Engineering Applications of Artificial Intelligence 25 (2012) 72–81

Contents lists available at SciVerse ScienceDirect

Engineering Applications of Artificial Intelligence

journal homepage: www.elsevier.com/locate/engappai

Multicriteria decision making under uncertainty: An advanced ordered weighted averaging operator for planning electric power systems

M.Q. Suo [1], Y.P. Li *, G.H. Huang [2]

MOE Key Laboratory of Regional Energy Systems Optimization, S-C Academy of Energy and Environmental Research, North China Electric Power University, Beijing 102206, China

ARTICLE INFO

Article history:
Received 11 February 2010
Received in revised form
24 May 2011
Accepted 22 August 2011
Available online 7 September 2011

Keywords:
Advanced OWA
Electric power systems
Fuzzy logical
Interval
MCDM
Uncertainty

ABSTRACT

In this study, an advanced ordered weighted averaging (AOWA) operator is proposed for tackling multicriteria decision making (MCDM) problems under uncertainties. The AOWA incorporates techniques of interval theory and a center of gravity (COG) method within a traditional ordered weighted averaging (OWA) operator. It can deal with the uncertain inputs under optimistic and pessimistic conditions without knowing their distribution information and linguistic important degrees of all inputs in MCDM systems. The results obtained help decision makers select the optimal alternative according to their optimism degrees. A case study of planning electric power problems is provided for demonstrating the applicability of the proposed method. The results indicate that reasonable solutions have been generated for both discrete intervals and linguistic inputs. For all criteria under consideration, corrective alternatives can be undertaken sensitively under various optimism degrees and thus can help resolve the conflicts in electric power systems under uncertainties.

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1. Introduction

Electric power problems are crucial for many cities throughout the world. The limited energy reserves, rising environmental- and health-impact concerns, increasing material prices, and different technological, geographic, political and economic conditions are having significant effects on electricity generation allocation problems (Cai et al., 2008). Moreover, development and application of renewable energy for electric power have also aroused more and more concerns. These issues are related to a multitude of impact factors and objectives. Furthermore, in electric power systems, there are lots of complicated factors need to be considered by decision makers, such as electricity production, service life of electricity generation facilities and the resulting greenhouse gas (GHG)/pollutant emissions. In addition, many system parameters (e.g., capital and operating cost, material sources, energy consumption, existing and potential capacities) may appear uncertain and may be given with optimistic and pessimistic data. The important degrees of criteria given by experts and used for evaluation are generally linguistic quantifiers. Such uncertainties may lead to further complexities in the related decision making processes and the generated decision support systems (Li et al., 2009; Chen et al., 2009; Kosugi, 2009). Therefore, effective technology for making tradeoffs and yielding optimal decision alternatives in electricity industry under various uncertainties is desired.

Previously, a number of studies were reported from the perspective of programming models to address multiple conflicting objectives (such as cost, environmental concerns and energy efficiency measures) in the electricity generation planning problems. For example, Martins et al. (1996) developed a multiobjective linear programming model for power generation expansion planning incorporating demand-side management (DSM), wherein DSM was the process of managing the consumption of energy, generally to optimize available and planned generation resources and was included by modeling it as a new generating group along with the generating alternatives from the supply side. Christian (1999) presented a decision support framework for environmental planning in developing countries, which was based on identifying the priorities of conflicting goals by working through and reducing the conflicts. Hsu and Chou (2000) proposed a multiobjective programming approach integrated with a Leontief inter-industry model to evaluate the impact of energy conservation policy on the cost of reducing CO_2 emissions and undertaking industrial adjustment in Taiwan. Liu et al. (2003) proposed a hybrid fuzzy-stochastic robust programming (FSRP) method for a case study of regional air quality management, which reflected complex tradeoffs between environmental and economic considerations. Antunes et al. (2004) made a multiobjective mixed integer linear programming for power generation expansion planning that allowed the consideration of modular expansion capacity values of supply-side options and also considered

* Corresponding author. Tel.: +86 10 6177 2977; fax: +86 10 6177 2982.
E-mail addresses: echorise201@hotmail.com (M.Q. Suo), yongping.li@iseis.org (Y.P. Li), huangg@iseis.org (G.H. Huang).
[1] Tel./fax: +86 10 6177 2982.
[2] Tel.: +86 10 6177 2981; fax: +86 10 6177 2982.

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doi:10.1016/j.engappai.2011.08.007

Nomenclature

AOWA	advanced ordered weighted averaging
COG	center of gravity
EOWA	extended ordered weighted averaging
MCDM	multicriteria decision making
OWA	ordered weighted averaging
α	the optimism degree of the decision maker
θ	the degree to which the aggregation is an *or* operation
l	the left benchmark of normalized triangular fuzzy number Z
m	the value of normalized triangular fuzzy number Z with the highest membership degree $f_Z(m)=1$
r	the right benchmark of normalized triangular fuzzy number Z
w	a non-negative weighting vector of dimension n
$x^{\pm}$	an interval number with known upper and lower bounds but unknown distribution information
x^{-}	lower bound of $x^{\pm}$
$x^{\pm}$	upper bound of $x^{\pm}$
x_i^{min}	the minimum value in criterion i
x_i^{max}	the maximum value in criterion i.

demand-side management, which was possible to perform an evaluation of the rate impact in order to further inform the decision process. Li et al. (2006a) developed an interval-parameter multi-stage stochastic linear programming method for water resources decision making under uncertainty, whose results are helpful for water resources managers in not only making decisions of water allocation but also gaining insight into the tradeoffs between environmental and economic objectives. Nasiri and Huang (2008) proposed a fuzzy environmental policy analysis approach based on multi-objective optimization and developed a post-optimization assessment based on fuzzy set theory concept to determine the best compromise plan for capacity planning in electricity generation.

Generally, most of the programming models for dealing with multi-objective problems included two major methods: compromise program and aspiration approach. In compromise programming (Merino et al., 2003; Prodanovic and Simonovic, 2003), a variety of distances between the solutions of the problem and the ideal value of objectives were examined for evaluating the optimal solution. In aspiration approach (Abel and Korhonen, 1996; Buchanan and Gardiner, 2003; Li et al., 2007; Qin et al., 2007; Billionnet, 2009; Yoshida and Matsuhashi, 2009), one of the multiple objectives was optimized while the others were converted to model constraints. Although the aspiration approach can provide a less complex and, consequently, a more practical methodology, especially for large-scale problems, the solutions may not be as preferred as those provided by the compromising approach when ignoring the tradeoffs of objectives (Nasiri and Huang, 2008). In addition, most of the optimization models may only be effective for the measurable parameters but ineffective for the parameters that cannot be quantified, such as the linguistic quantifiers.

Consequently, an ordered weighted averaging (OWA) operator proposed by Yager (1988) was a general technology in multicriteria decision making (MCDM) systems. The OWA solution process included three steps: (i) reorder the inputs in descending order; (ii) decide the weighting vector related with the OWA operators; and (iii) aggregative procedure. The OWA operator was applied to many environmental planning problems (Despic and Simonovic, 2000; McPhee and Yeh, 2004; Mysiak et al., 2005; Makropoulos and Butler, 2006; Sadiq and Tesfamariam, 2007). However, it assumed that the inputs were definite and the important degrees for all inputs were identical. In fact, in many real problems, inputs were often uncertain and their important degrees were generally unequal. These two facts limited its application to practical problems. As a result, a number of literatures focused on the study of ascertaining the weighting vector using different methods were proposed (Yager and Filev, 1994, 1999; Herrera et al., 1996; Torra, 1997; Xu and Da, 2002; Fullér and Majlender, 2003; Chen and Chen, 2003, 2005; Xu, 2005; Zarghami and Szidarovszky, 2009). Nevertheless, few studies are concerned with the inputs of uncertain nature. Actually, the inputs in OWA are always uncertain and even have their respective different important degrees, which are different from the weighting vector. In addition, the important degrees of all inputs may be generally shown in formulations of linguistic formats in MCDM systems. Both of these could make the MCDM process more difficult and complicated. Although the normalized triangular fuzzy numbers and the max-membership method were employed by Zarghami et al. (2008) to deal with the uncertainty when both of the inputs and the important degrees of criteria were linguistic quantifiers, they were unable to handle the uncertainty when the inputs had only the optimistic and pessimistic information but unknown distribution information. Moreover, the max-membership method can only be applied in some specific cases.

Therefore, this study will propose an advanced ordered weighted averaging (AOWA) operator to solve a MCDM problem containing uncertain inputs and the linguistic important degrees of criteria in electric power systems. In this study, interval parameters will be introduced into AOWA to reflect the uncertain inputs under optimistic and pessimistic conditions without knowing their distribution information. Moreover, to make the defuzzying procedure more simple and effective, vector method is adopted to solve the center of gravity (COG) of a normalized triangular fuzzy number to quantize the linguistic important degrees of criteria in plane right angle coordinate. On the other hand, the weighting vector in AOWA will be obtained by the minimal variability method here (Fullér and Majlender, 2003; Zarghami et al., 2008). In this case, the AOWA operator will be able to deal with uncertainties expressed as not only discrete intervals but also linguistic quantifiers, and facilitate the results analysis for decision maker to select the optimal alternative in a decision making support system.

This study firstly depicts the OWA operator and the extended OWA (EOWA) operator (Zarghami et al., 2008), and then advances the OWA operator from three steps, wherein the theory proofs and remarks about vector methods will be used for solving COG of a normalized triangular fuzzy number, which would enhance the feasibility and the reliability of AOWA. Secondly, a case study of planning electric power problems is provided for demonstrating the applicability of the developed AOWA. Afterwards, the results analysis is shown in detail to afford the decision maker with an optimal support solution for electric power systems. Finally, comparisons between AOWA and EOWA are made to further demonstrate the advantages of the proposed AOWA.

2. Methodology

2.1. Concept for OWA

In MCDM systems, there are two key factors: the important degrees of criteria and the being assessed objectives with respect to the criteria that need to be quantified. However, the first factor is generally given by experts and represented in verbal terms such as "high", "low" and "medium", and the second factor is not defined exactly; instead, it is known to fall within a certain interval. Both of these make further complexities in MCDM systems. Therefore, this study will advance the OWA operator to quantify the above two factors and enhance its application to MCDM problems. Definition and concept about OWA and EOWA will be, respectively, introduced as follows.

Definition 1. An OWA operator of dimension n is a mapping $F: I^n \to I$ (where $I=[0,1]$) with formulation below (Yager, 1988):

$$F(a_1,a_2,\ldots,a_n)=\sum_{i=1}^{n} w_i b_i = w_1b_1+w_2b_2+\ldots+w_nb_n \tag{1}$$

where b_i is the ith largest element in the inputs a_1, a_2, …, a_n, and $\mathbf{w}=(w_1, w_2, \ldots, w_n)$ is a non-negative weighting vector of dimension n so that $w_i \in [0,1]$ and $\sum w_i = 1$, F is the integrated goodness scoring of an alternative.

Remark 1. Zarghami et al. (2008) extended OWA (EOWA) from three steps as follows:(i) the weighting vector in OWA can be obtained by the minimal variability method; (ii) the linguistic inputs can be converted into crisp numbers by using equivalent normalized triangular fuzzy numbers and then be defuzzied using the max-membership method; and (iii) the inputs should be multiplied by their important degrees when the important degrees of all criteria are different.

OWA supposes that the important degrees of all criteria are identical and can deal only with the certain inputs, correspondingly, the inputs in OWA contain only one kind of parameters: b_i. In fact, the inputs should be comprised of two kinds of parameters: the important degrees of criteria and the being assessed objectives with respect to the criteria because the first parameter could not be equal to each other in real problems and are generally represented in linguistic terms. Moreover, the being assessed objectives with respect to the criteria (regarded as the inputs for OWA) in MCDM systems are often uncertain and contain optimistic and pessimistic data. Both of these make OWA unavailable. Although EOWA had done improvements in defuzzying the linguistic terms using the max-membership method, it has difficulties in dealing with the uncertain being assessed objectives, which include optimistic and pessimistic information. Moreover, the method for defuzzying the linguistic terms should be improved. Therefore, more effective method to deal with the uncertain data in MCDM systems is desired to be developed.

2.2. Development of advanced OWA

An advanced OWA method, which is based on EOWA from two main perspectives: introduction of interval theory to dispose the uncertain inputs, which contain only optimistic and pessimistic data and adoption of COG method to deal with the linguistic information, will be developed. Firstly, a number of useful definitions and theorems will be given for the algorithm of AOWA.

Definition 2. Let x denote a closed and bounded set of real numbers. An interval number $x^{\pm}$ is defined as an interval with known upper and lower bounds but unknown distribution information for x (Huang, 1996; Li et al., 2006b):

$$x^{\pm}=[x^-,x^+]=[t\in x \mid x^- \le t \le x^+] \tag{2}$$

where x^- and x^+ are the lower and upper bounds of $x^{\pm}$, respectively. When $x^-=x^+$, $x^{\pm}$ becomes a deterministic number, i.e. $x^{\pm}=x^-=x^+$.

Definition 3. A convex normalized fuzzy set $Z=\{(x,f_Z(x)) \mid x\in R\}$ on the real line R is called a fuzzy number if it satisfies that (Rommelfanger, 1996): (I) there exits exactly one $x_0\in R$ with the membership degree $f_Z(x_0)=1$; and (II) $f_Z(x)$ is piecewise continuous in R.

Definition 4. A fuzzy number $Z=\{(x,f_Z(x)) \mid x\in R\}$ is regarded as a normalized triangular fuzzy number if there exist reference functions and scalars $m,l,r>0$ such that

$$f_Z(x)=\begin{cases}(x-l)/(m-l) & l\le x\le m\\ (x-r)/(m-r) & m\le x\le r\\ 0 & \text{otherwise}\end{cases} \tag{3}$$

where l and r are the left and right benchmarks of normalized triangular fuzzy number Z, respectively, and $f_Z(m)=1$. Accordingly, the specific values of l, m and r are obtained based on the given triangular fuzzy number Z. For example, if Z is a normalized triangular fuzzy number Z (0.5, 0.4, 0.7), then $l=0.4$, $r=0.7$ and $m=0.5$ with $f_Z(0.5)=1$.

Based on Definition 4, a normalized triangular fuzzy number Z can be easily and effectively converted into crisp data by COG method. This COG approach aims to obtain the center of gravity of a normalized triangular fuzzy number Z using the vector method, whose specific calculation procedure can be seen in Theorem 1. Because in a plane right angle coordinate system, a normalized triangular fuzzy number Z has and only has one COG, it can be represented by its COG. Moreover, vector method is much easier to operate in solving COG than the integration method, which will be testified by Theorem 2.

Theorem 1. If the coordinates of the three apexes of a normalized triangular fuzzy number Z are $A(x_1,y_1)$, $B(x_2,y_2)$, $C(x_3,y_3)$ (see Fig. 1), and $f(x_2)=y_2=1$, $f(x_1)=y_1=f(x_3)=y_3=0$, then the coordinates of the COG about Z will be $G((x_1+x_2+x_3)/3, (y_1+y_2+y_3)/3)$ (means $G((x_1+x_2+x_3)/3,1/3)$).

Proof. Let the midpoint of segment AC be $E(x',y')$, and the center of gravity be $G(x,y)$. According to the midpoint formulations, the values of x' and y' are $(x_1+x_3)/2$ and $(y_1+y_3)/2$, respectively. In the center of gravity theorem, $\overrightarrow{BG}:\overrightarrow{GE}=2:1$. In this case, there are equations below:

$$\begin{cases}x_2-x=2(x-(x_1+x_3)/2)\\ y_2-y=2(y-(y_1+y_3)/2)\end{cases}, \text{ and the results are } \begin{cases}x=(x_1+x_2+x_3)/3\\ y=(y_1+y_2+y_3)/3\end{cases} \tag{4}$$

Theorem 2. In plane right angle coordinate, the COG of a normalized triangular fuzzy number Z obtained by vector method is the same as the one obtained by the integration method, but its calculation procedure is much easier than the latter one.

Proof. Since the COG of a normalized triangular fuzzy number Z obtained by vector method has been proved in Theorem 1, it only needs to verify the result obtained by integration method. Assuming that the density of Z is uniform, the integral expression for the center of gravity of Z is

$$\begin{cases}x_G=\left(\int_S x\,dS\right)/S\\ y_G=\left(\int_S y\,dS\right)/S\end{cases} \tag{5}$$

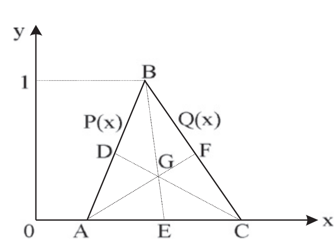

Fig. 1. A triangular fuzzy number Z (B, A, C) (symbols "A" and "C" denote the left and right benchmarks of fuzzy number Z, respectively, and $f_Z(B)=1$; symbols "D", "E" and "F" are the midpoint of segment "AB", "AC" and "BC", respectively; symbols "$P(x)$" and "$Q(x)$" mean the functions of segments "AB" and "BC", respectively).

where S is the area of triangle. Let $P(x)$ and $Q(x)$ be the functions of segments AB and BC, respectively, then the COG of Z can be formulated as (see Fig. 1):

$$\begin{cases} x_G = \left(\int_{x_1}^{x_2} xP(x)dx + \int_{x_2}^{x_3} xQ(x)dx\right) / \left(\int_{x_1}^{x_2} P(x)dx + \int_{x_2}^{x_3} Q(x)dx\right) \\ y_G = \left(\int_{y_3}^{y_2} yQ^{-1}(x)dy - \int_{y_1}^{y_2} yP^{-1}(x)dy\right) / \left(\int_{y_3}^{y_2} Q^{-1}(x)dy - \int_{y_1}^{y_2} P^{-1}(x)dy\right) \end{cases} \tag{6}$$

where

$$\begin{cases} P(x) = ((y_2 - y_1)/(x_2 - x_1))(x - x_1) + y_1 \\ Q(x) = ((y_2 - y_3)/(x_2 - x_3))(x - x_3) + y_3 \end{cases} \tag{7}$$

In addition, $P^{-1}(x)$ and $Q^{-1}(x)$ are the inverse functions of $P(x)$ and $Q(x)$, respectively. Through integration of Eqs. (7) and (8), Eq. (6) can be obtained as follows:

$$\begin{cases} x_G = (x_1 + x_2 + x_3)/3 \\ y_G = (y_1 + y_2 + y_3)/3 \end{cases} \tag{8}$$

It can be seen that Eq. (8) is the same as Eq. (4). Furthermore, the calculation procedure of integration method is more complicated than the vector method. At this point, this theorem is confirmed.

According to Theorems 1 and 2, the linguistic terms can be transformed into crisp values such that there are only crisp data in the important degree vector of criteria. If linguistic terms also exist in other impact vector, the same method can be used. Therefore, the developed AOWA integrated with the interval theory and COG method can effectively deal with the dual uncertainties: interval and fuzzy, and can be meaningfully and efficiently applied in MCDM systems. The specific calculation steps for AOWA operator will be described as follows:

Step 1. The uncertain inputs including optimistic and pessimistic data in OWA can be quantified as interval numbers based on Definition 2 and can be formulated as (Wang and Kerre, 2001)

$$x^{\pm} = [x^-, x^+] = \begin{cases} \{\alpha x^+ + (1-\alpha)x^-\}, \text{ for positive inputs} \\ \{\alpha x^- + (1-\alpha)x^+\}, \text{ for negative inputs} \end{cases} \tag{9}$$

where x^+ and x^- represent the optimistic and pessimistic data for the positive criterion in this study, respectively, vice versa. In addition, $\alpha \in [0,1]$ and α value represents the optimism degree of the decision maker, in which the higher α value the more optimistic the decision maker. Moreover, when the inputs have different units, they are necessary to be normalized as follows:

$$x_{ij}^* = \begin{cases} (x_{ij} - x_i^{min})/(x_i^{max} - x_i^{min}), \text{ for positive inputs} \\ (x_i^{max} - x_{ij})/(x_i^{max} - x_i^{min}), \text{ for negative inputs} \end{cases} \tag{10}$$

where x_i^{max} and x_i^{min} are the maximum and the minimum values in criterion i, respectively.

Step 2. For the linguistic important degrees, it is required to transfer the linguistic quantifiers into crisp data. Firstly, linguistic quantifiers should be converted into their equivalent normalized triangular fuzzy numbers according to Hwang and Chen (1992). Secondly, the normalized triangular fuzzy numbers can be defuzzied by the vector method of COG based on Theorem 1.

Step 3. Based on Steps 1 and 2, the new inputs of OWA operator can be obtained as follows:

$$a = (a_1, a_2, \ldots, a_n) = (c_1 d_1, c_2 d_2, \ldots, c_n d_n) \tag{11}$$

where $a_i = c_i d_i$, and c_i and d_i are the results of Steps 1 and 2, respectively. In addition, the weighting vector can be explored by the minimal variability method (Zarghami et al., 2008) with a fixed θ as follows:

$$\begin{aligned} \textit{Minimize } Var(w) &= \sum_{i=1}^{n} 1/n(w_i - E(w))^2 = 1/n \sum_{i=1}^{n} w_i^2 \\ &\quad - \left(1/n \sum_{i=1}^{n} w_i\right)^2 = 1/n \sum_{i=1}^{n} w_i^2 - 1/n^2 \end{aligned} \tag{12a}$$

subject to

$$1/(n-1) \sum_{j=1}^{n} (n-j) w_j = \theta \tag{12b}$$

$$\sum_{j=1}^{n} w_j = 1 \tag{12c}$$

$$w_j \geq 0 \tag{12d}$$

In order to obtain a unique weighting vector for optimization, the Kuhn – Tucker second-order sufficiency conditions can be used for solving the above model (Fullér and Majlender, 2003), and the results can be formulated as follows:

$$\begin{aligned} & w_1 = (2(2n-1) - 6(n-1)(1-\theta))/(n(n+1)) \\ & w_n = (6(n-1)(1-\theta) - 2(n-2))/(n(n+1)) \\ & w_j = ((n-j)/(n-1))w_1 + ((j-1)/(n-1))w_n, \text{ if } j \in \{2, \ldots, n-1\} \end{aligned} \tag{12e}$$

where $\theta \in [0,1]$ is an independent variable representing the degree to which the aggregation is an *or* operation. When $\theta = 0$, the weighting vector **w** is (0, 0, …, 1), which means that only the minimal element in the inputs can be satisfied and the OWA becomes a minimum operator. Conversely, when $\theta = 1$, the weighting vector **w** is (1, 0, …, 0), which means that only the maximal element in the inputs can be satisfied and the OWA becomes a maximum operator.

Apparently, the developed AOWA not only can deal with the uncertain data containing optimistic and pessimistic information in the being assessed objectives with respect to different criteria but also can dispose the linguistic important degrees of criteria. Moreover, the introduction of interval theory and COG method make the decision support systems proposed by AOWA more suited to the actual MCDM problems. For better understanding of the procedural steps of the developed AOWA, a framework is shown in Fig. 2.

3. Case study

The following electric power problem is used to demonstrate the applicability of the developed AOWA operator. In the study system, multiple power stations need to know how many generation shares they are permitted while the decision maker needs to decide which one should be first developed. Conventional and renewable power stations with different availabilities are employed. Among them, conventional power stations, such as a coal-fired power station, have larger capacity but more serious pollution, which may cause irreparable environmental problems under large-scale development. On the contrary, although most of renewable power stations have little pollution, their efficiency is relatively low and need many capital and time to develop which is not useful for rapid economic development. Moreover, renewable power stations would be greatly affected by many natural, spatial and temporal factors, such as precipitation and cloud variation, as well as temperature profiles and may not be able to meet the local demand for electricity (Cai et al., 2009; Chen et al., 2009). Therefore, the decision maker is expected to make a reasonable electricity generation allocation based on the current generation capacity, available resources and

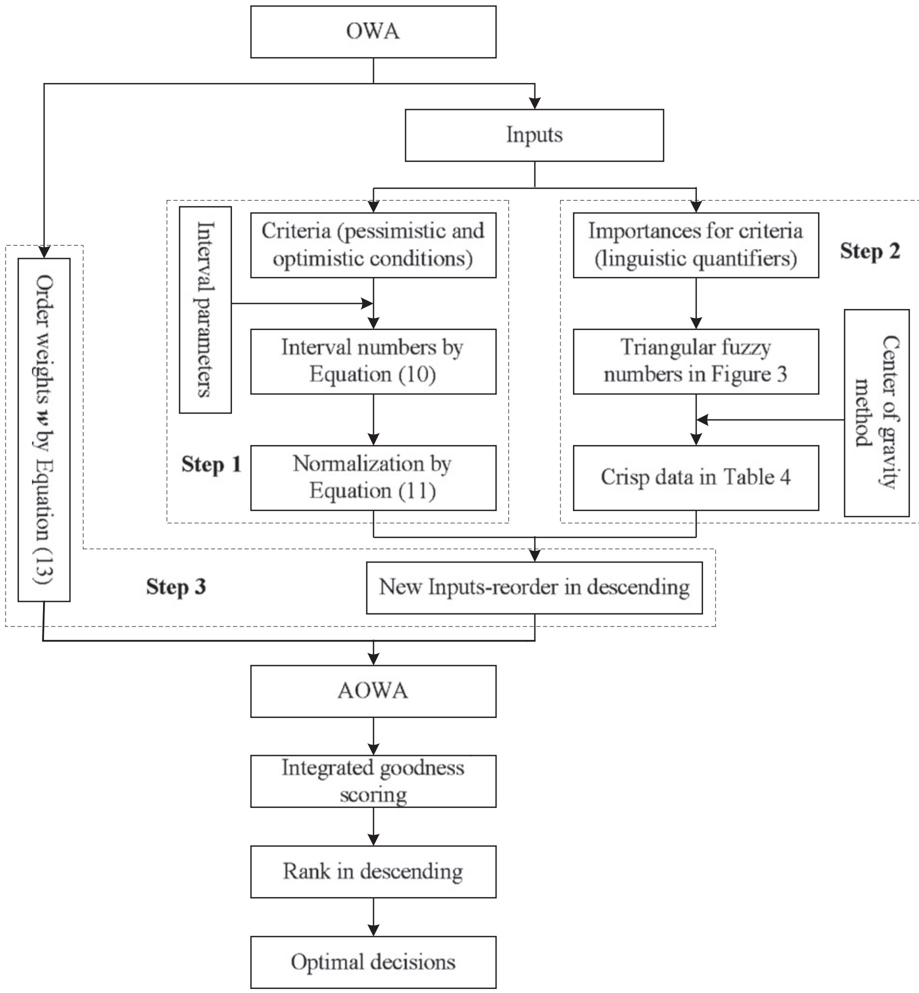

Fig. 2. Framework of AOWA (AOWA is the advanced ordered weighted averaging operator).

pollution severity about various power stations to satisfy electricity needs, economic development and environment requirements. Generally, most of the parameters (such as generation capacity and available resources) have the optimistic and pessimistic data but have not the distribution information. Meanwhile, the important degrees of various criteria are often different and represented as linguistic quantifiers, such as low (L), low to medium (LM), medium (M), medium to high (MH) and high (H).

The study system is composed of seven power stations: coal-fired power station, hydropower station, nuclear power station, natural gas-fired power station, wind power station, biomass power station and oil/diesel-fired power station, and eight criteria: operation and maintenance costs (OMC), capital cost (CC), energy intensity (EN), GHG intensity (GHG), retirement (R), current capacity (CUC), potential capacity (PC) and service life (SL) (Fig. 3). The specific meanings about eight criteria are shown in Table 1, and their important degrees (H, H, MH, MH, L, MH, LM and M) are represented in Table 2. Moreover, the current optimistic and pessimistic data of seven power stations with respect to eight criteria can be seen in Table 2. The objective is to make an electricity generation allocation about the seven power stations to decide how many share they can take to ensure the sustainable development of the electricity industry in this city in the next 5 years.

Firstly, the optimistic and pessimistic data in Table 2 should be quantified as interval numbers according to Step 1 in AOWA. The lower bounds of OMC_j, CC_j, EN_j, GHG_j and R_j represent the optimistic data, while their upper bounds denote the pessimistic data. These conditions are just the opposite for criteria CUC, PC and SL. For example, the value of OMC_5^- in Table 2 is 33.600, which means that the estimated levelized capital cost of wind power station based on discount rate 5% is \$ 33.6×10^3 per GWh under optimistic condition. Because the data of every power station on eight criteria have different units, they should be normalized using Eq. (11). Furthermore, the linguistic important degrees of eight criteria should be converted into their equivalent normalized triangular fuzzy numbers (Fig. 4) and then be transferred into crisp numbers based on Step 2, whose corresponding results are shown in Table 3, wherein real lines in Fig. 4 represent the normalized triangular fuzzy numbers in Table 3 while broken lines denote the equivalent normalized triangular fuzzy numbers for the changed linguistic important degrees.

Moreover, the values of x_{ij}^* should be multiplied by their different important degrees according to Step 3 and then the results are regarded as the new inputs of AOWA. The results under $\alpha = 0.5$ are shown in Table 4, and the other results corresponding to the α values can be achieved by the same calculation procedure. (Since the positive and negative criteria have been normalized, and the important degrees for different criteria also have been calculated, the same α value for all criteria means that all objectives are evaluated under the same optimism degree of the decision maker. Correspondingly, the other α values is used to conduct risk analysis for the decision maker.) The last step of AOWA is to solve the weighting vector. It is noted that the value of

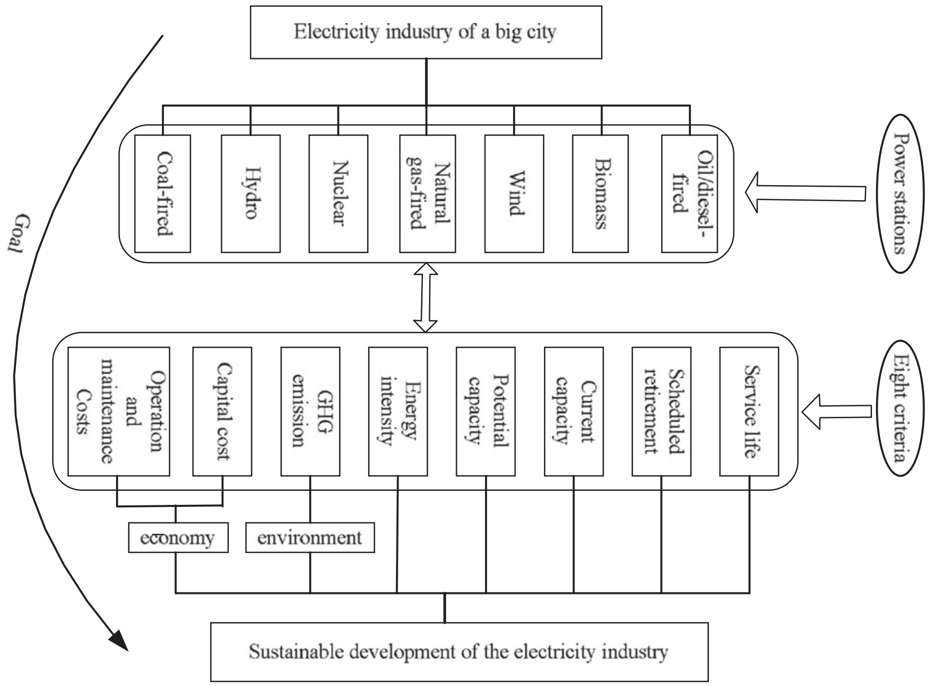

Fig. 3. Framework for the case study.

Table 1
Specific meanings of eight criteria.

Criteria	Specific meanings	Units
OMC	Levelized operation and maintenance costs of power stations based on discount rate 5%	\$1000/GWh
Capital cost	Levelized capital cost of power stations based on discount rate 5%	\$1000/GWh
Energy intensity	Unit energy consumption for power stations	TJ/GWh
GHG intensity	Unit greenhouse gas emission resulted from power stations	KT/GWh
Retirement	Scheduled retirement of generation capacity for power stations in the next five years	GW
Current capacity	Current generation capacity for power stations	GW
Potential capacity	Forecasted potential development capacity for power stations	GW
Service life	Estimated service life for power stations	Year

Note: GW, GWh, KT and TJ are gigawant, gigwant hour, kiloton and terra joule, respectively.

Table 2
Data for case study.

Criteria	Important degrees	Approximation	Seven power stations						
			$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$
			Coal-fired	Hydro	Nuclear	Natural gas	Wind	Biomass	Oil/diesel
OMC	*H*	Optimistic	26.100	22.150	24.770	29.200	33.600	50.100	38.620
(\$1000/GWh)		Pessimistic	31.200	33.920	30.100	46.600	93.300	63.200	70.080
Capital cost	*H*	Optimistic	9.920	17.360	16.330	3.610	27.300	35.660	11.360
(\$1000/GWh)		Pessimistic	15.680	30.500	18.720	7.220	76.700	44.270	22.840
Energy intensity	MH	Optimistic	10.620	3.490	11.540	8.590	0.098	10.390	8.078
(TJ/GWh)		Pessimistic	11.500	3.700	11.570	10.080	0.106	10.510	10.610
GHG intensity	MH	Optimistic	0.920	0.005	0.003	0.460	0.008	0.016	0.650
(KT/GWh)		Pessimistic	1.030	0.012	0.016	0.510	0.009	0.027	0.828
Retirement	*L*	Optimistic	9.435	0.197	1.000	3.206	0.06	0.597	6.881
(GW)		Pessimistic	15.901	15.607	9.425	4.528	1.000	1.532	7.426
Current capacity	MH	Optimistic	19.902	80.275	14.951	17.762	0.705	1.921	9.667
(GW)		Pessimistic	15.334	70.035	10.551	11.254	0.505	1.315	5.227
Potential capacity	LM	Optimistic	0.186	11.162	4.087	6.679	30.120	0.003	0.003
(GW)		Pessimistic	0.159	11.042	4.001	6.325	27.890	0.001	0.002
Service life	*M*	Optimistic	45	100	35	55	25	30	20
(year)		Pessimistic	25	60	20	35	12	15	13

θ in this study is supposed to be 0.33, representing that many criteria considered in this decision making system are satisfied. Under this assumption, the weighting vector **w** calculated by Eq. (13) is (0.0276, 0.0554, 0.0832, 0.1111, 0.1389, 0.1668, 0.1946, 0.2224), where n=number of criteria (n=8). Finally, the results of the shares taken by seven power stations in the case study can be calculated by Equation (1), and then the final decision support system can be obtained by arranging the integrated goodness scorings of seven power stations at different optimism degrees of the decision maker.

4. Result and discussion

4.1. Result from AOWA under different α values

AOWA can effectively deal with uncertainties presented as both linguistic quantifiers and intervals within a MCDM system. Solutions of AOWA provide an effective linkage between the optimism degrees and the final decision support systems. The obtained results under different optimism degrees can help the decision maker identify desired policies for a number of conflicting objectives. Table 5 shows the solutions obtained from AOWA for the case study and their corresponding ranking orders of seven power stations.

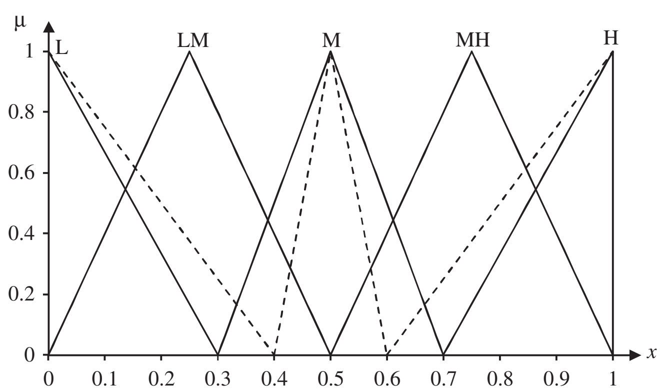

Fig. 4. Triangular fuzzy numbers with different left–right benchmarks (symbols "*L*", "LM", "*M*", "MH" and "*H*" denote "low", "low to medium", "medium", "medium to high" and "high", respectively).

Table 3
Linguistic quantifiers-equivalent normalized triangular fuzzy numbers-crisp data.

Linguistic Variables	Triangular fuzzy numbers	Crisp data	
		MAX	COG
Low	(0.00, 0.00, 0.30)	0.00	0.10
Low to medium	(0.25, 0.25, 0.25)	0.25	0.25
Medium	(0.50, 0.20, 0.20)	0.50	0.50
Medium to high	(0.75, 0.25, 0.25)	0.75	0.75
High	(1.00, 0.30, 0.00)	1.00	0.90

Note: MAX = membership method.

When $\alpha=0.3$, shares taken by coal-fired power station, hydropower station, nuclear power station, natural gas-fired power station, wind power station, biomass and oil/diesel-fired power stations over the next 5 years would be 0.0987, 0.3462, 0.1371, 0.1799, 0.0977, 0.0743 and 0.0660, respectively, and the ranking orders of them are 4, 1, 3, 2, 5, 6 and 7, respectively. This is a pessimistic scenario, which means that the decision maker keeps a conservative attitude, and thus relatively more capital and resources would be put to use in order to meet local demand for electricity.

When $\alpha=0.5$, it means that the decision maker keeps neutral attitude. The results and the ranking orders of seven power stations are (0.1006, 0.3490, 0.1381, 0.1834, 0.0993, 0.0635, 0.0661) and (4, 1, 3, 2, 5, 7, 6).

When $\alpha=0.7$, the integrated goodness scorings would be 0.1020 (for coal-fired power station), 0.3536 (for hydropower station), 0.1386 (for nuclear power station), 0.1879, 0.1132, 0.0373 and 0.0675 (for natural gas-fired power station, wind power station, biomass power station and oil/diesel-fired power station, respectively), and the ranking orders of seven power stations are 5, 1, 3, 2, 4, 7 and 6, respectively. This implies that the usage of less capital and resources can ensure local demand for electricity.

Based on the above description, it can be concluded that the three rank results are basically uniform for hydropower station, natural gas-fired power station and nuclear power station which are the first three ranks. The coal-fired power station and wind power station take the fourth and fifth place when $\alpha=0.3$ and 0.5, respectively, and exchange when $\alpha=0.7$. Oil/diesel-fired power station comes at the last place when $\alpha=0.3$, while biomass power station gets the least scoring when $\alpha=0.5$ and 0.7. In addition, the specific share taken by each power station would vary slightly. All of these result from the different α values, meaning different optimism degrees. In general, a higher α value represents a relatively high degree of optimism but a higher risk level. Under this condition, the decision maker can select the optimal alternative according to his/her optimism degree.

Table 5
Results from AOWA under different α values.

Power stations	Integrated goodness scoring			Ranking orders		
	$\alpha=0.3$	$\alpha=0.5$	$\alpha=0.7$	$\alpha=0.3$	$\alpha=0.5$	$\alpha=0.7$
Coal-fired	0.0987	0.1006	0.1020	4	4	5
Hydro	0.3462	0.3490	0.3536	1	1	1
Nuclear	0.1371	0.1381	0.1386	3	3	3
Natural gas-fired	0.1799	0.1834	0.1879	2	2	2
Wind	0.0977	0.0993	0.1132	5	5	4
Biomass	0.0743	0.0635	0.0373	6	7	7
Oil/diesel	0.0660	0.0661	0.0675	7	6	6

Table 4
New inputs of AOWA about seven power stations when $\alpha=0.5$.

Criteria	New inputs about seven power stations						
	Coal-fired	Hydro	Nuclear	Natural gas	Wind	Biomass	Oil/diesel
OMC	0.8378	0.8526	0.8670	0.6151	0.0000	0.1637	0.2191
Capital cost	0.7296	0.5224	0.6416	0.8670	0.0000	0.2240	0.6495
Energy intensity	0.0324	0.52130	0.0000	0.1454	0.7500	0.0724	0.1448
GHG intensity	0.0000	0.2496	0.2494	0.1265	0.2496	0.2459	0.2500
Retirement	0.0000	0.0521	0.0815	0.0962	0.1330	0.1269	0.0603
Current capacity	0.1712	0.7500	0.1222	0.1399	0.0000	0.0102	0.0688
Potential capacity	0.0051	0.3318	0.1208	0.1943	0.8670	0.0000	0.0000
Service life	0.1457	0.5000	0.0866	0.2244	0.0157	0.0472	0.0000

4.2. Comparison between AOWA and EOWA

The main difference between AOWA and EOWA is the process of converting normalized triangular fuzzy numbers into crisp data; this leads to the results of AOWA and EOWA are significantly different (as shown in Table 3). In EOWA, the max-membership method is used to defuzzy the normalized triangular fuzzy numbers; in comparison, COG method is used for defuzzying the normalized triangular fuzzy numbers in AOWA. In this part, there are two comparisons between AOWA and EOWA: one is reached under different α values and the other is obtained under changed criteria important degrees.

4.2.1. Results under different α values

Table 6 shows the solutions from EOWA for the corresponding ranking order of the seven power stations. The corresponding values of seven power stations are (0.1081, 0.3619, 0.1379, 0.1760, 0.0909, 0.0625, 0.0626) under $\alpha=0.3$, (0.1103, 0.3622, 0.1376, 0.1805, 0.0926, 0.0533, 0.0635) under $\alpha=0.5$, and (0.1137, 0.3647, 0.1365, 0.1864, 0.1059, 0.0269, 0.0659) under $\alpha=0.7$. Although the integrated goodness scorings of seven power stations under three α values are different, their ranking orders are the same. This means that the final decision support systems provided by EOWA cannot reflect the decision maker's optimism degree. Under this condition, it is not necessary for the existence of the optimism degrees, which is inconsistent with the actual situations. On the other hand, when $\alpha=0.3$, the ranking orders of biomass power station and oil/diesel power station are 7 and 6, respectively; when $\alpha=0.7$, the ranking orders of wind power station and coal-fired power station are 5 and 4, respectively. Both these are different from those obtained from AOWA. In addition, the specific generation shares obtained by EOWA and AOWA are also different for seven power stations under three α values. For example, when $\alpha=0.5$, the precise generation shares for coal-fired power station and hydropower station obtained from EOWA are higher than the results from AOWA, while the conditions are opposite for nuclear power station, natural gas-fired power station, wind power station, biomass power station and oil/diesel-fired power station. This is because different processes of defuzzying the normalized triangular fuzzy numbers in the two operators.

In EOWA, the crisp data of L, LM, M, MH and H are 0.00, 0.25, 0.50, 0.75 and 1.00, respectively. Compared with the ones in AOWA, the value of L is lower while the value of H is higher; thus the range between L and H is much larger. Furthermore, because of $H=1$, the important degrees of OMC and CC higher than the ones in AOWA. In this case, the integrated goodness scoring of coal-fired power station higher than the integrated goodness scoring of wind power station when $\alpha=0.7$, which is opposite for AOWA. On the other hand, due to $L=0$, the negative side of oil/diesel-fired power station is neglected when $\alpha=0.3$; thus biomass power station comes at the last place. Furthermore, the different crisp data about the triangular fuzzy numbers of two methods lead to the different generation shares for seven power stations, even under the same α value. Actually, the values of H and L resulted from the max-membership method are a relatively extreme case for practical problems. Comparatively, the crisp data of the normalized triangular fuzzy numbers defuzzied by COG method in Table 3 are more proper, and can more effectively reflect the decision maker's optimism degree. Hence the corresponding final solutions calculated by AOWA are more exact and sensitive. In comparison, AOWA can afford the decision maker with a more reasonable and sensitive decision support system than EOWA.

Table 6
Results from EOWA under different α values.

Power stations	Integrated goodness scoring			Ranking orders		
	$\alpha=0.3$	$\alpha=0.5$	$\alpha=0.7$	$\alpha=0.3$	$\alpha=0.5$	$\alpha=0.7$
Coal-fired	0.1081	0.1103	0.1137	4	4	4
Hydro	0.3619	0.3622	0.3647	1	1	1
Nuclear	0.1379	0.1376	0.1365	3	3	3
Natural gas-fired	0.1760	0.1805	0.1864	2	2	2
Wind	0.0909	0.0926	0.1059	5	5	5
Biomass	0.0625	0.0533	0.0269	7	7	7
Oil/diesel	0.0626	0.0635	0.0659	6	6	6

Table 7
Results from EOWA and AOWA under changed important degrees for criteria.

Power stations	EOWA ← Integrated goodness scoring → AOWA					
	$\alpha=0.3$	$\alpha=0.5$	$\alpha=0.7$	$\alpha=0.3$	$\alpha=0.5$	$\alpha=0.7$
Coal-fired	0.1081	0.1103	0.1137	0.0959	0.0976	0.0988
Hydro	0.3619	0.3622	0.3647	0.3419	0.3453	0.3496
Nuclear	0.1379	0.1376	0.1365	0.1371	0.1384	0.1419
Natural gas-fired	0.1760	0.1805	0.1864	0.1817	0.1846	0.1886
Wind	0.0909	0.0926	0.1059	0.1000	0.1014	0.1146
Biomass	0.0625	0.0533	0.0269	0.0765	0.0656	0.0385
Oil/diesel	0.0626	0.0635	0.0659	0.0668	0.0671	0.0681

4.2.2. Results under changed important degrees

Table 7 shows the results calculated by AOWA and EOWA under the changed important degrees for all criteria. It is pointed that the changed important degrees are only different in the left and right benchmarks of normalized triangular fuzzy numbers L, M and H, which are denoted by broken lines in Fig. 4. It can be obtained that neither the integrated goodness scorings nor ranking orders from EOWA have changed, which are just converse to the results from AOWA. The reason is that the changed important degrees for criteria don't change the max-membership degrees of normalized triangular fuzzy numbers. In this case, the crisp data of normalized triangular fuzzy numbers resulted from the max-membership method will not have any change; thus the solutions from EOWA are still same with the ones in Table 6. Conversely, since the COG method is sensitive to the changes of the normalized triangular fuzzy numbers, the crisp data of L and H have changed and caused the different solutions provided by AOWA. Compared with the original important degrees, the values of H and L become smaller and larger, respectively; correspondingly, the important degrees for criteria OMC, CC and R become lower, lower and higher, respectively. In this case, the ranking orders between wind power station and coal-fired power station have exchanged when $\alpha=0.3$ and 0.5.

In general, the results from AOWA are more effective than the ones from EOWA when the changes of important degrees for criteria have happened. Actually, these kinds of changes in MCDM problems indeed exist. For example, when the numbers of experts for making important degrees for criteria have changed or the local economic policy and/or other social factors have altered, changes as shown in Fig. 4 will happen and finally result in the changes of the decision making systems. Therefore, it can be concluded that AOWA is more sensitive and more suit for MCDM problems, and simultaneously supplies the decision maker with an optimal and reasonable decision support system.

5. Conclusions

In this study, an advanced ordered weighted averaging operator has been developed for dealing with multicriteria decision making problems under uncertainties. Based on the traditional OWA, this operator employs interval theory and center of gravity method. AOWA is capable of dealing with the linguistic quantifiers and the uncertain information which is given under optimistic and pessimistic conditions without knowing their distribution information. Therefore, it can make up the defects in OWA and EOWA. Meanwhile, the application of the vector method for solving COG of a normalized triangular fuzzy number in plane right angle coordinate makes the calculation procedure much simpler than the integration method. Furthermore, the adoption of COG method for defuzzying the normalized triangular fuzzy numbers makes AOWA more sensitive and exact in important degrees for criteria and optimism degrees. Consequently, AOWA can afford a reasonable and applicable decision support system under different optimism degrees.

The developed AOWA has then been applied to a case of planning electric power problems. The obtained results under different optimism degrees can be used for generating decision alternatives and thus help the decision maker identify desired policies for a number of conflicting objectives. In addition, two comparisons between AOWA and EOWA also further imply the superiority and sensitivity of AOWA. Although this study is the first attempt for planning electric power systems through the developed AOWA operator, the results suggest that this operator is applicable for other MCDM problems containing uncertainties and linguistic quantifiers. Furthermore, the developed method can also be used to advance the fuzzy logic controllers based on genetic algorithm. Since the factors effecting on controllers include not only the size of every fuzzy set membership function covering domains but also the every fuzzy set membership function shape, for simplicity and feasibility of the algorithm, the simplest isosceles triangle membership function is generally chosen and used as a defuzzying technique; this may neglect lots of useful information in real-case problems and reduce the effectiveness of the controllers. On the other hand, the AOWA can defuzzy triangular fuzzy numbers with simple algorithm but without assuming the triangular fuzzy number is isosceles; meanwhile, it can provide more sensitive results than the max-membership method for defuzzification. Therefore, the introduction of AOWA into the fuzzy logic controllers may be an interesting research work in future.

Acknowledgments

This research has been supported by the Program for New Century Excellent Talents in University (NCET-10-0376) and the Key Project of Chinese Ministry of Education. The authors are grateful to the editors and the anonymous reviewers for their insightful comments and suggestions.

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