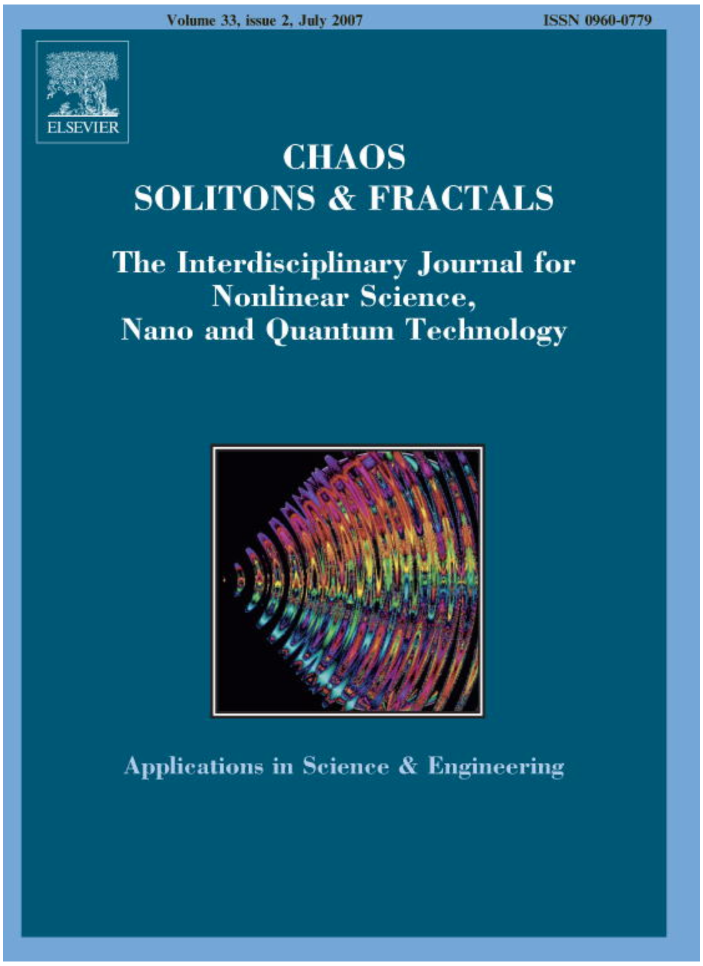
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Estimating parameters of chaotic systems synchronized by external driving signal

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Abstract

Noise-induced synchronization (NIS) has evoked great research interests recently. Two uncoupled identical chaotic systems can achieve complete synchronization (CS) by feeding a common noise with appropriate intensity. Actually, NIS belongs to the category of external feedback control (EFC). The significance of applying EFC in secure communication lies in fact that the trajectory of chaotic systems is disturbed so strongly by external driving signal that phase space reconstruction attack fails. In this paper, however, we propose an approach that can accurately estimate the parameters of the chaotic systems synchronized by external driving signal through chaotic transmitted signal, driving signal and their derivatives. Numerical simulation indicates that this approach can estimate system parameters and external coupling strength under two driving modes in a very rapid manner, which implies that EFC is not superior to other methods in secure communication.

1. Introduction

Chaotic synchronization has gradually become the focus in the field of nonlinear dynamics due to its potential application in secure communication, laser system, electronic chemistry, neurophysiology and ecology. Since Pecora and Carroll introduced the drive-response method in 1990 [1], many chaotic synchronization schemes have been proposed successively [2–5], among which noise-induced synchronization (NIS) has aroused great interests. In NIS, two uncoupled identical chaotic systems can achieve complete synchronization (CS) by feeding a common noise with appropriate intensity. Since this phenomenon of NIS was proposed by Maritan and Banavar in 1994 [2], there have been great controversies over NIS experimentally [6–8] and theoretically [9–11].

Compared to other variable feedback control (VFC) methods for achieving CS, NIS belongs to the category of external feedback control (EFC) method actually. The significance of applying EFC in secure communication lies in the fact that the trajectory of chaotic systems is disturbed so strongly by external driving signal that phase space reconstruction attack fails. Consequently, several secure communication schemes based on EFC or its variations have

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been proposed [12]. Following the idea in Ref. [13], we present an approach demonstrated by Lorenz system, which can accurately estimate the parameters of chaotic systems synchronized by external driving signal. We achieve this accurate estimation by first transforming the driven Lorenz equation to a linear equation of parameters, into which the values of chaotic transmitted signal, driving signal and their derivatives at different points are filled. The parameters can be obtained through solving the final equations. Compared with adaptive control schemes [15–18] and other parameter estimation approaches [19], the advantage of the proposed approach is that it can estimate system parameters and external coupling strength under two driving modes in a very rapid manner, which is demonstrated through numerical simulation. Moreover, this approach can be easily applied to the parameter estimation of Chua's circuits synchronized by external driving signal according to Ref. [14].

2. External feedback control (EFC)

Generally, there are two driving modes in EFC governed by Eqs. (1) and (2) respectively.

$$\begin{aligned} \dot{\mathbf{x}}_s &= \mathbf{F}(\mathbf{x}_s) + \mathbf{c}f(t) \\ \dot{\mathbf{x}}_r &= \mathbf{F}(\mathbf{x}_r) + \mathbf{c}f(t) \end{aligned} \tag{1}$$

$$\begin{aligned} \dot{\mathbf{x}}_s &= \mathbf{F}(x_s^1, x_s^2, \ldots, [\alpha x_s^i + \beta f(t)], \ldots) \\ \dot{\mathbf{x}}_r &= \mathbf{F}(x_r^1, x_r^2, \ldots, [\alpha x_r^i + \beta f(t)], \ldots) \end{aligned} \tag{2}$$

Here $\mathbf{c}$ denotes the vector of external coupling strength, and α and β denote the coupling strength.

Given that NIS was illustrated first by Lorenz system, we will use Lorenz system to explain our approach in the following. According to the conclusion of Ref. [10], the efficient driving variable is y. Eqs. (3) and (4) are the driven Lorenz systems under the above two modes respectively. Since the sender systems and receiver systems are the same, only the sender systems are given here.

$$\begin{cases} \dot{x} = \sigma(y - x) \\ \dot{y} = rx - y - xz + cf(t) \\ \dot{z} = xy - bz \end{cases} \tag{3}$$

$$\begin{cases} \dot{x} = \sigma([\alpha y + \beta f(t)] - x) \\ \dot{y} = rx - [\alpha y + \beta f(t)] - xz \\ \dot{z} = x[\alpha y + \beta f(t)] - bz \end{cases} \tag{4}$$

Here σ, r and b are the internal parameters of the Lorenz system. The following numerical simulation indicates that CS can be achieved under both modes with appropriately chosen coupling strength c, α and β.

3. Parameter estimation

3.1. Parameter estimation under the first mode

From Eq. (3) we get

$$\begin{aligned} & y = \frac{1}{\sigma}x' + x \\ & \frac{1}{\sigma}x'' + x' = rx - \frac{1}{\sigma}x' - x - xz + cf \\ & z = -\frac{1}{\sigma}\frac{x''}{x} - \left(1 + \frac{1}{\sigma}\right)\frac{x'}{x} + (r-1) + c\frac{f}{x} \quad \text{and } x \neq 0 \\ & \quad -\frac{1}{\sigma}\frac{x'''}{x} + \frac{1}{\sigma}\frac{x''x'}{x^2} - \left(1 + \frac{1}{\sigma}\right)\frac{x''}{x} + \left(1 + \frac{1}{\sigma}\right)\frac{(x')^2}{x^2} + c\frac{f'}{x} - c\frac{fx'}{x^2} \\ & \quad = \frac{1}{\sigma}x'x + x^2 + \frac{b}{\sigma}\frac{x''}{x} + b\left(1 + \frac{1}{\sigma}\right)\frac{x'}{x} - b(r-1) - bc\frac{f}{x} \quad \text{and } x \neq 0 \end{aligned} \tag{5}$$

Let

$$
\begin{aligned}
&p_1 = 1/\sigma, && q_1 = x'''x - x''x' + x''x - (x')^2 + x'x^3\\
&p_2 = b/\sigma, && q_2 = (x'' + x')x\\
&p_3 = b, && q_3 = x'x\\
&p_4 = b(r-1), && q_4 = -x^2\\
&p_5 = c, && q_5 = fx' - f'x\\
&p_6 = bc, && q_6 = -fx\\
&q_0 = (x')^2 - x''x - x^4, &&
\end{aligned}
$$

and

$$\mathbf{P} = (p_1, p_2, p_3, p_4, p_5, p_6)^{\mathrm{T}}, \quad \mathbf{Q} = (q_1, q_2, q_3, q_4, q_5, q_6)$$

From Eq. (5) we get

$$\mathbf{QP} = q_0 \tag{6}$$

Chaotic transmitted signal x and driving signal f are first sampled with high frequency. Then their derivatives x', x'', x''' and f' can be calculated through the conventional numerical derivative. The values at different points are filled into Eq. (6). By solving the final equations, we will obtain the parameters.

3.2. Parameter estimation under the second mode

From Eq. (4) we get

$$
\begin{aligned}
&y = \frac{1}{\alpha\sigma}x' + \frac{1}{\alpha}x - \frac{\beta}{\alpha}f\\
&\frac{1}{\alpha\sigma}x'' + \frac{1}{\alpha}x' - \frac{\beta}{\alpha}f' = rx - \frac{1}{\sigma}x' - x - xz\\
&z = -\frac{1}{\alpha\sigma}\frac{x''}{x} - \left(\frac{1}{\alpha} + \frac{1}{\sigma}\right)\frac{x'}{x} + (r-1) + \frac{\beta}{\alpha}\frac{f'}{x} \quad \text{and } x \neq 0\\
&\quad -\frac{1}{\alpha\sigma}\frac{x'''}{x} + \frac{1}{\alpha\sigma}\frac{x''x'}{x^2} - \left(\frac{1}{\alpha} + \frac{1}{\sigma}\right)\frac{x''}{x} + \left(\frac{1}{\alpha} + \frac{1}{\sigma}\right)\frac{(x')^2}{x^2} + \frac{\beta}{\alpha}\frac{f''}{x} - \frac{\beta}{\alpha}\frac{f'x'}{x^2}\\
&\quad = \frac{1}{\sigma}x'x + x^2 + \frac{b}{\alpha\sigma}\frac{x''}{x} + b\left(\frac{1}{\alpha} + \frac{1}{\sigma}\right)\frac{x'}{x} - b(r-1) - \frac{b\beta}{\alpha}\frac{f'}{x} \quad \text{and } x \neq 0
\end{aligned}
\tag{7}
$$

Let

$$
\begin{aligned}
&p_1 = 1/\alpha\sigma, && q_1 = x'''x - x''x'\\
&p_2 = 1/\alpha + 1/\sigma, && q_2 = x''x - (x')^2\\
&p_3 = \beta/\alpha, && q_3 = f'x' - f''x\\
&p_4 = 1/\sigma, && q_4 = x'x^3\\
&p_5 = b/\alpha\sigma, && q_5 = x''x\\
&p_6 = b(1/\alpha + 1/\sigma), && q_6 = x'x\\
&p_7 = b(r-1), && q_7 = -x^2\\
&p_8 = b\beta/\alpha, && q_8 = -f'x\\
&q_0 = -x^4, &&
\end{aligned}
$$

and

$$\mathbf{P} = (p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8)^{\mathrm{T}}, \quad \mathbf{Q} = (q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8)$$

From Eq. (7) we get

$$\mathbf{QP} = q_0 \tag{8}$$

The estimation process in the second mode is rather similar to the first one. Chaotic transmitted signal x and driving signal f are sampled with high frequency first. Then their derivatives x', x'', x''', f' and f'' can be calculated through the conventional numerical derivative. The values at different points are filled into Eq. (8). By solving the final equations, we will obtain the parameters.

4. Numerical simulation

In the following experiments, the internal parameters of Lorenz system are set at $\sigma = 16$, $r = 45.6$ and $b = 4$. According to Ref. [12], let $f(t) = A\sin(2\pi vt)$, and change the amplitude A and frequency v across each period with $A = 50\xi$ and $v = 0.8(0.5 + \xi')$. Here ξ and ξ' are pseudorandom numbers in range (0, 1). The coupling coefficients are set at $c = 20$, $\alpha = 0.9$ and $\beta = 20$. The sender takes $[1.465, 1.287, 0.156]$ as initial conditions, and those of the receiver are set at $[-1.968\ -1.345\ 1.453]$.

Eqs. (3) and (4) are solved using the Runge–Kutta method, where the step takes $\Delta t = 0.0001$ ($=10^4$ Hz). In parameter estimation, chaotic transmitted signal x and driving signal f are sampled with $h = 0.001$ ($=10^3$ Hz). Numerical derivatives are calculated through the conventional formulas below:

$$g'(t_0) = (\delta_3 - 9\delta_2 + 45\delta_1)/60h$$
$$g''(t_0) = (2\eta_3 - 27\eta_2 + 270\eta_1 - 490g(t_0))/180h^2$$
$$g'''(t_0) = (-\delta_3 + 8\delta_2 - 13\delta_1)/8h^3$$

Here $\delta_i = g(t_0 + ih) - g(t_0 - ih)$, and $\eta_i = g(t_0 + ih) + g(t_0 - ih)$.

4.1. Numerical simulation under the first mode

The numerical results of NIS determined by Eq. (3) are given here. Fig. 1a shows the driving signal f. Fig. 1b shows the chaotic transmitted signal x_s. The synchronizing error $e = x_s - x_r$ is shown in Fig. 1c, from which we can see that the chaotic systems of sender and receiver are completely synchronized by feeding the driving signal. Fig. 1d shows the attractor of sender system, where the driving signal disturbs the trajectory of chaotic systems to a certain degree.

We can get $\{f(n)\}$, $(n = -2, -1, 0, 1, \ldots, 6N + 3)$ by sampling a segment of driving signal indicated by the distance between dash lines shown in Fig. 1a. In the same way, we can also get $\{x(n)\}$, $(n = -2, -1, 0, 1, \ldots, 6N + 3)$. Their derivatives are calculated through the above numerical derivative formulas. For higher precision, filling the corresponding values numbered by $k_i = (i, N + i, 2N + i, 3N + i, 4N + i, 5N + i)$, $(i = 1, \ldots, N)$ into Eq. (6), we get a full rank equations $\mathbf{Q}(k_i)\mathbf{P} = q_0(k_i)$. Then the estimation of parameters can be written as

$$\mathbf{P}^* = \frac{1}{N}\sum_{i=1}^{N}\mathbf{Q}(k_i)^{-1}q_0(k_i)$$

Here $N = 50$. We get

$$\mathbf{P}^* = [0.062499985181, 0.250000063957, 3.999997250595, 178.399939301600, 19.999995470932, 80.000022936896]$$

Then

$$b^* = (p_2^*/p_1^* + p_6^*/p_5^* + p_3^*)/3 = 4.00000042500168 \quad (b = 4)$$
$$\sigma^* = (b^*/p_2^* + 1/p_1^*)/2 = 16.00000070028464 \quad (\sigma = 16)$$
$$r^* = p_4^*/b^* + 1 = 45.59998008663346 \quad (r = 45.6)$$
$$c^* = (p_6^*/b^* + p_5^*)/2 = 19.99999954007369 \quad (c = 20)$$

The relative estimating errors here are all less than 10^{-6}.

4.2. Numerical simulation under the second mode

The numerical results of NIS determined by Eq. (4) are given here. Fig. 2a shows the driving signal f. Fig. 2b shows the chaotic transmitted signal x_s. The synchronizing error $e = x_s - x_r$ is shown in Fig. 2c, from which we can see that the chaotic systems of sender and receiver are completely synchronized by feeding the driving signal. Fig. 2d shows the attractor of sender system, where the driving signal strongly disturbs the trajectory of chaotic systems.

We can get $\{f(n)\}$, $(n = -2, -1, 0, 1, \ldots, 8N + 3)$ by sampling a segment of driving signal indicated by dash lines shown in Fig. 1a. In the same way, we can also get $\{x(n)\}$, $(n = -2, -1, 0, 1, \ldots, 8N + 3)$. Their derivatives are calculated through the above numerical derivative formulas. For higher precision, filling the corresponding values numbered by $k_i = (i, N + i, 2N + i, 3N + i, 4N + i, 5N + i, 6N + i, 7N + i)$, $(i = 1, \ldots, N)$ into Eq. (8), we get a full rank equations $\mathbf{Q}(k_i)\mathbf{P} = q_0(k_i)$. Then the estimation of parameters can be written as

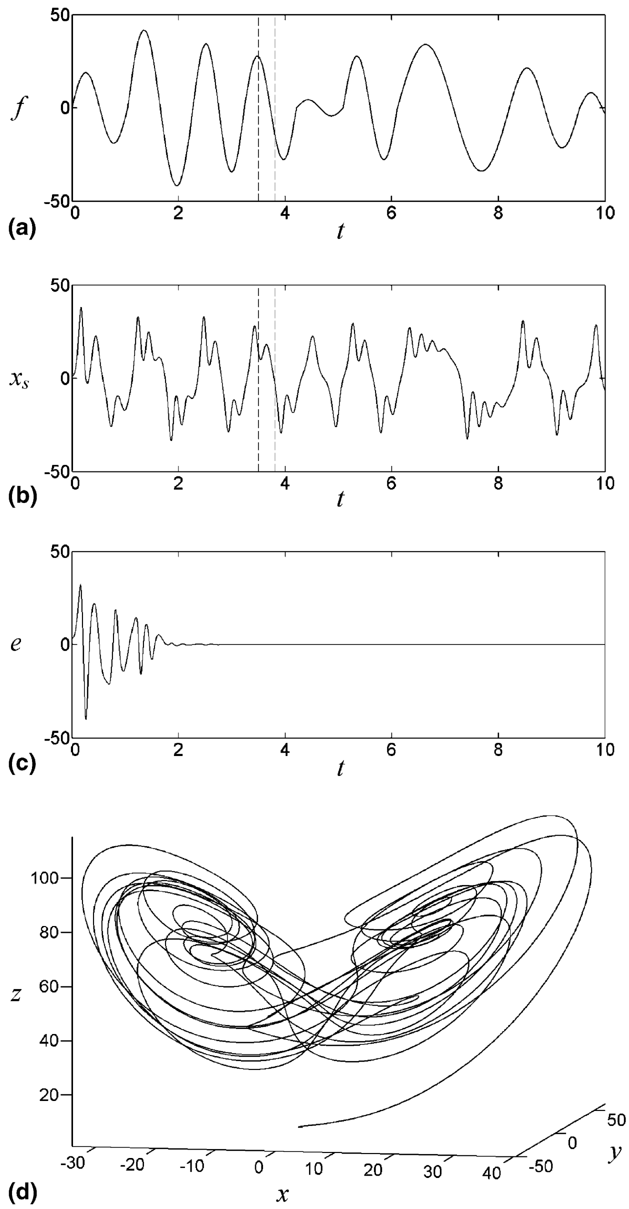

Fig. 1. The numerical results of NIS determined by Eq. (3). (a) The driving signal f, (b) the chaotic transmitted signal x_s, (c) the synchronizing error $e = x_s - x_r$ and (d) the attractor of sender system.

$$\mathbf{P}^* = \frac{1}{N}\sum_{i=1}^{N}\mathbf{Q}(k_i)^{-1}q_0(k_i)$$

Here $N = 50$. We get

$$\mathbf{P}^* = [0.069444334958, 1.173615238563, 22.222212612013, 0.062499716268, 0.277771874340, 4.694397518939, 178.400438228787, 88.888821060915]$$

Then

$$\begin{aligned}
b^* &= (p_5^*/p_1^* + p_6^*/p_2^* + p_8^*/p_3^*)/3 = 3.99995530773063 \quad (b = 4)\\
\sigma^* &= 1/p_4^* = 16.00007263562793 \quad (\sigma = 16)\\
r^* &= p_7^*/b^* + 1 = 45.60060788279216 \quad (r = 45.6)\\
\alpha^* &= p_4^*/p_1^* = 0.89999733320949 \quad (\alpha = 0.9)\\
\beta^* &= p_3^*\alpha^* = 19.99993208882647 \quad (\beta = 20)
\end{aligned}$$

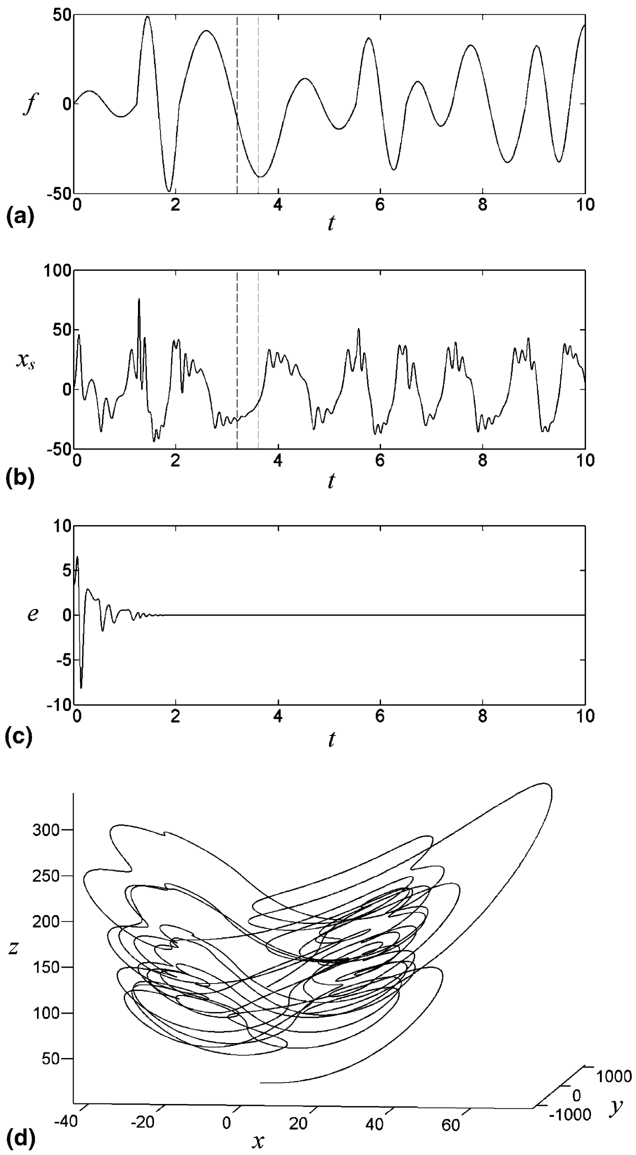

Fig. 2. The numerical results of NIS determined by Eq. (4). (a) The feeding signal f, (b) the chaotic transmitted signal x_s, (c) the synchronizing error $e = x_s - x_r$ and (d) the attractor of sender system.

The relative estimating errors here are all less than 10^{-4}.

5. Conclusions

In this paper, we present an accurate approach, as illustrated by Lorenz system, which can estimate the parameters of chaotic systems synchronized by external driving signal. We achieve this accurate estimation by first transforming the driven Lorenz equation to a linear equation of the substitutes of parameters, into which the values of chaotic transmitted signal, driving signal and their derivatives at different points are filled. The parameters can be obtained through solving the final equations. Numerical result shows that this approach can estimate system parameters and external coupling strength under two driving modes in a very rapid manner. Moreover, this approach can be easily applied to the parameter estimation of Chua's circuits synchronized by external driving signal according to Ref. [14].

To sum up, although driving signal may strongly disturb the trajectory of chaotic systems under EFC, and thus destroy the possibility of phase space reconstruction attack, EFC is not superior to other VFC methods for achieving CS in secure communication since parameters can be easily estimated by the proposed approach.

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