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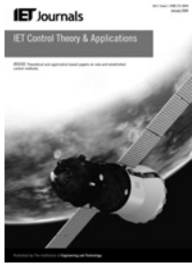

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Delay-dependent robust H_∞ control of time-delay systems

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Abstract: In this study, a delay-dependent H_∞ performance criterion that possess decoupling structure is derived for a class of time-delay systems. It is then extended to H_∞ state-feedback synthesis for time-delay systems with polytopic uncertainty and multichannel H_∞ dynamic output-feedback synthesis for time-delay systems. All the conditions are given in terms of the linear matrix inequalities. In some previous descriptor methods, the products of controller matrices and Lyapunov matrices are completely separated in performance analysis, whereas it is not the case in controller synthesis. However, with the method in the paper, the weakness is eliminated. Numerical examples illustrate the effectiveness of our solutions as compared to results obtained by other methods.

1 Introduction

Application of a single Lyapunov function to the analysis and design is investigated for systems with polytopic uncertainty and multichannel constraints [1]. The strict requirement of a single Lyapunov function for all admissible uncertainties and all multichannel constraints can lead to conservative results. To reduce the conservativeness, researchers turn to using parameter-dependent Lyapunov functions and have obtained many results [2–7]. Among these works, an effective idea is to separate the products of Lyapunov matrices and controller matrices in the given LMIs by introducing auxiliary slack variables. A breakthrough towards this direction is the work for discrete systems in [2] that is extended to continuous time case in [3–7] by different methods. However, time delay is not considered in the above references. Time delay is a source of instability in many cases. Therefore the stability and performance analysis for time-delay systems are of theoretical and practical importance [8–12]. Static output-feedback control problems are investigated in [11, 12]. However, time-delay systems considered are with norm-bounded uncertainty but not with polytopic uncertainty. Although stability and stabilisability conditions with parameter-dependent Lyapunov variables for time-delay system are presented in [13, 14]. These works are restricted to state-feedback and static output-feedback control, which are easier to solve than dynamic output-feedback control. In some situations, there is a strong need to construct a dynamic output-feedback controller instead of a static one in order to obtain a better performance and dynamical behaviour of the state response. The dynamic output-feedback control is considered in [15]; however, the multichannel H_∞ output-feedback synthesis still remains an open problem and the products of controller matrices and Lyapunov matrices are not completely separated in [15], and this motivates the present paper.

In this paper, we solve dynamic output-feedback H_∞ controller design problems for time-delay systems. We present new delay-dependent H_∞ performance conditions based on the Lyapunov method. Then we derive the H_∞ performance criterion, which decouples controller matrices and Lyapunov matrices. It can reduce conservatism inherent in the conventional method in solving both robust control problems of polytopic systems and multichannel synthesis problems by providing a parameter-dependent Lyapunov function. Then we design a robust H_∞ state-feedback controller for polytopic systems and a dynamic H_∞ output-feedback controller for systems with multichannel constraints. The advantages of the results over the conventional methods are shown by two numerical examples. For simplification, we use the symbol $\mathrm{Sym}\{\cdot\}$ to denote $\mathrm{Sym}\{X\} \stackrel{\mathrm{def}}{=} X + X^{\mathrm{T}}$, the symbol $*$ to denote the symmetric part.

2 H_∞ performance analysis

Consider the following time-delay system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-h) + Bw(t) \\ z(t) &= Cx(t) + Dw(t) \\ x(t) &= 0, \forall t \in [-h, 0] \end{aligned} \tag{1}$$

where, $x(t) \in R^n$ is the state, $w(t) \in R^{n_w}$ is the disturbance signal of finite energy in the space $L_2[0, \infty)$, $z(t) \in R^{n_z}$ is the exogenous output signal, and A, B, A_d, C, D are constant matrices of appropriate dimensions. The time-delay $h > 0$ is assumed to be known. For a prescribed scalar $\gamma > 0$, we define the performance index

$$J(w) = \int_0^\infty (z^T z - \gamma^2 w^T w)\, dt \tag{2}$$

Lemma 1 [16]: For any $2n \times 2n$ symmetric and positive-definite matrix $R > 0$ and matrix M, the following inequality holds

$$-2\int_{t-h}^{t} b^T(\alpha)a(\alpha)\, d\alpha \leq \int_{t-h}^{t} \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix}^T \begin{bmatrix} R & RM \\ * & (2,2) \end{bmatrix} \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix} d\alpha$$

for $a(\alpha) \in R^{2n}$, $b(\alpha) \in R^{2n}$ given for $s \in [t-h, t]$. Here $(2,2) = (M^T R + I)R^{-1}(RM + I)$.

Lemma 2 [13]: Let ϕ, a and b be given matrices of appropriate dimension. Then the two statements are equivalent:

(i) ϕ, a and b satisfy $\phi < 0$ and $\phi + ab^T + ba^T < 0$.

(ii) ϕ, a and b are such as the following LMI is feasible in the variable G.

$$\begin{bmatrix} \phi & a + bG^T \\ * & -G - G^T \end{bmatrix} = \begin{bmatrix} \phi & a \\ * & 0 \end{bmatrix} + \mathrm{Sym}\left\{ \begin{bmatrix} 0 \\ I \end{bmatrix} G \begin{bmatrix} b^T & -I \end{bmatrix} \right\} < 0$$

We first provide delay-dependent H_∞ performance conditions for the time-delay system (1).

Theorem 1: For prescribed positive scalars $\gamma > 0$, $h > 0$ and scalar λ, system (1) is stable and the cost function (2) achieves $J(w) < 0$ for all non-zero $w \in L_2[0, \infty)$, if there exist symmetric and positive-definite matrices $P > 0$, $S > 0$, $R_1 > 0$, $R_3 > 0$ and a matrix R_2 such that (see (3))

where $(1, 1) = \mathrm{Sym}\{P(A + A_d + \lambda A_d)\} + S$, $(1, 3) = h(A + (1+\lambda)A_d)^T A_d^T R_3$.

The proof can be referred to in the appendix, and hence is omitted here.

Corollary 1: For prescribed positive scalars $\gamma > 0$, $h > 0$ and scalar λ, system (1) is stable and the cost function (2) achieves $J(w) < 0$ for all non-zero $w \in L_2[0, \infty)$, if there exist symmetric and positive-definite matrices $Q > 0$, $\bar{S} > 0$, $\bar{R}_1 > 0$, $\bar{R}_3 > 0$ and a matrix $\bar{R}_2$ such that (see (4))

where

$$(1, 1) = \mathrm{Sym}\{(A + A_d + \lambda A_d)Q\}$$
$$(1, 3) = hQ(A + (1+\lambda)A_d)^T A_d^T$$

Proof: Pre- and post-multiplying both sides of the LMI (3) by $\mathrm{diag}\{Q, \bar{S}, \bar{R}_3, I, I, \bar{R}_3, \bar{R}_3\}$, denoting $Q = P^{-1}$, $\bar{S} = S^{-1}$, $\bar{R}_1 = R_3^{-1}R_1R_3^{-1}$, $\bar{R}_2 = R_3^{-1}R_2R_3^{-1}$, $\bar{R}_3 = R_3^{-1}$, making a series of congruence transformations and following the Schur complement Lemma, then we can obtain Corollary 1 immediately. □

3 H_∞ state-feedback synthesis

In this section, the results developed in the previous section are extended to the state-feedback synthesis problem for

$$\begin{bmatrix} (1,1) & -\lambda PA_d & (1,3) & PB & C^T & h(\lambda+1)P & h(\lambda+1)P \\ * & -S & -\lambda hA_d^T A_d^T R_3 & 0 & 0 & 0 & 0 \\ * & * & -hR_3 & hR_3A_dB & 0 & 0 & h^2(\lambda+1)R_3A_d \\ * & * & * & -\gamma^2 I & D^T & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -hR_1 & -hR_2 \\ * & * & * & * & * & * & -hR_3 \end{bmatrix} < 0 \tag{3}$$

$$\begin{bmatrix} (1,1) & -\lambda A_d\bar{S} & (1,3) & B & QC^T & h(\lambda+1)\bar{R}_3 & h(\lambda+1)\bar{R}_3 & Q \\ * & -\bar{S} & -\lambda h\bar{S}A_d^T A_d^T & 0 & 0 & 0 & 0 & 0 \\ * & * & -h\bar{R}_3 & hA_dB & 0 & 0 & h^2(\lambda+1)A_d\bar{R}_3 & 0 \\ * & * & * & -\gamma^2 I & D^T & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -h\bar{R}_1 & -h\bar{R}_2 & 0 \\ * & * & * & * & * & * & -h\bar{R}_3 & 0 \\ * & * & * & * & * & * & * & -\bar{S} \end{bmatrix} < 0 \tag{4}$$

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time-delay systems. Consider the following system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-h) + B_1 w(t) + B_2 u(t) \\ z(t) &= Cx(t) + D_{11} w(t) + D_{12} u(t) \end{aligned} \tag{5}$$

where $u(t) \in R^{n_u}$ and the other signals are the same with system (1), D_{12} is constant matrix of appropriate dimension.

We seek a state-feedback control law

$$u(t) = Kx(t) \tag{6}$$

that will asymptotically stabilise the system and achieve $J(w) < 0$ for all non-zero $w \in L_2[0, \infty)$.

Applying the state-feedback control law (6) to system (5), we can obtain the closed-loop system

$$\begin{aligned} \dot{x}_c(t) &= A_c x_c(t) + A_d x_c(t-h) + B_1 w(t) \\ z_c(t) &= C_c x_c(t) + D_{11} w(t) \end{aligned} \tag{7}$$

where

$$A_c = A + B_2 K, \quad C_c = C + D_{12} K \tag{8}$$

Then we present the following delay-dependent condition under which there exists a state-feedback H_∞ controller for system (7).

Theorem 2: For prescribed positive scalars $\gamma > 0$, $h > 0$, $\varepsilon > 0$ and scalar λ, there exists a state-feedback controller such that the closed-loop system (7) is asymptotically stable and the cost function achieves $J(w) < 0$ for all non-zero $w \in L_2[0, \infty)$ if there exist symmetric and positive-definite matrices $Q > 0$, $S > 0$, $\bar{R}_1 > 0$, $\bar{R}_3 > 0$ and matrix $\bar{R}_2$, V, U such that (see (9))

where

$$\begin{aligned} (1, 6) &= (1, 7) = h(\lambda + 1) R_3 \\ (1, 9) &= Q + (A + A_d + \lambda A_d) V + \frac{1}{2} \varepsilon V + B_2 U \\ (3, 7) &= h^2 (\lambda + 1) A_d \bar{R}_3 \\ (3, 9) &= h A_d (A + A_d + \lambda A_d) V + h A_d B_2 U \end{aligned}$$

If the above condition holds, a desired state-feedback controller can be given by $u(t) = UV^{-1} x(t)$.

The proof can be referred to in the appendix, and hence is omitted here.

Remark 1: The descriptor system method is applied in [15]. From [15, Theorem 2.1], we can see that the products of system matrices and Lyapunov matrices are completely separated. However, from [15, Theorem 3.1], we can see that the controller matrix is coupled with a symmetric and positive-definite matrix Q_1. Therefore, with the method in [15], the products of controller matrices and Lyapunov matrices are completely separated in performance analysis while it is not the case in controller synthesis. However, with our method, the weakness is eliminated. Therefore it is expected to obtain less conservative results.

The LMI in Theorem 2 is affine in the system matrices. It can thus be applied also to the case where these matrices are uncertain and are known to reside within a given polytope. Consider system (5) and denote

$$\Omega = \begin{bmatrix} A & B_1 & B_2 & A_d \\ C & D_{11} & D_{12} & \end{bmatrix}$$

we assume that $\Omega \in \mathcal{C}_o\{\Omega_i, i = 1, \ldots, N\}$, namely $\Omega = \sum_{j=1}^{N} f_j \Omega_j$ for some $0 \le f_j \le 1$, $\sum_{j=1}^{N} f_j = 1$, where the N vertices of the polytope are described by

$$\Omega_i = \begin{bmatrix} A_i & B_{1i} & B_{2i} & A_{di} \\ C_i & D_{11i} & D_{12i} & \end{bmatrix}$$

We obtain the following corollary.

Corollary 2: Consider system (5), where the system matrices reside within the polytope Ω. For a prescribed $\gamma > 0$, the state-feedback law of (6) achieves, $J(w) < 0$ for all non-zero $w \in L_2[0, \infty)$ and for all the matrices in Ω if for prescribed scalars $\varepsilon > 0$, λ, there exist symmetric and positive-definite matrices $Q_i > 0$, $\bar{S}_i > 0$, $\bar{R}_{1i} > 0$, $\bar{R}_{3i} > 0$ and matrices $\bar{R}_{2i}$, V, U that satisfy LMIs (9) for $i = 1, \ldots, N$, where the matrices

$$A_i, A_{di}, B_{1i}, B_{2i}, C_i, D_{11i}, D_{12i}, Q_i, \bar{S}_i, \bar{R}_{1i}, \bar{R}_{2i}, \bar{R}_{3i}$$

are taken with the index i. The state-feedback gain is then given by $K = UV^{-1}$.

$$\begin{bmatrix} -\varepsilon Q & -\lambda A_d \bar{S} & 0 & B & 0 & (1,6) & (1,7) & 0 & (1,9) \\ * & -\bar{S} & -\lambda h \bar{S} A_d^{\mathrm{T}} A_d^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -h\bar{R}_3 & hA_d B & 0 & 0 & (3,7) & 0 & (3,9) \\ * & * & * & -\gamma^2 I & D_{11}^{\mathrm{T}} & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 & CV + D_{12} U \\ * & * & * & * & * & -h\bar{R}_1 & -h\bar{R}_2 & 0 & 0 \\ * & * & * & * & * & * & -h\bar{R}_3 & 0 & 0 \\ * & * & * & * & * & * & * & -\bar{S} & V \\ * & * & * & * & * & * & * & * & -V - V^{\mathrm{T}} \end{bmatrix} < 0 \tag{9}$$

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Example 1: We consider the following time-delay system

$$\begin{aligned}\dot{x}(t) &= Ax(t) + A_d x(t-h) + B_1 w(t) + B_2 u(t)\\ z(t) &= Cx(t) + D_{12} u(t)\end{aligned} \tag{10}$$

where

$$A = \begin{bmatrix} 0 & e \\ f & 0.8 \end{bmatrix},\quad A_d = \begin{bmatrix} 0.1 & 0.02 \\ 0.03 & 0.1 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix},\quad B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\quad C = \begin{bmatrix} 1 & 0 \end{bmatrix},\quad D_{12} = 1 \tag{11}$$

where $0 \le e \le 0.1$, $-0.5 \le f \le -0.4$, therefore, the plant can be described as a polytope with four vertices. Our goal is to find a state-feedback gain K minimising the worst case H_∞ cost for all possible values of the parameters e and f.

From [15, Theorem 3.1], for $h = 0.5$, a minimum value of $\gamma = 1.8069$ is obtained with a corresponding state-feedback gain of $K = \begin{bmatrix} 8.3629 & -9.8057 \end{bmatrix} \times 10^4$. For $\gamma = 3$, the gain of $K = \begin{bmatrix} 398.5485 & -475.2630 \end{bmatrix}$ is obtained.

Applying Corollary 2 in this paper, we obtain, for the same h and for $\lambda = -0.1$, $\varepsilon = 100$, a minimum γ of 1.7698 with corresponding state-feedback gain of $K = [\,381.5342 \quad -446.1074\,]$. For $\gamma = 3$, the gain of $K = [\,175.0746 \quad -210.5423\,]$ is obtained.

It can be seen that the method in this paper achieves improvement of the H_∞ upper bounds. It also shows that the method in this paper arrives at much lower gain than that of the method in [15]. Therefore the method in this paper is less conservative.

4 H_∞ output-feedback synthesis

In this section, we use the criterion derived above to provide a new more accurate method for output-feedback controller synthesis with multichannel constraints. Let us consider the following time-delay system

$$\begin{aligned}\dot{x}(t) &= Ax(t) + A_d x(t-h) + B_1 w(t) + B_2 u(t)\\ z(t) &= C_1 x(t) + D_{11} w(t) + D_{12} u(t)\\ y(t) &= C_2 x(t) + D_{21} w(t)\end{aligned} \tag{12}$$

where $y(t) \in R^{n_y}$ is the measured output signal and the other signals are the same with system (5).

Denote the output-feedback controller by

$$\begin{aligned}\dot{\hat{x}}(t) &= A_K \hat{x}(t) + B_K y(t)\\ u(t) &= C_K \hat{x}(t)\end{aligned} \tag{13}$$

Applying the controller (13) to (12) will result in the following closed-loop system

$$\begin{aligned}\dot{\hat{x}}_c(t) &= A_c \hat{x}_c(t) + A_{dc} \hat{x}_c(t-h) + B_{1c} w(t)\\ \hat{z}_c(t) &= C_c \hat{x}_c(t) + D_c w(t)\end{aligned} \tag{14}$$

where

$$\hat{x}_c = \begin{bmatrix} x \\ \hat{x} \end{bmatrix},\quad A_c = \begin{bmatrix} A & B_2 C_K \\ B_K C_2 & A_K \end{bmatrix},\quad A_{dc} = \begin{bmatrix} A_d & 0 \\ 0 & 0 \end{bmatrix}$$
$$B_{1c} = \begin{bmatrix} B_1 \\ B_K D_{21} \end{bmatrix},\quad C_c = \begin{bmatrix} C_1 & D_{12} C_K \end{bmatrix},\quad D_c = D_{11} \tag{15}$$

Then we present a sufficient condition under which there exists an output-feedback H_∞ controller of form (13) for the closed-loop system (14).

Theorem 3: Define $\Gamma = \begin{bmatrix} W_{11}^{\mathrm{T}} & U^{\mathrm{T}} \\ I & V_{11} \end{bmatrix}$. For prescribed positive scalars $\gamma > 0$, $h > 0$, $\varepsilon > 0$ and scalar λ, there exists a dynamical output-feedback controller such that the closed-loop system (14) is asymptotically stable and the cost function achieves $J(w) < 0$ for all non-zero $w \in L_2[0, \infty)$ if there exist symmetric and positive-definite matrices $T > 0$, $S > 0$, $R = \begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} > 0$ and matrices V_{11}, V_{21}, W_{11}, U, $\hat{A}$, $\hat{B}$, $\hat{C}$ such that the following LMI holds (see (16))

$$\begin{bmatrix} -\varepsilon T & (1,2) & (1,3) & 0 & (1,5) & \Gamma \\ * & (2,2) & \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ hA_d B_1 & 0 \\ 0 & 0 \end{bmatrix} & (2,4) & \begin{bmatrix} -\lambda A_d^{\mathrm{T}} & -\lambda A_d^{\mathrm{T}} V_{11} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} & 0 \\ * & * & \begin{bmatrix} -\gamma^2 I & D_{11}^{\mathrm{T}} \\ * & -I \end{bmatrix} & 0 & \begin{bmatrix} B_1^{\mathrm{T}} & B_1^{\mathrm{T}} V_{11} + D_{21}^{\mathrm{T}} \hat{B}^{\mathrm{T}} \\ 0 & 0 \end{bmatrix} & 0 \\ * & * & * & (4,4) & h(\lambda+1)\begin{bmatrix} I & V_{11} \\ 0 & V_{21} \\ I & V_{11} \\ 0 & V_{21} \end{bmatrix} & 0 \\ * & * & * & * & -\Gamma - \Gamma^{\mathrm{T}} & 0 \\ * & * & * & * & * & (6,6) \end{bmatrix} < 0 \tag{16}$$

where

$$(1, 2) = h\begin{bmatrix} 0 & 0 & W_{11}^{\mathrm{T}}(A+(1+\lambda)A_d)^{\mathrm{T}}A_d^{\mathrm{T}} + \hat{C}^{\mathrm{T}}B_2^{\mathrm{T}}A_d^{\mathrm{T}} & 0 \\ 0 & 0 & (A+(1+\lambda)A_d)^{\mathrm{T}}A_d^{\mathrm{T}} & 0 \end{bmatrix}$$

$$(1, 3) = \begin{bmatrix} 0 & W_{11}^{\mathrm{T}}C_1^{\mathrm{T}} + \hat{C}^{\mathrm{T}}D_{12}^{\mathrm{T}} \\ 0 & C_1^{\mathrm{T}} \end{bmatrix}$$

(see equation at the bottom of the page)

$$(2, 2) = \begin{bmatrix} -S & \begin{bmatrix} -\lambda h A_d^{\mathrm{T}}A_d^{\mathrm{T}} & 0 \\ 0 & 0 \end{bmatrix} \\ * & -2hI + hR_3 \end{bmatrix}$$

$$(2, 4) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & h^2(\lambda+1)A_d & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$(4, 4) = -h\begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix}, \quad (6, 6) = -\begin{bmatrix} 2I & V_{11} \\ * & V_{21}+V_{21}^{\mathrm{T}} \end{bmatrix} + S$$

After we obtain the solution of the LMI (16), the corresponding controller gain matrices A_K, B_K and C_K can be obtained by the following formulas

$$A_K = V_{21}^{-\mathrm{T}}(\hat{A} - V_{11}^{\mathrm{T}}(A + A_d + \lambda A_d)W_{11} - \hat{B}C_2W_{11} - V_{11}^{\mathrm{T}}B_2\hat{C})W_{21}^{-1}$$

$$B_K = V_{21}^{-\mathrm{T}}\hat{B}, \; C_K = \hat{C}W_{21}^{-1}$$

The proof can be referred to in the appendix, and hence is omitted here.

Then we design an output-feedback controller with the form (13), which meets a family of input–output specifications. One such set of specifications is, for instance, $\|L_1T_{zw}(s)J_1\|_\infty < \gamma_1$, $\|L_2T_{zw}(s)J_2\|_\infty < \gamma_2$. Matrices L_i, J_i are selection matrices that specify which channel is involved in the corresponding constraint. With each channel is associated an LMI constraint of the form encountered in Theorem 3. The desired characterisation for output-feedback synthesis with multichannel specifications can be derived in the following three steps: (i) introduce different Lyapunov variables T_i, S_i, R_i for each channel; (ii) introduce a variable V common to all channels; (iii) perform adequate congruence transformations and use linearising changes of variables to end up with LMI synthesis condition.

Corollary 3: For prescribed positive scalars $\gamma > 0$, $h > 0$, $\varepsilon > 0$ and scalar λ, there exists a dynamical output-feedback controller such that the closed-loop system (14) is asymptotically stable and the cost function achieves $J(w) > 0$ for all non-zero $w \in L_2[0, \infty)$ if there exist symmetric and positive-definite matrices $T_i > 0$, $S_i > 0$, $R_i > 0$ and matrices V_{11}, V_{21}, W_{11}, U, $\hat{A}$, $\hat{B}$, $\hat{C}$ satisfy the LMI (16) for $i = 1, \ldots, N$, where the matrices T_i, S_i, R_i are taken with the index i. If the above conditions hold, the output-feedback gain can be obtained similarly from Theorem 3.

Remark 2: In contrast with earlier results, a different Lyapunov function is employed for each channel. Hence far better results can generally be expected.

Example 2: Consider the system (12) with multichannel constraints, where the system matrices are as follows

$$A = \begin{bmatrix} 0 & 1 \\ -0.8 & 0.6 \end{bmatrix}, \quad A_d = \begin{bmatrix} 0.1 & 0.02 \\ 0.03 & 0.1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0.2 \\ 0.5 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}, \quad D_{12} = 2, \quad D_{11} = D_{21} = 0$$

Then we design an output-feedback controller with the form (13) which meets $\|L_1T_{zw}(s)J_1\|_\infty < \gamma_1$, $\|L_2T_{zw}(s)J_2\|_\infty < \gamma_2$. Let $L_1 = 2$, $L_2 = -2$, $J_1 = \begin{bmatrix} 0.8 & 1 \\ -1 & 0.2 \end{bmatrix}$, $J_2 = \begin{bmatrix} 0.6 & 0.8 \\ 0 & 1 \end{bmatrix}$, $\gamma_1 = 2$.

Applying Corollary 3 we obtain, for $h = 0.5$, $\lambda = -0.9$, $\varepsilon = 300$, a minimum γ_2 of 1.3142. And we can obtain the gain matrices of the dynamic output-feedback controller (13) for system (12)

$$A_K = \begin{bmatrix} -2.6882 & -2.5190 \\ -2.7187 & -2.5479 \end{bmatrix} \times 10^6, \quad B_K = \begin{bmatrix} -5.3824 \\ -5.4325 \end{bmatrix} \times 10^3$$

$$C_K = \begin{bmatrix} 693.6611 & 683.7420 \end{bmatrix}$$

Then we use the output feedback controller design method derived directly from the LMI (3) without carrying out the process similarly to the proof of Theorem 2 (we omit the design process for simplification). Because it cannot separate the Lyapunov matrices and controller matrices, it should introduce the same Lyapunov function for each channel. We can obtain the minimum of γ_2 is 2.0981. And the corresponding gain matrices of the dynamic output-feedback controller are as follows

$$A_K = \begin{bmatrix} -1.9870 & -1.6742 \\ -1.7365 & -1.7590 \end{bmatrix} \times 10^7, \quad B_K = \begin{bmatrix} -2.0871 \\ -2.2245 \end{bmatrix} \times 10^4$$

$$C_K = \begin{bmatrix} 879.4389 & 902.6732 \end{bmatrix}$$

$$(1, 5) = T + \begin{bmatrix} W_{11}^{\mathrm{T}}(A + A_d + \lambda A_d)^{\mathrm{T}} + \hat{C}^{\mathrm{T}}B_2^{\mathrm{T}} & \hat{A}^{\mathrm{T}} \\ (A + A_d + \lambda A_d)^{\mathrm{T}} & (A + A_d + \lambda A_d)^{\mathrm{T}}V_{11} + C_2^{\mathrm{T}}\hat{B}^{\mathrm{T}} \end{bmatrix} + \frac{1}{2}\varepsilon\Gamma$$

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It can be seen that the method in Corollary 3 achieves better H_∞ upper bound and lower gain because it introduces a different Lyapunov function for each channel.

5 Conclusions

A new H_∞ performance criterion is proposed based on LMI approach for a class of time-delay systems with parameter-dependent Lyapunov variables. It possesses a decoupling structure which is then used to solve the multichannel output-feedback control problems and synthesise the polytopic uncertain systems. Unfortunately, the scalar ε non-linearly appears in the LMI conditions. Further results, therefore, includes how to eliminate the non-linear influence. Numerical results show that the proposed method does provide a further improvement in reducing conservativeness for time-delay systems with polytopic uncertainty.

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8 Appendix

8.1 Proof of Theorem 1

Proof: Consider a Lyapunov–Krasovskii functional candidate $V(t)$ as follows

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{17}$$

where

$$V_1(t) = x^{\mathrm{T}}(t)Px(t)$$

$$V_2(t) = \int_{t-h}^{t} x^{\mathrm{T}}(\alpha)Sx(\alpha)\,\mathrm{d}\alpha$$

$$V_3(t) = \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(\alpha)A_d^{\mathrm{T}}R_3A_d\dot{x}(\alpha)\,\mathrm{d}\alpha\,\mathrm{d}\theta$$

Note the following identity (Leibniz–Newton): $\int_a^b \dot{v}(t)\,\mathrm{d}t = v(b) - v(a)$. Then the time derivative of $V(t)$ along the solution of (1) is given by $\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t)$, where

$$\begin{aligned}\dot{V}_1(t) &= 2x^{\mathrm{T}}(t)P\left[(A + A_d)x(t) - A_d\int_{t-h}^{t}\dot{x}(\alpha)\,\mathrm{d}\alpha + Bw(t)\right]\\ &= 2x^{\mathrm{T}}(t)P(A + A_d)x(t) - 2\int_{t-h}^{t} x^{\mathrm{T}}(t)PA_d\dot{x}(\alpha)\,\mathrm{d}\alpha\\ &\quad + 2x^{\mathrm{T}}(t)PBw(t)\\ \dot{V}_2(t) &= x^{\mathrm{T}}(t)Sx(t) - x^{\mathrm{T}}(t-h)Sx(t-h)\\ \dot{V}_3(t) &= -\int_{t-h}^{t}\dot{x}^{\mathrm{T}}(\alpha)A_d^{\mathrm{T}}R_3A_d\dot{x}(\alpha)\,\mathrm{d}\alpha + h\dot{x}^{\mathrm{T}}(t)A_d^{\mathrm{T}}R_3A_d\dot{x}(t)\end{aligned}$$

Let

$$\beta = R_3A_d\dot{x}(t),\ \xi^{\mathrm{T}} = \begin{bmatrix} x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-h) & \beta^{\mathrm{T}}\end{bmatrix}$$

$$R = \begin{bmatrix} R_1 & R_2\\ * & R_3\end{bmatrix} > 0, \quad \delta = \begin{bmatrix} P & 0 & 0\\ P & 0 & hA_d^{\mathrm{T}}\end{bmatrix}.$$

In order to obtain an LMI we have to restrict ourselves to the case of $M = \lambda I$ in Lemma 1, where $\lambda \subset R$ is a scalar parameter. And let $a(\alpha) = \begin{bmatrix} 0 & A_d^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}\dot{x}(\alpha)$, $b(\alpha) = \delta\xi$ in Lemma 1. By Lemma 1, we can obtain

$$\begin{aligned}&-2\int_{t-h}^{t} x^{\mathrm{T}}(t)PA_d\dot{x}(\alpha)\,\mathrm{d}\alpha\\ &= -2\int_{t-h}^{t}\xi^{\mathrm{T}}\delta^{\mathrm{T}}\begin{bmatrix}0\\ A_d\end{bmatrix}\dot{x}(\alpha)\,\mathrm{d}\alpha + 2h\int_{t-h}^{t}\beta^{\mathrm{T}}A_dA_d\dot{x}(\alpha)\,\mathrm{d}\alpha\\ &\le \int_{t-h}^{t}\dot{x}^{\mathrm{T}}(\alpha)\begin{bmatrix}0 & A_d^{\mathrm{T}}\end{bmatrix}R\begin{bmatrix}0\\ A_d\end{bmatrix}\dot{x}(\alpha)\,\mathrm{d}\alpha\\ &\quad + 2\lambda\int_{t-h}^{t}\xi^{\mathrm{T}}\delta^{\mathrm{T}}\begin{bmatrix}0\\ A_d\end{bmatrix}\dot{x}(\alpha)\,\mathrm{d}\alpha\\ &\quad + (\lambda+1)^2\int_{t-h}^{t}\xi^{\mathrm{T}}\delta^{\mathrm{T}}R^{-1}\delta\xi\,\mathrm{d}\alpha\\ &\quad + 2h\int_{t-h}^{t}\beta^{\mathrm{T}}A_dA_d\dot{x}(\alpha)\,\mathrm{d}\alpha\\ &= \int_{t-h}^{t}\dot{x}^{\mathrm{T}}(\alpha)A_d^{\mathrm{T}}R_3A_d\dot{x}(\alpha)\,\mathrm{d}\alpha\\ &\quad + 2\lambda\int_{t-h}^{t}x^{\mathrm{T}}(t)PA_d\dot{x}(\alpha)\,\mathrm{d}\alpha\\ &\quad + 2(\lambda+1)h\int_{t-h}^{t}\beta^{\mathrm{T}}A_dA_d\dot{x}(\alpha)\,\mathrm{d}\alpha\\ &\quad + (\lambda+1)^2\int_{t-h}^{t}\xi^{\mathrm{T}}\delta^{\mathrm{T}}R^{-1}\delta\xi\,\mathrm{d}\alpha\end{aligned}$$

Note that

$$\begin{aligned}&2(\lambda+1)h\int_{t-h}^{t}\beta^{\mathrm{T}}A_dA_d\dot{x}(\alpha)\,\mathrm{d}\alpha + h\dot{x}^{\mathrm{T}}(t)A_d^{\mathrm{T}}R_3A_d\dot{x}(t)\\ &\quad = 2h\beta^{\mathrm{T}}A_d(A + (1+\lambda)A_d)x(t) - 2\lambda h\beta^{\mathrm{T}}A_dA_dx(t-h)\\ &\qquad - h\beta^{\mathrm{T}}R_3^{-1}\beta + 2h\beta^{\mathrm{T}}A_dBw(t)\end{aligned}$$

Then substituting $\int_{t-h}^{t}\dot{x}(\alpha) = x(t) - x(t-h)$ and $A_dx(t-h) = \dot{x}(t) - Ax(t) - Bw(t)$ into the above equation, we obtain

$$\frac{\mathrm{d}V(t)}{\mathrm{d}t} + z^{\mathrm{T}}(t)z(t) - \gamma^2w^{\mathrm{T}}(t)w(t) = \rho^{\mathrm{T}}\Omega\rho + z^{\mathrm{T}}(t)z(t) \tag{18}$$

where

$$\rho^{\mathrm{T}} = \begin{bmatrix}\xi^{\mathrm{T}} & w^{\mathrm{T}}(t)\end{bmatrix}$$

$$\Omega = \begin{bmatrix}\begin{bmatrix}\Psi_1 & -\lambda PA_d & h(A + A_d + \lambda A_d)^{\mathrm{T}}A_d^{\mathrm{T}}\\ * & -S & -h\lambda A_d^{\mathrm{T}}A_d^{\mathrm{T}}\\ * & * & -hR_3^{-1}\end{bmatrix} + \Omega_1 & \begin{bmatrix}PB\\ 0\\ hA_dB\end{bmatrix}\\ * & -\gamma^2 I\end{bmatrix}$$

$$\Omega_1 = h(\lambda+1)^2\delta^{\mathrm{T}}R^{-1}\delta$$

$$\Psi_1 = \mathrm{Sym}\{P(A + A_d + \lambda A_d)\} + S$$

Integrate (18) in t from 0 to ∞. Because $V(0) = 0, V(\infty) \ge 0$ and $\int_0^\infty z^{\mathrm{T}}(t)z(t) = \int_0^\infty (x^{\mathrm{T}}(t)C^{\mathrm{T}} + w^{\mathrm{T}}(t)D^{\mathrm{T}})(Cx(t) + Dw(t))$. From Schur complement Lemma, we can obtain that $J(w) < 0$ (and $\dot{V} < 0$) if the following LMI holds (see equation at the bottom of the page)

Then we can obtain the LMI (3) by Schur complement lemma. □

$$\begin{bmatrix}\Psi_1 & -\lambda PA_d & h(A + A_d + \lambda A_d)^{\mathrm{T}}A_d^{\mathrm{T}} & PB & C^{\mathrm{T}} & h(\lambda+1)P & h(\lambda+1)P\\ * & -S & -h\lambda A_d^{\mathrm{T}}A_d^{\mathrm{T}} & 0 & 0 & 0 & 0\\ * & * & -hR_3^{-1} & hA_dB & 0 & 0 & h^2(\lambda+1)A_d\\ * & * & * & -\gamma^2 I & D^{\mathrm{T}} & 0 & 0\\ * & * & * & * & -I & 0 & 0\\ * & * & * & * & * & -hR_1 & -hR_2\\ * & * & * & * & * & * & -hR_3\end{bmatrix} < 0$$

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8.2 Proof of Theorem 2

Proof: Applying Corollary 1 to the closed-loop system (7), we can obtain (see (19))

where

$$(1, 1) = \mathrm{Sym}\{(A_c + A_d + \lambda A_d)Q\}$$
$$(1, 3) = hQ(A_c + (1+\lambda)A_d)^{\mathrm{T}} A_d^{\mathrm{T}}$$

It is obvious that there exists a positive scalar ε such that the following equation holds (see (20))

Note that the LMI (19) can be rewritten as

$$\Pi + ab^{\mathrm{T}} + ba^{\mathrm{T}} < 0 \tag{21}$$

where

$$a = \begin{bmatrix} Q & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$
$$b = \Big[(A_c + A_d + \lambda A_d)^{\mathrm{T}} + \tfrac{1}{2}\varepsilon I \; 0 \; h(A_c + A_d + \lambda A_d)^{\mathrm{T}} A_d^{\mathrm{T}} \; 0 \; C_c^{\mathrm{T}} \; 0 \; 0 \; I\Big]^{\mathrm{T}}$$

Substitute (8) into (21) and define $U = KV$. From Lemma 2, we can obtain that the LMIs (20) and (21) are equivalent to the LMI (9). □

8.3 Proof of Theorem 3

Proof: Apply Theorem 1 to the closed-loop system (14) and carry out the process similar to the proof of Theorem 2, then we can obtain (see (22))

where

$$(1, 3) = h(A_c + (1+\lambda)A_{dc})^{\mathrm{T}} A_{dc}^{\mathrm{T}} R_3$$
$$(1, 8) = P + (A_c + (1+\lambda)A_{dc})^{\mathrm{T}} V + \frac{1}{2}\varepsilon V$$
$$(3, 7) = h^2(1+\lambda)R_3 A_{dc}$$

Partition V and its inverse V^{-1} in the LMI (22) as

$$V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}, \quad W = V^{-1} = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} \tag{23}$$

From $VW = I$, define

$$F_1 = \begin{bmatrix} W_{11} & I \\ W_{21} & 0 \end{bmatrix}, \quad F_2 = \begin{bmatrix} I & V_{11} \\ 0 & V_{21} \end{bmatrix} \tag{24}$$

$$\begin{bmatrix} (1,1) & -\lambda A_d \bar{S} & (1,3) & B & QC_c^{\mathrm{T}} & h(\lambda+1)\bar{R}_3 & h(\lambda+1)\bar{R}_3 & Q \\ * & -\bar{S} & -\lambda h \bar{S} A_d^{\mathrm{T}} A_d^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 \\ * & * & -h\bar{R}_3 & hA_d B & 0 & 0 & h^2(\lambda+1)A_d\bar{R}_3 & 0 \\ * & * & * & -\gamma^2 I & D_{11}^{\mathrm{T}} & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -h\bar{R}_1 & -h\bar{R}_2 & 0 \\ * & * & * & * & * & * & -h\bar{R}_3 & 0 \\ * & * & * & * & * & * & * & -\bar{S} \end{bmatrix} < 0 \tag{19}$$

$$\Pi = \begin{bmatrix} -\varepsilon Q & -\lambda A_d \bar{S} & 0 & B & 0 & h(\lambda+1)\bar{R}_3 & h(\lambda+1)\bar{R}_3 & 0 \\ * & -\bar{S} & -\lambda h \bar{S} A_d^{\mathrm{T}} A_d^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 \\ * & * & -h\bar{R}_3 & hA_d B & 0 & 0 & h^2(\lambda+1)A_d\bar{R}_3 & 0 \\ * & * & * & -\gamma^2 I & D_{11}^{\mathrm{T}} & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -h\bar{R}_1 & -h\bar{R}_2 & 0 \\ * & * & * & * & * & * & -h\bar{R}_3 & 0 \\ * & * & * & * & * & * & * & -\bar{S} \end{bmatrix} < 0 \tag{20}$$

$$\begin{bmatrix} -\varepsilon P & 0 & (1,3) & 0 & C_c^{\mathrm{T}} & 0 & 0 & (1,8) & I \\ * & -S & -\lambda h A_{dc}^{\mathrm{T}} A_{dc}^{\mathrm{T}} R_3 & 0 & 0 & 0 & 0 & -\lambda A_{dc}^{\mathrm{T}} V & 0 \\ * & * & -hR_3 & hR_3 A_{dc} B_{1c} & 0 & 0 & (3,7) & 0 & 0 \\ * & * & * & -\gamma^2 I & D_{11}^{\mathrm{T}} & 0 & 0 & B_{1c}^{\mathrm{T}} V & 0 \\ * & * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -hR_1 & -hR_2 & h(\lambda+1)V & 0 \\ * & * & * & * & * & * & -hR_3 & h(\lambda+1)V & 0 \\ * & * & * & * & * & * & * & -V - V^{\mathrm{T}} & 0 \\ * & * & * & * & * & * & * & * & -S^{-1} \end{bmatrix} < 0 \tag{22}$$

And also we define $U = V_{21}^{\mathrm{T}} W_{21} + V_{11}^{\mathrm{T}} W_{11}$. It is always possible to find the invertible matrices V_{21} and W_{21} satisfying $V_{21}^{\mathrm{T}} W_{21} = U - V_{11}^{\mathrm{T}} W_{11}$. This is verified by assuring the invertibility of $U - V_{11}^{\mathrm{T}} W_{11}$, which is proved below.

After performing the linearising congruence transformations and change of variables, we can obtain

$$\begin{aligned} F_1^{\mathrm{T}}(V^{\mathrm{T}} + V)F_1 &= F_1^{\mathrm{T}} V^{\mathrm{T}} F_1 + F_1^{\mathrm{T}} V F_1 \\ &= \begin{bmatrix} W_{11} + W_{11}^{\mathrm{T}} & I + U^{\mathrm{T}} \\ * & V_{11} + V_{11}^{\mathrm{T}} \end{bmatrix} > 0 \end{aligned}$$

The above inequality ensures the invertibility of the three matrices $F_1^{\mathrm{T}} V^{\mathrm{T}} F_1$, W_{11} and V_{11}. With W_{11}, we can define the invertible matrix $\Pi = \begin{bmatrix} I & I \\ 0 & -W_{11} \end{bmatrix}$. The product of the two matrices $F_1^{\mathrm{T}} V F_1$ and Π leads to

$$\begin{aligned} F_1^{\mathrm{T}} V^{\mathrm{T}} F_1 \Pi &= \begin{bmatrix} W_{11} & I \\ U & V_{11}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} I & I \\ 0 & -W_{11} \end{bmatrix} \\ &= \begin{bmatrix} W_{11} & 0 \\ U & U - V_{11}^{\mathrm{T}} W_{11} \end{bmatrix} \end{aligned} \tag{25}$$

which assures that $U - V_{11}^{\mathrm{T}} W_{11}$ is invertible.

Perform the following linearising changes of variables:

$$\begin{aligned} \hat{A} &= V_{11}^{\mathrm{T}}(A + A_d + \lambda A_d)W_{11} + V_{21}^{\mathrm{T}} B_K C_2 W_{11} \\ &\quad + V_{11}^{\mathrm{T}} B_2 C_K W_{21} + V_{21}^{\mathrm{T}} A_K W_{21} \\ \hat{B} &= V_{21}^{\mathrm{T}} B_K,\ \hat{C} = C_K W_{21},\ T = F_1^{\mathrm{T}} P F_1 \end{aligned} \tag{26}$$

Pre- and post-multiply the LMI (22) by $\mathrm{diag}\{F_1^{\mathrm{T}}, I, R_3^{-1}, I, I, I, I, F_1^{\mathrm{T}}, F_1^{\mathrm{T}} V^{\mathrm{T}}\}$ and its inverse, respectively, substitute (15), (23), (24) into the obtained equation. Obviously, $-F_1^{\mathrm{T}} V^{\mathrm{T}} S^{-1} V F_1 \leq -F_1^{\mathrm{T}} V^{\mathrm{T}} - V F_1 + S$, $-R^{-1} \leq -2I + R$. Using the above equations, the LMI (16) can be derived.

After we obtain the solution of the LMI (16), the corresponding controller of form (13) will be constructed as follows: (i) compute W_{21} from $U - V_{11}^{\mathrm{T}} W_{11}$; (ii) utilising the matrices A_j, B_j, C_j, V_{11}, W_{11}, V_{21} and W_{21} obtained above, compute the controller gain by reversing (26). □

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