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Minimization of sound radiation from baffled beams through optimization of partial constrained layer damping treatment

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Abstract

An optimization study is presented with aim to minimize the sound power radiated by a simply supported, baffled beam with constrained layer damping (CLD) treatment. The governing equation of motion for the calculation of time-harmonic response of a partially CLD covered beam is derived first on the basis of energy approach. Assumed-modes method is used to solve the equation with obtained frequency response functions at different beam locations, which are further used for the calculation of its radiated sound power into half free-space by using Rayleigh's integral. The optimization problem is then formulated to minimize the sound power radiated by the beam over a frequency range of interest covering multiple resonant modes. A genetic algorithm-based penalty function method is employed to search for the optimum of location/length of the CLD patch and the shear modulus of viscoelastic layer. Optimal results show that for a simply supported beam with a transverse force applied at its central location, it is not necessary to fully cover the structure using CLD patch in order to achieve the largest reduction in the sound power radiated by the beam over a frequency range. With inclusion of the amount of damping material to be minimized, the optimal CLD coverage length is only one-fourth of the base beam's. Moreover, the optima of three design variables, the CLD coverage length, location on the beam and the shear modulus of viscoelastic layer, are highly relevant to each other.
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Keywords: Beam; Sound radiation; Optimization; Viscoelastic material; Damping treatment; Constrained-layer damping

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1. Introduction

Constrained layer damping (CLD) treatment has been regarded as an effective way to suppress vibrations of and sound radiation from vibrating structures. Since the late 1950s, there have been a number of publications with reported formulations and techniques for vibration damping calculations of various structures with CLD treatment. A detailed description of these formulations can be found in the two books on vibration damping by Nashif et al. [1] and Sun and Lu [2]. Most of early works in the field dealt mainly with full CLD coverage. Among these formulations, one widely used is that developed by DiTaranto [3] and Mead and Markus [4], i.e., a sixth-order equation of motion about the transverse displacement of the damped structure. The theoretical model is basically the extension of Kerwin's analysis of sandwich beams with simply supported ends [5] to beams with general boundary conditions. The problem of computing damped natural frequencies and loss factors is explicitly solved for both beams and plates when different boundary conditions are assumed [6]. On the basis of design consideration of CLD treatments for complex structures, the finite element procedure has also been adopted [7].

Owing to the real constraints of cost and weight in most real-life situations, the partial CLD treatment where only a portion of the base structure is covered with CLD is obviously more practical. Nokes and Nelson [8] were among the earliest investigators to provide the solution to the problem of a partially covered sandwich beam. And a more thorough analytical study was carried out by Lall et al. who solved, by using Rayleigh–Ritz approach, the eigenvalue problem for a beam [9] and for a plate [10] with a single damping patch. Recently, Kung and Singh [11] presented a refined method for analyzing the modal damping of beams with multiple constrained-layer viscoelastic patches.

With aim to improve the damping performance, a number of efforts have been exerted to optimally design CLD treatments of vibrating structures as well. Lifshitz and Leibowitz [12] have determined the optimal CLD for sandwich beams with viscoelastic core with layer thickness as design variables. Baz and Ro [13] used univariate search method (USM) to optimize the passive damping of a active constrained layer damping treatment by selecting the optimal thickness and shear modulus of the viscoelastic layer. Lall et al. [14] carried out the optimum design studies for a sandwich plate with CLD. Their objective functions were the modal loss factor and displacement response, with design variables as the layer material densities, thicknesses and temperature. In another optimization study carried out by Lall et al. [14] for the partially covered plate, the objective function was to maximize the system loss factor of a specific natural mode, with design parameters as dimensions of the patch, and the thicknesses of constraining layer and viscoelastic layer. The patch coverage area and the added mass were both restricted. Chen and Huang [15] presented a study on optimal placement of CLD treatment for vibration suppression of plates. Their objective functions include structural damping ratios, resonant frequencies' shift and CLD thickness. It is found that the best damping performance occurs as the CL thickness is twice that of the VEM thickness. Marcelin et al. [16,17] used genetic algorithm (GA) and beam finite elements to maximize the modal damping loss factor for partially covered beam. The design variables were the dimensions and location of all the viscoelastic layers of multiple CLD patches. Results of these optimization studies showed that using the CLD patches with optimum parameters leads to significant saving in the CLD material used. Furthermore, it is possible to obtain higher values of modal damping factor for a partially covered beam compared to that obtained for a fully covered one.

The vibration damping of a CLD-treated beam is undoubtedly determined by a large number of parameters including material properties and thicknesses of both the constraining layer and viscoelastic layer. For a partially CLD covered beam, its vibration damping further depends on the patch location and coverage area. From above discussions, one may see that current optimum design studies selected part CLD parameters as the design variables to be optimized. Also, the objective function for the optimization study was chosen as to maximum the system loss factor of vibration modes or to minimize the vibration response at a specific resonant mode. Therefore, as pointed out by Nakra in his summary of studies on using viscoelastic damping for structural vibration control [18], it would be meaningful to carry out optimization studies of the dynamic response covering a large frequency range over a number of modes. Recently, Zheng et al. [19] performed such an optimization study as to minimize the vibrational energy (VE) of the vibrating beams over a broad frequency range covering multiple resonant modes. Also, in their study, three major parameters of the CLD patch involved in a partially covered beam were identified. These parameters are the CLD coverage length, the location of the patch on the beam and the shear modulus of viscoelastic layer. However, as the sound radiation of a structure depends highly on not only its vibration response magnitude but also the radiation efficiencies at resonant frequencies, the minimization of the VE of a beam does not warrant that a minimum sound power radiated from it could be achieved. The evaluation of reduction in noise radiation from structures with optimal damping treatment, to the authors' knowledge, has been few so far in open literature. With consideration of this, the attempt of this paper is made to present a multi-parameter optimization study aim to minimize the sound power radiated by a simply supported, baffled beam with constrained layer damping (CLD) treatment. The considered beam is excited by a time-harmonic transverse force at its center location and above-mentioned three parameters are treated as the design variables of the optimization problem. In particular, the total sound power at multiple resonant modes in a frequency range of interest is chosen as the objective function.

The whole paper comprises three major parts. First, the governing equation of motion for the calculation of time-harmonic response of a partially CLD covered beam is derived first on the basis of energy approach. Assumed-modes method is used to solve the equation with obtained frequency response functions at different beam locations, which are further used for the calculation of its radiated sound power into half free-space by using Rayleigh's integral. Second, the optimization problem is formulated to minimize the sound power radiated by the beam over a frequency range of interest under given excitation condition. A genetic algorithm (GA)-based penalty function method is employed to search for the optimum of location/length of the CLD patch and the shear modulus of its viscoelastic layer.

The optimization results with detailed discussions and conclusions are given in the third part.

2. Sound power radiated by a baffled beam with single CLD patch

In order to formulate our optimization problem, a numerical procedure has to be established first for calculation of the sound power radiated by a beam with partial CLD coverage. Fig. 1 shows schematically such a CLD-treated beam structure. For the sake of simplicity, single-patch treatment is considered here. The three layers, i.e., the base beam, constraining layer (CL) and viscoelastic layer (VL), are denoted by the subscripts s, c and v, respectively. Both CL and VL have the same width, b, as that of the base beam. The thicknesses of three layers are, respectively, denoted by h_s for the base beam, h_c for the CL, and h_v for the VL of the CLD patch. The coordinates of the patch's ends, x_1 and x_2, determine its length $l(l = x_1 - x_2)$ and location $x_0(x_0 = (x_1 + x_2)/2)$. The Young's moduli of the materials for base beam and CL are represented by E_s and E_c, respectively.

Those assumptions common to most studies on CLD treatment for structural vibration suppression are adopted here. They are [1–4]:

(1) Shear strains in the base beam and CL, and also the rotary inertia of all layers are negligible;.
(2) The VL only carries transverse shear but no normal stresses. A linear, frequency-dependent, complex shear modulus, $G_v(\omega)^* = G_v(\omega)[1 + j\eta_v(\omega)]$, where $\eta_v(\omega)$, the loss factor, is used for the description of the viscoelastic property of the layer.
(3) All displacements are small compared to the structural dimensions; thus, linear theories of elasticity and viscoelasticity are used.
(4) No slipping occurs at the interfaces of the layers.
(5) The plane transverse to the middle plane remains plane when bending.
(6) The three layers undergo the same transverse deflection.

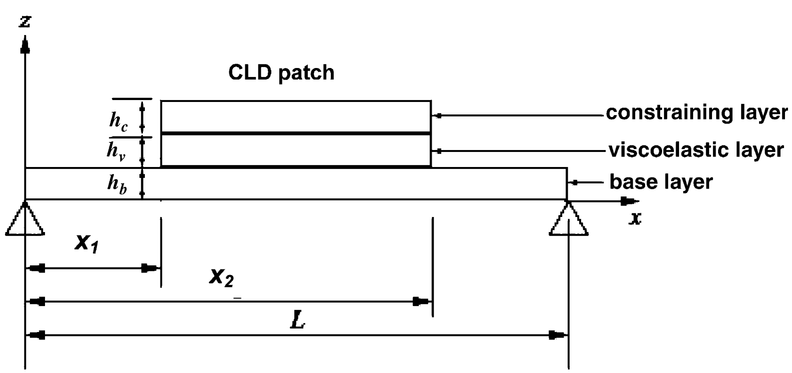

Fig. 1. A simply supported beam with a CLD patch.

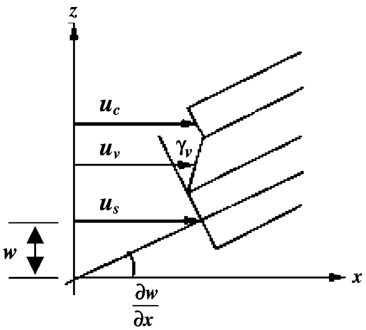

Fig. 2. The deformation pattern of the layers in the model of beam with CLD treatment.

The deformation pattern of the structure is illustrated in Fig. 2. Applying the procedure of energy-based approach [15], kinetic energy, strain energy of the vibrating beam with CLD patch, and work done by the external concentrated force to the beam are formulated. The in-plane inertia is included in the kinetic energy of the system but rotary inertia neglected. Using the relation implied by assumption (4) that requires the continuity of displacement at the interface between the layers, the kinetic energy of beam system becomes

$$
\begin{aligned}
T = {} & \frac{1}{2}\int_{x_1}^{x_2} \rho_c A_c \left[\left(\frac{\partial w}{\partial t} \right)^2 + \left(\frac{\partial u_c}{\partial t} \right)^2 \right] dx \\
& + \frac{1}{2}\int_{x_1}^{x_2} \rho_v A_v \left[\left(\frac{\partial w}{\partial t} \right)^2 + \left(\frac{1}{2}\left(\frac{\partial u_c}{\partial t} + \frac{\partial u_s}{\partial t} \right) + \frac{(h_s - h_c)}{4} \frac{\partial^2 w}{\partial t \, \partial x} \right)^2 \right] dx \\
& + \frac{1}{2}\int_{0}^{L} \rho_s A_s \left[\left(\frac{\partial w}{\partial t} \right)^2 + \left(\frac{\partial u_s}{\partial t} \right)^2 \right] dx, \qquad (1)
\end{aligned}
$$

where ρ_s, ρ_v and ρ_c are, respectively, the material density of the base beam, VL and CL for the CLD patch, while A_s, A_v and A_c are their section areas; u_c, u_v and u_s are, respectively, the mid-plane displacements of the CL, VL and the base beam along the x-axis; and x is the axial distance relative to one end of the beam.

With assumption (1), the strain energy of the base beam is given by

$$
\begin{aligned}
V_s &= \frac{1}{2}\int_V \sigma_s \varepsilon_s \, dV = \frac{1}{2}\int_0^L \int_{h_s/2}^{h_s/2} \sigma_s \varepsilon_s \, dz \, dx \\
&= \frac{1}{2}\int_0^L E_s \left[A_s \left(\frac{\partial u_s}{\partial x} \right)^2 + I_s \left(\frac{\partial^2 w}{\partial x^2} \right) \right] dx, \qquad (2)
\end{aligned}
$$

where I_s is the area moment of inertia about the mid-plane of base beam, $I_s = bh_s^3/12$.

Similarly, the strain energy of the CL is derived as

$$V_{\mathrm{c}} = \frac{1}{2}\int_V \sigma_{\mathrm{c}}\varepsilon_{\mathrm{c}}\,\mathrm{d}V = \frac{1}{2}\int_{x_1}^{x_2}\int_{-h_{\mathrm{c}}/2}^{h_{\mathrm{c}}/2} \sigma_{\mathrm{c}}\varepsilon_{\mathrm{c}}\,\mathrm{d}z\,\mathrm{d}x$$
$$= \frac{1}{2}\int_{x_1}^{x_2} E_{\mathrm{c}}\left[A_{\mathrm{c}}\left(\frac{\partial u_{\mathrm{c}}}{\partial x}\right)^2 + I_{\mathrm{c}}\left(\frac{\partial^2 w}{\partial x^2}\right)^2\right]\mathrm{d}x. \tag{3}$$

Since the VL is assumed to carry transverse shear only (assumption (2)), its strain energy is given by

$$V_{\mathrm{v}} = \frac{1}{2}\int_{x_1}^{x_2} G_{\mathrm{v}}^{*}A_{\mathrm{v}}(\gamma_{\mathrm{v}})^2\,\mathrm{d}x = \frac{1}{2}\int_{x_1}^{x_2} G_{\mathrm{v}}^{*}A_{\mathrm{v}}\left\{\frac{1}{h_{\mathrm{v}}}\left[(u_{\mathrm{c}} - u_{\mathrm{s}}) + d\frac{\partial w}{\partial x}\right]^2\right\}\mathrm{d}x, \tag{4}$$

where $d = (1/2)(h_{\mathrm{c}} + h_{\mathrm{s}}) + h_{\mathrm{v}}$ is the distance between the mid-plane of the base beam and that of the constraining layer and G_{v}^{*} denotes the complex shear modulus of the viscoelastic material.

Further assuming an external transverse load of $f(x,t)$ applied to the beam, the work done by it can be expressed as

$$P = \int_0^L f(x,t)w(x,t)\,\mathrm{d}x. \tag{5}$$

Further following the procedure of assumed-modes expansion, the displacements of the beam and the CL are approximated as:

$$w(x,t) = \sum_{i=1}^{n_W} W_i(x)\eta_i(t) = \eta^{\mathrm{T}}W = W^{\mathrm{T}}\eta, \tag{6}$$

$$u_{\mathrm{s}}(x,t) = \sum_{i=1}^{n_{U_{\mathrm{s}}}} U_{\mathrm{s}_i}(x)\xi_i(t) = \xi^{\mathrm{T}}U_{\mathrm{s}} = U_{\mathrm{s}}^{\mathrm{T}}\xi, \tag{7}$$

$$u_{\mathrm{c}}(x,t) = \sum_{i=1}^{n_{U_{\mathrm{c}}}} U_{\mathrm{c}_i}(x)\zeta_i(t) = \zeta^{\mathrm{T}}U_{\mathrm{c}} = U_{\mathrm{c}}^{\mathrm{T}}\zeta, \tag{8}$$

where W and U_{s} are the transverse and longitudinal shape functions of the base beam, respectively, while U_{c} is the longitudinal shape function of the CL for the CLD patch, each term of these shape functions satisfies the essential boundary conditions of the beam and CL, respectively; η, ξ and ζ are the generalized displacement vectors. n_W, $n_{U_{\mathrm{s}}}$ and $n_{U_{\mathrm{c}}}$ denote the number of the modes to be considered for appropriate numerical accuracy.

With the above energy expressions and the assumed shape functions of the base beam for the transverse and longitudinal displacements and of the CL for longitudinal displacements, Lagrange's equation is applied to derive the system equations of motion in terms of generalized displacements as

$$[M]\{\ddot{q}(t)\} + [K]\{q(t)\} = \{P(t)\}, \tag{9}$$

where $[M]$ and $[K]$ are the generalized mass and stiffness matrices of the whole system, of which $[K]$ is a complex matrix owing to the viscoelastic nature of the VL; $\{P\}$

is the vector of generalized force and $\{q(t)\}$ is the column vector containing $\eta(t)$, $\xi(t)$ and $\zeta(t)$.

The system's structural response is solved by restricting the analysis to steady-state harmonic excitation. Introducing the time dependence, $e^{j\omega t}$, in Eq. (9),

$$\{q(t)\} = \{\tilde{q}\}e^{j\omega t}; \quad \{P(t)\} = \{\tilde{P}\}e^{j\omega t},$$

the differential equation reduces to a linear system and can be expressed as

$$[-\omega^2 M + K]\{\tilde{q}\} = \{\tilde{P}\}, \tag{10}$$

where $\{\tilde{q}\}$ and $\{\tilde{P}\}$ represent the complex amplitudes of the generalized displacements and generalized force, respectively. This linear system is solved numerically for $\{\tilde{q}\}$ at each frequency of interest.

The details of the matrices, $[M]$ and $[K]$ and the column vectors, $\{\tilde{q}\}$ and $\{\tilde{P}\}$, and the expressions of elements of these sub-matrices and column vectors can be found in [18].

In the above equations, we use the shape functions that are easily evaluated to simplify the calculations, but are also similar to the exact mode shapes so that a large number of terms in the expansion is not required.

The present study is restricted to partial CLD treatment. In this case, the metallic sheet for the CL are free in the sense that it is attached only through the VL. So its response is expanded in the mode functions of a free-free beam with ith longitudinal mode shape given by

$$U_{c,i}(x) = \cos\left[(i-1)\frac{\pi x}{l}\right] \quad (i = 1, 2, \ldots, n_{U_c}). \tag{11}$$

In the case of simply supported beam, the mode shape function for expansion of its longitudinal displacement of the base beam, U_s, is given by Eq. (11), where l is replaced by L, the length of the beam and the transverse mode shape is given by

$$W_i(x) = \sin\left(\frac{i\pi x}{L}\right) \quad (i = 1, 2, \ldots, n_W). \tag{12}$$

Using these mode shape functions, all elements of the mass and stiffness matrices can be obtained through performing integration. Further substituting them into Eq. (10) and solving it, the generalized displacement vector, $\{\tilde{q}\} = [\{\tilde{\eta}\}^T, \{\tilde{\xi}\}^T, \{\tilde{\zeta}\}^T]^T$ and the transverse displacement at beam location $x, w(x\omega)$, can be obtained at any excitation frequency ω.

It is common to characterize the frequency-dependency of both shear modulus and damping loss factor of the viscoelastic materials by using temperature-frequency nomogram [1] or at a constant temperature, by a mathematical function as

$$G_v^*(\omega) = G_v(\omega)[1 + j\eta_v(\omega)], \tag{13}$$

where $G_v(\omega) = G_0 g(\omega)$ is the storage shear modulus and $\eta_v(\omega) = \eta_0 h(\omega)$ is the damping loss factor; $g(\omega)$ and $h(\omega)$ are two real-valued functions of the circular frequency, ω. The tests on simple beam structure normally provide the data for arriving at these frequency-dependent relations.

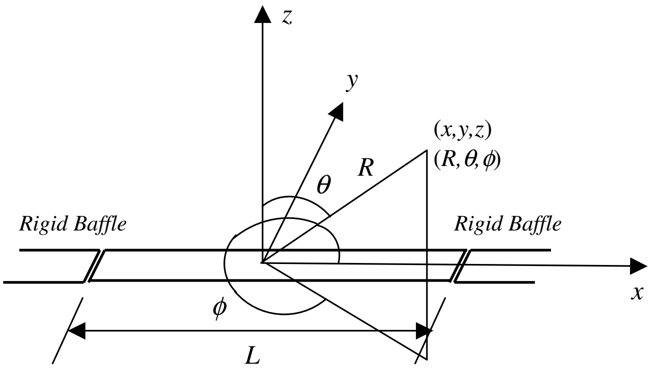

Fig. 3. The coordinate system for calculation of sound radiation from a baffled beam.

The obtained response of the beam by solving the matrix equation (10) is now used for the calculation of sound radiation. Consider the rectangular coordinate system (x, y, z) shown in Fig. 3, where the structure's plane is defined by specifying a single coordinate, $z = 0$, and the sound wave equation is separable along x, y and z directions. Using the simplified Kirchhoff–Helmholtz integral, the pressure radiated by a planar radiator mounted in an infinite baffle becomes [20]:

$$p(r) = \frac{\rho_0}{2\pi} \int \int_{S_0} \ddot{w}(r_0) \frac{\mathrm{e}^{-\mathrm{j}k|\vec{r}-\vec{r}_0|}}{|\vec{r}-\vec{r}_0|} \, \mathrm{d}S(r_0), \tag{14}$$

where ρ_0 is the density of surrounding medium, S_0 the area of structural surface, $\vec{r}_0$ the point on structural surface, $\vec{r}$ the field point and k the sound wave number, $k = \omega/c_0$, with c_0 the sound speed in the medium. The above expression constitutes the basic relation between structural response (out-of-plane acceleration) to radiated sound pressure. Note that this simplified Kirchhoff–Helmholtz integral formulation introduces no restriction on the location of the field point, $\vec{r}$, or the frequency, ω.

A simplified expression is obtained in the far field where the distance between the field point $\vec{r}$ and the center of the radiator is large compared to the characteristic dimension of the radiator. Taking the origin of the coordinate system near the center of the radiator, the distance $|\vec{r}-\vec{r}_0|$ in the denominator of the Green's function can be approximated by $R = |\vec{r}|$. The resulting equation is referred to as the Rayleigh's integral. For a beam radiator, the Rayleigh's integral is expressed by

$$p(r) = \frac{\rho_0}{2\pi R} \int_0^L b\ddot{w}(x_0) \, \mathrm{e}^{-\mathrm{j}k|\vec{r}-\vec{r}_0|} \, \mathrm{d}x_0. \tag{15}$$

Expressing $\vec{r}$ in spherical coordinates (R, θ, ϕ), where

$$\begin{aligned} x &= R \sin\theta \cos\phi, \\ y &= R \sin\theta \sin\phi, \\ z &= R \cos\theta, \end{aligned} \tag{16}$$

the distance $|\vec{r} - \vec{r}_0|$ can be approximated by

$$|\vec{r} - \vec{r}_0| \approx R - x_0 \sin\theta\cos\phi - y_0 \sin\theta\sin\phi. \tag{17}$$

Substituting Eq. (17) in Eq. (15) yields a simplified expression for the Rayleigh's integral for a baffled beam,

$$p(R,\theta,\phi) = \frac{\rho_0\, \mathrm{e}^{-\mathrm{j}kR} b}{2\pi R} J_0\left(\frac{bk}{2}\sin\theta\cos\phi\right)\int_0^L \ddot{w}(x_0)\,\mathrm{e}^{\mathrm{j}kx_0\sin\theta\cos\phi}\,\mathrm{d}x_0, \tag{18}$$

where J_0 represents the Bessel function of the zeroth order.

The sound power radiated from a source is defined as the integral, over a surface of the vibrating beam, of the component of the time-averaged intensity vector normal to the surface. For harmonic excitations, the time-averaged intensity at field point $\vec{r}$ is defined as

$$I(r) = \tfrac{1}{2}\mathrm{Re}\{p(\vec{r})v^*(\vec{r})\}, \tag{19}$$

where $p(\vec{r})$ is the sound pressure complex amplitude and $v^*(\vec{r})$ is the conjugation of the vector of fluid particle velocity components, $v(\vec{r})$.

In the far-field, the particle velocity tends to become normal to the hemisphere centered on the source and its amplitude is approximated by $p(\vec{r})/\rho_0 c_0$ as in the case of plane waves. Therefore, the time-averaged acoustic intensity in the far-field becomes

$$I(R,\theta,\phi) \approx \frac{1}{2\rho_0 c_0}|p(R,\theta,\phi)|^2, \quad kR \gg 1. \tag{20}$$

The integral of the average acoustic intensity over the hemisphere in the far field yields the total acoustic power radiated by the beam,

$$\begin{aligned}\Pi(\omega) &= \int_0^{2\pi}\int_0^{\pi/2} I(R,\theta,\phi)R^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi \\ &= \int_0^{2\pi}\int_0^{\pi/2}\frac{|p(R,\theta,\phi)|^2}{2\rho c}R^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi \\ &= \frac{R^2}{2\rho_0 c}\int_0^{2\pi}\int_0^{\pi/2}|p(R,\theta,\phi)|^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi. \end{aligned}\tag{21}$$

3. Formulation of optimization problem

3.1. Objective function

In any case at the beginning of the formulation of an optimization problem, objective function, a quantity to be minimized or maximized needs to be assigned to the problem. Also, design variables to be optimized and constraints representing the physical restrictions have to be chosen appropriately.

For the purpose of minimizing the sound radiation from the beam, a meaningful quantity is obviously its radiated sound power, which represents its sound radiation capability and is also independent upon the observing location in the sound field. As the sound power depends upon the frequency, an integral criterion over an appropriate frequency range is required for the case of the excitation force in a broad frequency range. A solution that meets technical interest is therefore

$$\Pi(\omega_{\min} \sim \omega_{\max}) = \frac{1}{\omega_{\max} - \omega_{\min}} \int_{\omega_{\min}}^{\omega_{\max}} \Pi(\omega)\,\mathrm{d}\omega. \tag{22}$$

Here, $\omega_{\min}$ and $\omega_{\max}$ are, respectively, the minimum and maximum excitation frequencies in radiation per second. Since the sound radiation at resonant frequencies of the CLD treated beam constitutes the dominant contributor to its sound power, only the sound power at these frequencies are considered. A summation of the sound power at these resonant frequencies is, therefore, performed replacing the integration over the frequency range of interest. As such, expression (22) is written as

$$\Pi(\omega_{\min} \sim \omega_{\min}) = \frac{\sum_{i=1}^{N} \Pi(\omega_i)\Delta\omega}{\omega_{\max} - \omega_{\min}}, \tag{23}$$

where N is the number of resonant modes in the frequency range of interest, ω_i the ith resonant frequency and $\Delta\omega$ is a constant frequency bandwidth. Here, $\Delta\omega$ is set equal to 2 Hz.

In the optimization that follows, the sound power of the CLD-treated beam defined by expression (23) is chosen as the objective function to be minimized for the case of broad frequency excitation, while expression (21) is as for the case of excitation at single resonant frequency. In the formulated optimization problem, it is

$$\text{Minimize } f_1 = \begin{cases} \Pi(\omega_i) & \text{for excitation at ith resonant frequency,} \\ \Pi(\omega_{\min} \sim \omega_{\min}) & \text{for broad frequency excitation.} \end{cases} \tag{24}$$

3.2. Constraints

In real-life vibration and noise control design, the added weight to the base beam owing to CLD treatment is always restricted to a certain amount of percentage of the base beam. Two cases of CLD coverage are, therefore, considered for optimization study here. The first case is where the CLD coverage length is allowed varying from zero to full coverage. In this case, the problem is constrained to ensure physical feasibility in which the patches are bounded within the area of the beam surface, i.e.,

$$x_2 - x_1 = l, \quad L \geqslant x_2 \geqslant l, \quad L - l \geqslant x_1 \geqslant 0. \tag{25}$$

For the second case, the CLD coverage length, l, is fixed at a certain percentage of the length of the base beam, L, so that the optimization problem with three design variables reduces to that of two variables. They are the starting position of the patch, x_1, and the shear modulus amplitude of the VL, G_v, i.e., $\boldsymbol{x} = [x_1, G_v]^{\mathrm{T}}$.

With the constraints defined by (25), one optimization is performed to look for maximum reduction in the sound power radiated by the beam through partial CLD treatment. In the other optimization performed, the objective function is to minimize beam's sound power meantime to minimize the amount of damping material used for the treatment. In this situation, the previous single criteria optimization problem becomes a bicriteria one. Therefore, a transformation is needed to convert the problem into a scalar optimization problem. Using weighting objectives method [22], the objective function becomes

$$\text{Minimize } f = w_1 f_1 + w_2 f_2 = w_1 \Pi + w_2 \frac{(x_2 - x_1)}{L}, \tag{26}$$

where $w_i \geqslant 0$ $(i = 1, 2)$ are the weighting coefficients representing the relative importance of the criteria. Note that the second term representing the additive weight of the CLD patch has been normalized about that of full CLD coverage. When (w_1, w_2) is set equal to $(1, 0)$, the above objective function becomes that expressed by (24).

Furthermore, the optimal solutions of the viscoelastic shear modulus must be in parameter ranges of viscoelastic materials which are available in commercial market, i.e.,

$$G_{\max} \geqslant G_{\mathrm{v}} \geqslant G_{\min}. \tag{27}$$

In the optimization study presented as follows, $G_{\min}$ and $G_{\max}$ are set equal to 0.1 and 10.0 MPa, respectively.

3.3. Design variables

In view of the large number of parameters involved in a beam with partial CLD treatment, a complete optimization study should take all parameters of the CLD patch as the design variables. However, those variables that do not significantly affect the amplitude of vibration response of the base structure as others should be taken away from the design variable list in the optimization. In this way the mathematical model is easier to solve compared to where all parameters are treated design variables.

Number of studies have shown that stiffer constraining layer warrants larger shear strain in VL which dissipates more vibrational energy of the vibrating beam [1]. So the elastic modulus of the CL will be fixed in the course of optimization study. Furthermore, parametric studies have been carried out [19] to examine the influence of varying the location and length of the CLD patch, thicknesses of both VL and CL, as well as the shear modulus of the VL, on the displacement response of the base beam. It is shown that when the elastic Young's modulus of the CL is fixed, the patch's location and length, and the shear modulus of the VL are more crucial than the thicknesses of both CL and VL in determining the structural damping loss factor. Therefore, there are three design variables for the optimization problem here, they are namely, the coordinates of two ends of the CLD patch, x_1 and x_2, and G_{v}^*, the shear modulus of VL. Therefore, the vector of design variables is defined for the problem as $\boldsymbol{x} = [x_1, x_2, G_{\mathrm{v}}^*]^{\mathrm{T}}$.

For the viscoelastic layer, the frequency-dependent shear modulus is adopted from the paper of Douglas and Yang [21]:

$$G_v^*(f) = G_v f^{0.494}(1 + 1.46j), \tag{28}$$

where f is the frequency, $f = \omega/2\pi$.

In the course of optimization, this frequency–dependency relationship is kept invariant but the real-valued amplitude, G_v, is taken as one of design variables together with other two parameters to be optimized. The vector of design variables simply becomes $\boldsymbol{x} = [x_1, x_2, G_v]^T$.

3.4. Optimization method

A number of optimization algorithms/methods are available to solve the problem defined as above. Among these algorithms, however, most are designed so far to find a local optimum. The real situation is that the optimization problem here probably have several local optima. So it is of interest to find the best optimum in the whole feasible design domain, i.e., the global optimum. The genetic algorithm, or shortly GA, is such a method that the approximation of global optimum is searched for. The GA has been used previously by a lot of researchers to solve various nonlinear optimization problems [23] and is selected here for solving the problem.

In order to apply the GA on our constrained optimization problem, the concept of penalty function has to be introduced to convert the problem from a constrained optimization to an unconstrained problem which the GA is able to solve. The basic idea of this so-called "GA-based penalty function method" is adding a penalty factor, R, that accounts for violation of both equality and inequality constraints, g_i $(i = 1, 2, \ldots, m)$, which leads an equivalent unconstrained optimization problem formulated as

$$\text{Minimize} \quad f + R\sum_{i=1}^{m} \Phi_i(g_i) \tag{29}$$

subject to constraints of feasible domain for each design variable's value. Here, Φ_i is the Heaviside operator such that $\Phi_i = -1$ for $g_i(\boldsymbol{x}) < 0$ and $\Phi_i = 0$ for $g_i(\boldsymbol{x}) \geqslant 0$. The GA, in contrast to conventional initial conditions-prone optimization methods, is forced to search from a population of points in order to search a set of feasible solutions. The solution solved by GA-based penalty function method is, therefore, guaranteed to be optimal or near optimal globally.

4. Optimization results and discussion

The beam used for the optimization study has simply supported boundary condition at its two ends. Its geometric parameters are the length of 0.4 m, width of 0.03 m and thickness of 0.004 m. The geometric and material properties of the base beam and CL for implementing the CLD treatment are shown in Table 1 with inclusion of the density parameter and thickness of the VL, of which the shear modulus is one of design variables to be optimized. According to Chen and Huang [15], the VL's thickness is fixed as the half of the CL's for better damping performance of the

Table 1
Properties of materials of base beam and CLD

Properties	Base beam (aluminum)	Constraining material	Viscoelastic material
Elastic modulus, E (GPa)	$70(1+\mathrm{j}0.0001)$	$49(1+\mathrm{j}0.0001)$	–
Density, ρ (kg/m^3)	2.71×10^3	7.50×10^3	1.00×10^3
Thickness, h (m)	0.004	0.002	0.001

Table 2
Beam modal frequencies below 1 kHz

Mode n	Natural frequency (Hz)
1	57.6
2	230.5
3	518.5
4	921.8

damped beam. A small structural damping is introduced in the form of a complex elastic modulus for the base beam and the CL: $E_i = \tilde{E}_i(1+\mathrm{j}\eta_i)$ $(i = \mathrm{s}, \mathrm{c})$, where η_i is the structural loss factor.

A unit harmonic transverse force is applied at the center of beam, i.e, $x_\mathrm{f}^* = 0.5L$, and the excitation frequency is from $f = 0$ Hz to $f = 1.0$ kHz. Before performing the optimization, the analytical model and associated solution procedure are validated by comparing the natural frequencies of bare beam with the theoretical predictions. For the beam with CLD treatment, the analytical solution of frequency response at the force location is compared to results obtained by a multi-physics finite element code. Good agreements between the values are observed for both cases [19].

Modal frequencies of the first four modes of the considered beam within the excitation frequency range are listed in Table 2. Among them, only two odd modes would be excited if the CLD patch covers the beam symmetrically.

4.1. Minimization of sound power over a frequency range

The first optimization run is executed to minimize the sound power of the vibrating beam over a frequency range from 0 Hz to 1 kHz under constraints defined by (25). The additive weight of the CLD patch for damping treatment is not of concern here. A set of GA solution obtained after running 100 generations with max population size equal 900 and maximum number of chromosomes (binary bits) per individual equal 1000 is given in Table 3. It can be seen that the optimal length of this patch is 74.2% of the beam length, and the beam is covered by CLD patch from $x_1^* = 0.258L$ to $x_2^* = 1.0L$. In this optimal CLD length and location, the optimal shear modulus amplitude of the VL is $G_\mathrm{v} = 0.445$ MPa. With this optimal CLD arrangement, the sound power is reduced from 9.263×10^{-2} W for the bare beam to 0.877×10^{-5} W, which means that a 40.2 dB reduction in sound power level is achieved. An interested result in Table 3 is the second subset solution, i.e., although the beam is treated by a longer CLD patch, therefore, a larger amount of damping material is used, the achieved sound power reduction is about 40.1 dB, even slightly

Table 3
GA solutions of optimal CLD patch with exclusion of minimum added weight

x_1/L	x_2/L	Coverage (%)	G_v (MPa)	Sound power (W)	SP at 1st resonant frequency	SP at 3rd resonant frequency
0.140	0.876	73.6	0.346	1.192×10^{-5}	0.739×10^{-5}	0.357×10^{-5}
0.017	0.858	84.1	0.371	0.904×10^{-5}	0.614×10^{-5}	0.195×10^{-5}
0.258	1.000	74.2	0.445	0.877×10^{-5}	0.475×10^{-5}	0.294×10^{-5}

smaller than that for 74.2% coverage (40.2 dB). This indicates that a longer CLD coverage does not always ensure larger reduction of sound radiation from the vibrating beam. By optimum choice of the patch location, the sound power reduction achieved by a shorter patch may be larger than by a longer one. A comparison of the sound power spectra of the beam with the above optimal three CLD coverage are depicted in Fig. 4 together with comparison to that of the bare beam.

4.2. *Minimization of sound power over a frequency range considering minimization of damping material used*

With inclusion the amount of CLD material used to be minimized in the objective function defined by (27), the optimum of three CLD parameters are searched for again by using GA program. Since the thicknesses and mass densities of both con-

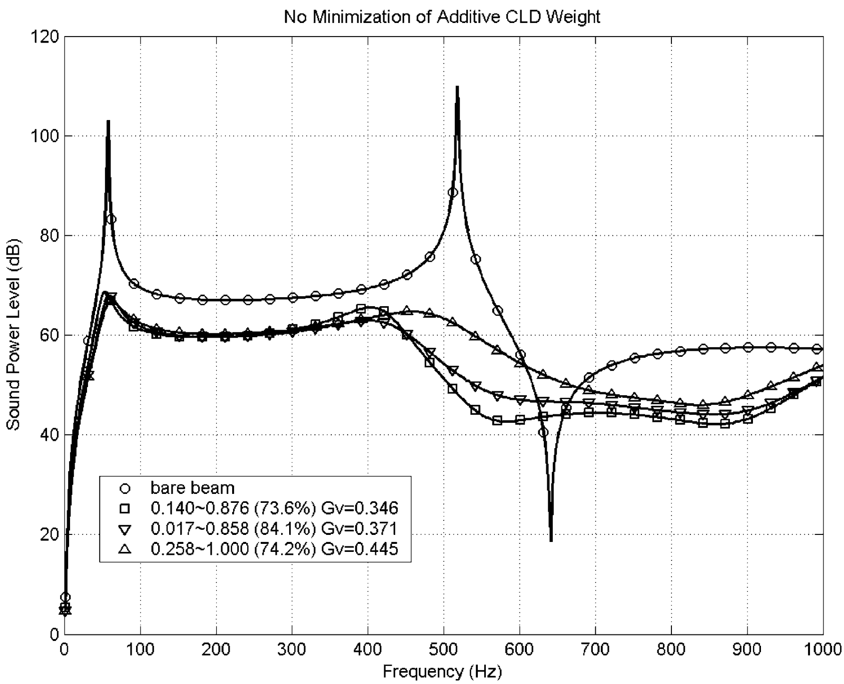

Fig. 4. Comparison of sound power levels of beam with optimized CLD treatments to that of bare beam.

Table 4
Optimal solutions of CLD treatment with inclusion of minimum coverage length

x_1/L	x_2/L	(w_1, w_2)	Coverage (%)	G_v (MPa)	Sound power (W)
0.425	0.909	(0.5, 0.5)	48.4	1.510	1.202×10^{-5}
0.161	0.393	(0.25, 0.75)	23.2	1.703	3.001×10^{-5}

straining and core layers fixed, the minimization of CLD material amount is equivalent to minimizing CLD coverage length, l, for a single patch treatment. In this case, the previous single criteria optimization problem becomes bi-criteria one. A Pareto optimal solution should be obtained. Two solution sets for minimization of the sound power over the frequency range and damping material used are obtained as given in Table 4. First, the weighting coefficients for objective functions of both radiated sound power and the amount of the damping material used are chosen equal to each other, i.e., $(w_1, w_2) = (0.50.5)$, the optimal solutions indicate that coverage length is 48.4%L and the reduction of sound power level is 39.0 dB, which is comparable to that of the beam with 84.1%L coverage, although the used amount of damping material for the former treatment is less than 60%L of that for the case of 84.1%L coverage. $x_1^* = 0.425L$ and $x_2^* = 0.909L$ represent the optimal location of the CLD patch for 48.4%L coverage length. Note that optimal shear modulus here is $G_v = 1.510$ MPa which is much larger than that for 84.1%L coverage.

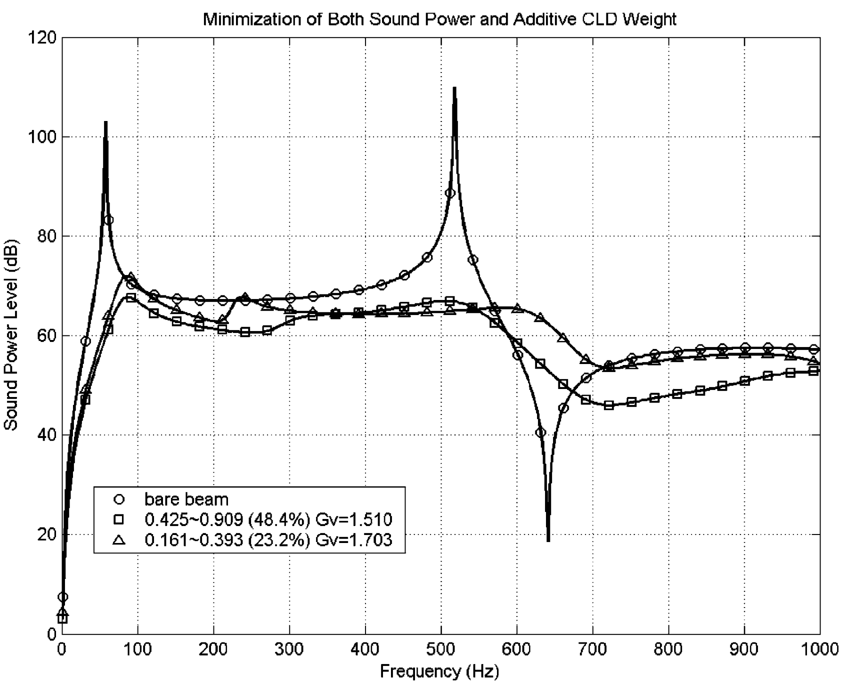

Fig. 5. Comparison of sound power levels of the beam with optimized CLD treatments when CLD weight is considered in objective function.

Changing the weighting coefficients to (0.25, 0.75) which means that the additive damping weight is more of concern than the previous optimization run, the obtained optimal length is 23.2%L of the beam length. The patch's optimal location is from $x_1^* = 0.161L$ to $x_2^* = 0.393L$. With this optimal CLD arrangement, the sound power level reduction is 35.1 dB, i.e., 3.9 dB less than the sound power reduction by 48.4%L coverage. However, damping material used for 23.2%L coverage is only less than half of that for 48.2%L coverage.

Fig. 5 shows the sound power spectra of the damped beam with, respectively, 48.4%L and 23.2%L coverage length and comparison with that of the bare beam. With 48.4%L coverage length, the sound power of the beam over the frequency range from 0 Hz to 1 kHz is reduced from 9.263×10^{-2} W for the bare beam to 1.202×10^{-5} W (39.0 dB reduction). On the other hand, with 23.2%L CLD coverage, the sound power of the beam is reduced to 3.001×10^{-5} W (35.1 dB reduction). This sound power level reduction is quite close to the case where the beam is covered by 84.1%L beam length, although in the former CLD arrangement the used damping material amount is only about one-fourth for the latter arrangement.

For maximum damping in the damped structure, the CLD patch should be located in the areas where shear strain energy is maximum. In spite of slight difference between the optimal coverage lengths for above two problems, the optima of the CLD location obtained are always that it starts from a location close to one beam's end and ends at a location nearby the beam's center (Fig. 6).

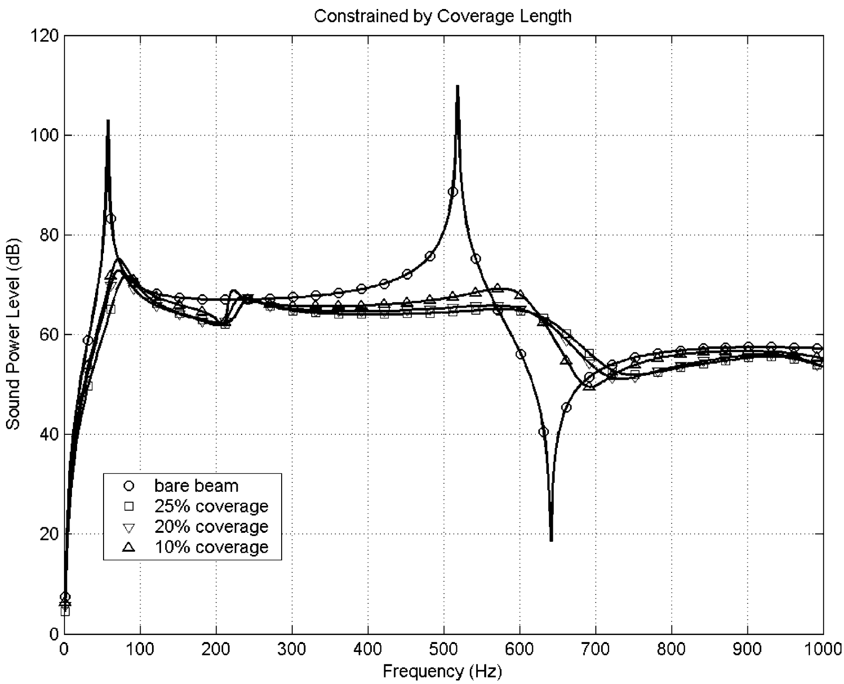

Fig. 6. Comparison of sound power levels of beam with different CLD coverage lengths.

Furthermore, it can be observed that the optimal shear modulus of the viscoelastic layer is highly relevant to the optimal length for minimizing beam's broadband sound radiation. A generic trend is that the shorter the coverage length of the CLD patch, the larger the shear modulus of the VL. This means that a stiffer viscoelastic material is needed for shorter CLD coverage to ensure system's energy-dissipating capability.

4.3. Minimization of sound power over a frequency range under restriction of additive weight of CLD patch

In real situation of noise control design, the added weight allowed for damping treatment is always restricted. Under constraint of the CLD coverage length set equal to a certain percentage of the beam length, the previous optimization problem with three design variables, i.e., the patch coverage length and location and the amplitude of VL's shear modulus, is reduced to a problem with two variables. They are namely, the VL's shear modulus and the location of one patch end on the beam, since this end location of the patch plus the pre-determined length decide the location of the other patch end. GA solutions are given in Table 5. With the CLD coverage length set equal to 25%L of beam length first and objective function to minimization of the beam's sound power over frequency from 0 Hz to 1 kHz. The obtained solutions are: $x_1 = 0.165L$ and $G_v = 1.575$ MPa. The sound power of the beam over the frequency range of interest is reduced from 9.263×10^{-2} W for the bare beam to 0.210×10^{-4} W, a 36.6 dB reduction in sound power level is achieved. The broadband sound power level reductions for fixed 20%L coverage and 10%L coverage are, respectively, 36.0 and 33.9 dB. Although the used CLD material amount is doubled from 10%L coverage to 20%L coverage, the achieved sound power reduction using the optimized damping arrangement is 2.1 dB more. Moreover, the results illustrate again that the optimal location of CLD patch for beam vibration control is still in-between one end of the beam and its center. As for the VL's shear modulus, the trend of optimum, except for 20%L coverage, is very close to those solutions obtained in previous optimizations, i.e., an optimal combination of the CLD patch's parameters is that a shorter coverage requires a stiffer VL.

4.4. Minimization of sound power at a specific resonant frequency

All the above optimizations are performed to minimize the sound power of the damped beam over a frequency range covering several resonant modes. Some results

Table 5
Optimal location of CLD patch and shear modulus of the VL with different coverage length

Coverage length (%)	x_1/L	G_v (MPa)	Sound power (W)
25	0.165	1.575	1.19×10^{-5}
20	0.604	1.183	3.33×10^{-5}
10	0.236	1.583	4.04×10^{-5}

Table 6
GA solutions for minimizing sound power at the first resonant frequency

	x_1/L	x_2/L	Coverage (%)	G_v (MPa)	SP at 1st resonant frequency
Case 1	0.010	0.643	63.3	0.757	0.406×10^{-5}
Case 2	0.105	0.629	52.4	0.672	0.581×10^{-5}

of sound powers at the first and third resonant frequencies (both are odd modes) for three different partial CLD treatments have been given in Table 3. From the table, one can see that the sound power reduction at a single resonant frequency does not always follow the trend of sound power reduction in a broad frequency range when the optimized CLD arrangement is adopted. This is easily to be understood: as the shear strain induced in the viscoelastic layer is mode-shape dependent, at a specific resonant frequency, the optimum of CLD patch parameters for minimization of the beam's sound power are certainly dependent upon its resonant mode shape. With defined objective function to minimize the sound power at the first resonant frequency, two more GA runs are executed to search for optimum of CLD patch location and the core shear modulus and the solutions are given in Table 6. The first run is to search for the optimal CLD patch to minimize the sound power from the damped beam without concern about damping material used. The GA solutions are: $x_1 = 0.010L$, $0.643L$; $G_v = 0.757$ MPa. With this optimal CLD arrangement indicated in the table, the sound power of the beam at the first resonant frequency could be reduced from 0.221×10^{-1} W for bear beam to 0.406×10^{-5} W (37.4 dB reduction in sound power level). Including the minimization of the amount of CLD material used, the second run is executed by defined weighting coefficients for objective function, $(w_1, w_2) = (0.5, 0.5)$. The GA solutions are, for minimization of beam's sound power at the first resonant frequency, $x_1 = 0.105L$, $0.629L$; $G_v = 0.672$ MPa. With this optimal CLD arrangement, the sound power at the first resonant frequency is reduced to 0.581×10^{-5} W (35.8 dB reduction in sound power level). Both solutions show that the optimum of CLD patch location and the VL's shear modulus for minimizing the sound power at the first resonant frequency are similar to that for the minimization of the sound power over a frequency range covering the first several resonant modes. However, some differences are observed in optimum of the CLD patch length.

5. Conclusions

Major concluding remarks can be made as follows:

1. In order to achieve the biggest reduction in the sound power radiated by a simply supported beam with a transverse force applied at its central location, it is not necessary to fully cover the beam with CLD patch. An optimal coverage of the CLD treatment is 74.2% of the beam length from one beam's end for optimally suppressing the sound radiation over the frequency range covering the first four resonant modes.

2. The optimal shear modulus of the viscoelastic layer is highly relevant to the CLD coverage length and location on the beam. A general trend is that the shorter CLD patch needs a stiffer viscoelastic material for an optimal damping treatment.
3. The optimal location of the CLD patch to reduce the sound radiation from the beam is always that the coordinate of one end of the patch is close to one end of the beam while the other patch's end is nearby the beam's center.
4. The combination of optimal CLD patch location and the VL's shear modulus for minimizing the sound power at the first resonant frequency is quite similar to that for the minimization of the sound power over a frequency range covering the first several resonant modes. Some differences can be observed in optimum of the CLD patch length for the two cases.

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