/°C. Using the heating process analysis code, the 2 cases which having initial temperature of 52°C and 64°C were simulated by adjusting the weighting factor under the condition of maintaining the pattern shown in Table 4. The weighting factor means the ratio that the total heater power of the baseline case was divided by the total heater power of each case. The working rate for each position was just adjusted using the weighting factors(1.05 and 0.95) which have been calculated using the gradient shown in Fig. 5. In this adjustment, the working rate was 100% when the working rate was larger than 100%. Figure 6 and 7 show the analysis results of the 2 cases using the corrected heater power distributions. The mean temperature of the sheets of the 2 cases was similar to the baseline case.

## 5. Conclusion

An analytic solution for simulating the storing stage was derived. Using the analytic solution, the time-dependent temperature distribution of the total sheets was found out under the condition of assuming the temperature-dependent specific heat of the ABS sheets as a certain constant value. A new method for control using the heating time and heater power in the heating process under the condition of considering the initial temperature of the sheet was developed for a successful thermoforming. The method for analysis in this study will be used to improve the quality of the final products.

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