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2008 J. Phys. B: At. Mol. Opt. Phys. 41 215504

(http://iopscience.iop.org/0953-4075/41/21/215504)

IOP Publishing JOURNAL OF PHYSICS B: ATOMIC, MOLECULAR AND OPTICAL PHYSICS

J. Phys. B: At. Mol. Opt. Phys. **41** (2008) 215504 (8pp) doi:10.1088/0953-4075/41/21/215504

Ultraslow temporal vector optical solitons in a cold five-state atomic medium under Raman excitation

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Received 5 August 2008, in final form 9 September 2008
Published 23 October 2008
Online at stacks.iop.org/JPhysB/41/215504

Abstract

We theoretically investigate the formation of ultraslow temporal vector optical solitons in a lifetime-broadened five-state atomic system under Raman excitation. By analysing the interaction between the atomic system and two strong, linear-polarized continuous wave control fields and a weak, linear-polarized pulsed signal field, having two orthogonally polarized components, we find that the Maxwell equations of describing the propagation of two orthogonal polarization components can evolve into two coupled nonlinear Schrödinger equations, which admit solutions describing vector solitons, including bright–bright, bright–dark, dark–bright and dark–dark vector solitons with ultraslow group velocity. More importantly, the Manakov temporal vector solitons can be easily realized in our system by adjusting the corresponding parameters such as the Rabi frequencies of the control fields, one- and two-photon detunings. The unique features of the two coupled nonlinear Schrödinger equations are also discussed.

1. Introduction

The soliton represents a fascinating manifestation of nonlinear phenomena in nature and occurs in many states of matter ranging from solid media such as optical fibres [1, 2] or molecular magnets [3–5] to Bose–Einstein condensed atomic vapour [6–8]. In the context of optics, the soliton describes the particle-like properties of an optical pulse envelope in various nonlinear systems. Under certain conditions, a perfect balance between the nonlinear and dispersive effects can result in optical pulses maintaining their shapes not only during propagation but also during mutual collisions. Due to the potential applications in quantum information processing and transmission, optical solitons forming inside cold atomic media [9–14], semiconductor quantum wells [15] and other nonlinear media [16, 17] have attracted a great deal of attention in recent years.

Recently, among the various optical solitons studied so far, a novel class of solitons, namely vector optical solitons, which are the solutions of two coupled nonlinear Schrödinger (NLS) equations describing the envelope evolution of two polarization components of an electromagnetic field, have received much attention because of the promising applications for the design of new types of all-optical switches and logic gates [18]. The formation of vector optical solitons arises from the vector nature of light propagating in a nonlinear medium. According to the interaction between either two waves with different frequencies, or two waves with same frequency but belonging to two different polarizations, the optical waves propagating in nonlinear media may evolve into two types of vector optical solitons, the temporal [19–26] and spatial [27–31] vector optical solitons because of self- and cross-phase modulation effects. However, up to now, most vector optical solitons have been produced in passive optical media such as optical fibres [18, 21, 22, 24–31] and far-off-resonance excitation schemes are generally employed in order to avoid unmanageable attenuation and distortion of the optical field. Due to the lack of distinctive energy levels and strong nonlinear effects, high intense electromagnetic fields and substantial propagation distance are required in order to generate optical solitons. As a consequence, vector optical solitons produced in this way generally travel with a speed very close to c, the speed of light in vacuum, and require an extended propagation distance.

In the past few years the technique of electromagnetically induced transparency (EIT) [32–36] has been used to obtain vanishing linear absorption and large nonlinear effects as well as ultraslow light speed. Based on the EIT effect in nonlinear media, lots of nonlinear phenomena including soliton [3, 5, 37] and four-wave mixing [11, 38, 39] (FWM) have been realized. For example, on the basis of the EIT in a crystal of molecular magnets, Wu and Yang [3] have shown the formation of microwave solitons. Hang and Huang [37] have investigated the ultraslow temporal vector optical solitons generated in an atomic medium via EIT. However, as shown in [10], there are difficulties of soliton formation in the conditions of the usual EIT configuration in a three-state cold atomic system which can support the propagation of such ultraslow optical solitons under Raman excitation with nonvanishing one- and two-photon detunings. Motivated by the work in [10], we naturally want to ask if the ultraslow temporal vector optical solitons can also be formed in a lifetime-broadened five-state atomic system under Raman excitation.

In the present paper, we will examine a low-intensity linear-polarized signal light pulse propagation in a lifetime-broadened five-state cold atomic system under Raman excitation with nonvanishing one- and two-photon detunings. As an important departure from the conventional EIT scheme used in [40, 41] where zero one-photon detuning is required, the system is a Raman scheme with a large one-photon detuning, which has the advantage of being broadly tunable. We use the standard Fourier transform technique and the method developed by Wu and Deng [9, 10] to obtain two coupled NLS equations describing the envelopes evolution of two polarization components of a signal field. Under reasonable and realistic parameters conditions, we demonstrate that the system can support the existence of ultraslow temporal vector solitons, including bright–bright, bright–dark, dark–bright and dark–dark vector solitons, and the Manakov temporal vector solitons can be easily realized.

Our paper is organized as follows. In section 2, we first describe the theoretical model and investigate the dispersion properties of the system. In section 3, by taking the reasonable and realistic approximate conditions, we derive the system's two coupled NLS equations describing the envelopes evolution of two polarization components of a signal field. In section 4, vector soliton solutions, including bright–bright, bright–dark, dark–bright and dark–dark vector solitons with ultraslow group velocity, are provided and the Manakov temporal vector solitons in the system are also discussed. We conclude with a brief summary in section 5.

2. Model and linearity solutions of the system

We consider a lifetime-broadened five-state atomic system, whose two upper atomic sublevels are Zeeman split due to the applied magnetic field interacting with three optical waves $C1$, $C2$ and S, as depicted in figure 1, to study the formation of temporal vector optical solitons with ultraslow group velocity. $C1$ and $C2$ describe two strong continuous wave (CW) control

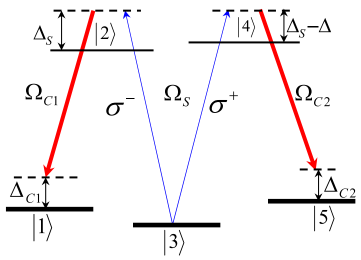

Figure 1. Energy level diagram and excitation scheme of the lifetime-broadened five-state atomic system interacting with two CW control fields $C1$ and $C2$ of the frequencies ω_{C1} and ω_{C2} and the Rabi frequencies Ω_{C1} and Ω_{C2}, respectively, and a weak, linear-polarized signal field of the frequency ω_S and the Rabi frequency Ω_S. The $\sigma^-(\sigma^+)$ component of the weak signal field couples to the energy levels $|3\rangle$ and $|2\rangle(|3\rangle$ and $|4\rangle)$, while the control field $C1(C2)$ couples to $|1\rangle$ and $|2\rangle(|4\rangle$ and $|5\rangle)$. $\Delta_S(\Delta_S+\Delta)$ and $\Delta_{C1}(\Delta_{C2})$ are one- and two-photon detunings, respectively. $\Delta = 2\mu_B g B/\hbar$ is the Zeeman shift of the upper atomic sublevel with B the applied magnetic field, μ_B the Bohr magneton and g the gyromagnetic factor.

(This figure is in colour only in the electronic version)

fields, respectively, and S denotes a weak, linear-polarized pulsed (pulse length of τ_0 at the entrance of the medium) signal field, having two orthogonally polarized components. Two control fields with optical frequencies ω_{C1}, ω_{C2} and Rabi frequencies Ω_{C1}, Ω_{C2} drive the transitions from $|1\rangle$ to $|2\rangle$ and $|4\rangle$ to $|5\rangle$, respectively, while two polarization components of the weak signal field with optical frequencies ω_S and Rabi frequency Ω_S drive the transitions from $|3\rangle$ to $|2\rangle$ and $|3\rangle$ to $|4\rangle$, respectively. It is worth pointing out that in our system the one-photon detuning is large because the lifetime-broadened five-state atomic system we study is under Raman excitation which is very different from the conventional EIT scheme used in [40, 41] where zero one-photon detuning is required. As we show below, the introduction of a large one-photon detuning is critical to parameter selections in demonstrating the formation of ultraslow temporal vector optical solitons.

Suppose the electric fields of the weak pulsed signal field and two strong CW control fields have the form of $\vec{E}_S = \vec{E}_{S+} + \vec{E}_{S-} = (\vec{e}_+ E_{S+} + \vec{e}_- E_{S-})\exp(-\mathrm{i}\omega_S t + \mathrm{i}k_S z) + c.c.$ and $\vec{E}_{C1(C2)} = \vec{e}_{C1(C2)} E_{(C1)C2} \exp(-\mathrm{i}\omega_{C1(C2)} t + \mathrm{i}\vec{k}_{C1(C2)}\cdot\vec{r}) + c.c.$, respectively. Here $c.c.$ stands for complex conjugate, $\vec{e}_+ = (\vec{x}+\mathrm{i}\vec{y})/\sqrt{2}$ and $\vec{e}_- = (\vec{x}-\mathrm{i}\vec{y})/\sqrt{2}$ are the unit vectors of the σ^+ and σ^- circular polarization components with the envelopes E_+ and E_-, which drive the transitions $|3\rangle \leftrightarrow |4\rangle$ and $|3\rangle \leftrightarrow |2\rangle$, respectively. $\vec{e}_{C1}(\vec{e}_{C2})$ is the unit vector of the control field with the envelopes $E_{C1}(E_{C2})$, which drives the transitions $|1\rangle \leftrightarrow |2\rangle(|4\rangle \leftrightarrow |5\rangle)$. It is obvious that the five-state atomic system is composed of two Raman Λ schemes which share the ground-state level $|3\rangle$ [10, 41–43]. Thus, under electric-dipole and rotating-wave approximations, we have the interaction Hamiltonian of the system in the interaction picture as follows [38, 44, 45]:

$$\begin{aligned}\hat{H}_{\mathrm{int}} = &-\hbar\Delta_{C1}|1\rangle\langle 1| - \hbar\Delta_S|2\rangle\langle 2| - \hbar\Delta_{C2}|5\rangle\langle 5| - \hbar(\Delta_S \\ &- \Delta)|4\rangle\langle 4| - \hbar[\Omega_{C1}\,\mathrm{e}^{\mathrm{i}\vec{k}_{C1}\cdot\vec{r}}|2\rangle\langle 1| + \Omega_{S1}\,\mathrm{e}^{\mathrm{i}k_S z}|2\rangle\langle 3| \\ &+ \Omega_{S2}\,\mathrm{e}^{\mathrm{i}k_S z}|4\rangle\langle 3| + \Omega_{C2}\,\mathrm{e}^{\mathrm{i}\vec{k}_{C2}\cdot\vec{r}}|4\rangle\langle 5| + H.c.],\end{aligned} \quad (1)$$

where $\Delta_S = \omega_S - \omega_{23}, \Delta_S - \Delta$ and $\Delta_{C1} = \omega_S - \omega_{C1} - \omega_{23}, \Delta_{C2} = \omega_S - \omega_{C2} - \omega_{53}$ are, respectively, one- and two-photon detunings. $\Omega_{C1} = (\vec{\mu}_{21} \cdot \vec{e}_{C1})E_{C1}/\hbar, \Omega_{C2} = (\vec{\mu}_{45} \cdot \vec{e}_{C2})E_{C2}/\hbar, \Omega_{S1} = (\vec{\mu}_{23} \cdot \vec{e}_{-})E_{S-}/\hbar$, and $\Omega_{S2} = (\vec{\mu}_{43} \cdot \vec{e}_{+})E_{S+}/\hbar$ are Rabi frequencies with $\vec{\mu}_{ij}$ being the dipole moment for the relevant transitions $|i\rangle \leftrightarrow |j\rangle$. Here $\omega_{jn} = |\varepsilon_j - \varepsilon_n|/\hbar$ denoting the corresponding transition frequencies with ε_j the energy of state $|j\rangle$ and $\Delta = 2\mu_B g B/\hbar$ the Zeeman shift of the upper atomic sublevel with B the applied magnetic field, μ_B the Bohr magneton and g the gyromagnetic factor.

In order to study the kinetics of such a five-level system, we assume the state of the atomic system has the form of $|\Psi\rangle = C_1(t)\,e^{ik_S z - i\vec{k}_{C1}\cdot\vec{r}}|1\rangle + C_2(t)\,e^{ik_S z}|2\rangle + C_3(t)|3\rangle + C_4(t)\,e^{ik_S z}|4\rangle + C_5(t)\,e^{ik_S z - i\vec{k}_{C2}\cdot\vec{r}}|5\rangle$ and substitute this wavefunction expression into the Schrödinger equation $i\hbar\partial|\Psi\rangle/\partial t = \hat{H}_{\rm int}|\Psi\rangle$ in the interaction picture; then the evolution equations for the probability amplitudes $C_j(t)$ can be obtained:

$$\frac{\partial C_1}{\partial t} = i(\Delta_{C1} + i\gamma_1)C_1 + i\Omega_{C1}^* C_2, \qquad (2a)$$

$$\frac{\partial C_2}{\partial t} = i(\Delta_S + i\gamma_2)C_2 + i\Omega_{C1}C_1 + i\Omega_{S1}C_3, \qquad (2b)$$

$$\frac{\partial C_4}{\partial t} = i(\Delta_S - \Delta + i\gamma_4)C_4 + i\Omega_{S2}C_3 + i\Omega_{C2}C_5, \qquad (2c)$$

$$\frac{\partial C_5}{\partial t} = i(\Delta_{C2} + i\gamma_5)C_5 + i\Omega_{C2}^* C_4, \qquad (2d)$$

In the above equations we add the population decay rate, $2\gamma_k$ $(k = 1, 2, 4, 5)$, of the state $|k\rangle$ phenomenologically and C_3 can be decided by the relation $\sum_{j=1}^{5} |C_j|^2 = 1$.

In our system, two control fields $C1, C2$ are strong CW and the probe field S is weak signal wave. Consequently the envelopes $\vec{E}_{C1}$ and $\vec{E}_{C2}$ are independent on the spacetime variables z and t while the slowly-varying envelopes $\vec{E}_{S1}(z,t)$ and $\vec{E}_{S1}(z,t)$ of two polarization components of the weak signal field depend on the spacetime variables z and t. It is noted that two continuous electromagnetic waves $C1$ and $C2$ are always on and, hence, under the slowly varying envelope approximation the Rabi frequencies Ω_{C1} and Ω_{C2} are time- and space-independent constants. And the equations of the Rabi frequencies Ω_{S1} and Ω_{S2} for two polarization components of the weak signal field can be readily derived from Maxwell's equations and they read as

$$\frac{\partial \Omega_{S1}}{\partial z} + \frac{1}{c}\frac{\partial \Omega_{S1}}{\partial t} = i\kappa_{23} C_2 C_3^*, \qquad (3a)$$

$$\frac{\partial \Omega_{S2}}{\partial z} + \frac{1}{c}\frac{\partial \Omega_{S2}}{\partial t} = i\kappa_{43} C_4 C_3^*, \qquad (3b)$$

where $\kappa_{23(43)} = N\omega_S|\vec{\mu}_{23(43)} \cdot \vec{e}_{-(+)}|^2/(2\hbar\epsilon_0 c)$ with N and ϵ_0 being the concentration and vacuum dielectric constant, respectively.

To provide a clear picture of the interplay between the group-velocity dispersion and nonlinear (self-phase modulation, SPM, and cross-phase modulation, CPM) effects of the atomic system interacting with two CW optical fields and a pulsed signal field, we first investigate the dispersion properties of the system. This requires a perturbation treatment of the system response to the first order of two polarization components Ω_{S1} and Ω_{S2} of the weak signal field S while keeping all orders due to control fields Ω_{C1} and Ω_{C2}. In the following section, we demonstrate SPM and CPM effects that are due to higher order Ω_{S1} and Ω_{S2} that are required for balancing the group-velocity dispersion effect so that the formation of ultraslow temporal vector optical solitons can occur.

We assume that the signal field is weak compared with the other two control fields and that all the atoms are in their ground states before the signal field enters the medium at $t = 0$. With the assumptions above, we can make the asymptotic expansion $C_j = \sum_k C_j^{(k)}$, where $C_j^{(k)}$ is the kth order part of C_j in terms of ϵ. Within an adiabatic following framework it can be shown that $C_j^{(0)} = \delta_{j3}$ and $C_3^{(1)} = 0$. Considering the first order of ϵ and taking time Fourier transform of equations (2*a*), (2*d*), (3*a*) and (3*b*)

$$C_j^{(1)}(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \beta_j^{(1)}(\omega)\,e^{-i\omega t}\,d\omega, \quad j = 1, \ldots, 5, \qquad (4a)$$

$$\Omega_k(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \Lambda_k(\omega)\,e^{-i\omega t}\,d\omega, \quad k = S1, S2, \qquad (4b)$$

with ω being the Fourier-transform variable, we have

$$\beta_1^{(1)} = -\frac{\Omega_{C1}^*}{D_1(\omega)}\Lambda_{S1}, \qquad (5a)$$

$$\beta_2^{(1)} = \frac{\omega + \Delta_{C1} + i\gamma_1}{D_1(\omega)}\Lambda_{S1}, \qquad (5b)$$

$$\beta_4^{(1)} = \frac{\omega + \Delta_{C2} + i\gamma_5}{D_2(\omega)}\Lambda_{S2}, \qquad (5c)$$

$$\beta_5^{(1)} = -\frac{\Omega_{C2}^*}{D_2(\omega)}\Lambda_{S2} \qquad (5d)$$

and

$$\frac{\partial \Lambda_{S1}}{\partial z} - i\frac{\omega}{c}\Lambda_{S1} = i\kappa_{23}\beta_2^{(1)}, \qquad (6a)$$

$$\frac{\partial \Lambda_{S2}}{\partial z} - i\frac{\omega}{c}\Lambda_{S2} = i\kappa_{43}\beta_4^{(1)}, \qquad (6b)$$

where $D_1(\omega) = |\Omega_{C1}|^2 - (\omega + \Delta_{C1} + i\gamma_1)(\omega + \Delta_S + i\gamma_2)$ and $D_2(\omega) = |\Omega_{C2}|^2 - (\omega + \Delta_{C2} + i\gamma_5)(\omega + \Delta_S - \Delta + i\gamma_4)$.

Equations (6*a*) and (6*b*) can be solved analytically by using equations (5*b*) and (5*c*), yielding

$$\Lambda_{S1}(z,\omega) = \Lambda_{S1}(0,\omega)\exp[iK_1(\omega)z], \qquad (7a)$$

$$\Lambda_{S2}(z,\omega) = \Lambda_{S2}(0,\omega)\exp[iK_2(\omega)z], \qquad (7b)$$

where the propagation constants $K_1(\omega)$ and $K_2(\omega)$, corresponding to σ^- and σ^+ components of the signal field, respectively, are denoted by

$$K_1(\omega) = \frac{\omega}{c} + \kappa_{23}\frac{\omega + \Delta_{C1} + i\gamma_1}{D_1(\omega)} = K_1 + K_1'\omega + K_1''\omega^2 + \cdots, \qquad (8a)$$

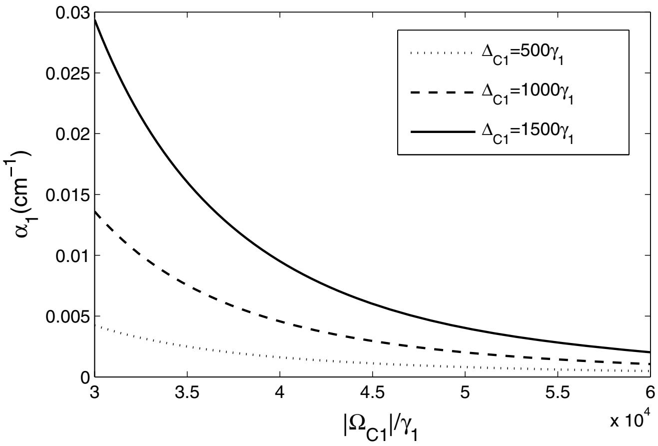

Figure 2. Absorption coefficients α_1 versus dimensionless Rabi frequency $|\Omega_{C1}|/\gamma_1$ for several different values of the two-photon detuning Δ_{C1}. The other parameters are $2\gamma_1 \simeq 4.0 \times 10^3\ \mathrm{s}^{-1}$, $\gamma_2 \simeq 10^4\gamma_1$, $\kappa_{23} \simeq 1.0 \times 10^9(\mathrm{cm}\cdot\mathrm{s})^{-1}$ and $\Delta_S \simeq 1.0 \times 10^7\ \mathrm{s}^{-1}$.

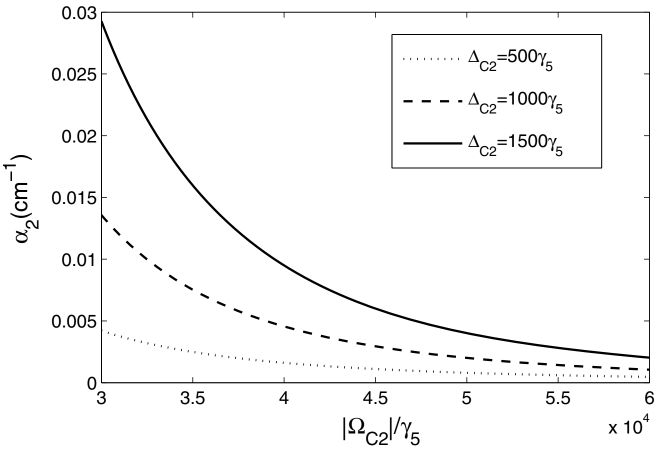

Figure 3. Absorption coefficients α_2 versus dimensionless Rabi frequency $|\Omega_{C2}|/\gamma_5$ for several different values of the two-photon detuning Δ_{C2}. The parameter values are $\gamma_5 \approx \gamma_1$, $\gamma_4 \approx \gamma_2$, $\kappa_{43} \approx \kappa_{23}$ and $\Delta \simeq 2.0 \times 10^6\ \mathrm{s}^{-1}$.

$$K_2(\omega) = \frac{\omega}{c} + \kappa_{43}\frac{\omega + \Delta_{C2} + \mathrm{i}\gamma_5}{D_2(\omega)}$$
$$= K_2 + K_2'\omega + K_2''\omega^2 + \cdots, \tag{8b}$$

where $K_{1(2)} = K_{1(2)}(0)$, $K'_{1(2)} = \mathrm{d}K_{1(2)}(\omega)/\mathrm{d}\omega|_{\omega=0}$, $K''_{1(2)} = 2\mathrm{d}^2K_{1(2)}(\omega)/\mathrm{d}\omega^2|_{\omega=0}$, which have clear physical signification. $K_{1(2)} = \phi_{1(2)} + \mathrm{i}\alpha_{1(2)}/2$ describes the phase shift $\phi_{1(2)}$ per unit length and absorption coefficient $\alpha_{1(2)}$ (see figures 2 and 3) of the $\sigma^-(\sigma^+)$ component of the signal field. $K'_{1(2)} = 1/V_{g1(g2)}$ gives the propagation group velocity, and $K''_{1(2)}$ represents the group-velocity dispersion that contributes to the pulse's shape change and additional loss of field intensity. It should be emphasized that the vector optical soliton pairs produced in this way generally travel with, respectively, a group velocity given by $V_{g1} = 1/K_1'$ and $V_{g2} = 1/K_2'$(see figures 4 and 5).

Figures 2 and 3 illustrate, respectively, the absorption coefficients $\alpha_1 = 2\mathrm{Im}(K_1)$ and $\alpha_2 = 2\mathrm{Im}(K_2)$ of the σ^- and σ^+ components of the signal field versus the dimensionless Rabi frequencies $|\Omega_{C1}|/\gamma_1$ and $|\Omega_{C2}|/\gamma_5$ for several different values of two-photon detunings Δ_{C1} and Δ_{C2}. In these two figures, the corresponding parameters are chosen as $2\gamma_1 \approx 2\gamma_5 \simeq 4.0\times10^3\ \mathrm{s}^{-1}$, $\gamma_4 \approx \gamma_2$, $\gamma_2 \simeq 10^4\gamma_1$, $\kappa_{23} \approx \kappa_{43} \simeq 1.0\times 10^9\ (\mathrm{cm}\cdot\mathrm{s})^{-1}$, $\Delta_S \simeq 1.0\times10^7\ \mathrm{s}^{-1}$ and $\Delta \simeq 2.0\times10^6\ \mathrm{s}^{-1}$. These two figures clearly demonstrate that the absorptions of two polarization components of the weak signal field are affected by the intensities of the control fields and the two-photon detunings. We see that there exist parameter regimes with small absorption coefficients $\alpha_{1(2)}$, which means that the absorption of the signal field is almost completely suppressed in this five-level Raman system. As we demonstrate below, a reasonable and realistic set of parameters can be surely found, which satisfy that $\alpha_{1(2)} \approx 0$.

The purpose of our paper is to search for the formation of ultraslow temporal vector optical solitons which are evolved from two polarization components of the signal field. Therefore, we need to systematically keep terms up to ω^2 in equations (8a) and (8b). For a Gaussian input of the σ^- component of the signal field, namely, $\Omega_{S1}(0,t) = \Omega_{S1}(0,0)\exp\left(-t^2/\tau_0{}^2\right)$, we obtain from equation (7a) after carrying out the inverse Fourier transformation

$$\Omega_{S1}(z,t) = \frac{\Omega_{S1}(0,0)}{\sqrt{b_1(z) - \mathrm{i}b_2(z)}} \times \exp\left[\mathrm{i}zK_1 - \frac{(zK_1' - t)^2}{[b_1(z) - \mathrm{i}b_2(z)]\tau_0{}^2}\right], \tag{9}$$

where $b_1(z) = 1 + 4z\,\mathrm{Re}(K_1'')/\tau_0{}^2$ and $b_2(z) = 4z\,\mathrm{Im}(K_1'')/\tau_0{}^2$. A similar analytical expression for $\Omega_{S2}(z,t)$ can also be obtained if the initial condition of the σ^+ component of the signal field is a Gaussian pulse. From equation (9), we clearly see that the linear and quadratic dispersion effects contribute to the pulse attenuation, phase shift, group velocity and propagation-dependent pulse spreading. In this case, it would be possible to generate ultraslow temporal vector optical solitons in the lifetime-broadened five-state atomic system under Raman excitation only if we search for an effective remedy to balance the rapid increase in pulse width. This is the main objective of the following section where two

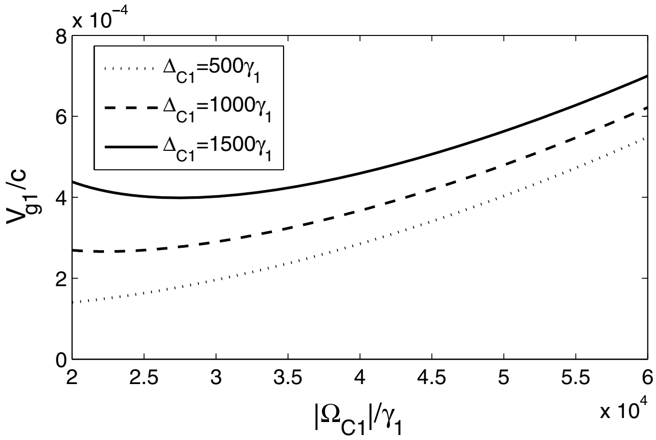

Figure 4. The relative group velocity V_{g1}/c versus dimensionless Rabi frequency $|\Omega_{C1}|/\gamma_1$ for several different values of the two-photon detuning Δ_{C1}. The other parameters are the same as in figure 2 except the one-photon detuning $\Delta_S \simeq -1.0 \times 10^9\ \mathrm{s}^{-1}$.

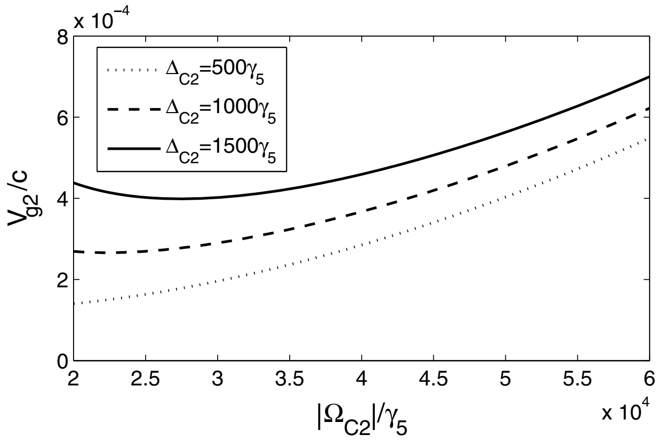

Figure 5. The relative group velocity V_{g2}/c versus dimensionless Rabi frequency $|\Omega_{C2}|/\gamma_5$ for several different values of the two-photon detuning Δ_{C2}. The other parameters are the same as in figure 3 except the one-photon detuning $\Delta_S \simeq -1.0 \times 10^9\ \mathrm{s}^{-1}$.

coupled NLS equations describing the envelopes evolution of two polarization components of the signal field are derived and the SPM and CPM effects are investigated for balancing the group-velocity dispersion.

3. Two coupled NLS equations

In this section we will investigate the nonlinear evolution of two polarization components of the signal field. In the following we show that a reasonable and realistic set of parameters can be found, the SPM and CPM effects of two components of the signal field can balance the group-velocity dispersion and lead to the formation of ultraslow temporal vector optical solitons in the lifetime-broadened five-state atomic system under Raman excitation.

As a first step in getting a quantitative description for the formation and dynamics of ultraslow temporal vector optical solitons, we now derive the nonlinear envelope equations describing the evolution of two polarization components of the signal field by using the method developed by [2, 3, 5, 9–11]. Taking a trial function $\Lambda_{S1(S2)}(z,\omega) = \Lambda_{1(2)}(z,\omega)\exp[\mathrm{i}zK_{1(2)}]$ and substituting them into the wave equations

$$\frac{\partial}{\partial z}\Lambda_{S1}(z,\omega) = \mathrm{i}K_1(\omega)\Lambda_{S1}(z,\omega), \tag{10a}$$

$$\frac{\partial}{\partial z}\Lambda_{S2}(z,\omega) = \mathrm{i}K_2(\omega)\Lambda_{S2}(z,\omega), \tag{10b}$$

we then obtain

$$\frac{\partial\Lambda_1(z,\omega)}{\partial z}\,\mathrm{e}^{\mathrm{i}zK_1} = \mathrm{i}(K_1'\omega + K_1''\omega^2)\Lambda_1(z,\omega)\,\mathrm{e}^{\mathrm{i}zK_1}, \tag{11a}$$

$$\frac{\partial\Lambda_2(z,\omega)}{\partial z}\,\mathrm{e}^{\mathrm{i}zK_2} = \mathrm{i}(K_2'\omega + K_2''\omega^2)\Lambda_2(z,\omega)\,\mathrm{e}^{\mathrm{i}zK_2}. \tag{11b}$$

Here we only keep terms up to order ω^2 in expanding the propagation constants $K_1(\omega)$ and $K_2(\omega)$. In order to balance the interplay between group-velocity dispersion and nonlinear effect, it is necessary for us to consider the terms of nonlinear polarization on the right-hand sides of equations (3a) and (3b), namely,

$$\mathrm{i}\kappa_{23}C_2C_3^* \approx \mathrm{i}\kappa_{23}C_2^{(1)}\left[C_3^{(0)}\right]^* + \mathrm{iNLT}_1, \tag{12a}$$

$$\mathrm{i}\kappa_{43}C_4C_3^* \approx \mathrm{i}\kappa_{43}C_4^{(1)}\left[C_3^{(0)}\right]^* + \mathrm{iNLT}_2, \tag{12b}$$

where the nonlinear terms NLT_1 and NLT_2 are, respectively, given by $\mathrm{NLT}_1 = -\kappa_{23}C_2^{(1)}\left[\left|C_1^{(1)}\right|^2+\left|C_2^{(1)}\right|^2+\left|C_4^{(1)}\right|^2+\left|C_5^{(1)}\right|^2\right]$ and $\mathrm{NLT}_2 = -\kappa_{43}C_4^{(1)}\left[\left|C_1^{(1)}\right|^2+\left|C_2^{(1)}\right|^2+\left|C_4^{(1)}\right|^2+\left|C_5^{(1)}\right|^2\right]$. With the help of the explicit expressions $C_j^{(1)} (j = 1, 2, 4, 5)$, we can immediately obtain from equations (5a)–(5d) by setting $\omega = 0$ and replacing $\beta_j^{(1)}$, $\Lambda_{S1(S2)}$ with $\beta_j^{(1)}$ and $\Omega_{S1(S2)}$, respectively

$$C_1^{(1)} = -\frac{\Omega_{C1}^*}{D_1}\Omega_{S1}, \qquad C_2^{(1)} = \frac{\Delta_{C1}+\mathrm{i}\gamma_1}{D_1}\Omega_{S1}, \tag{13a}$$

$$C_5^{(1)} = -\frac{\Omega_{C2}^*}{D_2}\Omega_{S2}, \qquad C_4^{(1)} = \frac{\Delta_{C2}+\mathrm{i}\gamma_5}{D_2}\Omega_{S2}, \tag{13b}$$

with $D_1 = |\Omega_{C1}|^2 - (\Delta_{C1}+\mathrm{i}\gamma_1)(\Delta_S+\mathrm{i}\gamma_2)$ and $D_2 = |\Omega_{C2}|^2 - (\Delta_{C2}+\mathrm{i}\gamma_5)(\Delta_S-\Delta+\mathrm{i}\gamma_4)$.

With the nonlinear polarization terms (12a) and (12b), we now turn to the investigation of the nonlinear effect of the system. Performing the inverse Fourier transformation for the above evolution equations (11a) and (11b)

$$\Omega_k(z,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\Lambda_k(z,\omega)\,\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{d}\omega, \qquad k = 1,2, \tag{14}$$

we can straightforwardly obtain the following nonlinear evolution equations, namely, two coupled NLS equations for the slowly varying envelopes $\Omega_1(z,t)$ and $\Omega_2(z,t)$,

$$\mathrm{i}\left(\frac{\partial}{\partial z}+\frac{1}{V_{g1}}\frac{\partial}{\partial t}\right)\Omega_1(z,t) - K_1''\frac{\partial^2}{\partial t^2}\Omega_1(z,t) = [U_{11}\,\mathrm{e}^{-\alpha_1 z}|\Omega_1(z,t)|^2 + U_{12}\,\mathrm{e}^{-\alpha_2 z}|\Omega_2(z,t)|^2]\Omega_1(z,t), \tag{15a}$$

$$\mathrm{i}\left(\frac{\partial}{\partial z}+\frac{1}{V_{g2}}\frac{\partial}{\partial t}\right)\Omega_2(z,t) - K_2''\frac{\partial^2}{\partial t^2}\Omega_2(z,t) = [U_{21}\,\mathrm{e}^{-\alpha_1 z}|\Omega_1(z,t)|^2 + U_{22}\,\mathrm{e}^{-\alpha_2 z}|\Omega_2(z,t)|^2]\Omega_2(z,t), \tag{15b}$$

with $\alpha_1 = 2\mathrm{Im}(K_1)$ and $\alpha_2 = 2\mathrm{Im}(K_2)$ being the absorption coefficients, $V_{g1} = 1/K_1'$ and $V_{g2} = 1/K_2'$ being the group velocities, K_1'' and K_2'' characterizing the group-velocity dispersion, U_{11} and U_{22} characterizing the SPM, and U_{12} and U_{21} characterizing the CPM of the σ^- and σ^+ components of the signal field, respectively. The expressions of these corresponding coefficients are shown in the appendix.

We define $\delta = (1/V_{g1} - 1/V_{g2})/2$, $1/V_g = (1/V_{g1} + 1/V_{g2})/2$, $\xi = z$ and $\tau = t - z/V_g$, according to $\partial/\partial z \sim \partial/\partial\xi - \partial/(V_g\partial\tau)$ and $\partial/\partial t \sim \partial/\partial\tau$, the nonlinear evolution equations of equations (15*a*) and (15*b*) can be simplified as

$$\mathrm{i}\left(\frac{\partial}{\partial\xi}+\delta\frac{\partial}{\partial\tau}\right)\Omega_1-K_1''\frac{\partial^2}{\partial\tau^2}\Omega_1$$
$$=(U_{11}\,\mathrm{e}^{-\alpha_1\xi}|\Omega_1|^2+U_{12}\,\mathrm{e}^{-\alpha_2\xi}|\Omega_2|^2)\Omega_1, \qquad (16a)$$
$$\mathrm{i}\left(\frac{\partial}{\partial\xi}-\delta\frac{\partial}{\partial\tau}\right)\Omega_2-K_2''\frac{\partial^2}{\partial\tau^2}\Omega_2$$
$$=(U_{22}\,\mathrm{e}^{-\alpha_2\xi}|\Omega_2|^2+U_{21}\,\mathrm{e}^{-\alpha_1\xi}|\Omega_1|^2)\Omega_2. \qquad (16b)$$

The two coupled NLS equations (16*a*) and (16*b*) generally have complex coefficients and hence do not allow soliton solutions. However, as we show below, the absorption of the signal field is largely suppressed by adjusting the intensities of the control fields and the one- and two-photon detunings. If we can choose a reasonable and realistic set of parameters for the present system so that the imaginary parts of the complex coefficients are much smaller than the corresponding real parts, i.e., $\alpha_1\approx\alpha_2\approx 0$, $K_n''=K_{n,r}''+\mathrm{i}K_{n,i}''\approx K_{n,r}''(n=1,2)$,and $U_{lm}''=U_{lm,r}''+\mathrm{i}U_{lm,i}''\approx U_{lm,r}''(l,m=1,2)$. In this situation equations (16*a*) and (16*b*) can be written in the dimensionless form

$$\mathrm{i}\frac{\partial u}{\partial s}+\mathrm{i}G_\delta\frac{\partial u}{\partial\sigma}-G_1\frac{\partial^2 u}{\partial\sigma^2}-(G_{11}|u|^2+G_{12}|v|^2)u=0, \quad (17a)$$
$$\mathrm{i}\frac{\partial v}{\partial s}-\mathrm{i}G_\delta\frac{\partial v}{\partial\sigma}-G_2\frac{\partial^2 v}{\partial\sigma^2}-(G_{22}|v|^2+G_{21}|u|^2)v=0, \quad (17b)$$

which admit solutions describing various vector solitons [1, 19, 37, 46–49], including bright–bright, bright–dark, dark–bright and dark–dark vector solitons, as will be seen in the following section. Here $s=\xi/L_D$, $\sigma=\tau/\tau_0$, $u=\Omega_1/U_0$, $v=\Omega_2/U_0$, $G_\delta=\mathrm{sgn}(\delta)L_D/L_\delta$, $G_n=K_{n,r}''/|K_{2,r}''|(n=1,2)$ and $G_{lm}=U_{lm,r}/|U_{22,r}|(l,m=1,2)$ with τ_0 being the characteristic pulse length of the signal field. $U_0=|K_{2,r}''/U_{22,r}|^{1/2}/\tau_0$ is the typical Rabi frequency of the signal field. $L_D={\tau_0}^2/|K_{2,r}''|$ is the characteristic dispersion length. $L_N=1/(|U_{22,r}|{U_0}^2)$ is the characteristic nonlinear length. And $L_\delta=\tau_0/|\delta|$ is the characteristic group velocity mismatch length of the system. Besides, we have set $L_D=L_N$, which means the balance between the group-velocity dispersion and nonlinearity effects in our system, to obtain soliton solutions from equations (17*a*) and (17*b*).

4. Vector soliton solutions and Manakov equations

Based on the analysis above, we find that equations (17*a*) and (17*b*) are nearly integrable, the SPM and CPM coefficients in equations (15*a*) and (15*b*) defined by equations (A.4*a*), (A.4*b*), (A.5*a*) and (A.5*b*) satisfy the relation $U_{11}U_{22}=U_{12}U_{21}$, i.e., $G_{11}G_{22}=G_{12}G_{21}$, which represents the balance between the dispersion and the SPM and CPM effects, and hence shape-preserving temporal vector optical soliton solutions are possible that can propagate for an extended distance without significant deformation in the system.

When the parameters fulfil the condition $G_{22}G_1=G_{12}G_2$, we can easily obtain bright–bright, bright–dark, dark–bright and dark–dark vector soliton solutions of equations (17*a*) and (17*b*) as shown below.

(i) *Bright–bright vector soliton solution.*

$$u=C_1\mathrm{sech}(\sigma)\exp[\mathrm{i}(F_{11}\sigma+F_{12}s)], \qquad (18a)$$
$$v=C_2\mathrm{sech}(\sigma)\exp[\mathrm{i}(F_{21}\sigma+F_{22}s)], \qquad (18b)$$

where $\mathrm{sech}(\sigma)$ is the hyperbolic secant function. And we have defined $F_{11}=G_\delta/(2G_1)$, $F_{12}=-G_1-{G_\delta}^2/(4G_1)$, $F_{21}=-G_\delta/(2G_2)$, $F_{22}=-G_2-{G_\delta}^2/(4G_2)$ and $C_2=\left[\left(2G_1-G_{11}{C_1}^2\right)/G_{12}\right]^{1/2}$ with C_1 being a free parameter.

(ii) *Bright–dark vector soliton solution.*

$$u=C_1\mathrm{sech}(\sigma)\exp[\mathrm{i}(F_{11}\sigma+F_{12}s)], \qquad (19a)$$
$$v=C_2\tanh(\sigma)\exp[\mathrm{i}(F_{21}\sigma+F_{22}s)], \qquad (19b)$$

where $\tanh(\sigma)$ is the hyperbolic tangent function. And we also have defined $F_{11}=G_\delta/(2G_1)$, $F_{12}=-F_{11}G_\delta-G_1(1-{F_{11}}^2)-G_{12}{C_2}^2$, $F_{21}=-G_\delta/(2G_2)$, $F_{22}=F_{21}G_\delta+G_2{F_{21}}^2-G_{22}{C_2}^2$ and $C_2=[(G_{11}{C_1}^2-2G_1)/G_{12}]^{1/2}$ with C_1 being a free parameter.

(iii) *Dark–bright vector soliton solution.*

$$u=C_1\tanh(\sigma)\exp[\mathrm{i}(F_{11}\sigma+F_{12}s)], \qquad (20a)$$
$$v=C_2\mathrm{sech}(\sigma)\exp[\mathrm{i}(F_{21}\sigma+F_{22}s)], \qquad (20b)$$

where $F_{11}=G_\delta/(2G_1)$, $F_{12}=-F_{11}G_\delta+G_1{F_{11}}^2-G_{11}{C_1}^2$, $F_{21}=-G_\delta/(2G_2)$, $F_{22}=F_{21}G_\delta+G_2\left(1-{F_{21}}^2\right)-G_{21}{C_1}^2$ and $C_2=\left[\left(G_{11}{C_1}^2+2G_1\right)/G_{12}\right]^{1/2}$ with C_1 being a free parameter.

(iv) *Dark–dark vector soliton solution.*

$$u=C_1\tanh(\sigma)\exp[\mathrm{i}(F_{11}\sigma+F_{12}s)], \qquad (21a)$$
$$v=C_2\tanh(\sigma)\exp[\mathrm{i}(F_{21}\sigma+F_{22}s)], \qquad (21b)$$

where $F_{11}=G_\delta/(2G_1)$, $F_{12}=-F_{11}G_\delta+G_1(2+{F_{11}}^2)$, $F_{21}=-G_\delta/(2G_2)$, $F_{22}=F_{21}G_\delta+G_2\left(2+{F_{21}}^2\right)$ and $C_2=\left[-\left(G_{11}{C_1}^2+2G_1\right)/G_{12}\right]^{1/2}$ with C_1 being a free parameter. It is worth pointing out that all four types of temporal vector optical soliton pairs described by equations (18*a*), (18*b*), (19*a*), (19*b*), (20*a*), (20*b*), (21*a*) and (21*b*) are allowed in our system and travel with ultraslow group velocity V_g.

We now present numerical examples to demonstrate the existence of ultraslow bright–bright and dark–dark vector optical solitons described above in the atomic system studied through equations (16*a*) and (16*b*). We consider a realistic atomic system where the decay rates are $2\gamma_1\approx2\gamma_5\approx4.0\times10^3\ \mathrm{s}^{-1}$ and $2\gamma_2\approx2\gamma_4\approx4.0\times10^7\ \mathrm{s}^{-1}$.

We first consider the case of ultraslow dark–dark vector optical solitons. Taking $\kappa_{23}\approx\kappa_{43}=1.0\times10^9\ (\mathrm{cm\ s})^{-1}$, $2\Omega_{C1}\approx2\Omega_{C2}=2.0\times10^8\ \mathrm{s}^{-1}$, $\Delta_{C1}\approx\Delta_{C2}\approx2.0\times10^6\ \mathrm{s}^{-1}$, $\Delta_S\approx-1.0\times10^9\ \mathrm{s}^{-1}$ and $\Delta=2.0\times10^6\ \mathrm{s}^{-1}$, we can obtain $K_1\approx0.1667+0.0007\mathrm{i}\ \mathrm{cm}^{-1}$, $K_2\approx0.1666+0.0007\mathrm{i}\ \mathrm{cm}^{-1}$, $K_1'\approx(6.950+0.044\mathrm{i})\times10^{-8}\ \mathrm{s\ cm}^{-1}$, $K_2'\approx(6.946+0.044\mathrm{i})\times10^{-8}\ \mathrm{s\ cm}^{-1}$, $K_1''\approx(-5.765+0.061\mathrm{i})\times10^{-15}\ \mathrm{s}^2\ \mathrm{cm}^{-1}$, $K_2''\approx(-5.770+0.061\mathrm{i})\times10^{-15}\ \mathrm{s}^2\ \mathrm{cm}^{-1}$, $U_{11}\approx(1.158+0.0048\mathrm{i})\times10^{-17}\ \mathrm{s}^2\ \mathrm{cm}^{-1}$, $U_{22}\approx(1.157+$

0.0048i) $\times$ 10^{-17} s^2 cm^{-1}, $U_{12} \approx (1.157 + 0.0048i) \times 10^{-17}$ s^2 cm^{-1}, $U_{21} \approx (1.157 + 0.0048i) \times 10^{-17}$ s^2 cm^{-1}, $V_{g1}/c \approx 4.796 \times 10^{-4}$, $V_{g2}/c \approx 4.800 \times 10^{-4}$ and $\alpha_1 \approx \alpha_2 \approx 0.0014$ cm^{-1}. Note that the imaginary parts of these quantities are indeed much smaller than their relevant real parts. With these quantities, we have $L_D = 0.624$ cm and $L_\delta = 1296.2$ cm with $\tau_0 = 6.0 \times 10^{-8}$ s and $U_0 = 3.723 \times 10^8$ s^{-1} and the dimensionless coefficients read $G_\delta = 4.81 \times 10^{-4}$, $G_1 = -0.999$, $G_2 = -1$ and $G_{11} \approx G_{12} \approx G_{21} \approx G_{22} = 1$, then the two coupled NLS equations (17*a*) and (17*b*) are well characterized, and hence we have demonstrated the existence of dark–dark vector optical solitons that are evolved from two polarization components of the weak signal field with nearly matched (see figures 4 and 5), ultraslow propagating velocities comparing with *c* in a cold atomic medium.

For ultraslow bright–bright vector optical solitons we adjust one-photon detuning $\Delta_S \approx 1.0 \times 10^9$ s^{-1} and Rabi frequencies $2\Omega_{C1} \approx 2\Omega_{C2} = 4.2 \times 10^7$ s^{-1} with all other parameters given above unchanged. In this case we obtain $K_1 \approx -1.282 + 0.033i$ cm^{-1}, $K_2 \approx -1.289 + 0.034i$ cm^{-1}, $K_1' \approx (1.827 + 0.098i) \times 10^{-7}$ s cm^{-1}, $K_2' \approx (1.846 + 0.100i) \times 10^{-7}$ s cm^{-1}, $K_1'' \approx (-1.165 + 0.071i) \times 10^{-13}$ s^2 cm^{-1}, $K_2'' \approx (-1.179 + 0.072i) \times 10^{-13}$ s^2 cm^{-1}, $U_{11} \approx (-2.346 + 0.061i) \times 10^{-16}$ s^2 cm^{-1}, $U_{22} \approx (-2.382 + 0.062i) \times 10^{-16}$ s^2 cm^{-1}, $U_{12} \approx (-2.370 + 0.061i) \times 10^{-16}$ s^2 cm^{-1}, $U_{21} \approx (-2.358 + 0.061i) \times 10^{-16}$ s^2 cm^{-1}, $V_{g1}/c \approx 1.819 \times 10^{-4}$, $V_{g2}/c \approx 1.800 \times 10^{-4}$ and $\alpha_1 \approx \alpha_2 \approx 0.067$ cm^{-1}. Note that the imaginary parts of these quantities are also indeed much smaller than their relevant real parts. With these quantities, we have $L_D = 0.34$ cm and $L_\delta = 105.7$ cm with $\tau_0 = 2.0 \times 10^{-7}$ s and $U_0 = 1.112 \times 10^8$ s^{-1} and the dimensionless coefficients read $G_\delta = -0.003$, $G_1 = -0.989$, $G_2 = -1$ and $G_{11} \approx G_{12} \approx G_{21} \approx G_{22} = -1$. These parameters and results again show that the two coupled NLS equations (17*a*) and (17*b*) are well characterized and that the bright–bright vector optical solitons which are evolved from two polarization components of the signal field with nearly matched, ultraslow propagating velocities comparing with *c* indeed can be formed in this atomic system.

It is worth noting that the above-described parameter sets also lead to realize a temporal Manakov system, which is completely integrable and can be determined by Hirota's method [49, 50]. In fact, with the quantities obtained in the case of ultraslow bright–bright vector optical solitons, two coupled NLS equations (17*a*) and (17*b*) can be written as the standard integrable Manakov equations,

$$i\frac{\partial}{\partial s}u + \frac{\partial^2}{\partial \sigma^2}u + (|u|^2 + |v|^2)u = 0, \qquad (22a)$$

$$i\frac{\partial}{\partial s}v + \frac{\partial^2}{\partial \sigma^2}v + (|v|^2 + |u|^2)v = 0, \qquad (22b)$$

which admits of exact *N*-soliton solutions [50]. And the bright–bright vector soliton solutions of equations (22*a*) and (22*b*) are given by

$$u = \sqrt{2}\cos(\theta)\,\mathrm{sech}(\sigma)\,e^{is}, \qquad (23a)$$

$$v = \sqrt{2}\sin(\theta)\,\mathrm{sech}(\sigma)\,e^{is}, \qquad (23b)$$

where θ is a free parameter. Here we have $\theta = \pi/4$ due to the fact that the injected signal field is linearly polarized and two polarization components have equal amplitudes. Thus, we have demonstrated the existence of the Manakov temporal vector solitons with ultraslow group velocity ($V_g \sim 10^{-4}c$) in a lifetime-broadened five-state atomic system under Raman excitation.

In addition, we point out that the general solutions of two coupled NLS equations (17*a*) and (17*b*) which govern the propagation of two pules with same frequency but belonging to two different polarizations not only include one-soliton solutions (18*a*)–(21*b*), but depending on the choice of corresponding parameters such as the Rabi frequencies of the control fields, one- and two-photon detunings, they also exhibit different types of soliton solutions [49]. For instance, by adjusting the parameters, the dark–bright vector soliton may experience a breakup into another dark–bright soliton and an oscillating soliton, and even its reverse process, the fusion of a dark–bright vector soliton and an oscillating soliton into another dark–bright vector soliton, is also possible. This provides the possibility of the promising applications for the design of new types of all-optical switches and logic gates [18, 49].

5. Conclusions

In conclusion, we have analysed nonlinear dynamics of a weak, linear-polarized pulsed signal field, having two orthogonally polarized components, in a lifetime-broadened five-state atomic system under Raman excitation. In the presence of two coherent driving control fields, the linear as well as nonlinear dispersion are dramatically enhanced while simultaneously the absorptions of two polarization components of the signal field are suppressed in the medium. We have derived the corresponding nonlinear evolution equations, i.e., two coupled nonlinear Schrödinger equations, and have shown that the dispersion and the SPM and CPM effects can achieve perfect balance, and there are parameter regimes in which the temporal vector solitons, including bright–bright, bright–dark, dark–bright and dark–dark vector solitons with ultraslow group velocity, can propagate through the medium of cold atom. Besides, the ultraslow Manakov temporal vector optical solitons can be easily realized in the system by adjusting the corresponding parameters such as the Rabi frequencies of the control fields, one- and two-photon detunings. We also discuss the unique features of the two coupled NLS equations. The Raman scheme described may lead to other new phenomena that manifest themselves under well-controlled balance of dispersion and nonlinear effects. These include, but are not limited to, simultaneous formation of multiple solitons and soliton–soliton interactions in the ultraslow propagation regime.

Acknowledgments

We would like to acknowledge Professor Y Wu for his enlightening suggestions and encouragement. The work is supported in part by the NSF of China (grant nos. 60478029, 10575040, 10634060 and 90503010) and by the National Fundamental Research Program of China (grant no. 2005CB724508).

Appendix

The explicit expressions of K_1, K_2, K_1', K_2', K_1'', K_2'', U_{11}, U_{22}, U_{12} and U_{21} are written as follows:

$$K_1 = \frac{\kappa_{23}(\Delta_{C1} + i\gamma_1)}{D_1}, \qquad K_2 = \frac{\kappa_{43}(\Delta_{C2} + i\gamma_5)}{D_2}, \tag{A.1}$$

$$K_1' = \frac{1}{c} + \kappa_{23}\frac{|\Omega_{C1}|^2 + (\Delta_{C1} + i\gamma_1)^2}{{D_1}^2}, \tag{A.2a}$$

$$K_2' = \frac{1}{c} + \kappa_{43}\frac{|\Omega_{C2}|^2 + (\Delta_{C2} + i\gamma_5)^2}{{D_2}^2}, \tag{A.2b}$$

$$K_1'' = \kappa_{23}[2|\Omega_{C1}|^2(\Delta_{C1} + i\gamma_1) + (\Delta_{C1} + i\gamma_1)^3 + |\Omega_{C1}|^2(\Delta_S + i\gamma_2)] \div {D_1}^3, \tag{A.3a}$$

$$K_2'' = \kappa_{43}[2|\Omega_{C2}|^2(\Delta_{C2} + i\gamma_5) + (\Delta_{C2} + i\gamma_5)^3 + |\Omega_{C2}|^2(\Delta_S - \Delta + i\gamma_4)] \div {D_2}^3, \tag{A.3b}$$

$$U_{11} = \frac{\kappa_{23}(\Delta_{C1} + i\gamma_1)(|\Omega_{C1}|^2 + {\Delta_{C1}}^2 + {\gamma_1}^2)}{D_1|D_1|^2}, \tag{A.4a}$$

$$U_{22} = \frac{\kappa_{43}(\Delta_{C2} + i\gamma_5)(|\Omega_{C2}|^2 + {\Delta_{C2}}^2 + {\gamma_5}^2)}{D_2|D_2|^2}, \tag{A.4b}$$

$$U_{12} = \frac{\kappa_{23}(\Delta_{C1} + i\gamma_1)(|\Omega_{C2}|^2 + {\Delta_{C2}}^2 + {\gamma_5}^2)}{D_1|D_2|^2}, \tag{A.5a}$$

$$U_{21} = \frac{\kappa_{43}(\Delta_{C2} + i\gamma_5)(|\Omega_{C1}|^2 + {\Delta_{C1}}^2 + {\gamma_1}^2)}{D_2|D_1|^2}, \tag{A.5b}$$

with $D_1 = |\Omega_{C1}|^2 - (\Delta_{C1} + i\gamma_1)(\Delta_S + i\gamma_2)$ and $D_2 = |\Omega_{C2}|^2 - (\Delta_{C2} + i\gamma_5)(\Delta_S - \Delta + i\gamma_4)$.

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