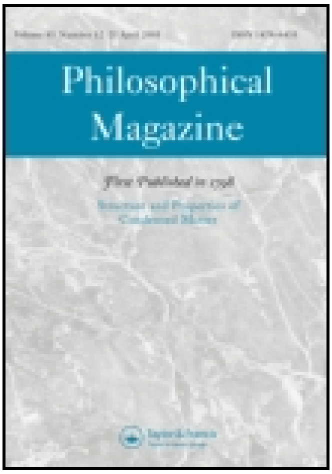

Philosophical Magazine
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tphm20

Non-local theory solution of two collinear mode-I permeable cracks in a magnetoelectroelastic composite material plane

Zhen-Gong Zhou [a] , Pei-Wei Zhang [a] & Lin-Zhi Wu [a]
[a] Center for Composite Materials and Structures, Harbin Institute of Technology , P.O. Box 3010, No. 2 Yikuang Street, Harbin 150080, China
Published online: 20 Apr 2010.

Philosophical Magazine
Vol. 90, No. 13, 7 May 2010, 1743–1769

Non-local theory solution of two collinear mode-I permeable cracks in a magnetoelectroelastic composite material plane

Zhen-Gong Zhou*, Pei-Wei Zhang and Lin-Zhi Wu

Center for Composite Materials and Structures, Harbin Institute of Technology, P.O. Box 3010, No. 2 Yikuang Street, Harbin 150080, China

(*Received 19 August 2009; final version received 6 November 2009*)

The non-local theory solution of two collinear mode-I permeable cracks in a magnetoelectroelastic composite material plane was investigated using the generalized Almansi's theorem and the Schmidt method. The problem was formulated through Fourier transform into two pairs of dual integral equations, in which the unknown variables are the jumps in displacements across the crack surfaces. To solve the dual integral equations, the displacement jumps across the crack surfaces were directly expanded as a series of Jacobi polynomials. Numerical examples were provided to show the effects of crack length, the distance between two collinear cracks and the lattice parameter on the stress field, the electric displacement field and the magnetic flux field near the crack tips. Unlike the classical elasticity solutions, it is found that no stress, electric displacement or magnetic flux singularities are present at the crack tips in a magnetoelectroelastic composite material plane. The non-local elastic solutions yield a finite hoop stress at the crack tip, thus allowing us to use the maximum stress as a fracture criterion.

Keywords: magnetoelectroelastic composite materials; two collinear cracks; non-local theory; Schmidt method; lattice parameter

1. Introduction

Combining two or more distinct piezoelectric and piezomagnetic (magnetostrictive) constituents, the resulting piezoelectric/piezomagnetic composite material can assume the advantages of each constituent and, consequently, have superior coupling magnetoelectric properties compared to conventional piezoelectric or piezomagnetic materials. Magnetoelectric coupling is a new product property of the composite, since it is absent in each constituent. Consequently, they are extensively used as electric packaging, sensors and actuators, magnetic field probes, acoustic/ultrasonic devices, hydrophones, and transducers with the responsibility of electro-magneto-mechanical energy conversion [1]. Therefore, the study of fracture mechanics of magnetoelectroelastic composite materials is important in the design of magnetoelectroelastic composite devices and magnetoelectroelastic composite structures, such as dampers in controlling structural vibration, sensors and actuators

*Corresponding author. Email: zhouzhg@hit.edu.cn

ISSN 1478–6435 print/ISSN 1478–6443 online

DOI: 10.1080/14786430903473007
http://www.informaworld.com

in adhesively bonded joints and sensors in non-destructive testing. There have been numerous studies on fracture parameters, stress intensity factors, electric displacement intensity factors, magnetic flux intensity factors and energy release rate [2–12] under different electric boundary conditions on crack surfaces. It is interesting to note that very different results were obtained when changing the boundary conditions [13]. However, these solutions contain stress singularities at the crack tips, which is not reasonable according to physical nature. In fact, the stresses at the crack tips are finite. As a result, beginning with Griffith, all fracture criteria in practice today are based on other considerations, such as energy and the *J*-integral [14].

In contrast to the local approach of zero-range internal interactions, modern non-local continuum mechanics, developed over the last four decades, postulates that the local state at a point of a body is influenced by the action of all particles of that body. This theory was constructed primarily by Edelen [15], Eringen [16], and Green and Rivlin [17]. Pan and Takeda [18] described the basic theory of non-local elasticity with emphasis on the difference between the non-local theory and classical continuum mechanics. To overcome the stress singularity at the crack tips in the classical elastic theory, Eringen [19–21] used non-local theory to investigate the stress near the tip of a sharp line crack in an isotropic elastic plate subject to uniform tension, shear and anti-plane shear loading. His solutions did not contain any stress singularities at the crack tips. This enables us to employ the maximum stress criterion to predict fracture in a more natural way. However, it is well-known that there is disagreement between Eringen and co-workers [19–21] and Atkinson [22,23] over the non-local mixed boundary value problem of crack. As discussed by Cheng [24], the non-local theory can be used to solve brittle fracture problems, i.e. it can be used to solve fracture problems in piezoelectric and magnetoelectroelastic materials. Recently, the non-local theory was used to study fracture problems in piezoelectric materials [25,26]. As expected, the solutions did not contain any stress and electric displacement singularities at the crack tips. However, to the best of our knowledge, the magneto-electro-elastic behavior of magnetoelectroelastic composite materials with two collinear mode-I permeable cracks has not been studied by the non-local theory.

2. Basic equations of the non-local magnetoelectroelastic composite materials

For the plane problem, the basic equations of linear, homogeneous, non-local transversely isotropic magnetoelectroelastic composite materials, with vanishing body force, are written as follows [6,10,25–27]:

$$\begin{cases} \dfrac{\partial \tau_{xx}(x,z)}{\partial x} + \dfrac{\partial \tau_{xz}(x,z)}{\partial z} = 0 \\ \dfrac{\partial \tau_{xz}(x,z)}{\partial x} + \dfrac{\partial \tau_{zz}(x,z)}{\partial z} = 0 \\ \dfrac{\partial D_x(x,z)}{\partial x} + \dfrac{\partial D_z(x,z)}{\partial z} = 0 \\ \dfrac{\partial B_x(x,z)}{\partial x} + \dfrac{\partial B_z(x,z)}{\partial z} = 0 \end{cases} \tag{1}$$

$$\tau_{xx}(x,z)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[c_{11}^{*}\left(|x'-x|,|z'-z|\right)\frac{\partial u(x',z')}{\partial x}+c_{13}^{*}\left(|x'-x|,|z'-z|\right)\frac{\partial w(x',z')}{\partial z}\right.$$
$$\left.+\,e_{31}^{*}\left(|x'-x|,|z'-z|\right)\frac{\partial \phi(x',z')}{\partial z}-f_{31}^{*}\left(|x'-x|,|z'-z|\right)\frac{\partial \psi(x',z')}{\partial z}\right]\mathrm{d}x'\mathrm{d}z' \tag{2}$$

$$\tau_{zz}(x,z)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[c_{13}^{*}\left(|x'-x|,|z'-z|\right)\frac{\partial u(x',z')}{\partial x}+c_{33}^{*}\left(|x'-x|,|z'-z|\right)\frac{\partial w(x',z')}{\partial z}\right.$$
$$\left.+\,e_{33}^{*}\left(|x'-x|,|z'-z|\right)\frac{\partial \phi(x',z')}{\partial z}-f_{33}^{*}\left(|x'-x|,|z'-z|\right)\frac{\partial \psi(x',z')}{\partial z}\right]\mathrm{d}x'\mathrm{d}z' \tag{3}$$

$$\tau_{xz}(x,z)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[c_{44}^{*}\left(|x'-x|,|z'-z|\right)\left(\frac{\partial u(x',z')}{\partial z}+\frac{\partial w(x',z')}{\partial x}\right)\right.$$
$$\left.+\,e_{15}^{*}\left(|x'-x|,|z'-z|\right)\frac{\partial \phi(x',z')}{\partial x}-f_{15}^{*}\left(|x'-x|,|z'-z|\right)\frac{\partial \psi(x',z')}{\partial x}\right]\mathrm{d}x'\mathrm{d}z' \tag{4}$$

$$D_{x}(x,z)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[e_{15}^{*}\left(|x'-x|,|z'-z|\right)\left(\frac{\partial u(x',z')}{\partial z}+\frac{\partial w(x',z')}{\partial x}\right)\right.$$
$$\left.-\,\varepsilon_{11}^{*}\left(|x'-x|,|z'-z|\right)\frac{\partial \phi(x',z')}{\partial x}-g_{11}^{*}\left(|x'-x|,|z'-z|\right)\frac{\partial \psi(x',z')}{\partial x}\right]\mathrm{d}x'\mathrm{d}z' \tag{5}$$

$$D_{z}(x,z)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[e_{31}^{*}\left(|x'-x|,|z'-z|\right)\frac{\partial u(x',z')}{\partial x}+e_{33}^{*}\left(|x'-x|,|z'-z|\right)\frac{\partial w(x',z')}{\partial z}\right.$$
$$\left.-\,\varepsilon_{33}^{*}\left(|x'-x|,|z'-z|\right)\frac{\partial \phi(x',z')}{\partial z}-g_{33}^{*}\left(|x'-x|,|z'-z|\right)\frac{\partial \psi(x',z')}{\partial z}\right]\mathrm{d}x'\mathrm{d}z' \tag{6}$$

$$B_{x}(x,z)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[f_{15}^{*}\left(|x'-x|,|z'-z|\right)\left(\frac{\partial u(x',z')}{\partial z}+\frac{\partial w(x',z')}{\partial x}\right)\right.$$
$$\left.+\,g_{11}^{*}\left(|x'-x|,|z'-z|\right)\frac{\partial \phi(x',z')}{\partial x}-\mu_{11}^{*}\left(|x'-x|,|z'-z|\right)\frac{\partial \psi(x',z')}{\partial x}\right]\mathrm{d}x'\mathrm{d}z' \tag{7}$$

$$B_{z}(x,z)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[f_{31}^{*}\left(|x'-x|,|z'-z|\right)\frac{\partial u(x',z')}{\partial x}+f_{33}^{*}\left(|x'-x|,|z'-z|\right)\frac{\partial w(x',z')}{\partial z}\right.$$
$$\left.+\,g_{33}^{*}\left(|x'-x|,|z'-z|\right)\frac{\partial \phi(x',z')}{\partial z}-\mu_{33}^{*}\left(|x'-x|,|z'-z|\right)\frac{\partial \psi(x',z')}{\partial z}\right]\mathrm{d}x'\mathrm{d}z', \tag{8}$$

where the only difference from classical magneto-electro-elastic theory is in the stress, electric displacement and magnetic flux constitutive equations, i.e. Equations (2)–(8), in which the stress $\tau_{ik}(x,z)$ $(i,k=x,z)$, electric displacement $D_k(x,z)$ $(k=x,z)$ and magnetic flux $B_k(x,z)$ $(k=x,z)$ at a point (x,z) depend on $u_{,k}(x,z)$, $w_{,k}(x,z)$, $\phi_{,k}(x,z)$ and $\psi_{,k}(x,z)$ $(k=x,\ y)$, at all points of the body. $\tau_{ik}(x,z)$, $D_k(x,z)$ and $B_k(x,z)$ $(i=x,z,\ k=x,z)$ are the plane stresses, in-plane electric displacements and in-plane magnetic fluxes, respectively. $u(x,z)$ and $w(x,z)$ are the mechanical displacements. $\phi(x,z)$ and $\psi(x,z)$ are the electric potential and magnetic potential, respectively. $c^*_{11}(|x'-x|,|z'-z|)$, $c^*_{13}(|x'-x|,|z'-z|)$, $c^*_{33}(|x'-x|,|z'-z|)$ and $c^*_{44}(|x'-x|,|z'-z|)$ are the non-local elastic stiffness parameters, $\varepsilon^*_{11}(|x'-x|,|z'-z|)$ and $\varepsilon^*_{33}(|x'-x|,|z'-z|)$ are the non-local dielectric parameters, $e^*_{15}(|x'-x|,|z'-z|)$, $e^*_{31}(|x'-x|,|z'-z|)$ and $e^*_{33}(|x'-x|,|z'-z|)$ are the non-local piezoelectric parameters, $f^*_{15}(|x'-x|,|z'-z|)$, $f^*_{31}(|x'-x|,|z'-z|)$ and $f^*_{33}(|x'-x|,|z'-z|)$ are the non-local piezomagnetic parameters, $g^*_{11}(|x'-x|,|z'-z|)$ and $g^*_{33}(|x'-x|,|z'-z|)$ are the non-local electromagnetic parameters, and $\mu^*_{11}(|x'-x|,|z'-z|)$ and $\mu^*_{33}(|x'-x|,|z'-z|)$ are the non-local magnetic permeability parameters, which are functions of the distance $d=\sqrt{(x'-x)^2+(y'-y)^2}$ between the point of interest and an arbitrary point in the body. As discussed in [28], the forms of $c^*_{11}(|x'-x|,|z'-z|)$, $c^*_{13}(|x'-x|,|z'-z|)$, $c^*_{33}(|x'-x|,|z'-z|)$, $c^*_{44}(|x'-x|,|z'-z|)$, $\varepsilon^*_{11}(|x'-x|,|z'-z|)$, $\varepsilon^*_{33}(|x'-x|,|z'-z|)$, $e^*_{15}(|x'-x|,|z'-z|)$, $e^*_{31}(|x'-x|,|z'-z|)$, $e^*_{33}(|x'-x|,|z'-z|)$, $f^*_{15}(|x'-x|,|z'-z|)$, $f^*_{31}(|x'-x|,|z'-z|)$, $f^*_{33}(|x'-x|,|z'-z|)$, $g^*_{11}(|x'-x|,|z'-z|)$, $g^*_{33}(|x'-x|,|z'-z|)$, $\mu^*_{11}(|x'-x|,|z'-z|)$ and $\mu^*_{33}(|x'-x|,|z'-z|)$ can be assumed as follows:

$$\begin{cases} [c^*_{11}(|x'-x|,|z'-z|),c^*_{13}(|x'-x|,|z'-z|),c^*_{33}(|x'-x|,|z'-z|)] \\ \quad =\alpha(|x'-x|,|z'-z|)[c_{11},c_{13},c_{33}] \\ [c^*_{44}(|x'-x|,|z'-z|),\varepsilon^*_{11}(|x'-x|,|z'-z|),\varepsilon^*_{33}(|x'-x|,|z'-z|)] \\ \quad =\alpha(|x'-x|,|z'-z|)[c_{44},\varepsilon_{11},\varepsilon_{33}] \\ [e^*_{15}(|x'-x|,|z'-z|),e^*_{31}(|x'-x|,|z'-z|),e^*_{33}(|x'-x|,|z'-z|)] \\ \quad =\alpha(|x'-x|,|z'-z|)[e_{15},e_{31},e_{33}] \\ [f^*_{15}(|x'-x|,|z'-z|),f^*_{31}(|x'-x|,|z'-z|),f^*_{33}(|x'-x|,|z'-z|)] \\ \quad =\alpha(|x'-x|,|z'-z|)[f_{15},f_{31},f_{33}] \\ [g^*_{11}(|x'-x|,|z'-z|),g^*_{33}(|x'-x|,|z'-z|),\mu^*_{11}(|x'-x|,|z'-z|)] \\ \quad =\alpha(|x'-x|,|z'-z|)[g_{11},g_{33},\mu_{11}] \\ \mu^*_{33}(|x'-x|,|z'-z|)=\alpha(|x'-x|,|z'-z|)\mu_{33}, \end{cases} \tag{9}$$

$$\alpha(|x'-x|,|z'-z|)=\alpha_0\exp\{-(\beta/a)^2[(x'-x)^2+(z'-z)^2]\}, \tag{10}$$

where $\alpha(|x'-x|,|z'-z|)$ is known as the influential function. β and a are material constants. Normally, a is a known constant as the characteristic length of a material, which may be selected according to the range and sensitivity of the physical phenomena to be investigated. For perfect crystals, a may be taken as the lattice parameter. For granular materials, a may be considered to be the average granular distance and, for fiber composites, the fiber distance, etc. The material constant

β may be determined experimentally. $c_{11}, c_{13}, c_{33}, c_{44}$, $\varepsilon_{11}, \varepsilon_{33}$, e_{15}, e_{31}, e_{33}, f_{15}, f_{31}, f_{33}, g_{11}, g_{33}, μ_{11} and μ_{33} are elastic, dielectric, piezoelectric, piezomagnetic, electromagnetic and magnetic constants, respectively. α_0 is determined by the normalization:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \alpha(|x'-x|, |z'-z|)\mathrm{d}x'\mathrm{d}z' = 1. \tag{11}$$

Substituting Equations (9) and (10) into Equation (11), we can obtain, in a three-dimensional space,

$$\alpha_0 = \frac{1}{\pi}(\beta/a)^2. \tag{12}$$

Substituting Equations (2)–(12) into Equation (1) and using the Green–Gauss theorem, we have

$$\begin{aligned}
&\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \alpha(|x'-x|, |z'-z|)\Bigg[c_{11}\frac{\partial^2 u(x',z')}{\partial x^2} + c_{44}\frac{\partial^2 u(x',z')}{\partial z^2} + (c_{13}+c_{44})\frac{\partial^2 w(x',z')}{\partial x \partial z}\\
&\quad + (e_{31}+e_{15})\frac{\partial^2 \phi(x',z')}{\partial x \partial z} - (f_{31}+f_{15})\frac{\partial^2 \psi(x',z')}{\partial x \partial z}\Bigg]\mathrm{d}x'\mathrm{d}z'\\
&\quad - \left[\int_{-1}^{-l} + \int_{l}^{1}\right]\alpha(|x'-x|, |0|)[\sigma_{xz}(x',0^+) - \sigma_{xz}(x',0^-)]\mathrm{d}x' = 0,
\end{aligned} \tag{13}$$

$$\begin{aligned}
&\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \alpha(|x'-x|, |z'-z|)\Bigg[(c_{13}+c_{44})\frac{\partial^2 u(x',z')}{\partial x \partial z} + c_{44}\frac{\partial^2 w(x',z')}{\partial x^2} + c_{33}\frac{\partial^2 w(x',z')}{\partial z^2}\\
&\quad + e_{15}\frac{\partial^2 \phi(x',z')}{\partial x^2} + e_{33}\frac{\partial^2 \phi(x',z')}{\partial z^2} - f_{15}\frac{\partial^2 \psi(x',z')}{\partial x^2} - f_{33}\frac{\partial^2 \psi(x',z')}{\partial z^2}\Bigg]\mathrm{d}x'\mathrm{d}z'\\
&\quad - \left[\int_{-1}^{-l} + \int_{l}^{1}\right]\alpha(|x'-x|, |0|)[\sigma_{zz}(x',0^+) - \sigma_{zz}(x',0^-)]\mathrm{d}x' = 0,
\end{aligned} \tag{14}$$

$$\begin{aligned}
&\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \alpha(|x'-x|, |z'-z|)\Bigg[(e_{15}+e_{31})\frac{\partial^2 u(x',z')}{\partial x \partial z} + e_{15}\frac{\partial^2 w(x',z')}{\partial x^2} + e_{33}\frac{\partial^2 w(x',z')}{\partial z^2}\\
&\quad - \varepsilon_{11}\frac{\partial^2 \phi(x',z')}{\partial x^2} - \varepsilon_{33}\frac{\partial^2 \phi(x',z')}{\partial z^2} - g_{11}\frac{\partial^2 \psi(x',z')}{\partial x^2} - g_{33}\frac{\partial^2 \psi(x',z')}{\partial z^2}\Bigg]\mathrm{d}x'\mathrm{d}z'\\
&\quad - \left[\int_{-1}^{-l} + \int_{l}^{1}\right]\alpha(|x'-x|, |0|)[D_z(x',0^+) - D_z(x',0^-)]\mathrm{d}x' = 0,
\end{aligned} \tag{15}$$

$$\begin{aligned}
&\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \alpha(|x'-x|, |z'-z|)\Bigg[(f_{15}+f_{31})\frac{\partial^2 u(x',z')}{\partial x \partial z} + f_{15}\frac{\partial^2 w(x',z')}{\partial x^2} + f_{33}\frac{\partial^2 w(x',z')}{\partial z^2}\\
&\quad + g_{11}\frac{\partial^2 \phi(x',z')}{\partial x^2} + g_{33}\frac{\partial^2 \phi(x',z')}{\partial z^2} - \mu_{11}\frac{\partial^2 \psi(x',z')}{\partial x^2} - \mu_{33}\frac{\partial^2 \psi(x',z')}{\partial z^2}\Bigg]\mathrm{d}x'\mathrm{d}z'\\
&\quad - \left[\int_{-1}^{-l} + \int_{l}^{1}\right]\alpha(|x'-x|, |0|)[B_z(x',0^+) - B_z(x',0^-)]\mathrm{d}x' = 0,
\end{aligned} \tag{16}$$

where

$$\begin{cases} \sigma_{zz}(x,z) = c_{13}\dfrac{\partial u(x,z)}{\partial x} + c_{33}\dfrac{\partial w(x,z)}{\partial z} + e_{33}\dfrac{\partial \phi(x,z)}{\partial z} - f_{33}\dfrac{\partial \psi(x,z)}{\partial z} \\ D_z(x,z) = e_{31}\dfrac{\partial u(x,z)}{\partial x} + e_{33}\dfrac{\partial w(x,z)}{\partial z} - \varepsilon_{33}\dfrac{\partial \phi(x,z)}{\partial z} - g_{33}\dfrac{\partial \psi(x,z)}{\partial z}. \\ B_z(x,z) = f_{31}\dfrac{\partial u(x,z)}{\partial x} + f_{33}\dfrac{\partial w(x,z)}{\partial z} + g_{33}\dfrac{\partial \phi(x,z)}{\partial z} - \mu_{33}\dfrac{\partial \psi(x,z)}{\partial z} \end{cases} \tag{17}$$

The expressions in Equation (17) are the classical Hooke's law. $1-l$ is the length of the crack, which will be discussed shortly. It is worth noting that the surface integrals in Equations (13)–(16) may be dropped, since the displacement, electric displacement and magnetic flux fields vanish at infinity.

As discussed in [19], we have that $\sigma_{zz}(x,0^+) - \sigma_{zz}(x,0^-) = 0$, $D_z(x,0^+) - D_z(x,0^-) = 0$ and $B_z(x,0^+) - B_z(x,0^-) = 0$. Therefore, it can be shown that the general solutions of Equations (13)–(16) are identical to those of the following four differential equations almost everywhere over the entire plane:

$$\begin{cases} \left(c_{11}\frac{\partial^2 u(x,z)}{\partial x^2} + c_{44}\frac{\partial^2 u(x,z)}{\partial z^2}\right) + (c_{13}+c_{44})\frac{\partial^2 w(x,z)}{\partial x \partial z} \\ \quad + (e_{31}+e_{15})\frac{\partial^2 \phi(x,z)}{\partial x \partial z} - (f_{31}+f_{15})\frac{\partial^2 \psi(x,z)}{\partial x \partial z} = 0 \\ (c_{13}+c_{44})\frac{\partial^2 u(x,z)}{\partial x \partial z} + c_{44}\frac{\partial^2 w(x,z)}{\partial x^2} + c_{33}\frac{\partial^2 w(x,z)}{\partial z^2} \\ \quad + e_{15}\frac{\partial^2 \phi(x,z)}{\partial x^2} + e_{33}\frac{\partial^2 \phi(x,z)}{\partial z^2} - f_{15}\frac{\partial^2 \psi(x,z)}{\partial x^2} - f_{33}\frac{\partial^2 \psi(x,z)}{\partial z^2} = 0, \end{cases} \tag{18}$$

$$\begin{cases} (e_{15}+e_{31})\frac{\partial^2 u(x,z)}{\partial x \partial z} + e_{15}\frac{\partial^2 w(x,z)}{\partial x^2} + e_{33}\frac{\partial^2 w(x,z)}{\partial z^2} \\ \quad - \varepsilon_{11}\frac{\partial^2 \phi(x,z)}{\partial x^2} - \varepsilon_{33}\frac{\partial^2 \phi(x,z)}{\partial z^2} - g_{11}\frac{\partial^2 \psi(x,z)}{\partial x^2} - g_{33}\frac{\partial^2 \psi(x,z)}{\partial z^2} = 0 \\ (f_{15}+f_{31})\frac{\partial^2 u(x,z)}{\partial x \partial z} + f_{15}\frac{\partial^2 w(x,z)}{\partial x^2} + f_{33}\frac{\partial^2 w^{(j)}(x,z)}{\partial z^2} \\ \quad + g_{11}\frac{\partial^2 \phi(x,z)}{\partial x^2} + g_{33}\frac{\partial^2 \phi(x,z)}{\partial z^2} - \mu_{11}\frac{\partial^2 \psi(x,z)}{\partial x^2} - \mu_{33}\frac{\partial^2 \psi(x,z)}{\partial z^2} = 0. \end{cases} \tag{19}$$

3. Mode-I crack

It was assumed that there are two mode-I Griffith cracks of length $1-l$ along the x-axis in the magnetoelectroelastic composite material plane, as shown in Figure 1. $2l$ is the distance between the two cracks. (The solution of two collinear cracks of length $r-l$ in magnetoelectroelastic composite materials can easily be obtained by a simple change in the numerical values of the present paper for crack length $1-l/r$. $r > l > 0$.) As discussed in [29], the thickness of the crack is very small. So, in the present paper, the electric potential, magnetic potential, normal electric displacement and the normal magnetic flux were assumed to be continuous across the crack surfaces, i.e. a permeable crack mode was adopted. It was assumed that a distributed normal stress loading $\tau_{zz}(x,0) = -\tau_0$ (here, τ_0 is the magnitude of the uniform tension stress loading) was directly applied on the upper crack and the lower crack surfaces, which is equivalent to investigating the perturbation fields for a remotely loaded cracked-body through the standard superposition technique in

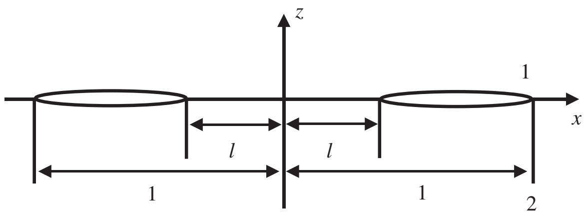

Figure 1. Geometry and coordinate system for two collinear cracks.

fracture mechanics. Therefore, the boundary conditions can be written, respectively, as follows:

$$\left\{\begin{array}{ll}\tau_{xz}^{(1)}(x,0^+)=\tau_{xz}^{(2)}(x,0^-)=0, & \tau_{zz}^{(1)}(x,0^+)=\tau_{zz}^{(2)}(x,0^-)=-\tau_0\\ \phi^{(1)}(x,0^+)=\phi^{(2)}(x,0^-), & \psi^{(1)}(x,0^+)=\psi^{(2)}(x,0^-)\\ D_z^{(1)}(x,0^+)=D_z^{(2)}(x,0^-), & B_z^{(1)}(x,0^+)=B_z^{(2)}(x,0^-)\end{array}\right., \quad l<|x|\leq 1 \tag{20}$$

$$\left\{\begin{array}{ll}u^{(1)}(x,0^+)=u^{(2)}(x,0^-), & w^{(1)}(x,0^+)=w^{(2)}(x,0^-)\\ \tau_{zz}^{(1)}(x,0^+)=\tau_{zz}^{(2)}(x,0^-), & \tau_{xz}^{(1)}(x,0^+)=\tau_{xz}^{(2)}(x,0^-)\\ \phi^{(1)}(x,0^+)=\phi^{(2)}(x,0^-), & \psi^{(1)}(x,0^+)=\psi^{(2)}(x,0^-)\\ D_z^{(1)}(x,0^+)=D_z^{(2)}(x,0^-), & B_z^{(1)}(x,0^+)=B_z^{(2)}(x,0^-)\end{array}\right., |x|>1 \quad \text{and} \quad 0\leq|x|<l \tag{21}$$

$$u^{(j)}(x,z)=w^{(j)}(x,z)=\phi^{(j)}(x,z)=\psi^{(j)}(x,z)=0 \text{ for } \sqrt{x^2+z^2}\rightarrow\infty, \tag{22}$$

where $\tau_{ik}^{(j)}(x,z)$ $(i=x,z,\ k=x,z,\ j=1,2)$ are the stresses, $u^{(j)}(x,z)$ and $w^{(j)}(x,z)$ represent the displacement components in the x- and z-axis directions, in which all quantities with superscript j correspond to the upper half plane 1 (for $j=1$) and the lower half plane 2 (for $j=2$) as shown in Figure 1, respectively.

4. Solution procedures

Equations (18) and (19) can be solved using of the generalized Almansi's theorem as given by Yang [30]. As expression in Yang's work [30], Equations (18) and (19) can be rewritten as follows:

$$[MD]\left\{\begin{array}{c}u(x,z)\\ w(x,z)\\ \phi(x,z)\\ \psi(x,z)\end{array}\right\}=0, \tag{23}$$

where the operator is

$$[MD] = \begin{bmatrix} c_{11}\frac{\partial^2}{\partial x^2} + c_{44}\frac{\partial^2}{\partial z^2} & (c_{13}+c_{44})\frac{\partial^2}{\partial x\partial z} & (e_{31}+e_{15})\frac{\partial^2}{\partial x\partial z} & -(f_{31}+f_{15})\frac{\partial^2}{\partial x\partial z} \\ (c_{13}+c_{44})\frac{\partial^2}{\partial x\partial z} & c_{44}\frac{\partial^2}{\partial x^2} + c_{33}\frac{\partial^2}{\partial z^2} & e_{15}\frac{\partial^2}{\partial x^2} + e_{33}\frac{\partial^2}{\partial z^2} & -f_{15}\frac{\partial^2}{\partial x^2} - f_{33}\frac{\partial^2}{\partial z^2} \\ (e_{15}+e_{31})\frac{\partial^2}{\partial x\partial z} & e_{15}\frac{\partial^2}{\partial x^2} + e_{33}\frac{\partial^2}{\partial z^2} & -\varepsilon_{11}\frac{\partial^2}{\partial x^2} - \varepsilon_{33}\frac{\partial^2}{\partial z^2} & -g_{11}\frac{\partial^2}{\partial x^2} - g_{33}\frac{\partial^2}{\partial z^2} \\ (f_{15}+f_{31})\frac{\partial^2}{\partial x\partial z} & f_{15}\frac{\partial^2}{\partial x^2} + f_{33}\frac{\partial^2}{\partial z^2} & g_{11}\frac{\partial^2}{\partial x^2} + g_{33}\frac{\partial^2}{\partial z^2} & -\mu_{11}\frac{\partial^2}{\partial x^2} - \mu_{33}\frac{\partial^2}{\partial z^2} \end{bmatrix}.$$

The determinant of $[MD]$ is

$$\det[MD] = a\frac{\partial^8}{\partial z^8} + b\frac{\partial^8}{\partial x^2\partial z^6} + c\frac{\partial^8}{\partial x^4\partial z^4} + d\frac{\partial^8}{\partial x^6\partial z^2} + e\frac{\partial^8}{\partial x^8},$$

in which a, b, c, d and e are given in Appendix 1. They are constants that only depend on the properties of the materials.

Based on the cofactors Δ_{ik} of det $[MD]$ (i, $k=1$, 2, 3, 4), and the method developed in [8,30], the general solutions of Equation (23) are expressed as follows:

$$[u(x,z), w(x,z), \phi(x,z), \psi(x,z)]^T = [\Delta_{i1}, \Delta_{i2}, \Delta_{i3}, \Delta_{i4}]^T F(x,z), \tag{24}$$

with $F(x,z)$ satisfying the equation

$$\det[MD]F(x,z) = 0. \tag{25}$$

In the following analysis, we use only ($\Delta_{21}, \Delta_{22}, \Delta_{23}, \Delta_{24}$) for problems symmetric about the z-axis:

$$\begin{cases} \Delta_{21} = \alpha_{11}\frac{\partial^6}{\partial x^5\partial z} + \alpha_{12}\frac{\partial^6}{\partial x^3\partial z^3} + \alpha_{13}\frac{\partial^6}{\partial x\partial z^5} \\ \Delta_{22} = \alpha_{21}\frac{\partial^6}{\partial x^6} + \alpha_{22}\frac{\partial^6}{\partial x^4\partial z^2} + \alpha_{23}\frac{\partial^6}{\partial x^2\partial z^4} + \alpha_{24}\frac{\partial^6}{\partial z^6} \\ \Delta_{23} = \alpha_{31}\frac{\partial^6}{\partial x^6} + \alpha_{32}\frac{\partial^6}{\partial x^4\partial z^2} + \alpha_{33}\frac{\partial^6}{\partial x^2\partial z^4} + \alpha_{34}\frac{\partial^6}{\partial z^6} \\ \Delta_{24} = \alpha_{41}\frac{\partial^6}{\partial x^6} + \alpha_{42}\frac{\partial^6}{\partial x^4\partial z^2} + \alpha_{43}\frac{\partial^6}{\partial x^2\partial z^4} + \alpha_{44}\frac{\partial^6}{\partial z^6}, \end{cases} \tag{26}$$

where α_{ik} $(i=1,2,3,4; k=1,2,3,4)$ can be obtained in Appendix 2. They are constants that only depend on the properties of the materials.

Using its symmetry on the z-axis and the Fourier transform on x, $F(x,z)$ can be expressed as follows:

$$F(x,z) = \frac{2}{\pi}\int_0^\infty f(s,z)\cos(sx)\mathrm{d}s. \tag{27}$$

Substituting Equation (27) into Equation (25) yields

$$a\frac{\partial^8 f(s,z)}{\partial z^8} - bs^2\frac{\partial^6 f(s,z)}{\partial z^6} + cs^4\frac{\partial^4 f(s,z)}{\partial z^4} - ds^6\frac{\partial^2 f(s,z)}{\partial z^2} + es^8 f(s,z) = 0, \tag{28}$$

which is a homogeneous equation. The solution of $f(s,z)$ is a function of $\exp(-\lambda sz)$, in which λ is the root of the algebraic equation:

$$a\lambda^8 - b\lambda^6 + c\lambda^4 - d\lambda^2 + e = 0 \tag{29}$$

whose roots (λ^2) are

$$\begin{cases} \lambda_1^2 = \dfrac{b}{4a} - \dfrac{1}{2}\sqrt{R_5+R_6} - \dfrac{1}{2}\sqrt{2R_5 - R_6 - \dfrac{\frac{b^3}{a^3} - \frac{4bc}{a^2} + \frac{8d}{a}}{4\sqrt{R_5+R_6}}} \\ \lambda_2^2 = \dfrac{b}{4a} - \dfrac{1}{2}\sqrt{R_5+R_6} + \dfrac{1}{2}\sqrt{2R_5 - R_6 - \dfrac{\frac{b^3}{a^3} - \frac{4bc}{a^2} + \frac{8d}{a}}{4\sqrt{R_5+R_6}}} \\ \lambda_3^2 = \dfrac{b}{4a} + \dfrac{1}{2}\sqrt{R_5+R_6} - \dfrac{1}{2}\sqrt{2R_5 - R_6 + \dfrac{\frac{b^3}{a^3} - \frac{4bc}{a^2} + \frac{8d}{a}}{4\sqrt{R_5+R_6}}} \\ \lambda_4^2 = \dfrac{b}{4a} + \dfrac{1}{2}\sqrt{R_5+R_6} + \dfrac{1}{2}\sqrt{2R_5 - R_6 + \dfrac{\frac{b^3}{a^3} - \frac{4bc}{a^2} + \frac{8d}{a}}{4\sqrt{R_5+R_6}}}, \end{cases} \tag{30}$$

where $R_1 = 2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace$, $R_2 = c^2 - 3bd + 12ae$, $R_3 = \sqrt{-4(R_2)^3 + (R_1)^2}$, $R_4 = \frac{(R_1+R_3)^{\frac{1}{3}}}{2^{\frac{1}{3}}}$, $R_5 = \frac{b^2}{4a^2} - \frac{2c}{3a}$, $R_6 = \frac{R_2}{3aR_4} + \frac{R_4}{3a}$.

Depending on the properties of λ^2, the function $f(s,z)$ has five different general solutions, as follows:

(a) If $\lambda_1^2 \neq \lambda_2^2 \neq \lambda_3^2 \neq \lambda_4^2 > 0$, then

$$\begin{cases} f^{(1)}(s,z) = A_1(s)e^{-\lambda_1 sz} + A_2(s)e^{-\lambda_2 sz} + A_3(s)e^{-\lambda_3 sz} + A_4(s)e^{-\lambda_4 sz}, & z \geq 0 \\ f^{(2)}(s,z) = B_1(s)e^{\lambda_1 sz} + B_2(s)e^{\lambda_2 sz} + B_3(s)e^{\lambda_3 sz} + B_4(s)e^{\lambda_4 sz}, & z \leq 0 \end{cases} \tag{31}$$

(b) If $\lambda_1^2 \neq \lambda_2^2 \neq \lambda_3^2 = \lambda_4^2 > 0$, then

$$\begin{cases} f^{(1)}(s,z) = A_1(s)e^{-\lambda_1 sz} + A_2(s)e^{-\lambda_2 sz} + A_3(s)e^{-\lambda_3 sz} + A_4(s)ze^{-\lambda_3 sz}, & z \geq 0 \\ f^{(2)}(s,z) = B_1(s)e^{\lambda_1 sz} + B_2(s)e^{\lambda_2 sz} + B_3(s)e^{\lambda_3 sz} + B_4(s)ze^{\lambda_3 sz}, & z \leq 0 \end{cases} \tag{32}$$

(c) If $\lambda_1^2 \neq \lambda_2^2 = \lambda_3^2 = \lambda_4^2 > 0$, then

$$\begin{cases} f^{(1)}(s,z) = A_1(s)e^{-\lambda_1 sz} + A_2(s)e^{-\lambda_2 sz} + A_3(s)ze^{-\lambda_2 sz} + A_4(s)z^2e^{-\lambda_2 sz}, & z \geq 0 \\ f^{(2)}(s,z) = B_1(s)e^{\lambda_1 sz} + B_2(s)e^{\lambda_2 sz} + B_3(s)ze^{\lambda_2 sz} + B_4(s)z^2e^{\lambda_2 sz}, & z \leq 0 \end{cases} \tag{33}$$

(d) If $\lambda_1^2 = \lambda_2^2 = \lambda_3^2 = \lambda_4^2 > 0$, then

$$\begin{cases} f^{(1)}(s,z) = A_1(s)e^{-\lambda_2 sz} + A_2(s)ze^{-\lambda_2 sz} + A_3(s)z^2e^{-\lambda_2 sz} + A_4(s)z^3e^{-\lambda_2 sz}, & z \geq 0 \\ f^{(2)}(s,z) = B_1(s)e^{\lambda_2 sz} + B_2(s)ze^{\lambda_2 sz} + B_3(s)z^2e^{\lambda_2 sz} + B_4(s)z^3e^{\lambda_2 sz}, & z \leq 0 \end{cases} \tag{34}$$

(e) If $\lambda_1^2 > 0$, $\lambda_2^2 > 0$, $\lambda_1^2 \neq \lambda_2^2$ and $\lambda_3^2, \lambda_4^2 < 0$ or λ_3^2 and λ_4^2 being a pair of conjugate complex roots, then, in this case, the λ_3 and λ_4 are a pair of conjugate complexes $-\delta \pm i\omega$. The solution of the function $f(s,z)$ is

$$\begin{cases} f^{(1)}(s,z) = A_1(s)e^{-\lambda_1 sz} + A_2(s)e^{-\lambda_2 sz} + A_3(s)e^{-\delta sz}\cos(s\omega z) + A_4(s)e^{-\delta sz}\sin(s\omega z), & z \geq 0 \\ f^{(2)}(s,z) = B_1(s)e^{\lambda_1 sz} + B_2(s)e^{\lambda_2 sz} + B_3(s)e^{\delta sz}\cos(s\omega z) + B_4(s)e^{\delta sz}\sin(s\omega z), & z \leq 0, \end{cases} \tag{35}$$

where δ and $\omega > 0$ and $A_i(s)$, $B_i(s)$ ($i = 1, 2, 3, 4$) are two functions of s to be determined by the boundary conditions.

Due to the symmetry, it suffices to consider the problem for $x \geq 0, |z| < \infty$. Based on the solution of the auxiliary function $f(s,z)$ for the case of $\lambda_1^2 \neq \lambda_2^2 \neq \lambda_3^2 \neq \lambda_4^2 > 0$, the displacements, stresses, electric displacements, electric potentials, magnetic fluxes and the magnetic potentials satisfying Equation (22) were calculated using Mathematica and via Equations (2)–(8), (24), (26), (27), (31)–(35) for the problems, as follows (the other cases can be obtained using a similar method; they were omitted in the present paper):

$$\begin{cases} u^{(1)}(x,z) = \dfrac{2}{\pi}\sum_{i=1}^{4} \chi_k^{(1)} \displaystyle\int_0^\infty A_i(s)s^6 \sin(sx)e^{-\lambda_i sz}\mathrm{d}s \\ w^{(1)}(x,z) = \dfrac{2}{\pi}\sum_{i=1}^{4} \chi_k^{(2)} \displaystyle\int_0^\infty A_i(s)s^6 \cos(sx)e^{-\lambda_i sz}\mathrm{d}s \\ \phi^{(1)}(x,z) = \dfrac{2}{\pi}\sum_{i=1}^{4} \chi_k^{(3)} \displaystyle\int_0^\infty A_i(s)s^6 \cos(sx)e^{-\lambda_i sz}\mathrm{d}s \\ \psi^{(1)}(x,z) = \dfrac{2}{\pi}\sum_{i=1}^{4} \chi_k^{(4)} \displaystyle\int_0^\infty A_i(s)s^6 \cos(sx)e^{-\lambda_i sz}\mathrm{d}s, \end{cases} \tag{36}$$

$$\begin{cases} u^{(2)}(x,z) = \dfrac{2}{\pi}\sum_{i=1}^{4} \chi_k^{(1^*)} \displaystyle\int_0^\infty B_i(s)s^6 \sin(sx)e^{\lambda_i sz}\mathrm{d}s \\ w^{(2)}(x,z) = \dfrac{2}{\pi}\sum_{i=1}^{4} \chi_k^{(2^*)} \displaystyle\int_0^\infty B_i(s)s^6 \cos(sx)e^{\lambda_i sz}\mathrm{d}s \\ \phi^{(2)}(x,z) = \dfrac{2}{\pi}\sum_{i=1}^{4} \chi_k^{(3^*)} \displaystyle\int_0^\infty B_i(s)s^6 \cos(sx)e^{\lambda_i sz}\mathrm{d}s \\ \psi^{(2)}(x,z) = \dfrac{2}{\pi}\sum_{i=1}^{4} \chi_k^{(4^*)} \displaystyle\int_0^\infty B_i(s)s^6 \cos(sx)e^{\lambda_i sz}\mathrm{d}s, \end{cases} \tag{37}$$

where $\chi_k^{(1)} = \lambda_i(\alpha_{11} - \alpha_{12}\lambda_i^2 + \alpha_{13}\lambda_i^4) = -\chi_k^{(1^*)}$, $\chi_k^{(2)} = -\alpha_{21} + \alpha_{22}\lambda_i^2 - \alpha_{23}\lambda_i^4 + \alpha_{24}\lambda_i^6 = \chi_k^{(2^*)}$, $\chi_k^{(3)} = -\alpha_{31} + \alpha_{32}\lambda_i^2 - \alpha_{33}\lambda_i^4 + \alpha_{34}\lambda_i^6 = \chi_k^{(3^*)}$, $\chi_k^{(4)} = -\alpha_{41} + \alpha_{42}\lambda_i^2 - \alpha_{43}\lambda_i^4 + \alpha_{44}\lambda_i^6 = \chi_k^{(4^*)}$,

$$
\begin{aligned}
\tau_{zz}(x,z) &= \begin{cases} \int_0^\infty \int_{-\infty}^\infty \alpha(|x'-x|,|z'-z|)\sigma_{zz}^{(1)}(x',z')\mathrm{d}x'\mathrm{d}z' \\ +\int_{-\infty}^0 \int_{-\infty}^\infty \alpha(|x'-x|,|z'-z|)\sigma_{zz}^{(2)}(x',z')\mathrm{d}x'\mathrm{d}z' \end{cases} \\
&= \int_0^\infty \int_{-\infty}^\infty [\alpha(|x'-x|,|z'-z|)+\alpha(|x'-x|,|z'+z|)]\sigma_{zz}^{(1)}(x',z')\mathrm{d}x'\mathrm{d}z' \\
&= \int_{-\infty}^0 \int_{-\infty}^\infty [\alpha(|x'-x|,|z'-z|)+\alpha(|x'-x|,|z'+z|)]\sigma_{zz}^{(2)}(x',z')\mathrm{d}x'\mathrm{d}z', \qquad (38)
\end{aligned}
$$

$$
\begin{aligned}
\tau_{xz}(x,z) &= \begin{cases} \int_0^\infty \int_{-\infty}^\infty \alpha(|x'-x|,|z'-z|)\sigma_{xz}^{(1)}(x',z')\mathrm{d}x'\mathrm{d}z' \\ +\int_{-\infty}^0 \int_{-\infty}^\infty \alpha(|x'-x|,|z'-z|)\sigma_{xz}^{(2)}(x',z')\mathrm{d}x'\mathrm{d}z' \end{cases} \\
&= \int_0^\infty \int_{-\infty}^\infty [\alpha(|x'-x|,|z'-z|)+\alpha(|x'-x|,|z'+z|)]\sigma_{xz}^{(1)}(x',z')\mathrm{d}x'\mathrm{d}z' \\
&= \int_{-\infty}^0 \int_{-\infty}^\infty [\alpha(|x'-x|,|z'-z|)+\alpha(|x'-x|,|z'+z|)]\sigma_{xz}^{(2)}(x',z')\mathrm{d}x'\mathrm{d}z', \qquad (39)
\end{aligned}
$$

$$
\begin{aligned}
D_z(x,z) &= \begin{cases} \int_0^\infty \int_{-\infty}^\infty \alpha(|x'-x|,|z'-z|)D_z^{(1)}(x',z')\mathrm{d}x'\mathrm{d}z' \\ +\int_{-\infty}^0 \int_{-\infty}^\infty \alpha(|x'-x|,|z'-z|)D_z^{(2)}(x',z')\mathrm{d}x'\mathrm{d}z' \end{cases} \\
&= \int_0^\infty \int_{-\infty}^\infty [\alpha(|x'-x|,|z'-z|)+\alpha(|x'-x|,|z'+z|)]D_z^{(1)}(x',z')\mathrm{d}x'\mathrm{d}z' \\
&= \int_{-\infty}^0 \int_{-\infty}^\infty [\alpha(|x'-x|,|z'-z|)+\alpha(|x'-x|,|z'+z|)]D_z^{(2)}(x',z')\mathrm{d}x'\mathrm{d}z', \qquad (40)
\end{aligned}
$$

$$
\begin{aligned}
B_z(x,z) &= \begin{cases} \int_0^\infty \int_{-\infty}^\infty \alpha(|x'-x|,|z'-z|)B_z^{(1)}(x',z')\mathrm{d}x'\mathrm{d}z' \\ +\int_{-\infty}^0 \int_{-\infty}^\infty \alpha(|x'-x|,|z'-z|)B_z^{(2)}(x',z')\mathrm{d}x'\mathrm{d}z' \end{cases} \\
&= \int_0^\infty \int_{-\infty}^\infty [\alpha(|x'-x|,|z'-z|)+\alpha(|x'-x|,|z'+z|)]B_z^{(1)}(x',z')\mathrm{d}x'\mathrm{d}z' \\
&= \int_{-\infty}^0 \int_{-\infty}^\infty [\alpha(|x'-x|,|z'-z|)+\alpha(|x'-x|,|z'+z|)]B_z^{(2)}(x',z')\mathrm{d}x'\mathrm{d}z', \qquad (41)
\end{aligned}
$$

where

$$
\begin{cases}
\sigma_{zz}^{(1)}(x,z) = \dfrac{2}{\pi}\displaystyle\sum_{k=1}^{4} \beta_k^{(1)} \int_0^\infty A_k(s,t)s^7\cos(sx)e^{-\lambda_k sz}\mathrm{d}s \\
\sigma_{xz}^{(1)}(x,z) = \dfrac{2}{\pi}\displaystyle\sum_{k=0}^{4} \beta_k^{(2)} \int_0^\infty A_k(s,t)s^7\sin(sx)e^{-\lambda_k sz}\mathrm{d}s \\
D_z^{(1)}(x,z) = \dfrac{2}{\pi}\displaystyle\sum_{k=0}^{4} \beta_k^{(3)} \int_0^\infty A_k(s,t)s^7\cos(sx)e^{-\lambda_k sz}\mathrm{d}s \\
B_z^{(1)}(x,z) = \dfrac{2}{\pi}\displaystyle\sum_{k=0}^{4} \beta_k^{(4)} \int_0^\infty A_k(s,t)s^7\cos(sx)e^{-\lambda_k sz}\mathrm{d}s,
\end{cases} \qquad (42)
$$

$$\begin{cases} \sigma_{zz}^{(2)}(x,z) = \dfrac{2}{\pi}\displaystyle\sum_{k=1}^{4}\beta_k^{(1^*)}\int_0^\infty A_k(s,t)s^7\cos(sx)e^{\lambda_k sz}\mathrm{d}s \\ \sigma_{xz}^{(2)}(x,z) = \dfrac{2}{\pi}\displaystyle\sum_{k=0}^{4}\beta_k^{(2^*)}\int_0^\infty A_k(s,t)s^7\sin(sx)e^{\lambda_k sz}\mathrm{d}s \\ D_z^{(2)}(x,z) = \dfrac{2}{\pi}\displaystyle\sum_{k=0}^{4}\beta_k^{(3^*)}\int_0^\infty A_k(s,t)s^7\cos(sx)e^{\lambda_k sz}\mathrm{d}s \\ B_z^{(2)}(x,z) = \dfrac{2}{\pi}\displaystyle\sum_{k=0}^{4}\beta_k^{(4^*)}\int_0^\infty A_k(s,t)s^7\cos(sx)e^{\lambda_k sz}\mathrm{d}s, \end{cases} \tag{43}$$

and

$$\begin{cases} \beta_k^{(1)} = c_{13}\chi_k^{(1)} - c_{33}\lambda_k\chi_k^{(2)} - e_{33}\lambda_k\chi_k^{(3)} + f_{33}\lambda_k\chi_k^{(4)} \\ \beta_k^{(2)} = -c_{44}[\lambda_k\chi_k^{(1)} + \chi_k^{(2)}] - e_{15}\chi_k^{(3)} + f_{15}\chi_k^{(4)} \\ \beta_k^{(3)} = e_{31}\chi_k^{(1)} - e_{33}\lambda_k\chi_k^{(2)} + \varepsilon_{33}\lambda_k\chi_k^{(3)} + g_{33}\lambda_k\chi_k^{(3)} \\ \beta_k^{(4)} = f_{31}\chi_k^{(1)} - f_{33}\lambda_k\chi_k^{(2)} - g_{33}\lambda_k\chi_k^{(3)} + \mu_{33}\lambda_k\chi_k^{(4)} \\ \beta_k^{(1^*)} = c_{13}\chi_k^{(1^*)} + c_{33}\lambda_k\chi_k^{(2^*)} + e_{33}\lambda_k\chi_k^{(3^*)} - f_{33}\lambda_k\chi_k^{(4^*)} = -\beta_k^{(1)} \\ \beta_k^{(2^*)} = c_{44}[\lambda_k\chi_k^{(1^*)} - \chi_k^{(2^*)}] - e_{15}\chi_k^{(3^*)} + f_{15}\chi_k^{(4^*)} = \beta_k^{(2)} \\ \beta_k^{(3^*)} = e_{31}\chi_k^{(1)} + e_{33}\lambda_k\chi_k^{(2^*)} - \varepsilon_{33}\lambda_k\chi_k^{(3^*)} - g_{33}\lambda_k\chi_k^{(3^*)} = -\beta_k^{(3)} \\ \beta_k^{(4^*)} = f_{31}\chi_k^{(1^*)} + f_{33}\lambda_k\chi_k^{(2^*)} + g_{33}\lambda_k\chi_k^{(3^*)} - \mu_{33}\lambda_k\chi_k^{(4^*)} = -\beta_k^{(4)}. \end{cases}$$

Substituting for α from Equation (10) and applying Equations (42) and (43), the integrations may be performed with respect to x' and z' in Equations (38)–(41) by noting the integrals [31]:

$$\int_0^\infty \exp(-py^2 - \gamma y)\mathrm{d}y = \frac{1}{2}(\pi/p)^{1/2}\exp(\gamma^2/4p)[1 - \Phi(\gamma/2\sqrt{p})]$$

$$\int_{-\infty}^{\infty} \exp(-px'^2)\begin{Bmatrix} \sin\xi(x'+x) \\ \cos\xi(x'+x) \end{Bmatrix}\mathrm{d}x' = (\pi/p)^{1/2}\exp\left(-\frac{\xi^2}{4p}\right)\begin{Bmatrix} \sin(\xi x) \\ \cos(\xi x) \end{Bmatrix}$$

$$\Phi(z) = \frac{2}{\sqrt{\pi}}\int_0^z \exp(-t^2)\mathrm{d}t.$$

Therefore, Equations (38)–(41) can be expressed as follows:

$$\begin{cases} \tau_{zz}^{(1)}(x,z) = \dfrac{2}{\pi}\displaystyle\sum_{k=1}^{4}\beta_k^{(1)}\int_0^\infty h_k(a,\beta,s,z)A_k(s)\cos(sx)e^{-(\beta/a)^2z^2}\mathrm{d}s \\ \tau_{xz}^{(1)}(x,z) = \dfrac{2}{\pi}\displaystyle\sum_{k=0}^{4}\beta_k^{(2)}\int_0^\infty h_k(a,\beta,s,z)A_k(s)\sin(sx)e^{-(\beta/a)^2z^2}\mathrm{d}s \\ D_z^{(1)}(x,z) = \dfrac{2}{\pi}\displaystyle\sum_{k=1}^{4}\beta_k^{(3)}\int_0^\infty h_k(a,\beta,s,z)A_k(s)\cos(sx)e^{-(\beta/a)^2z^2}\mathrm{d}s \\ B_z^{(1)}(x,z) = \dfrac{2}{\pi}\displaystyle\sum_{k=1}^{4}\beta_k^{(4)}\int_0^\infty h_k(a,\beta,s,z)A_k(s)\cos(sx)e^{-(\beta/a)^2z^2}\mathrm{d}s, \end{cases} \tag{44}$$

$$\begin{cases} \tau_{zz}^{(2)}(x,z) = \dfrac{2}{\pi}\sum_{k=1}^{4}\beta_k^{(1^*)}\int_0^\infty h_k(a,\beta,s,z)B_k(s)\cos(sx)e^{-(\beta/a)^2z^2}\mathrm{d}s \\ \tau_{xz}^{(2)}(x,z) = \dfrac{2}{\pi}\sum_{k=0}^{4}\beta_k^{(2^*)}\int_0^\infty h_k(a,\beta,s,z)B_k(s)\sin(sx)e^{-(\beta/a)^2z^2}\mathrm{d}s \\ D_z^{(2)}(x,z) = \dfrac{2}{\pi}\sum_{k=1}^{4}\beta_k^{(3^*)}\int_0^\infty h_k(a,\beta,s,z)B_k(s)\cos(sx)e^{-(\beta/a)^2z^2}\mathrm{d}s \\ B_z^{(2)}(x,z) = \dfrac{2}{\pi}\sum_{k=1}^{4}\beta_k^{(4^*)}\int_0^\infty h_k(a,\beta,s,z)B_k(s)\cos(sx)e^{-(\beta/a)^2z^2}\mathrm{d}s, \end{cases} \tag{45}$$

where $h_k(a,\beta,s,z) = \begin{cases} \frac{1}{2}\exp(-\frac{s^2a^2}{4\beta^2}) \times \{\exp(\frac{[\lambda_k s-2(\beta/a)^2z]^2}{4(\beta/a)^2})[1-\Phi(\frac{\lambda_k s-2(\beta/a)^2z}{2(\beta/a)})] \\ +\exp(\frac{[\lambda_k s+2(\beta/a)^2z]^2}{4(\beta/a)^2})[1-\Phi(\frac{\lambda_k s+2(\beta/a)^2z}{2(\beta/a)})]\} \end{cases}$ and we have $\lim\limits_{a\to 0} h_k(a,\beta,s,0) = 1$.

To solve the problem, the jumps in displacements across the crack surfaces were defined as follows:

$$\begin{cases} f_1(x) = u^{(1)}(x,0^+) - u^{(2)}(x,0^-) \\ f_2(x) = w^{(1)}(x,0^+) - w^{(2)}(x,0^-). \end{cases} \tag{46}$$

It can be verified that $f_1(x)$ is an odd function about the variable x and $f_2(x)$ is an even functions about the variable x.

Substituting Equations (36) and (37) into Equation (46), then applying the Fourier transform along with the Equations (44) and (45) and the boundary conditions Equations (20)–(22), we have

$$\begin{cases} \sum_{k=1}^{4}\chi_k^{(1)}A_k(s) - \sum_{k=1}^{4}\chi_k^{(1^*)}B_k(s) = \bar{f}_1(s)/s^6 \\ \sum_{k=1}^{4}\chi_k^{(2)}A_k(s) - \sum_{k=1}^{4}\chi_k^{(2^*)}B_k(s) = \bar{f}_2(s)/s^6 \\ \sum_{k=1}^{4}\chi_k^{(3)}A_k(s) - \sum_{k=1}^{4}\chi_k^{(3^*)}(s)B_k(s) = 0 \\ \sum_{k=1}^{4}\chi_k^{(4)}A_k(s) - \sum_{k=1}^{4}\chi_k^{(4^*)}(s)B_k(s) = 0, \end{cases} \tag{47}$$

$$\begin{cases} \sum_{k=1}^{4}\beta_k^{(1)}h_k^*(a,\beta,s)A_k(s) - \sum_{k=1}^{4}\beta_k^{(1^*)}h_k^*(a,\beta,s)B_k(s) = 0 \\ \sum_{k=1}^{4}\beta_k^{(2)}h_k^*(a,\beta,s)A_k(s) - \sum_{k=1}^{4}\beta_k^{(2*)}h_k^*(a,\beta,s)B_k(s) = 0 \\ \sum_{k=1}^{4}\beta_k^{(3)}h_k^*(a,\beta,s)A_k(s) - \sum_{k=1}^{4}\beta_k^{(3^*)}h_k^*(a,\beta,s)B_k(s) = 0 \\ \sum_{k=1}^{4}\beta_k^{(4)}h_k^*(a,\beta,s)A_k(s) - \sum_{k=1}^{4}\beta_k^{(4^*)}h_k^*(a,\beta,s)B_k(s) = 0, \end{cases} \tag{48}$$

where $h_k^*(a,\beta,s) = h_k(a,\beta,s,0)$. A top bar indicates one-dimensional Fourier transform with respect to x, as shown in Equation (27).

Solving the eight equations of Equations (47) and (48) with eight unknown functions, and substituting the results into Equations (44) and (45) and applying the boundary conditions Equations (20) and (21), we can obtain

$$\tau_{zz}^{(1)}(x,0) = \frac{1}{\pi}\int_0^\infty s g_1(a,\beta,s)\bar{f}_2(s)\cos(sx)\mathrm{d}s = -\tau_0, \quad l \le x \le 1, \tag{49}$$

$$\tau_{xz}^{(1)}(x,0) = \frac{1}{\pi}\int_0^\infty s g_2(a,\beta,s)\bar{f}_1(s)\sin(sx)\mathrm{d}s = 0, \quad l \le x \le 1, \tag{50}$$

$$\int_0^\infty \bar{f}_1(s)\sin(sx)\mathrm{d}s = 0, \qquad \int_0^\infty \bar{f}_2(s)\cos(sx)\mathrm{d}s = 0, \qquad x > 1 \text{ and } 0 \le x < l, \tag{51}$$

where $g_k(a,\beta,s)(k = 1,2)$ is a known function, as shown in Appendix 3. Here, we just give these constants for $\lambda_1^2 \ne \lambda_2^2 \ne \lambda_3^2 \ne \lambda_4^2 > 0$. The other cases can be obtained using the same method. To determine the unknown functions $\bar{f}_1(s)$ and $\bar{f}_2(s)$, the above two pairs of dual integral equations (49)–(51) must be solved.

5. Solution of the dual integral equations

As the only difference between the classical and non-local equations is in the influence function $g_k(a,\beta,s)(k = 1,2)$, it is logical to utilize the classical solution to convert the system equations (49)–(51), to an integral equation of the second kind that is generally better behaved. However, the dual integral equations (49)–(51) cannot be transformed into a Fredholm integral equation of the second kind because $g_k(a,\beta,s)(k = 1,2)$ does not tend to a constant C ($C \ne 0$) for $s \to \infty$. This is explained in Eringen's papers [19,20]. Of course, the dual integral equations (49)–(51) can be considered as a single integral equation of the first kind with discontinuous kernel. It is well-known in the literature that integral equations of the first kind are generally ill-posed in the sense of Hadamard, i.e. small perturbations of the data can arbitrarily yield large changes in the solution. This makes the numerical solution of such equations quite difficult. To overcome the difficulty, the Schmidt method [32–34] was used to solve the dual integral equations (49)–(51). The displacement jumps across the crack surfaces were expanded as the following series:

$$f_1(x) = \begin{cases} \sum_{n=0}^{\infty} a_n P_n^{(1/2,1/2)}\left(\frac{x-\frac{1+l}{2}}{\frac{1-l}{2}}\right)\left(1-\frac{\left(x-\frac{1+l}{2}\right)^2}{\left(\frac{1-l}{2}\right)^2}\right)^{\frac{1}{2}}, & l \le x \le 1, \\ 0, \quad x > 1 \quad \text{and} \quad 0 \le x < l \end{cases} \tag{52}$$

$$f_2(x) = \begin{cases} \sum_{n=0}^{\infty} b_n P_n^{(1/2,1/2)}\left(\frac{x-\frac{1+l}{2}}{\frac{1-l}{2}}\right)\left(1-\frac{\left(x-\frac{1+l}{2}\right)^2}{\left(\frac{1-l}{2}\right)^2}\right)^{\frac{1}{2}}, & l \le x \le 1, \\ 0, \quad x > 1 \quad \text{and} \quad 0 \le x < l, \end{cases} \tag{53}$$

where a_n and b_n are unknown coefficients to be determined, and $P_n^{(1/2,1/2)}(x)$ is a Jacobi polynomial [31]. The Fourier transforms [35] of Equations (51) and (52) are expresses as follows:

$$\bar{f}_1(s) = \sum_{n=0}^{\infty} a_n F_n G_n^{(1)}(s) \frac{1}{s} J_{n+1}\left(s\frac{1-l}{2}\right), \quad G_n^{(1)}(s) = \begin{cases} (-1)^{\frac{n}{2}} \sin\left(s\frac{1+l}{2}\right), & n = 0, 2, 4, 6, \ldots \\ (-1)^{\frac{n-1}{2}} \cos\left(s\frac{1+l}{2}\right), & n = 1, 3, 5, 7, \ldots \end{cases}, \tag{54}$$

$$\bar{f}_2(s) = \sum_{n=0}^{\infty} b_n F_n G_n^{(2)}(s) \frac{1}{s} J_{n+1}\left(s\frac{1-l}{2}\right), \quad G_n^{(2)}(s) = \begin{cases} (-1)^{\frac{n}{2}} \cos\left(s\frac{1+l}{2}\right), & n = 0, 2, 4, 6, \ldots \\ (-1)^{\frac{n+1}{2}} \sin\left(s\frac{1+l}{2}\right), & n = 1, 3, 5, 7, \ldots \end{cases}, \tag{55}$$

where $F_n = 2\sqrt{\pi}\frac{\Gamma(n+1+\frac{1}{2})}{n!}$, $\Gamma(x)$ and $J_n(x)$ are the Gamma and Bessel functions of order n, respectively.

Substituting Equations (54) and (55) into Equations (49)–(51), it can be shown that Equation (51) was automatically satisfied. Equations (49) and (50) were reduced to forms, as follows:

$$\frac{1}{\pi}\sum_{n=0}^{\infty} b_n F_n \int_0^{\infty} g_1(a, \beta, s) G_n^{(2)}(s) J_{n+1}\left(s\frac{1-l}{2}\right) \cos(sx)\mathrm{d}s = -\tau_0, \quad l \le x \le 1, \tag{56}$$

$$\frac{1}{\pi}\sum_{n=0}^{\infty} a_n F_n \int_0^{\infty} g_2(a, \beta, s) G_n^{(1)}(s) J_{n+1}\left(s\frac{1-l}{2}\right) \sin(sx)\mathrm{d}s = 0, \quad l \le x \le 1. \tag{57}$$

From Equation (57), it can be derived that $a_n = 0$ $(n = 0, 1, 2, 3, \ldots.)$, i.e. $f_1(x) = 0$. Equation (56) can now be solved for the coefficients b_n using the Schmidt method [32–34]. For brevity, Equation (56) can be rewritten as follows:

$$\sum_{n=0}^{\infty} b_n E_n(x) = U(x), \quad l < x < 1, \tag{58}$$

where $E_n(x)$ and $U(x)$ are known functions; coefficients b_n are unknown and to be determined. A set of functions $P_n(x)$ which satisfy the orthogonality conditions

$$\int_l^1 P_m(x) P_n(x)\mathrm{d}x = N_n \delta_{mn}, \quad N_n = \int_l^1 P_n^2(x)\mathrm{d}x \tag{59}$$

can be constructed from the function, $E_n(x)$, such that

$$P_n(x) = \sum_{i=0}^{n} \frac{M_{in}}{M_{nn}} E_i(x), \tag{60}$$

where M_{ij} is the cofactor of the element d_{ij} of D_n, defined as

$$D_n = \begin{bmatrix} d_{00}, d_{01}, d_{02}, \ldots, d_{0n} \\ d_{10}, d_{11}, d_{12}, \ldots, d_{1n} \\ d_{20}, d_{21}, d_{22}, \ldots, d_{2n} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ d_{n0}, d_{n1}, d_{n2}, \ldots, d_{nn} \end{bmatrix}, \quad d_{ij} = \int_l^1 E_i(x)E_j(x)\mathrm{d}x. \tag{61}$$

Using Equations (58)–(61), we obtain

$$b_n = \sum_{j=n}^{\infty} q_j \frac{M_{nj}}{M_{jj}} \text{ with } q_j = \frac{1}{N_j}\int_l^1 U(x)P_j(x)\mathrm{d}x. \tag{62}$$

6. Numerical calculations and discussion

Once we determine the coefficients b_n, we can obtain the entire stress fields. However, in the fracture mechanics, it is important to determine the stresses $\sigma_{zz}^{(1)}$, $\sigma_{xz}^{(1)}$, electric displacement $D_z^{(1)}$ and magnetic flux $B_z^{(1)}$ near the vicinity of crack tips. In the present study, $\sigma_{zz}^{(1)}$, $\sigma_{xz}^{(1)}$, $D_z^{(1)}$ and $B_z^{(1)}$ along the crack line can be, respectively, expressed as follows:

$$\tau_{zz}^{(1)}(x,0) = \frac{1}{\pi}\sum_{n=0}^{\infty} b_n F_n \int_0^{\infty} g_1(a,\beta,s)G_n^{(2)}(s)J_{n+1}\left(s\frac{1-l}{2}\right)\cos(sx)\mathrm{d}s, \tag{63}$$

$$\tau_{xz}^{(1)}(x,0) = 0, \tag{64}$$

$$D_z^{(1)}(x,0) = \frac{1}{\pi}\sum_{n=0}^{\infty} b_n F_n \int_0^{\infty} g_3(a,\beta,s)G_n^{(2)}(s)J_{n+1}\left(s\frac{1-l}{2}\right)\cos(sx)\mathrm{d}s, \tag{65}$$

$$B_z^{(1)}(x,0) = \frac{1}{\pi}\sum_{n=0}^{\infty} b_n F_n \int_0^{\infty} g_4(a,\beta,s)G_n^{(2)}(s)J_{n+1}\left(s\frac{1-l}{2}\right)\cos(sx)\mathrm{d}s, \tag{66}$$

where $g_k(a,\beta,s)$ $(k=3,4)$ is a known function, as shown in Appendix 3.

As discussed in [25,26,34], it can be seen that the Schmidt method is performed satisfactorily if the first 10 terms of the infinite series in Equation (58) are retained. From the expressions of functions $g_k(a,\beta,s)$ $(k=1,3,4)$, we see that the semi-infinite integration and the series in Equations (63)–(66) are convergent for any variable x for $a/\beta \neq 0$. Equations (63)–(66) give a finite stress field, a finite electric displacement field and a finite magnetic flux field all along $z=0$; therefore, there is no stress, electric displacement and magnetic flux singularity at the crack tips. However, for $a/\beta = 0$, the classical stress, electric displacement and the magnetic flux singularities will be present at crack tips because $\lim_{a\to 0} h_k(a,\beta,s,0) = 1$. At $l < x < 1$, $\tau_{zz}^{(1)}(x,0)/\tau_0$ is very close to negative unity, and for $x > 1$, $\tau_{zz}^{(1)}(x,0)/\tau_0$ possesses finite values diminishing from finite values at $x = l$ and $x = 1$ to zero at $x = \infty$. The semi-infinite numerical integrals, which occur, were evaluated easily by Filon's method [36] due to the rapid diminution of the integrands. In all computations, the material properties were assumed as given in Table 1 based on [27,37] and the variables of all were selected as dimensionless.

The calculated stress, electric displacement and magnetic flux fields near crack tips are plotted in Figures 2–11. Now, we will discuss the results as follows:

(i) In the present paper, the traditional concept of the non-local theory was extended to deal with the fracture problem of magnetoelectroelastic composite materials by means of the generalized Almansi's theorem and the Schmidt method [32–34].

(ii) In the present paper, the generalized Almansi's theorem was applied and the basic non-local theory solution was obtained for potential functions, stress, electric displacement and magnetic flux fields near the crack tips in magnetoelectroelastic composite materials. This method in the present paper is feasible for general cases, as discussed in Equations (31)–(35); thus, the obtained solution is valid for general cases. However, the Eshelby–Stroh method, as adopted in [10], is valid only for the cases of non-degenerate materials.

(iii) For $a/\beta \neq 0$, it can be proved that the semi-infinite integration and the series in Equations (63), (65) and (66) are convergent for any variable x. So, the stress, electric displacement and magnetic flux fields give finite values along the crack line, as shown in Figures 2–5. Contrary to the classical electro-elastic theory solution, it is found that no stress, electric displacement or magnetic flux singularities are present at the crack tips; also, the present results converge to the classical results when far away from the crack tip. The maximum stress does not occur at the crack tip, but slightly away from it, as shown in Figures 2 and 3. This phenomenon has been thoroughly substantiated in Eringen's paper [38]. The distance between the crack tip and the maximum stress point is very small, and is dependent on crack length and the lattice parameter.

(iv) The stress at the crack tip becomes infinite as the lattice parameter $a \to 0$. This is the classical continuum limit of square root singularity, which can be shown from Equations (49)–(51). For $\lim_{a\to 0} h_k(a, \beta, s, 0) = 1$, Equations (49)–(51) will reduce to the dual integral equations for the

Table 1. Material properties of magnetoelectroelastic composite materials.

c_{11} (10^{10}N/m^2)	c_{12} (10^{10}N/m^2)	c_{13} (10^{10}N/m^2)	C_{33} (10^{10}N/m^2)	c_{44} (10^{10}N/m^2)
22.6	12.5	12.4	21.6	4.4
e_{31} (C/m^2)	e_{33}(C/m^2)	e_{15}(C/m^2)	ε_{11}(10^{-10} C^2/Nm^2)	ε_{33}(10^{-10} C^2/Nm^2)
–2.2	9.3	5.8	56.4	63.5
f_{31} (N/Am)	f_{33} (N/Am)	F_{15} (N/Am)	μ_{11}(10^{-6} Ns^2/C^2)	μ_{33}(10^{-6} Ns^2/C^2)
290.2	350.0	275.0	297.0	83.5
g_{11} (10^{-9} Ns/VC)	g_{33} (10^{-9} Ns/VC)			
0.005	0.008			

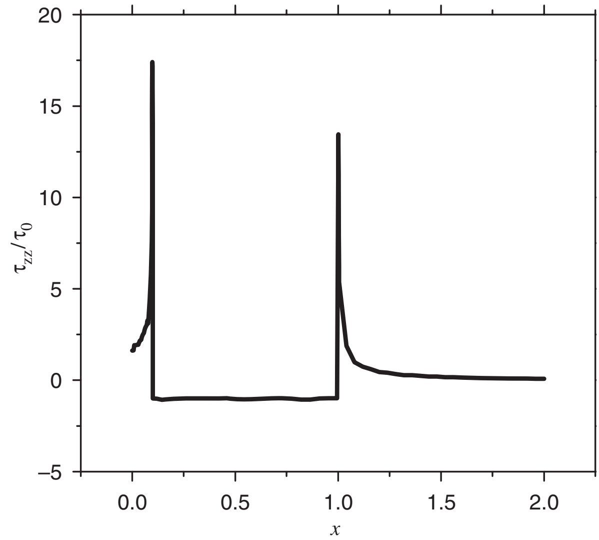

Figure 2. Stress along the crack line versus x for $l = 1.0$ and $a/\beta = 0.001$.

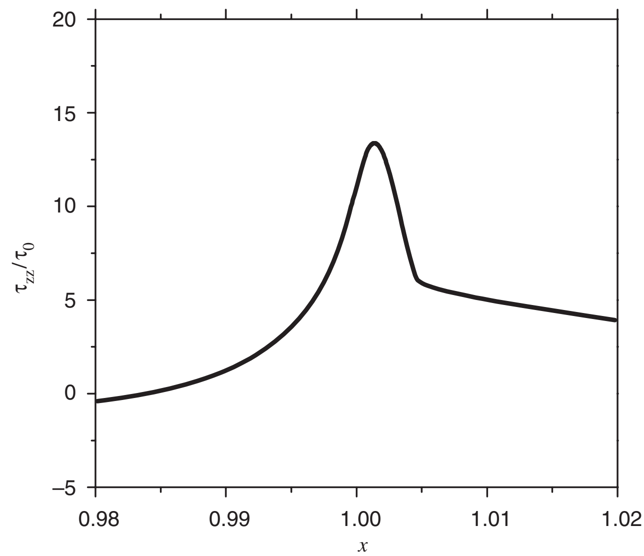

Figure 3. Locally enlarged graph of Figure 2 near the right crack tip.

same problem in classical magnetoelectroelastic composite materials. These dual integral equations can be solved by using the singular integral equation for the same problem in the local magnetoelectroelastic composite materials. However, the stress, electric displacement and magnetic flux singularities are present at the crack tips in local magnetoelectroelastic composite materials, as is well known.

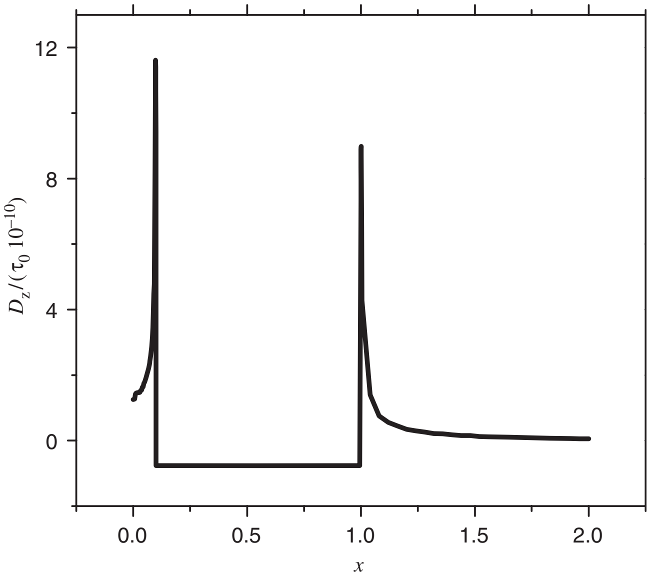

Figure 4. Electric displacement along the crack line versus x for $l = 1.0$ and $a/\beta = 0.001$.

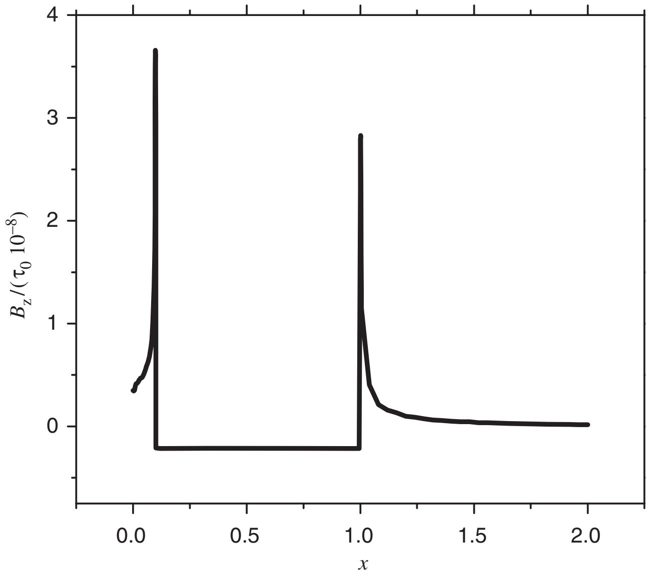

Figure 5. Magnetic flux along the crack line versus x for $l = 1.0$ and $a/\beta = 0.001$.

(v) The results of the stress, electric displacement and magnetic flux fields at the crack tips tend to decrease with an increase in the lattice parameter, as shown in Figures 6–8.

(vi) For the $a/\beta =$ constant, viz. the lattice parameter does not change, the value of the stress, electric displacement and magnetic flux concentrations (at the

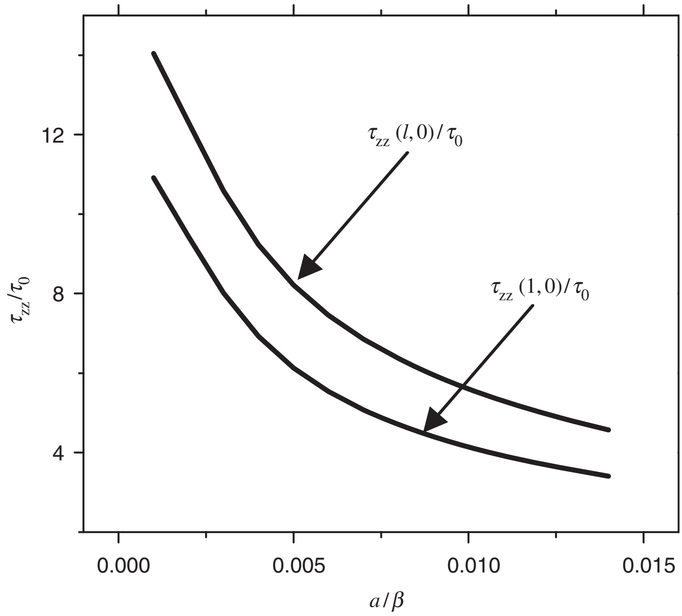

Figure 6. Stress at the crack tips versus a/β.

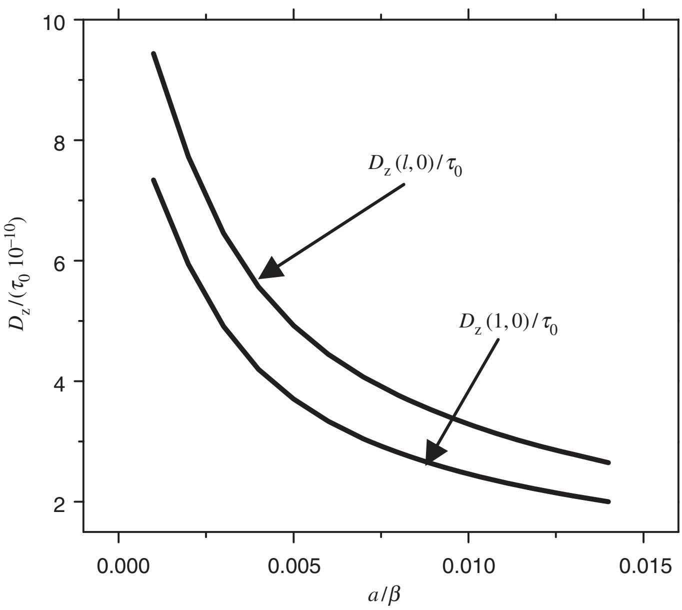

Figure 7. Electric displacement at the crack tip versus a/β.

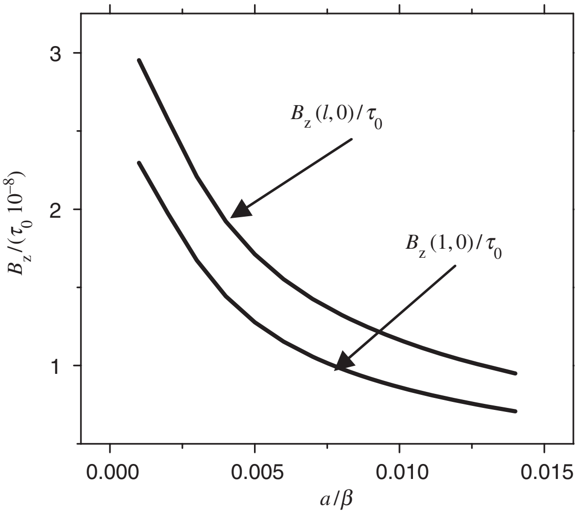

Figure 8. Magnetic flux at the crack tip versus a/β.

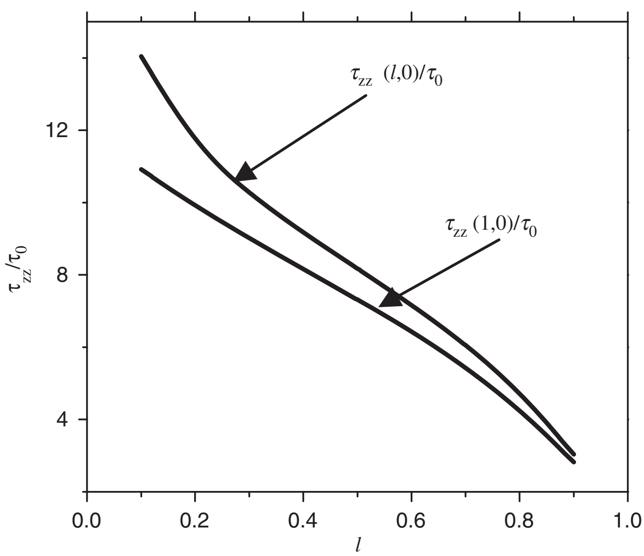

Figure 9. Stress at the crack tip versus l.

crack tip) increase with increasing crack length, as shown in Figures 9–11. Noting this fact, experiments indicate that magnetoelectroelastic composite materials with smaller cracks are more resistant to fracture than those with larger cracks.

(vii) It can be ascertained that the stress fields near inner crack tips are greater than those near outer crack tips, as shown in Figures 2–11.

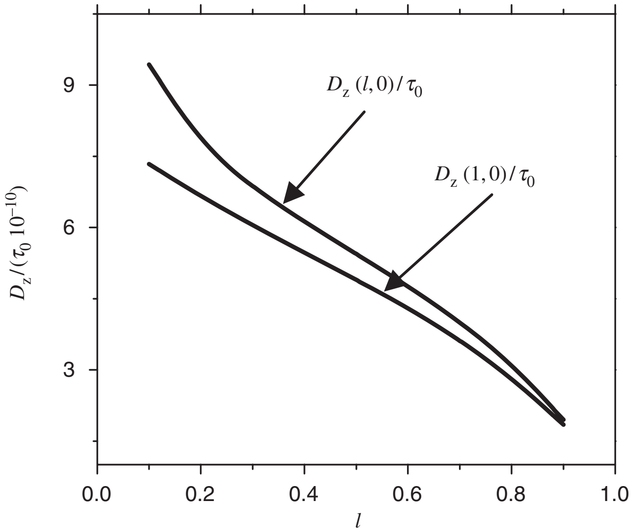

Figure 10. Electric displacement at the crack tip versus l.

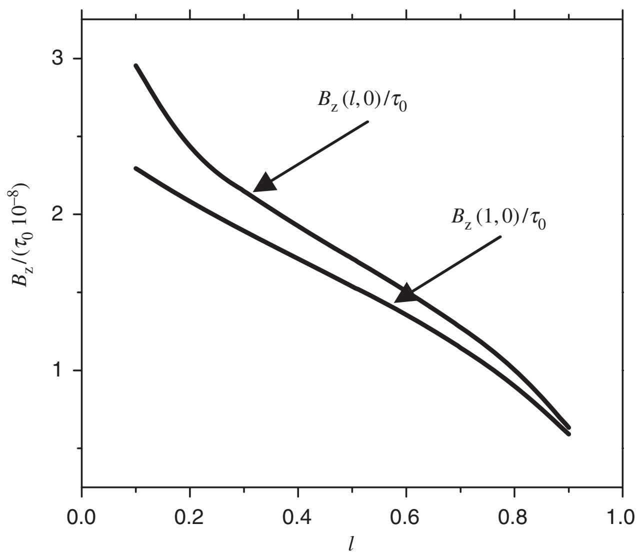

Figure 11. Magnetic flux at the crack tip versus l.

(viii) For the electric displacement and magnetic flux fields near the crack tips, they have the same changing tendency as stress fields, as indicated in Figures 2–11. However, the amplitude values of the electric displacement and magnetic flux fields are very small, as shown in Figures 4–11, because the perturbation electric displacement and the perturbation magnetic flux fields near the crack tips are cased only by the mechanical loading.

7. Conclusions

The non-local theory solution of two collinear mode-I cracks in a magnetoelectroelastic composite material plane was given by the generalized Almansi's theorem and the Schmidt method. Numerical examples were provided to show the effects of crack length, the distance between two collinear cracks and lattice parameter on the stress field, electric displacement field and the magnetic flux field near the crack tips. Unlike classical elasticity solutions, it was found that no stress, electric displacement and magnetic flux singularities are present near the crack tips in a magnetoelectroelastic composite material plane. The non-local elastic solutions yield a finite hoop stress at the crack tip, thus allowing us to use the maximum stress as a fracture criterion.

Acknowledgements

The authors are grateful for financial support by the National Natural Science Foundation of China (10572043, 10872057), the Natural Science Foundation with Excellent Young Investigators of Hei Long Jiang Province (JC04-08), and the Natural Science Foundation of Heilongjiang Province (A2007-05).

References

[1] T.L. Wu and J.H. Huang, Int. J. Solids Struct. 37 (2000) p.2981.
[2] G.C. Sih and Z.F. Song, Theor. Appl. Fract. Mech. 39 (2003) p.209.
[3] B.L. Wang and Y.W. Mai, Mech. Res. Commun. 31 (2003) p.65.
[4] C.F. Gao, H. Kessler and H. Balke, Eur. J. Mech. A 22 (2003) p.433.
[5] C.F. Gao, H. Kessler and H. Balke, Int. J. Eng. Sci. 41 (2003) p.969.
[6] C.P. Spyropoulos, G.C. Sih and Z.F. Song, Theor. Appl. Fract. Mech. 39 (2003) p.281.
[7] X. Wang and Y.P. Shen, Int. J. Eng. Sci. 40 (2002) p.1069.
[8] W.Q. Chen, K.Y. Lee and H.J. Ding, Int. J. Eng. Sci. 42 (2004) p.1361.
[9] M.H. Zhao, C.Y. Fan, F. Yang and T. Liu, Int. J. Solids Struct. 44 (2007) p.4505.
[10] J.X. Liu, X.L. Liu and Y.B. Zhao, Int. J. Eng. Sci. 39 (2001) p.1405.
[11] P.W. Zhang, Z.G. Zhou and L.Z. Wu, Phil. Mag. 87 (2007) p.1.
[12] Z.G. Zhou, P.W. Zhang and G.Q. Li, Phil. Mag. 88 (2008) p.1153.
[13] A.K. Soh, D.N. Fang and K.L. Lee, Eur. J. Mech. A 19 (2000) p.961.
[14] J.R. Rice, ASME J. Appl. Mech. 35 (1968) p.379.
[15] D.G.B. Edelen, *Non-local field theory*, in *Continuum Physics*, Vol. 4, A.C. Eringen, ed., Academic Press, New York, 1976, p.75.
[16] A.C. Eringen, *Non-local polar field theory*, in *Continuum Physics*, Vol. 4, A.C. Eringen, ed., Academic Press, New York, 1976, p.205.
[17] A.E. Green and R.S. Rivilin, Proc. R. Soc. Lond. A 284 (1965) p.303.
[18] K.L. Pan and N. Takeda, Arch. Appl. Mech. 68 (1998) p.179.
[19] A.C. Eringen, C.G. Speziale and B.S. Kim, J. Mech. Phys. Solids 25 (1977) p.339.
[20] A.C. Eringen, Int. J. Fract. 14 (1978) p.367.
[21] A.C. Eringen, Eng. Fract. Mech. 12 (1979) p.211.
[22] C. Atkinson, Arch. Mech. 32 (1980) p.317.
[23] C. Atkinson, 1980, Arch. Mech. 32 (1980) p.567.
[24] P.S. Cheng, Acta Mech. Sin. 24, (in Chinese) (1992) p.329.

[25] Z.G. Zhou, L.Z. Wu and S.Y. Du, Eur. J. Mech. A 25 (2006) p.793.
[26] Z.G. Zhou, L.Z. Wu and S.Y. Du, Mech. Adv. Mater. Struct. 14 (2007) p.191.
[27] Z.F. Song and G.C. Sih, Theor. Appl. Fract. Mech. 39 (2003) p.189.
[28] A.C. Eringen and B.S. Kim, Cryst. Lattice Def. 7 (1977) p.51.
[29] V.S. Parton, Acta Astronautra 3 (1976) p.671.
[30] F.Q. Yang, Int. J. Solids Struct. 38 (2001) p.3813.
[31] I.S. Gradshteyn and I.M. Ryzhik, *Tables of Integral, Series and Products*, Academic Press, New York, 1980, p.361, 333, 483.
[32] P.M. Morse and H. Feshbach, *Methods of Theoretical Physics*, McGraw-Hill, New York, 1958, p.828.
[33] W.F. Yan, SIAM J. Appl. Math. 15 (1967) p.219.
[34] S. Itou, J. Appl. Mech. 45 (1978) p.807.
[35] A. Erdelyi, *Tables of Integral Transforms*, Vol. 1, McGraw-Hill, New York, 1954, p.34.
[36] A. Amemiya and T. Taguchi, *Numerical Analysis and Fortran*, Maruzen, Tokyo, 1969, p.425.
[37] W.Y. Tian and U. Gabbert, Eur. J. Mech. A 23 (2004) p.599.
[38] A.C. Eringen, J. Appl. Phys. 54 (1983) p.6811.

Appendix 1

$$
\begin{aligned}
a &= c_{44}[-2e_{33}f_{33}g_{33} + e_{33}^2\mu_{33} - f_{33}^2\varepsilon_{33} + c_{33}(g_{33}^2 + \mu_{33}\varepsilon_{33})],\\
b &= -\{e_{31}^2f_{33}^2 + 2c_{33}e_{31}f_{15}g_{33} + 2c_{33}e_{31}f_{31}g_{33} - 2c_{13}e_{31}f_{33}g_{33} - 2c_{44}e_{31}f_{33}g_{33} - 2c_{33}c_{44}g_{11}g_{33}\\
&\quad + c_{13}^2g_{33}^2 - c_{11}c_{33}g_{33}^2 + 2c_{13}c_{44}g_{33}^2 - c_{33}e_{31}^2\mu_{33} + e_{33}^2(f_{15}^2 + 2f_{15}f_{31} + f_{31}^2 - c_{44}\mu_{11} - c_{11}\mu_{33})\\
&\quad + e_{15}^2(f_{33}^2 - c_{33}\mu_{33}) + 2e_{15}\{[c_{33}(f_{15} + f_{31}) - c_{13}f_{33}]g_{33} + e_{31}(f_{33}^2 - c_{33}\mu_{33})\}\\
&\quad - 2e_{33}\{-c_{44}f_{33}g_{11} + c_{13}f_{15}g_{33} + c_{13}f_{31}g_{33} + c_{44}f_{31}g_{33} - c_{11}f_{33}g_{33}\\
&\quad + e_{15}(f_{15}f_{33} + f_{31}f_{33} - c_{13}\mu_{33}) + e_{31}[f_{15}f_{33} + f_{31}f_{33} - \mu_{33}(c_{13} + c_{44})]\}\\
&\quad + c_{44}f_{33}^2\varepsilon_{11} - c_{33}c_{44}\mu_{33}\varepsilon_{11} + c_{33}f_{15}^2\varepsilon_{33} + 2c_{33}f_{15}f_{31}\varepsilon_{33} + c_{33}f_{31}^2\varepsilon_{33}\\
&\quad - 2c_{13}f_{15}f_{33}\varepsilon_{33} - 2c_{13}f_{31}f_{33}\varepsilon_{33}\\
&\quad - 2c_{44}f_{31}f_{33}\varepsilon_{33} + c_{11}f_{33}^2\varepsilon_{33} - c_{33}c_{44}\mu_{11}\varepsilon_{33} + c_{13}^2\mu_{33}\varepsilon_{33} - c_{11}c_{33}\mu_{33}\varepsilon_{33} + 2c_{13}c_{44}\mu_{33}\varepsilon_{33}\},\\
c &= 2c_{13}e_{33}f_{15}g_{11} + 2c_{13}e_{33}f_{31}g_{11} + 2c_{44}e_{33}f_{31}g_{11} - 2c_{11}e_{33}f_{33}g_{11} + c_{33}c_{44}g_{11}^2 - 2c_{11}e_{33}f_{15}g_{33}\\
&\quad - 2c_{13}^2g_{11}g_{33} + 2c_{11}c_{33}g_{11}g_{33} - 4c_{13}c_{44}g_{11}g_{33} + c_{11}c_{44}g_{33}^2 + c_{11}e_{33}^2\mu_{11}\\
&\quad + e_{15}^2(2f_{31}f_{33} + c_{33}\mu_{11} - 2c_{13}\mu_{33}) + e_{31}^2(-2f_{15}f_{33} + c_{33}\mu_{11} + c_{44}\mu_{33})\\
&\quad - 2e_{15}[c_{33}(f_{15} + f_{31})g_{11} - c_{13}f_{33}g_{11} - 2c_{13}f_{15}g_{33} - c_{13}f_{31}g_{33} + c_{11}f_{33}g_{33}\\
&\quad + e_{33}(f_{15}f_{31} + f_{31}^2 + c_{13}\mu_{11} - c_{11}\mu_{33})]\\
&\quad + 2e_{31}\{-c_{33}f_{15}g_{11} - c_{33}f_{31}g_{11} + c_{13}f_{33}g_{11} + c_{44}f_{33}g_{11} + c_{13}f_{15}g_{33} - c_{44}f_{31}g_{33}\\
&\quad + e_{33}[f_{15}^2 + f_{15}f_{31} - (c_{13} + c_{44})\mu_{11}] + e_{15}(-f_{15}f_{33} + f_{31}f_{33} + c_{33}\mu_{11} - c_{13}\mu_{33})\}\\
&\quad - c_{33}f_{15}^2\varepsilon_{11} - 2c_{33}f_{15}f_{31}\varepsilon_{11} - c_{33}f_{31}^2\varepsilon_{11} + 2c_{13}f_{15}f_{33}\varepsilon_{11} + 2c_{13}f_{31}f_{33}\varepsilon_{11} + 2c_{44}f_{31}f_{33}\varepsilon_{11}\\
&\quad - c_{11}f_{33}^2\varepsilon_{11} + c_{33}c_{44}\mu_{11}\varepsilon_{11} - c_{13}^2\mu_{33}\varepsilon_{11} + c_{11}c_{33}\mu_{33}\varepsilon_{11} - 2c_{13}c_{44}\mu_{33}\varepsilon_{11} + 2c_{13}f_{15}^2\varepsilon_{33}\\
&\quad + 2c_{13}f_{15}f_{31}\varepsilon_{33} - c_{44}f_{31}^2\varepsilon_{33} - 2c_{11}f_{15}f_{33}\varepsilon_{33} - c_{13}^2\mu_{11}\varepsilon_{33} + c_{11}c_{33}\mu_{11}\varepsilon_{33}\\
&\qquad - 2c_{13}c_{44}\mu_{11}\varepsilon_{33} + c_{11}c_{44}\mu_{33}\varepsilon_{33},
\end{aligned}
$$

$$
\begin{aligned}
d = &-2c_{11}e_{33}f_{15}g_{11} - c_{13}^2 g_{11}^2 + c_{11}c_{33}g_{11}^2 - 2c_{13}c_{44}g_{11}^2 + 2c_{11}c_{44}g_{11}g_{33} + e_{31}^2(-f_{15}^2 + c_{44}\mu_{11}) \\
&+ 2e_{31}(e_{15}f_{15}f_{31} + c_{13}f_{15}g_{11} - c_{44}f_{31}g_{11} - c_{13}e_{15}\mu_{11}) \\
&+ 2e_{15}[c_{13}(2f_{15} + f_{31})\, g_{11} - c_{11}(f_{33}g_{11} + f_{15}g_{33} - e_{33}\mu_{11})] \\
&- e_{15}^2(f_{31}^2 + 2c_{13}\mu_{11} - c_{11}\mu_{33}) + 2c_{13}f_{15}^2\varepsilon_{11} + 2c_{13}f_{15}f_{31}\varepsilon_{11} - c_{44}f_{31}^2\varepsilon_{11} - 2c_{11}f_{15}f_{33}\varepsilon_{11} \\
&- c_{13}^2\mu_{11}\varepsilon_{11} + c_{11}c_{33}\mu_{11}\varepsilon_{11} - 2c_{13}c_{44}\mu_{11}\varepsilon_{11} + c_{11}c_{44}\mu_{33}\varepsilon_{11} - c_{11}f_{15}^2\varepsilon_{33} + c_{11}c_{44}\mu_{11}\varepsilon_{33}, \\
e = &-2c_{11}e_{15}f_{15}g_{11} + c_{11}c_{44}g_{11}^2 + c_{11}e_{15}^2\mu_{11} - c_{11}f_{15}^2\varepsilon_{11} + c_{11}c_{44}\mu_{11}\varepsilon_{11}.
\end{aligned}
$$

Appendix 2

$$
\begin{aligned}
\alpha_{11} = &-[-e_{31}f_{15}g_{11} + c_{13}g_{11}^2 + c_{44}g_{11}^2 + e_{15}^2\mu_{11} + e_{15}(-2f_{15}g_{11} - f_{31}g_{11} + e_{31}\mu_{11}) \\
&- f_{15}^2\varepsilon_{11} - f_{15}f_{31}\varepsilon_{11} + c_{13}\mu_{11}\varepsilon_{11} + c_{44}\mu_{11}\varepsilon_{11}], \\
\alpha_{12} = &\, e_{31}f_{33}g_{11} + e_{31}f_{15}g_{33} - 2c_{13}g_{11}g_{33} - 2c_{44}g_{11}g_{33} + e_{33}[f_{15}g_{11} + f_{31}g_{11} - (e_{15} + e_{31})\mu_{11}] \\
&- e_{15}^2\mu_{33} + e_{15}(f_{33}g_{11} + 2f_{15}g_{33} + f_{31}g_{33} - e_{31}\mu_{33}) + f_{15}f_{33}\varepsilon_{11} + f_{31}f_{33}\varepsilon_{11} \\
&- c_{13}\mu_{33}\varepsilon_{11} - c_{44}\mu_{33}\varepsilon_{11} \\
&+ f_{15}^2\varepsilon_{33} + f_{15}f_{31}\varepsilon_{33} - c_{13}\mu_{11}\varepsilon_{33} - c_{44}\mu_{11}\varepsilon_{33}, \\
\alpha_{13} = &-\{-e_{15}f_{33}g_{33} - e_{31}f_{33}g_{33} + c_{13}g_{33}^2 + c_{44}g_{33}^2 + e_{33}[-f_{15}g_{33} - f_{31}g_{33} + (e_{15} + e_{31})\mu_{33}] \\
&- f_{15}f_{33}\varepsilon_{33} - f_{31}f_{33}\varepsilon_{33} + c_{13}\mu_{33}\varepsilon_{33} + c_{44}\mu_{33}\varepsilon_{33}\}, \\
\alpha_{21} = &\, c_{11}(g_{11}^2 + \mu_{11}\varepsilon_{11}), \\
\alpha_{22} = &-2e_{31}(f_{15} + f_{31})\, g_{11} + c_{44}g_{11}^2 + 2c_{11}g_{11}g_{33} + e_{15}^2\mu_{11} + e_{31}^2\mu_{11} \\
&- 2e_{15}(f_{15}g_{11} + f_{31}g_{11} - e_{31}\mu_{11}) \\
&- f_{15}^2\varepsilon_{11} - 2f_{15}f_{31}\varepsilon_{11} - f_{31}^2\varepsilon_{11} + c_{44}\mu_{11}\varepsilon_{11} + c_{11}\mu_{33}\varepsilon_{11} + c_{11}\mu_{11}\varepsilon_{33}, \\
\alpha_{23} = &-2e_{31}(f_{15} + f_{31})\, g_{33} + 2c_{44}g_{11}g_{33} + c_{11}g_{33}^2 + e_{15}^2\mu_{33} + e_{31}^2\mu_{33} \\
&- 2e_{15}(f_{15}g_{33} + f_{31}g_{33} - e_{31}\mu_{33}) \\
&+ c_{44}\mu_{33}\varepsilon_{11} - f_{15}^2\varepsilon_{33} - 2f_{15}f_{31}\varepsilon_{33} - f_{31}^2\varepsilon_{33} + c_{44}\mu_{11}\varepsilon_{33} + c_{11}\mu_{33}\varepsilon_{33}, \\
\alpha_{24} = &\, c_{44}(g_{33}^2 + \mu_{33}\varepsilon_{33}), \quad \alpha_{31} = -c_{11}(f_{15}g_{11} - e_{15}\mu_{11}), \\
\alpha_{32} = &-\{-c_{13}f_{15}g_{11} - c_{13}f_{31}g_{11} - c_{44}f_{31}g_{11} + c_{11}f_{33}g_{11} + c_{11}f_{15}g_{33} - c_{11}f_{33}g_{11} \\
&+ e_{31}[-f_{15}^2 - f_{15}f_{31} + (c_{13} + c_{44})\mu_{11}] + e_{15}(f_{15}f_{31} + f_{31}^2 + c_{13}\mu_{11} - c_{11}\mu_{33})\}, \\
\alpha_{33} = &-[-e_{15}f_{15}f_{33} - e_{31}f_{15}f_{33} - e_{15}f_{31}f_{33} - e_{31}f_{31}f_{33} + c_{44}f_{33}g_{11} - c_{13}f_{15}g_{33} \\
&- c_{13}f_{31}g_{33} - c_{44}f_{31}g_{33} + c_{11}f_{33}g_{33} + c_{13}e_{15}\mu_{33} + c_{13}e_{31}\mu_{33} + c_{44}e_{31}\mu_{33} \\
&+ e_{33}(f_{15}^2 + 2f_{15}f_{31} + f_{31}^2 - c_{44}\mu_{11} - c_{11}\mu_{33})], \\
\alpha_{34} = &-c_{44}(f_{33}g_{33} - e_{33}\mu_{33}), \quad \alpha_{41} = c_{11}(e_{15}g_{11} + f_{15}\varepsilon_{11}), \\
\alpha_{42} = &-\{-e_{31}^2f_{15} + e_{15}^2f_{31} + (c_{13} + c_{44})e_{31}g_{11} - c_{11}e_{33}g_{11} \\
&+ e_{15}[e_{31}(-f_{15} + f_{31}) + c_{13}g_{11} - c_{11}g_{33}] \\
&+ c_{13}f_{15}\varepsilon_{11} + c_{13}f_{31}\varepsilon_{11} + c_{44}f_{31}\varepsilon_{11} - c_{11}f_{33}\varepsilon_{11} - c_{11}f_{15}\varepsilon_{33}\}, \\
\alpha_{43} = &\, e_{15}^2f_{33} + e_{31}^2f_{33} + c_{44}e_{33}g_{11} + c_{11}e_{33}g_{33} - e_{15}[e_{33}(f_{15} + f_{31}) - 2e_{31}f_{33} + c_{13}g_{33}] \\
&- e_{31}[e_{33}(f_{15} + f_{31}) + (c_{13} + c_{44})\, g_{33}] + c_{44}f_{33}\varepsilon_{11} - c_{13}f_{15}\varepsilon_{33} - c_{13}f_{31}\varepsilon_{33} \\
&- c_{44}f_{31}\varepsilon_{33} + c_{11}f_{33}\varepsilon_{33}, \\
\alpha_{44} = &\, c_{44}(e_{33}g_{33} + f_{33}\varepsilon_{33}).
\end{aligned}
$$

Appendix 3

Equations (47) and (48) can be rewritten as follows:

$$Ma + Mb = \begin{bmatrix} \bar{f}_1/s^6 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad Na - Nb = \begin{bmatrix} \bar{f}_2/s^6 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{67}$$

where

$$M = \begin{bmatrix} \chi_1^{(1)} & \chi_2^{(1)} & \chi_3^{(1)} & \chi_4^{(1)} \\ \beta_1^{(1)} h_1^* & \beta_2^{(1)} h_2^* & \beta_3^{(1)} h_3^* & \beta_4^{(1)} h_4^* \\ \beta_1^{(3)} h_1^* & \beta_2^{(3)} h_2^* & \beta_3^{(3)} h_3^* & \beta_4^{(3)} h_4^* \\ \beta_1^{(4)} h_1^* & \beta_2^{(4)} h_2^* & \beta_3^{(4)} h_3^* & \beta_4^{(4)} h_4^* \end{bmatrix}, \quad N = \begin{bmatrix} \chi_1^{(2)} & \chi_2^{(2)} & \chi_3^{(2)} & \chi_4^{(2)} \\ \chi_1^{(3)} & \chi_2^{(3)} & \chi_3^{(3)} & \chi_4^{(3)} \\ \chi_1^{(4)} & \chi_2^{(4)} & \chi_3^{(4)} & \chi_4^{(4)} \\ \beta_1^{(2)} h_1^* & \beta_2^{(2)} h_2^* & \beta_3^{(2)} h_3^* & \beta_4^{(2)} h_4^* \end{bmatrix},$$

$$a = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix}, \quad b = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix}.$$

So, it can be obtained that

$$a = \frac{1}{2} M^{-1} \begin{bmatrix} \bar{f}_1/s^6 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \frac{1}{2} N^{-1} \begin{bmatrix} \bar{f}_2/s^6 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad b = \frac{1}{2} M^{-1} \begin{bmatrix} \bar{f}_1/s^6 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \frac{1}{2} N^{-1} \begin{bmatrix} \bar{f}_2/s^6 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{68}$$

So, the unknown functions A_i and B_i can be expressed as follows:

$$A_i = \frac{1}{2}(m_{i1}\bar{f}_1 + n_{i1}\bar{f}_2)/s^6, \quad B_i = \frac{1}{2}(m_{i1}\bar{f}_1 - n_{i1}\bar{f}_2)/s^6, \tag{69}$$

where $[m_{ij}]_{4\times4} = [M]^{-1}$, $[n_{ij}]_{4\times4} = [N]^{-1}$.

Substituting Equation (69) into Equation (67), it can be obtained:

$$\bar{f}_1 \begin{bmatrix} \chi_1^{(1)} & \chi_2^{(1)} & \chi_3^{(1)} & \chi_4^{(1)} \\ \beta_1^{(1)} h_1^* & \beta_2^{(1)} h_2^* & \beta_3^{(1)} h_3^* & \beta_4^{(1)} h_4^* \\ \beta_1^{(3)} h_1^* & \beta_2^{(3)} h_2^* & \beta_3^{(3)} h_3^* & \beta_4^{(3)} h_4^* \\ \beta_1^{(4)} h_1^* & \beta_2^{(4)} h_2^* & \beta_3^{(4)} h_3^* & \beta_4^{(4)} h_4^* \end{bmatrix} \begin{bmatrix} m_{11} \\ m_{21} \\ m_{31} \\ m_{41} \end{bmatrix} = \begin{bmatrix} \bar{f}_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{70}$$

$$\bar{f}_2 \begin{bmatrix} \chi_1^{(2)} & \chi_2^{(2)} & \chi_3^{(2)} & \chi_4^{(2)} \\ \chi_1^{(3)} & \chi_2^{(3)} & \chi_3^{(3)} & \chi_4^{(3)} \\ \chi_1^{(4)} & \chi_2^{(4)} & \chi_3^{(4)} & \chi_4^{(4)} \\ \beta_1^{(2)} h_1^* & \beta_2^{(2)} h_2^* & \beta_3^{(2)} h_3^* & \beta_4^{(2)} h_4^* \end{bmatrix} \begin{bmatrix} n_{11} \\ n_{21} \\ n_{31} \\ n_{41} \end{bmatrix} = \begin{bmatrix} \bar{f}_2 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{71}$$

Therefore, we have

$$\begin{bmatrix} \chi_1^{(1)} & \chi_2^{(1)} & \chi_3^{(1)} & \chi_4^{(1)} \\ \beta_1^{(1)}h_1^* & \beta_2^{(1)}h_2^* & \beta_3^{(1)}h_3^* & \beta_4^{(1)}h_4^* \\ \beta_1^{(3)}h_1^* & \beta_2^{(3)}h_2^* & \beta_3^{(3)}h_3^* & \beta_4^{(3)}h_4^* \\ \beta_1^{(4)}h_1^* & \beta_2^{(4)}h_2^* & \beta_3^{(4)}h_3^* & \beta_4^{(4)}h_4^* \end{bmatrix} \begin{bmatrix} m_{11} \\ m_{21} \\ m_{31} \\ m_{41} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} \sum_{i=1}^{4} \beta_i^{(1)} h_i^* m_{i1} = 0 \\ \sum_{i=1}^{4} \beta_i^{(3)} h_i^* m_{i1} = 0\,, \\ \sum_{i=1}^{4} \beta_i^{(4)} h_i^* m_{i1} = 0 \end{cases} \tag{72}$$

$$\begin{bmatrix} \chi_1^{(2)} & \chi_2^{(2)} & \chi_3^{(2)} & \chi_4^{(2)} \\ \chi_1^{(3)} & \chi_2^{(3)} & \chi_3^{(3)} & \chi_4^{(3)} \\ \chi_1^{(4)} & \chi_2^{(4)} & \chi_3^{(4)} & \chi_4^{(4)} \\ \beta_1^{(2)}h_1^* & \beta_2^{(2)}h_2^* & \beta_3^{(2)}h_3^* & \beta_4^{(2)}h_4^* \end{bmatrix} \begin{bmatrix} n_{11} \\ n_{21} \\ n_{31} \\ n_{41} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} \sum_{i=1}^{4} \chi_i^{(3)} n_{i1} = 0 \\ \sum_{i=1}^{4} \chi_i^{(4)} n_{i1} = 0 \\ \sum_{i=1}^{4} \beta_i^{(2)} h_i^* n_{i1} = 0 \end{cases}. \tag{73}$$

Substituting Equation (69) into Equation (44) and applying Equations (72) and (73), we have

$$\begin{aligned} \tau_{zz}^{(1)}(x,0) &= \frac{1}{\pi}\int_0^\infty \sum_{k=1}^{4} s\beta_k^{(1)} h_k^*(a,\beta,s)(m_{k1}\bar{f}_1 + n_{k1}\bar{f}_2)\cos(sx)\mathrm{d}s \\ &= \frac{1}{\pi}\int_0^\infty s\sum_{k=1}^{4} \beta_k^{(1)} h_k^*(a,\beta,s) n_{k1}\bar{f}_2\cos(sx)\mathrm{d}s = \frac{1}{\pi}\int_0^\infty sg_1(a,\beta,s)\bar{f}_2\cos(sx)\mathrm{d}s \end{aligned} \tag{74}$$

$$\begin{aligned} \tau_{xz}^{(1)}(x,0) &= \frac{1}{\pi}\int_0^\infty \left[s\sum_{k=1}^{4} \beta_k^{(2)} h_k^*(a,\beta,s)(m_{k1}\bar{f}_1 + n_{k1}\bar{f}_2)\right]\sin(sx)\mathrm{d}s \\ &= \frac{1}{\pi}\int_0^\infty s\sum_{k=1}^{4} \beta_k^{(2)} h_k^*(a,\beta,s) m_{k1}\bar{f}_1\sin(sx)\mathrm{d}s = \frac{1}{\pi}\int_0^\infty sg_2(a,\beta,s)\bar{f}_1\sin(sx)\mathrm{d}s \end{aligned} \tag{75}$$

$$\begin{aligned} D_z^{(1)}(x,0) &= \frac{1}{\pi}\int_0^\infty s\sum_{k=1}^{4} \beta_k^{(3)} h_k^*(a,\beta,s)(m_{k1}\bar{f}_1 + n_{k1}\bar{f}_2)\cos(sx)\mathrm{d}s \\ &= \frac{1}{\pi}\int_0^\infty s\sum_{k=1}^{4} \beta_k^{(3)} h_k^*(a,\beta,s) n_{k1}\bar{f}_2\cos(sx)\mathrm{d}s = \frac{1}{\pi}\int_0^\infty sg_3(a,\beta,s)\bar{f}_2\cos(sx)\mathrm{d}s \end{aligned} \tag{76}$$

$$\begin{aligned} B_z^{(1)}(x,0) &= \frac{1}{\pi}\int_0^\infty s\sum_{k=1}^{4} \beta_k^{(4)} h_k^*(a,\beta,s)(m_{k1}\bar{f}_1 + n_{k1}\bar{f}_2)\cos(sx)\mathrm{d}s \\ &= \frac{1}{\pi}\int_0^\infty s\sum_{k=1}^{4} \beta_k^{(4)} h_k^*(a,\beta,s) n_{k1}\bar{f}_2\cos(sx)\mathrm{d}s = \frac{1}{\pi}\int_0^\infty sg_4(a,\beta,s)\bar{f}_2\cos(sx)\mathrm{d}s. \end{aligned} \tag{77}$$