Space-time uncertainty relation from quantum and gravitational principles

Yi-Xin Chen^{*} and Yong Xiao[†] Zhejiang Institute of Modern Physics and Zhejiang University, Hangzhou 310027, China

By collecting both quantum and gravitational principles, a space-time uncertainty relation $(\delta t)(\delta r)^3 \ge \pi r^2 l_p^2$ is derived. It can be used to facilitate the discussion of several profound questions, such as computational capacity and thermodynamic properties of the universe and the origin of holographic dark energy. The universality and validity of the proposed relation are illustrated via these examples.

PACS numbers: 04.60.-m, 04.70.Dy, 95.36.+x, 03.67.Lx

I. INTRODUCTION

Any physical system in the quantum world is undergoing quantum fluctuations, which introduce space and time uncertainties to the system. Interestingly, general relativity (GR) conspired with quantum mechanics (QM) have determined the severity of these space-time uncertainties.

We firstly start from a theoretical question: What is the maximal spatial resolution when using photons uniformly spread in a box of size l for observation. Statistical mechanics tells that the photon gas has the energy $E \sim \frac{k_B^4}{c^3 h^3} l^3 T^4$. To insure that the system will not collapse to form a black hole, there must be a GR limitation that $E \sim \frac{k_B^4}{c^3 h^3} l^3 T^4 \leqslant E_{bh} \sim \frac{c^4}{G} l$, where E_{bh} is the energy of a black hole of the same size. This limitation leads to $k_B T \leqslant \left(\frac{c^7 h^3}{G}\right)^{1/4} l^{-1/2}$. The corresponding thermal wavelength is thus $\lambda = \frac{hc}{k_B T} \ge l^{1/2} \left(\frac{hG}{c^3}\right)^{1/4} = l^{1/2} l_p^{1/2}$, where $l_p \equiv \left(\frac{hG}{c^3}\right)^{1/2}$ is the Planck length. It gives the uncertainties in the positions of the photons themselves, so one can not detect the system by virtue of these photons with more precision than their shortest wavelength $l^{1/2}l_p^{1/2}$. Such a limitation is firstly given by 't Hooft in [1]. Since the size of a physical system should be larger than l_p , there is $\delta l \ge l^{1/2} l_p^{1/2} \ge l_p$. It implies that when limited to a smaller region, one can always employ higher energy photons and detect finer structures until the Planck scale.

The corresponding resolution of the system with $\delta l \ge l^{1/2} l_p^{1/2}$ reaches $\frac{l^3}{(\delta l)^3} \sim A^{3/4}$, where A is the boundary area of the system. To find a holographic resolution [2, 3] which is proportional to the area, namely $\frac{l^3}{(\delta l)^3} \sim A$, we have to require $\delta l \ge l^{1/3} l_p^{2/3}$. The latter uncertainty is consistent with holographic principle and related to the unknown details of black hole physics [9]. And it is different from the uncertainties of radiation systems which

are described by local quantum field theory (LQFT).

Notice that both the two uncertainty relation $\delta l \geq l^{1/2} l_p^{1/2}$ and $\delta l \geq l^{1/3} l_p^{2/3}$ have spread in the literature with various derivations and applications [1, 4, 5, 6, 8, 9, 10]. Firstly the two relations can be written with a form of ultraviolet-infrared (UV-IR) relations. Cohen et al [5] have proposed that the UV-IR relation for LQFT systems and for holographic systems are respectively $l^3 \Lambda^4 \leq l$ and $l^3 \Lambda^3 \leq l^2$, or written as $\Lambda \leq l^{-1/2}$ and $\Lambda \leq l^{-1/3}$. (Hereafter we shall set $G, \hbar, c, k_B = 1$ and not write l_p or m_p explicitly.) Since the UV cutoff Λ determines the minimal detectable lengths, that is $\Lambda = (\delta l)^{-1}$, they directly correspond to the two uncertainty relations above. Again, the different UV-IR relations play an important role in the strict verification of the entropy gap between LQFT and holographic systems, from $A^{3/4}$ to A [33].

On the other hand, the two type of uncertainties could be understood as the uncertainties in the measurement of a distance l. Realizing such distance measurements through Gedanken experiments using clocks and light signals, by carefully evaluating the quantum and gravitational corrections to the measurement procedure, both $\delta l \ge l^{1/2}$ and $\delta l \ge l^{1/3}$ can be derived consistently [34] [8]. Obviously, which δl should be applied depends on which tools one use for observation or transmitting signals: conventional photons or some unknown holographic "particles".

Moreover, Ng has treated these uncertainties from a viewpoint called quantum foam [9, 10]. That is treating the space-time geometry as undergoing quantum fluctuations which manifest themselves by the accuracy with which one can measure a distance l, generally written as $\delta l \ge l^{\alpha} l_p^{1-\alpha}$ with $\alpha = \frac{1}{2}$ and $\alpha = \frac{1}{3}$ as its special cases.

The two type of quantum fluctuations also find applications in cosmology to establish dark energy models. It has been found that both of them can yield an energy density of the form $\rho_{\Lambda} \sim l^{-2}$. Choosing the IR cutoff l to be a proper cosmological scale such as the size of cosmological horizons, the energy density of this type defines the so-called "holographic dark energy (HDE)" models [20, 21, 22, 23, 24, 25].

Generally speaking, different systems are subjected to different quantum fluctuations, up to what they are composed of. The characteristic quantum fluctuation

^{*}Electronic address: yxchen@zimp.zju.edu.cn

[†]Electronic address: yongxiao2008@live.cn

 $\delta l \ge l^{1/2}$ is applicable to LQFT systems with entropy bound $A^{3/4}$, while $\delta l \ge l^{1/3}$ is applicable to strongly gravitational systems like black holes with holographic entropy. Meanwhile, δl not only characterizes the quantum fluctuations and then the space-time uncertainties within the interior of a physical system, but also characterizes the measurement error to the size l of the entire system.

In this Letter, we devote ourselves to gain more insights into the significance of the quantum uncertainties rooted in a system. In Section II, from a quantum computational perspective, combining principles from GR and QM, we find a space-time uncertainty relation $(\delta t) (\delta r)^3 \ge \pi r^2$, with δt and δr representing the severity of space-time fluctuations of the constituents of the system at small scales. We find the relation could be very useful in the cases where the analysis of fundamental degrees of freedom plays an essential role. Several examples of this kind will be included in Section III. The main physical characteristics of them are extracted directly through the space-time uncertainty relation and compared to the known results in the literature.

II. SPACE-TIME UNCERTAINTY RELATION

A physical system could always be reduced to independent degrees of freedom doing Boolean calculations. The shift of states on these self-governed quantum bits leads to the evolvement of the entire system. In this section, we shall start from such a quantum computational perspective and derive a space-time uncertainty relation.

Without loss of generality, we consider a globular computer of radius r. Assume the computer is made up of $\frac{r^3}{(\delta r)^3} = r^3 \Lambda^3$ independent functional units which are employed to store information and execute instructions, with $\delta r \equiv \Lambda^{-1}$ representing the size of a single functional unit. Every unit has an energy ε for executing operations. GR requires that the computer as a whole cannot has an energy exceeding the mass of a black hole of the same size. Thus

$$\varepsilon r^3 \Lambda^3 \leqslant E_{bh} = r/2.$$
 (1)

For any independent degree of freedom or a quantum bit, the Margolus-Levitin theorem [11, 12, 13, 14] determines the minimal time it takes to finish an operation (a shift of states) is $\delta t = \frac{\pi}{2\varepsilon}$, where ε is the energy distributed in this degree of freedom for it to execute Boolean calculations. Together with Eq.(1), we find a space-time uncertainty relation

$$(\delta t)(\delta r)^3 \geqslant \pi r^2. \tag{2}$$

The above derivation to Eq.(2) is a heuristic one. Actually the simplified picture above is closely related to realistic physical systems. Taking the photon gas for example, the photons within have an uncertainty in position which is $\delta r \ge r^{1/2}$, as we have shown at the beginning of

this Letter. In addition, none of these $A^{3/4}$ uncorrelated photons (without wave-packet overlapped) is fixed and permanently unchangeable. Actually every of them is described by a quantum state evolving with time according to the QM laws, specifically Margolus-Levitin theorem here. The time interval it takes for a quantum bit or an independent photon here evolving from one state to its orthogonal states is typically $\delta t \ge r^{1/2}$. Obviously, it characterizes the uncertainty or randomness of the quantum states of an isolated photon in time direction. δr and δt together conform to Eq.(2). They characterize the quantum uncertainties of the constituents of the system within the space-time, but not directly the fluctuations of the space-time itself, unless one has introduced some microscopic mechanism of quantum gravity and then deals with a bulk of gravitons.

Whatever, it should be emphasized that Eq.(2) is not a space-time uncertainty relation of usual type. Though the physical system is composed of a large amount of fundamental units or degrees of freedom, in determining their uncertainties within space-time, GR has to be employed to limit the energy of the system as a whole. Thus Eq.(2) can only be applied to the cases taking a global viewpoint of measurement on the system, that is simultaneously taking all of its fundamental units into consideration. Furthermore, the deduced uncertainties δr and δt are relevant to the size l of the entire system. Actually it means that the combination of GR an QM principles would inevitably cause an unusual correlation between the global and local properties of a system like the UV-IR relations [5]. For general discussions about the limit on space-time measurements, one may refer to [8, 16] and references therein.

For LQFT systems like the conventional matter and radiation, one usually require the space and time are treated equally [9, 10]. Combining with the relation (2), the uncertainties of $\delta r \sim \delta t \ge r^{1/2}$ can be easily obtained. It can also be gained by directly comparing the energy formula $E \sim r^3 T^4 \sim T (r^3 T^3)$ for the photon gas system with Eq.(1). By contrast, to count the holographic degrees of freedom or entropy that is applicable to "black hole computers" [15], we have $\delta r \sim r^{1/3}$ and thus $\delta t \sim r$. The corresponding UV cutoffs of these systems are obtained from $\Lambda = (\delta r)^{-1}$.

UV cutoff
$$\Lambda$$
: 0 $\xrightarrow{\text{LQFT}}_{S \leqslant A^{3/4}} (rl_p)^{-1/2} \xrightarrow{\text{New physics?}}_{S \leqslant A} (rl_p^2)^{-1/3}$

As argued by Cohen et al, the physics with energy below the UV cutoff $\Lambda \leq r^{-1/2}$ is well-described by LQFT. It was also pointed out by Hsu [22] that when $\Lambda > r^{-1/2}$, the gravitational corrections to the energy of a LQFT system will be too large and lead it to undergo gravitational collapse, which makes a LQFT description invalid. The physics beyond LQFT from $\Lambda \sim r^{-1/2}$ to $\Lambda \sim r^{-1/3}$ is still obscure now. It might be some new physics constituting a necessary part of quantum gravity.

Note that the entropy bound $A^{3/4}$ is for conventional matter configurations, while A is for the back holes of the

same energy and is considered as the maximum entropy contained in the region in the spirit of holographic principle. Consider a system or a star composed of conventional matter with lesser entropy undergoes gravitational collapse to form a black hole with the area entropy A. It involves a drastic change in its interior metric from nearflat to an extreme one. What happens in such a collapse process in the context of quantum gravity? And what could fill such an entropy gap between $A^{3/4}$ and A? They are both interesting but difficult questions to be further explored and are beyond the scope of this Letter. It is worth to note that the so-called "monster configurations" which originated from a curved space consideration seem to be a candidate for filling up the gap between $A^{3/4}$ and A. See the original works of Hsu et al [18, 19] for details.

III. EXAMPLES

A. Universe as a supercomputer

Lloyd has investigated the universe from a quantum computational viewpoint in [12, 13]. It was found that the universe has performed about 10^{120} "ops" or "ticks and clicks" since the big bang. In this section, we shall explain and clarify this idea using the space-time uncertainty relation (2).

Since a computer has $\frac{r^3}{(\delta r)^3}$ working bits with each responding $\frac{t}{\delta t}$ times within a time interval t. Thus the total number of operations that can be performed in a supercomputer of radius r over time t, in other words, the number of events that can occur in this volume of space-time is

$$\sharp \equiv \frac{r^3}{(\delta r)^3} \frac{t}{(\delta t)} \leqslant \frac{rt}{\pi}.$$
(3)

The universe, like any computer produced in human factories, is obeying GR and QM and should be restricted by the above quantum-gravitational limit. It is a rather huge number when completed as $\frac{rt}{\pi l_{\perp}^2}$ and will be able to support any miracle that has happened in the history of the universe. This computational limit was obtained by Lloyd [14] from $\sharp = \frac{t}{\pi/(2E_{total})} \leqslant \frac{rt}{\pi}$. The equivalence of our derivation to this one is easily proved, by noticing that $E_{total} = \varepsilon \frac{r^3}{(\delta r)^3}$. The derivation in [14] is directly based on the energy limitation from GR. By contrast, our derivation reveals more subtleties on this issue, making one compare easily the differences between LQFT systems and holographic systems in computational aspect. That is, LQFT systems have at most $A^{3/4}$ computational units with each processing at a rate $r^{-1/2}$, while holographic objects like black holes have A units processing at the rate r^{-1} . Though these systems as whole are subject to the same bound on the information processing rate, which is $A^{3/4}r^{-1/2} \sim Ar^{-1} \sim r$ as found in [17], a single quantum bit of a LQFT system runs more efficiently than these in holographic systems.

Now consider a spatial-flat universe which is homogeneous and isotropic and is described by the FRW metric. The corresponding Friedmann equations read

$$3\left(\frac{\dot{a}}{a}\right)^2 = 8\pi\rho,\tag{4}$$

$$\dot{\rho} + 3(1+w)\frac{\dot{a}}{a}\rho = 0.$$
 (5)

Here w is defined by the effective equation of state (EoS) of the constituents of the universe, $p = w\rho$.

The first Friedmann equation (4) can be written as $\rho = \frac{3}{8\pi}r_a^{-2}$, where r_a is the radius of the dynamical apparent horizon of the universe. It is equal to the Hubble radius $r_h \equiv (\frac{\dot{a}}{a})^{-1}$ in the flat universe. Without confusion we shall write them both as r. The total energy confined within the apparent horizon is thus $\rho(\frac{4\pi}{3}r^3) = \frac{r}{2}$, as the same amount as the critical energy to form a black hole of the same size. Since the apparent horizon is argued to be a causal horizon [30], it is natural to view the FRW universe as a supercomputer of radius r which is running at its ceiling running speed determined by GR and QM. It is easy to compute out the number of "ops" performed by the universe since the big bang, through the formula $\sharp = \frac{1}{\pi l_{\perp}^2} \int r(t) dt$. From the Friedmann equations, the energy used to execute computations is just these spreading in the universe: radiation, dusts, dark energy, or others. Thus we know the universe keeps doing computations and what it computes is just the dynamical evolution of itself and its constituents [13]. Only little amount of its energy is employed by human beings to perform digital computations. For more details of the computational aspects of the universe, see [10, 13]. A point worthy of remark is that the EoS of the constituent greatly affects the calculation of the "ops" numbers executed by the universe, since it determines the behavior of r(t) by virtue of $\dot{r}(t) = \frac{3}{2}(1+w)$ which can be derived from the Friedman equations.

Knowing $E_{total} = \frac{r}{2}$ for the universe, the space-time uncertainty relation is saturated as

$$(\delta t) (\delta r)^3 = \pi r^2. \tag{6}$$

It characterizes the quantum fluctuations of the constituents uniformly distributed in the universe. We generally write $\delta r = cr^{\alpha}$, $\delta t = c^{-3}r^{2-3\alpha}$ obeying the relation, where c is a parameter of order 1. Different constituents of the universe such as local quantum fields or some unknown holographic contents shall involve different type of space-time fluctuations, with the choice of $\alpha = \frac{1}{2}$ for radiation and $\alpha = \frac{1}{3}$ for holographic constituents [9]. To the universe as a supercomputer, the size of any

To the universe as a supercomputer, the size of any functional cell must be smaller than the whole size r of the supercomputer, thus there is $\delta r \leq r$. Meanwhile, QM requires the lowest definable energy for an independent degree of freedom of the system confined in a box is the IR cutoff energy r^{-1} , thus $\varepsilon \geq r^{-1}$. The two limitations lead to the upper and lower bound on the space-time uncertainties. By applying Eq.(6), we determine the range of the parameter as $\alpha \in [\frac{1}{3}, 1]$. The total information storage capacity $\frac{r^3}{(\delta r)^3} \sim r^{3-3\alpha}$ of the computer can only range from 1 to A, as expected by the holographic principle. Furthermore, Eq.(6) directly reveals a space-time noncommutative property usually emerging in quantumgravitational models [26]. An evidence is that the spatial size δr of an independent operational unit of the system increases with its energy $\varepsilon \sim (\delta t)^{-1}$, which is a typical sign of the space-time noncommutativity.

B. Thermodynamics properties of the universe

In this section, we show that when employing the space-time relation (6), some of the thermodynamics properties of the universe can be extracted directly. Barrow [7] has investigated in detail the cosmological counterpart of the entropy gap between matter and radiation entropies and the Bekenstein-Hawking entropy. We implement such a cosmological entropy gap in our consideration. Both the radiation-dominated universe with information storage capacity $A^{3/4}$ and the holographic universe are referred to. More interestingly, when limited to the holographic case, the deduced expressions have a similar form with these in [31, 32], which aimed to explore the profound physical connections between thermodynamics and gravity.

Consider a spatial-flat FRW universe filled with a perfect fluid with EoS $p = w\rho$. The energy-momentum tensor of the fluid is of the form

$$T_{ab} = (\rho + p) u_a u_b + p g_{ab}, \tag{7}$$

where u_a satisfies $u_a u^a = -1$. The energy U is the integral of the energy density over the volume enveloped by the apparent horizon

$$U \equiv \int T_{ab} u^a u^b dV = \int \rho dV. \tag{8}$$

From the Friedmann equation (4), we find $U = \frac{r}{2}$ and $dU = \frac{1}{2}dr = \frac{1}{3}\rho dV$. This relation is different from the conventional one $dU = \rho dV$, due to the fact that the energy here is not proportional to the 3-dimensional volume of the system, but to its radius. On the other hand, the entropy of the universe is evaluated as

$$S = r^3 \left(\delta r\right)^{-3} = c^{-3} r^{3-3\alpha}.$$
 (9)

Having energy U and entropy S, the temperature can be obtained from the thermodynamical law dU = TdS - pdV, that is [35]

$$T = \frac{dU + pdV}{dS} = \frac{1}{3 - 3\alpha} \left(1 + 3w\right) \frac{1}{2} c^3 r^{-2 + 3\alpha}.$$
 (10)

Combining Eq.(9) and Eq.(10), we find $TS = \frac{1+3w}{3-3\alpha}U$.

Write it in a more general form

$$TS = \frac{1}{3 - 3\alpha} \int (\rho + 3p) dV$$

= $\frac{2}{3 - 3\alpha} \int \left(T_{ab} - \frac{1}{2} T_c^c g_{ab} \right) u^a u^b dV.$ (11)

Then the free energy of the system is given by

$$F = U - TS = \frac{1}{3 - 3\alpha} \int \left[\frac{1}{8\pi}R + (1 - 3\alpha)T_{ab}u^{a}u^{b}\right]dV.$$
(12)

Here the Einstein equation has been used.

Now we consider a radiation-dominate universe. Radiation has the EoS $w = \frac{1}{3}$. Together with its characteristic parameter $\alpha = \frac{1}{2}$ and the formula above Eq.(11), we obtain $TS = \frac{1+3w}{3-3\alpha}U = \frac{4}{3}U$. So the free energy is $F = U - TS = -\frac{1}{3}U$, exactly consistent with the thermodynamical property of photon gas. This observation is interesting, since it reveals that the application of $\alpha = \frac{1}{2}$ is essential for radiation. Surely, $T \sim r^{-2+3\alpha} \sim r^{-1/2}$ coincides with a familiar temperature-time relation for the radiation-dominated universe in standard cosmology. [36] In addition, having the temperature $T \sim r^{-1/2}$, the thermodynamics law dU = TdS - pdV immediately requires the entropy should be $A^{3/4}$ to make sure that TdSis comparable to $dU = \frac{1}{2}dr$. This gives another definite illustration that the information storage capacity of LQFT is $A^{3/4}$ other than in a holographic form. Obviously the entropy contained within the apparent horizon increases with the cosmic expansion.

For a holographic universe with $\alpha = \frac{1}{3}$, the temper-ature and entropy are respectively $T \sim r^{-2+3\alpha} \sim r^{-1}$ and $S \sim r^{3-3\alpha} \sim A$, the same as the thermodynamics for the de Sitter universe. Moreover, in this holographic case the $(1 - 3\alpha)$ term in Eq.(12) vanishes, thus the free energy becomes an integral of the scalar curvature. We notice that for this case our formulae (8), (11) and (12)have similar forms with these in [31, 32], which asserted that there is a close relationship between thermodynamical variables and geometrical variables, such as entropy density and gravitational acceleration [37], free energy density and scalar curvature. Inspired by the black hole thermodynamics [27, 28], many works have been devoted to explore more profound physical connections between thermodynamics and gravity, and to associate the notions of temperature and entropy with the spacetimes having horizons [29, 30, 31, 32]. Whatever, the discussions in [31, 32] are mainly around static spacetimes and based on an ansatz for gravitational entropy. So we expect these expressions of thermodynamical variables here and in [31, 32] should imply some general properties of the connections between thermodynamics and gravity.

C. Holographic dark energy

The quantum fluctuations in our scenario have a close relation with previous HDE models [20, 21, 22, 23, 24, 25]. Both the LQFT type and holographic type of quantum fluctuations have shown up in the derivation of the energy density behavior $\rho_{\Lambda} \sim r^{-2}$ which is essential for these models. The derivations take the similar forms as

$$\rho_{\Lambda} \sim \frac{\varepsilon}{\left(\delta r\right)^3} \sim \left(r^{-1/2}\right)^4 \sim r^{-2},\tag{13}$$

$$\rho_{\Lambda} \sim \frac{\varepsilon}{(\delta r)^3} \sim \frac{r^{-1}}{(r^{1/3})^3} \sim r^{-2}.$$
 (14)

It has led to puzzles there why the same density behavior arises from different derivations [24, 25]. Here we point out the differences are superficial. Since the maximum realizable energy of a system is always the critical energy to form a black hole [38], one always has $\rho_{\Lambda} \sim r/r^3 \sim$ r^{-2} , despite where one starts from. In other words, we can generally compute the energy density associated with the space-time uncertainty relation as

$$\rho_{\Lambda} \sim \frac{\varepsilon}{\left(\delta r\right)^3} \sim \frac{1}{\left(\delta t\right) \left(\delta r\right)^3} \sim r^{-2}.$$
 (15)

Obviously, according to Eq.(6), the derivation to this type of energy density is independent of certain choices of δr and δt .

One may find ρ_{Λ} is of the same order of the totoal energy density $\rho = \frac{3}{8\pi}r^{-2}$ of the universe, and thus think it may account for the density of dark energy. Actually, directly taking such an ρ_{Λ} as dark energy will lead to problems. Since ρ_{Λ}/ρ is a constant, due to $d \ln \frac{\rho_{\Lambda}}{\rho} = -3 (w_{\Lambda} - w) d \ln a$, the EoS of the constituent Λ will always trace the behavior of the effective EoS of the entire universe. As pointed out by Hsu [22], there is $w_{\Lambda} = 0$ in a matter-dominated universe, thus Λ cannot lead to an accelerating universe. To solve this problem and get a dark energy model having $w_{\Lambda} < -\frac{1}{3}$, afterwards infrared cutoffs other than the size of apparent horizon are widely suggested, such as the size of future event horizon [20], the age of the universe [24, 25] or even a mixture of them [21].

For the rest, though we have shown the derivation of $\rho_{\Lambda} \sim r^{-2}$ is independent of certain type of quantum fluctuations, it should be pointed out that Ng [9, 10] suggested it should be the holographic type of fluctuations

 $\delta r \sim r^{1/3}$ responsible for the unconventional dark energy/matter, because it is different from these of conventional matter and radiation. This kind of dark energy can be called "holographic", for that it is related to the holographic entropy. Ng has considered the data from Hubble Space Telescope to test the existence of such an unconventional holographic space-time fluctuation.

IV. CONCLUSION AND IMPLICATIONS

By combining GR and QM, the space-time uncertainty relation $(\delta t)(\delta r)^3 \ge \pi r^2$ has been derived from a quantum computational perspective. The case of $\delta r \ge r^{1/2}$ describes the distant fluctuations related to the wellestablished LQFT. By contrast, the case of $\delta r \ge r^{1/3}$ leads to the holographic entropy and is attached with some unknown microscopic physics of gravitational systems. Thought it could be introduced by various approaches, and Ng has tried to suggest it to account for the unconventional dark energy/matter, this type of fluctuations has not yet been understood at a deep lever. The holographic entropy is a special property attached with the space-times having horizons. And we also have shown its induced free energy is relevant to the integral of scalar curvature without matter term present, like that in [31] which aimed to explore the possible thermodynamical properties of gravity. Does $\delta r \ge r^{1/3}$ really characterize the quantum fluctuations of gravitons or say space-time itself? The question is surely worthy of further study.

We give several examples where the space-time uncertainty can be used, including the information storage and computational capacity of the universe, the thermodynamics properties of the universe, and the origin of holographic dark energy. Each topic is of interest in itself. Here we don't intend to discuss the details of them, but only focus on extracting typical characteristics of these topics by virtue of the space-time uncertainty relation and comparing them with the known results there. The universality and validity of the proposed relation are thus exemplified.

acknowledgments

We would like to thank J. L. Li for useful discussions. The work is supported in part by the NNSF of China Grant No. 90503009, No. 10775116, and 973 Program Grant No. 2005CB724508.

- [1] G. 't Hooft, gr-qc/9310026.
- [2] L. Susskind, J. Math. Phys. 36 (1995) 6377, hep-th/9409089.
- [3] R. Bousso, Rev. Mod. Phys. 74 (2002) 825, hep-th/0203101.
- [4] See the discussions in J. Baez's blog,

http://math.ucr.edu/home/baez/week167.html.

- [5] A. Cohen, D. Kaplan and A. Nelson, Phys. Rev. Lett. 82 (1999) 4971, hep-th/9803132.
- [6] Y.X. Chen and Y. Xiao, Phy. Lett. B 662 (2008) 71, arXiv:0705.1414 [hep-th].
- [7] J. D. Barrow, New Astron. 4 (1999) 333,

arXiv:astro-ph/9903225.

- [8] M. Maziashvili, arXiv:0709.0898 [gr-qc].
- [9] W.A. Christiansen, Y.J. Ng and H. van Dam, Phys. Rev. Lett. 96 (2006) 051301, gr-qc/0508121.
- [10] Y.J. Ng, Phys. Lett. B 657 (2007) 10, gr-qc/0703096.
- [11] N. Margolus and L.B. Levitin, Physica D 120 (1998) 188, quant-ph/9710043.
- [12] V. Giovannetti, S. Lloyd and L. Maccone, Science 306 (2004) 1330, quant-ph/0412078.
- [13] S. Lloyd, Phys. Rev. Lett. 88 (2002) 237901.
- [14] S. Lloyd, quant-ph/0505064.
- [15] S. Lloyd and Y.J. Ng, Sci. Am. 291 (2004) 52.
- [16] X. Calmet, M. Graesser and S. D. H. Hsu, Phys. Rev. Lett. 93 (2004) 211101, hep-th/0405033.
- [17] S. D. H. Hsu, Phys. Lett. B 641 (2006) 99, hep-th/0607082.
- [18] S. D. H. Hsu and D. Reeb, arXiv:0706.3239 [hep-th].
- [19] S. D. H. Hsu and D. Reeb, arXiv:0803.4212 [hep-th].
- [20] M. Li, Phys. Lett. B 603 (2004) 1, hep-th/0403127.
- [21] S. Nojiri and S.D. Odintsov, Gen. Rel. Grav. 38 (2006) 1285, hep-th/0506212.
- [22] S.D.H. Hsu, Phy. Lett. B 594 (2004) 13, hep-th/0403052.
- [23] M. Maziashvili, Int. J. Mod. Phys. D 16 (2007) 1531, gr-qc/0612110.
- [24] R.G. Cai, Phys. Lett. B 657 (2007) 228, arXiv:0707.4049 [hep-th].
- [25] H. Wei and R.G. Cai, Phys. Lett. B 660 (2008) 113, arXiv:0708.0884 [astro-ph].
- [26] J. McGreevy, L. Susskind and N. Toumbas, JHEP 06 (2000) 008, hep-th/0003075.
- [27] S.W. Hawking, Commun. Math. Phys. 43 (1975) 199.
- [28] J.D. Bekenstein, Phys. Rev. D 7 (1973) 2333.
- [29] T. Jacobson, Phys. Rev. Lett. 75 (1995) 1260, gr-qc/9504004.
- [30] R.G. Cai and S.P. Kim, JHEP 0502 (2005) 050, hep-th/0501055.
- [31] T. Padmanabhan, Class. Quant. Grav. 21 (2004) 4485, gr-qc/0308070.
- [32] T. Padmanabhan, Phys. Rept. 406 (2005) 49, gr-qc/0311036.

- [33] The entropy bound $A^{3/4}$ can be obtained by a strict counting of non-gravitational collapse quantum states where the UV-IR relation naturally presents itself [6]. The entropy gap from $A^{3/4}$ to A also has its cosmological counterpart [7], which brings about a huge numerical differences from 10^{90} to 10^{120} at the present era of our universe. The existence of such an entropy gap will be further verified in Section III B.
- [34] Note here it refers to the uncertainty in which a distance l can be measured. Such an uncertainty is different from that in the measurement of a position located with a macroscopic object. The latter is undoubtedly the minimal length of nature, i.e. the Planck length l_p [16], while the former can be derived as a cumulative effect of $\pm l_p$ over the distant l [8].
- [35] Here the temperature can be negative when w < -1/3leads to an accelerating universe. One who is not accustomed to a negative temperature can define the temperature to be positive like that in [31], i.e., $T \equiv |\beta|^{-1}$, where $\beta F = \beta U - S$. For implications of $\beta \geq 0$, see [31]. Here it is simply an indication of the accertaion or deceleration of the universe expansion.
- [36] For a radiation-dominated universe, there is r = 2tand $\rho \sim a^{-4}$. Thus the temperature is evaluated as $T \sim a(t)^{-1} \sim \rho^{1/4} \sim r^{-1/2} \sim t^{-1/2}$.
- [37] For gravitational systems we can not only define the internal energy $U \equiv \int T_{ab}u^a u^b dV$, but also the energy source of gravitational acceleration $E \equiv$ $\int (T_{ab} - \frac{1}{2}T_c^c g_{ab}) u^a u^b dV$, thus a close relation E = $(3 - 3\alpha)TS$ between TS and E can be observed here. Recall that for a Schwarzschild black hole, we have the thermodynamics description: dM = TdS, where $M = \frac{r}{2}$, $T = \frac{1}{8\pi M}$, $S = 4\pi M^2$. This gives 2TS = M = E like that in [31], corresponding to the holographic case where $\alpha = \frac{1}{3}$.
- [38] To introduce it as a choice of dark energy, one often conjectures that the total quantum zero-point energy in a region should not succeed the energy of a black hole of the same size [20].