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A novel stabilization approach for small signal disturbance of power system with time-varying delay

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Abstract: Small signal instability may cause severe accidents for power system if it can not be dealt correctly and timely. How to maintain power system stable under small signal disturbance is a big challenge for power system operators and dispatchers. Time delay existing in signal transmission process makes the problem more complex. Conventional eigenvalue analysis method neglects time delay influence and can not precisely describe power system dynamic behaviors. In this work, a modified small signal stability model considering time varying delay influence was constructed and a new time delay controller was proposed to stabilize power system under disturbance. By Lyapunov-Krasovskii function, the control law in the form of nonlinear matrix inequality (NLMI) was derived. Considering synthesis method limitation for time delay controller at present, both parameter adjustment method by using linear matrix inequality (LMI) solver and iteration searching method by solving nonlinear minimization problem were suggested to design the controller. Simulation tests were carried out on synchronous-machine infinite-bus power system. Satisfactory test results verify the correctness of the proposed model and the feasibility of the stabilization approach.

Key words: power system stability; small signal disturbance; time-varying delay; power system stabilizer

1 Introduction

When power system is subjected to small signal disturbances such as load fluctuation, load shedding, and conductor galloping, power system may become unstable and should be stabilized with appropriate control input [1]. Small signal instability takes the following forms: 1) steady increase in generator rotor angle due to the lack of synchronizing torque; 2) rotor oscillations with increasing amplitude due to the lack of sufficient damping torque [2]. Small signal instability endangers power system security and operation [3], and even causes power system splitting and blackout [4]. In order to improve power system stability performance under small signal disturbance, many interests have been focused on the study of power system stability criterion and stabilization controller design approach.

Conventional small signal stability model is a set of nonlinear differential equations and algebraic equations for power system, which is transformed into a linear control system using Taylor's formula at equilibrium point, and eigenvalues of the system state matrix are used to analyze power system small signal stability [5−8]. Based on the linear control theory, small signal stability criterion is given below: power system is stable when it is subjected to small signal disturbance, if and only if real parts of all eigenvalues for the state matrix are negative [1].

Recently, with the successful applications of phasor measurement unit (PMU) and wide area measurement system (WAMS) in power system, time delay in signal transmission process can not be neglected [9]. According to time delay control system theory [10], the delay would deteriorate power system damping performance, or even cause power system instability. Time delay in wide area control exists in the transmission of data from measurement location to a control center and the communication of these data to control devices [11]. It usually varies in the range of milliseconds based on communication media or routing algorithm [12], or even hundreds of milliseconds under unusual circumstances such as communication congestion [13]. It can be regarded as stochastic process [14], or time-varying delay [15]. In the WAMS environment, the conventional small signal stability model dose not consider time delay influence and can not precisely depict power system dynamic behaviors. Therefore, a novel small signal stability model for time delay power system should be constructed and a new stabilization controller design approach for preventing power system instability should be studied further. Pade approximation method [16],

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characteristic root method [17], Lyapunov-Krasovskii functional [18], and gain scheduling method [19] are usually used to analyze the stability criterion of time delay systems. Comparatively, Lyapunov-Krasovskii functional method is better than others because it can deal with both constant delay and time-varying delay, and provide the system stability margin with less conservativeness.

The contribution of the work is to propose a new stabilization controller design methodology to improve stability performance of time delay power system. In order to fit practical power system operation environment, the feedback controller itself also considers time delay influence. The control law can be achieved by solving LMI or NLMI according to the new small signal stability criterion derived by Lyapunov-Krasovskii functional method proposed in Ref. [18]. Both parameter adjustment method by using LMI solver and iteration searching method by solving nonlinear minimization problem can make the controller design more simple and feasible. Simulation results verify the correctness of the proposed model and the feasibility of the stabilization approach.

2 Problem description

Dynamic equations of generator with exciter and power system stabilizer (PSS) are given below:

$$\dot{\delta}=\omega_{\mathrm{s}}(\omega-1) \tag{1}$$

$$\dot{\omega}=\frac{1}{M}[P_{\mathrm{m}}-P_{\mathrm{e}}-D(\omega-1)] \tag{2}$$

$$P_{\mathrm{e}}=E'_q I_q-(x'_d-x_q)I_d I_q \tag{3}$$

$$\dot{E}'_q=\frac{1}{T'_{d0}}[E_{\mathrm{f}}-E'_q-(x_d-x'_d)I_d] \tag{4}$$

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix}=\frac{1}{x'_d x_q}\begin{bmatrix} 0 & x_q \\ -x'_d & 0 \end{bmatrix}\begin{bmatrix} -U_d \\ E'_q-U_q \end{bmatrix} \tag{5}$$

$$\dot{E}_{\mathrm{f}}=\frac{K_{\mathrm{e}}}{T_{\mathrm{e}}}(U_{\mathrm{ref}}-U_t+U_{\mathrm{pss}})-\frac{E_{\mathrm{f}}}{T_{\mathrm{e}}} \tag{6}$$

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix}=\begin{bmatrix} 0 & 1 \\ -\dfrac{1}{T_2T_3} & -\dfrac{T_2+T_3}{T_2T_3} \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}+\begin{bmatrix} 0 \\ \dfrac{1}{T_2T_3} \end{bmatrix}\Delta\omega(t-d(t)) \tag{7}$$

$$U_{\mathrm{pss}}=\frac{-K_{\mathrm{pss}}}{K_{\mathrm{e}}}\{\begin{bmatrix} \dfrac{-T_1}{T_2} & \dfrac{T_3-T_1T_2-T_1T_3}{T_2} \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}+\frac{T_1}{T_2}\Delta\omega(t-d(t))\} \tag{8}$$

where δ is generator rotor angle, rad; ω_{s} is generator synchronous angular speed, rad/s; ω is generator rotor angle speed, rad/s; M is mechanical starting time, s; P_{m} is mechanical active power, kW; P_{e} is electrical active power, kW; D is damping coefficient; E'_q is generator q-axis transient voltage, kV; I_q is generator q-axis circuit, A; x'_d is generator d-axis transient reactance, Ω; x_q is generator q-axis synchronous reactance, Ω; I_d is generator d-axis circuit, A; T'_{d0} is generator d-axis transient short circuit time constant, s; E_{f} is exciter excitation voltage, kV; x_d is generator d-axis synchronous reactance, Ω; U_d is generator d-axis voltage, kV; U_q is generator q-axis voltage, kV; K_{e} is exciter gain constant; T_{e} is exciter time constant, s; U_{ref} is reference bus voltage, kV; U_{t} is generator terminal voltage, kV; U_{pss} is PSS control voltage, kV; y_1 and y_2 are PSS state variables; T_1, T_2 and T_3 are PSS time constants, s; and K_{pss} is PSS gain constant.

It is assumed that $d(t)$ denotes PSS input signal transmission delay which at least includes signal measurement delay, signal delivery delay and PSS control delay [11]. The delay $d(t)$ is time-varying and complies with the rules of $0\le d(t)\le\tau$ and $\dot{d}(t)\le\mu$ where τ denotes delay upper bound or delay margin, and μ denotes delay variation ratio. Based on Eqs. (1)−(8) and power system network equations, a novel power system small signal stability model considering delay influence can be constructed.

To clearly demonstrate basic principle and simplify analysis process, synchronous-machine infinite-bus power system is chosen to derive small signal stability model. It is assumed that power system state $\boldsymbol{x}$ is denoted by $\boldsymbol{x}=[\Delta\delta\ \Delta\omega\ \Delta E'_q\ \Delta E_{\mathrm{f}}\ \Delta y_1\ \ \Delta y_2]$ and control input $\boldsymbol{u}$ is denoted by $\boldsymbol{u}=[\Delta\boldsymbol{P}_{\mathrm{m}}]$. U is infinite-bus voltage magnitude, kV; and x_{e} is transmission line reactance between generator and infinite bus, Ω. After linearizing Eqs. (1)−(8) at the equilibrium point $\boldsymbol{x}_0=[\delta_0\ \omega_0\ E'_{q0}\ E_{f0}\ y_{10}\ y_{20}]$, small signal stability model with time-varying delay is given below:

$$\dot{\boldsymbol{x}}(t)=\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{A}_d\boldsymbol{x}(t-d(t))+\boldsymbol{B}\boldsymbol{u}(t-d(t)) \tag{9}$$

where state matrix $\boldsymbol{A}$ is given by

$$\boldsymbol{A}=\begin{bmatrix} 0 & \omega_s & 0 & 0 & 0 & 0 \\ \dfrac{-K_1}{M} & \dfrac{-D}{M} & \dfrac{-K_2}{M} & 0 & 0 & 0 \\ \dfrac{-K_3}{T'_{d0}} & 0 & \dfrac{-K_4}{T'_{d0}} & \dfrac{1}{T'_{d0}} & 0 & 0 \\ \dfrac{-K_{\mathrm{e}}K_5}{T_{\mathrm{e}}} & 0 & \dfrac{-K_{\mathrm{e}}K_6}{T_{\mathrm{e}}} & -\dfrac{1}{T_{\mathrm{e}}} & \dfrac{K_{\mathrm{pss}}T_1}{T_{\mathrm{e}}T_2} & a_{46} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\dfrac{1}{T_2T_3} & a_{66} \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} 0 \\ I_q \end{bmatrix} + \begin{bmatrix} \dfrac{U\sin\delta}{x_e + x'_d} & \dfrac{U\cos\delta}{x_e + x_q} \\ \dfrac{1}{x_e + x'_d} & 0 \end{bmatrix} \begin{bmatrix} (x_q - x'_d)I_q \\ E'_q + (x_q - x'_d)I_d \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} K_3 \\ K_4 \end{bmatrix} = \begin{bmatrix} \dfrac{(x_d - x'_d)}{x_e + x'_d} U\sin\delta \\ 1 + \dfrac{x_d - x'_d}{x_e + x'_d} \end{bmatrix} \tag{12}$$

$$\begin{bmatrix} K_5 \\ K_6 \end{bmatrix} = \begin{bmatrix} 0 \\ \dfrac{U_q}{U_t} \end{bmatrix} + \begin{bmatrix} \dfrac{U\sin\delta}{x_e + x'_d} & \dfrac{U\cos\delta}{x_e + x_q} \\ \dfrac{1}{x_e + x'_d} & 0 \end{bmatrix} \begin{bmatrix} -x'_d \dfrac{U_q}{U_t} \\ x_q \dfrac{U_d}{U_t} \end{bmatrix} \tag{13}$$

$$a_{46} = \frac{K_{pss} T_1 (T_3 - T_1 T_2 - T_1 T_3)}{T_e T_2} \tag{14}$$

$$a_{66} = -\frac{T_2 + T_3}{T_2 T_3} \tag{15}$$

Time delay state matrix A_d is given by

$$\boldsymbol{A}_d = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\dfrac{K_{pss}T_1}{T_e T_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \dfrac{1}{T_2 T_3} & 0 & 0 & 0 & 0 \end{bmatrix} \tag{16}$$

and control input matrix $\boldsymbol{B}$ is given by

$$\boldsymbol{B} = [0 \quad \frac{1}{M} \quad 0 \quad 0 \quad 0 \quad 0]^{\mathrm{T}} \tag{17}$$

Compared with the conventional small signal stability model, the proposed model has following features: 1) The time delay item $\boldsymbol{A}_d\boldsymbol{x}(t-d(t))$ is added, which means that eigenvalue analysis method based on the linear control theory can not be used to analyze the stability of the newly constructed time delay power system; 2) The feedback controller $\boldsymbol{u}(t-d(t))$ also considers time delay influence and can fit for wide area control needs.

3 Stabilization approach

In order to maintain power system stable when it is subjected to small signal disturbance, the following controller can be used to stabilize Eq. (9) with appropriate control input.

$$\boldsymbol{u}(t-d(t)) = \boldsymbol{K}\boldsymbol{x}(t-d(t)) \tag{18}$$

As mentioned above, Lyapunov-Krasovskii functional method is better than Pade approximation and characteristic root methods. By analyzing features of the Eq. (9) without control input $\boldsymbol{u}$, a delay-dependent Lyapunov-Krasovskii functional which considers time delay influence on stability performance is constructed below:

$$\boldsymbol{V}(t,x_t) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{M}_1\boldsymbol{x}(t) + \int_{t-d(t)}^{t} \boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{M}_2\boldsymbol{x}(s)\mathrm{d}s + \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{\boldsymbol{x}}(s)^{\mathrm{T}}\boldsymbol{M}_3\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta \tag{19}$$

where $\boldsymbol{M}_1 = \boldsymbol{M}_1^{\mathrm{T}} > 0$, $\boldsymbol{M}_2 = \boldsymbol{M}_2^{\mathrm{T}} \geq 0$ and $\boldsymbol{M}_3 = \boldsymbol{M}_3^{\mathrm{T}} > 0$ are to be determined. It can be used to derive a new delay-dependent small signal stability criterion for power system when $\boldsymbol{u}(t-d(t))=0$. For any appropriately dimensioned $\boldsymbol{X} = \begin{bmatrix} \boldsymbol{X}_{11} & \boldsymbol{X}_{12} \\ * & \boldsymbol{X}_{22} \end{bmatrix} \geq 0$, the following inequality holds by Leibniz-Newton formula:

$$\tau\xi_1^{\mathrm{T}}(t)\boldsymbol{X}\xi_1(t) - \int_{t-d(t)}^{t} \xi_1^{\mathrm{T}}(t)\boldsymbol{X}\xi_1(t)\mathrm{d}s \geq 0 \tag{20}$$

where $\xi_1(t) = [\boldsymbol{x}^{\mathrm{T}}(t) \quad \boldsymbol{x}^{\mathrm{T}}(t-d(t))]^{\mathrm{T}}$.

Calculating the derivative of Eq. (19) along the solutions of Eq. (9) yields

$$\begin{aligned} \dot{V}(t,x_t) = {} & \boldsymbol{x}^{\mathrm{T}}(t)[\boldsymbol{M}_1\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{M}_1]\boldsymbol{x}(t) + \\ & 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{M}_1\boldsymbol{A}_d\boldsymbol{x}(t-d(t)) + \boldsymbol{x}(t)\boldsymbol{M}_2\boldsymbol{x}^{\mathrm{T}}(t) - \\ & (1-\dot{d}(t))\boldsymbol{x}^{\mathrm{T}}(t-d(t))\boldsymbol{M}_2\boldsymbol{x}(t-d(t)) + \\ & \tau\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{M}_3\dot{\boldsymbol{x}}(t) - \int_{t-\tau}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{M}_3\dot{\boldsymbol{x}}(s)\mathrm{d}s \leq \boldsymbol{x}^{\mathrm{T}}(t)\cdot \\ & [\boldsymbol{M}_1\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{M}_1]\boldsymbol{x}(t) + 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{M}_1\boldsymbol{A}_d\boldsymbol{x}(t-d(t)) + \\ & \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{M}_2\boldsymbol{x}(t) - (1-\mu)\boldsymbol{x}^{\mathrm{T}}(t-d(t))\boldsymbol{M}_2\boldsymbol{x}\cdot \\ & (t-d(t)) + \tau\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{M}_3\dot{\boldsymbol{x}}(t) - \\ & \int_{t-d(t)}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{M}_3\dot{\boldsymbol{x}}(s)\mathrm{d}s + \tau\xi_1^{\mathrm{T}}(t)\boldsymbol{X}\xi_1(t) - \\ & \int_{t-d(t)}^{t} \xi_1^{\mathrm{T}}(t)\boldsymbol{X}\xi_1(t)\mathrm{d}s + 2[\boldsymbol{x}(t) - \boldsymbol{x}(t-d(t))]\cdot \\ & [\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{N}_1 + \boldsymbol{x}^{\mathrm{T}}(t-d(t))\boldsymbol{N}_2] - \\ & 2\int_{t\text{-}d(t)}^{t} \dot{\boldsymbol{x}}(s)ds[\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{N}_1 + \boldsymbol{x}^{\mathrm{T}}(t-d(t))\boldsymbol{N}_2] = \\ & \xi_1^{\mathrm{T}}(t)\boldsymbol{\Gamma}_1\xi_1(t) - \int_{t-d(t)}^{t} \xi_2^{\mathrm{T}}(t,s)\boldsymbol{\Gamma}_2\xi_2(t,s)\mathrm{d}s \end{aligned} \tag{21}$$

where

$$\xi_2(t,s) = [\boldsymbol{x}^{\mathrm{T}}(t) \quad \boldsymbol{x}^{\mathrm{T}}(t-d(t)) \quad \dot{\boldsymbol{x}}^{\mathrm{T}}(s)] \tag{22}$$

$$\boldsymbol{\Gamma}_1 = \begin{bmatrix} \boldsymbol{\Gamma}_{11} + \tau\boldsymbol{A}^{\mathrm{T}}\boldsymbol{M}_3\boldsymbol{A} & \boldsymbol{\Gamma}_{12} + \tau\boldsymbol{A}^{\mathrm{T}}\boldsymbol{M}_3\boldsymbol{A}_d \\ \boldsymbol{\Gamma}_{12}^{\mathrm{T}} + \tau\boldsymbol{A}_d^{\mathrm{T}}\boldsymbol{M}_3\boldsymbol{A} & \boldsymbol{\Gamma}_{22} + \tau\boldsymbol{A}_d^{\mathrm{T}}\boldsymbol{M}_3\boldsymbol{A}_d \end{bmatrix} \tag{23}$$

$$\boldsymbol{\Gamma}_{11} = \boldsymbol{M}_1\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{M}_1 + \boldsymbol{N}_1 + \boldsymbol{N}_1^{\mathrm{T}} + \boldsymbol{M}_2 + \tau\boldsymbol{X}_{11} \tag{24}$$

$$\boldsymbol{\Gamma}_{12} = \boldsymbol{M}_1 \boldsymbol{A}_d - \boldsymbol{N}_1 + \boldsymbol{N}_2^{\mathrm{T}} + \tau \boldsymbol{X}_{12} \tag{25}$$

$$\boldsymbol{\Gamma}_{22} = -\boldsymbol{N}_2 - \boldsymbol{N}_2^{\mathrm{T}} - (1-\mu)\boldsymbol{M}_2 + \tau \boldsymbol{X}_{22} \tag{26}$$

$$\boldsymbol{\Gamma}_2 = \begin{bmatrix} \boldsymbol{X}_{11} & \boldsymbol{X}_{12} & \boldsymbol{N}_1 \\ \boldsymbol{X}_{12}^{\mathrm{T}} & \boldsymbol{X}_{22} & \boldsymbol{N}_2 \\ \boldsymbol{N}_1^{\mathrm{T}} & \boldsymbol{N}_2^{\mathrm{T}} & \boldsymbol{M}_3 \end{bmatrix} \tag{27}$$

By Schur complement, the LMI $\boldsymbol{\Gamma}_1<0$ is equivalent to:

$$\boldsymbol{\Gamma}_3 = \begin{bmatrix} \boldsymbol{\Gamma}_{11} & \boldsymbol{\Gamma}_{12} & \tau \boldsymbol{A}^{\mathrm{T}} \boldsymbol{M}_3 \\ \boldsymbol{\Gamma}_{12}^{\mathrm{T}} & \boldsymbol{\Gamma}_{22} & \tau \boldsymbol{A}_d^{\mathrm{T}} \boldsymbol{M}_3 \\ \tau \boldsymbol{M}_3 \boldsymbol{A} & \tau \boldsymbol{M}_3 \boldsymbol{A}_d & -\tau \boldsymbol{M}_3 \end{bmatrix} < 0 \tag{28}$$

If $\boldsymbol{\Gamma}_2 \geq 0$ and $\boldsymbol{\Gamma}_3<0$ hold, then $\dot{V}(t,x_t) < -\gamma \|\boldsymbol{x}(t)\|^2$ for a sufficiently small $\gamma>0$. Therefore, small signal stability criterion for power system with time-varying delay is given below.

Theorem 1: Given scalars $\tau>0$ and μ, power system is stable if there exist matrices $\boldsymbol{M}_1 = \boldsymbol{M}_1^{\mathrm{T}} > 0$, $\boldsymbol{M}_2 = \boldsymbol{M}_2^{\mathrm{T}} \geq 0$ and $\boldsymbol{M}_3 = \boldsymbol{M}_3^{\mathrm{T}} > 0$, $\boldsymbol{X} \geq 0$, and any appropriately dimensioned matrices $\boldsymbol{N}_1$ and $\boldsymbol{N}_2$ such that $\boldsymbol{\Gamma}_2 \geq 0$ and $\boldsymbol{\Gamma}_3<0$ hold.

Next, we will analyze control law when $\boldsymbol{u}(t-d(t)) = \boldsymbol{K}\boldsymbol{x}(t-d(t))$. Replacing $\boldsymbol{A}_d$ with $\boldsymbol{A}_d+\boldsymbol{BK}$, defining

$$\begin{aligned} &\boldsymbol{\Pi}_1 = \boldsymbol{M}_1^{-1},\ \boldsymbol{\Pi}_2 = \boldsymbol{M}_1^{-1}\boldsymbol{M}_2\boldsymbol{M}_1^{-1},\ \boldsymbol{\Pi}_3 = \boldsymbol{M}_3^{-1} \\ &\boldsymbol{\Theta}_1 = \boldsymbol{M}_1^{-1}\boldsymbol{N}_1\boldsymbol{M}_1^{-1},\ \boldsymbol{\Theta}_2 = \boldsymbol{M}_1^{-1}\boldsymbol{N}_2\boldsymbol{M}_1^{-1},\ \boldsymbol{\Theta}_3 = \boldsymbol{K}\boldsymbol{M}_1^{-1} \\ &\boldsymbol{\Phi} = \mathrm{diag}\{\boldsymbol{M}_1^{-1} \quad \boldsymbol{M}_1^{-1}\}\boldsymbol{X}\mathrm{diag}\{\boldsymbol{M}_1^{-1} \quad \boldsymbol{M}_1^{-1}\} \end{aligned} \tag{29}$$

and pre- and post-multiplying left and right sides of Eq. (27) by $\mathrm{diag}\{\boldsymbol{M}_1^{-1}\ \boldsymbol{M}_1^{-1}\ \boldsymbol{M}_1^{-1}\}$ and pre- and post-multiplying left and right sides of Eq. (28) by $\mathrm{diag}\{\boldsymbol{M}_1^{-1}\ \boldsymbol{M}_1^{-1}\ \boldsymbol{M}_3^{-1}\}$ respectively yield

$$\boldsymbol{\Xi}_1 = \begin{bmatrix} \boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{12} & \boldsymbol{\Theta}_1 \\ \boldsymbol{\Phi}_{12}^{\mathrm{T}} & \boldsymbol{\Phi}_{22} & \boldsymbol{\Theta}_2 \\ \boldsymbol{\Theta}_1^{\mathrm{T}} & \boldsymbol{\Theta}_2^{\mathrm{T}} & \boldsymbol{\Pi}_1\boldsymbol{\Pi}_3^{-1}\boldsymbol{\Pi}_1 \end{bmatrix} \geq 0 \tag{30}$$

$$\boldsymbol{\Xi}_2 = \begin{bmatrix} \boldsymbol{\Xi}_{11} & \boldsymbol{\Xi}_{12} & \tau\boldsymbol{\Pi}_1\boldsymbol{A}^{\mathrm{T}} \\ \boldsymbol{\Xi}_{12}^{\mathrm{T}} & \boldsymbol{\Xi}_{22} & \tau(\boldsymbol{\Pi}_1\boldsymbol{A}_d^{\mathrm{T}} + \boldsymbol{\Theta}_3^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}) \\ \tau\boldsymbol{A}\boldsymbol{\Pi}_1 & \tau(\boldsymbol{A}_d\boldsymbol{\Pi}_1 + \boldsymbol{B}\boldsymbol{\Theta}_3) & -\tau\boldsymbol{\Pi}_3 \end{bmatrix} < 0 \tag{31}$$

where

$$\begin{aligned} &\boldsymbol{\Xi}_{11} = \boldsymbol{A}\boldsymbol{\Pi}_1 + \boldsymbol{\Pi}_1\boldsymbol{A}^{\mathrm{T}} + \boldsymbol{\Theta}_1 + \boldsymbol{\Theta}_1^{\mathrm{T}} + \boldsymbol{\Pi}_2 + \tau\boldsymbol{\Phi}_{11} \\ &\boldsymbol{\Xi}_{12} = \boldsymbol{A}_d\boldsymbol{\Pi}_1 + \boldsymbol{B}\boldsymbol{\Theta}_3 - \boldsymbol{\Theta}_1 + \boldsymbol{\Theta}_2^{\mathrm{T}} + \tau\boldsymbol{\Phi}_{12} \\ &\boldsymbol{\Xi}_{22} = -\boldsymbol{\Theta}_2 - \boldsymbol{\Theta}_2^{\mathrm{T}} - (1-\mu)\boldsymbol{\Pi}_2 + \tau\boldsymbol{\Phi}_{22} \end{aligned} \tag{32}$$

Theorem 2: For given scalars $\tau>0$ and μ, if there exist matrices $\boldsymbol{\Pi}_1 = \boldsymbol{\Pi}_1^{\mathrm{T}} > 0$, $\boldsymbol{\Pi}_2 = \boldsymbol{\Pi}_2^{\mathrm{T}} \geq 0$, $\boldsymbol{\Pi}_3 = \boldsymbol{\Pi}_3^{\mathrm{T}} > 0$, $\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{12} \\ \boldsymbol{\Phi}_{12}^{T} & \boldsymbol{\Phi}_{22} \end{bmatrix} \geq 0$, and any appropriately dimensioned matrices $\boldsymbol{\Theta}_1$, $\boldsymbol{\Theta}_2$ and $\boldsymbol{\Theta}_3$ such that Eqs. (30) and (31) hold, then power system can be stabilized and the state feedback control law is given by $\boldsymbol{u}(t) = \boldsymbol{\Theta}_3\boldsymbol{\Pi}_1^{-1}\boldsymbol{x}(t-d(t))$.

Remark 1: Since eigenvalue analysis method is not applicable for Eq. (9), a new small signal stability criterion and controller design approach for time delay power system, which overcomes the disadvantages of eigenvalue analysis method, is discussed. Note that if $\tau=0$, the model can be transformed into small signal stability model without time delay, and can still be analyzed by eigenvalue analysis method. Hence, it is clear that the proposed model and stability criterion extends application field of eigenvalue analysis method.

Remark 2: The controller gain, $\boldsymbol{K}$, in the proposed controller design method can be solved by two ways: 1) parameter adjustment by using LMI solver while setting $\boldsymbol{\Pi}_3 = \varepsilon\boldsymbol{\Pi}_1$; 2) iteration searching by solving nonlinear minimization problem. Both methods can only find sub-optimum value of $\boldsymbol{K}$.

4 Numerical example

To validate the correctness of the proposed model and the feasibility of the stabilization approach, simulation tests are carried out on synchronous-machine infinite-bus power system [2]. In the test, the controller gain $\boldsymbol{K}$ with respect to delay variation rate μ is also investigated. Parameter adjustment method is adopted to obtain $\boldsymbol{K}$ by using Matlab LMI Toolbox. In order to demonstrate power system stability performance after small signal disturbance, the simulation model for small signal stability analysis is also constructed by using Matlab Simulink Toolbox.

It is assumed that the delay measured by practical power system is as large as 0.1 s. In order to maintain power system stability under small signal disturbance, it is required that the delay margin for the power system with the stabilization controller should satisfy the following rule: $\tau \geq 0.1$ s. Therefore, the objective of the stabilization controller design is to find an appropriate controller gain $\boldsymbol{K}$ which can ensure power system stability under small signal disturbance even if the input signal is delayed for 0.1 s.

By using **Theorem 2**, delay margin τ and controller gain $\boldsymbol{K}$ are listed in Table 1. During the computation, ε is a factor that can be adjusted. By setting different ε, the relatively larger τ can be obtained and its corresponding controller gain $\boldsymbol{K}$ can also be calculated by $\boldsymbol{K} = \boldsymbol{\Theta}_3\boldsymbol{\Pi}_1^{-1}$. It is general that delay margin τ decreases when delay variation rate μ increases [10, 18]. However, from Table 1, it is clear that delay variation rate μ has little influence on delay margin τ if the controller gain $\boldsymbol{K}$ is adjusted properly, which means that μ does not affect power system stability under small signal disturbance only if an appropriate controller is designed. Note that because $\boldsymbol{\Pi}_3$ is limited to the form of $\varepsilon\boldsymbol{\Pi}_1$, the gain $\boldsymbol{K}$ obtained is only the sub-optimum value.

Table 1 Delay margin and controller gain

μ	τ	$\boldsymbol{K}$			ε
0	0.102	[0.003 4 −0.002 3	0.458 5 −0.145 4	0.009 7 0.215 4]	17
0.25	0.101	[0.006 1 −0.003 3	−1.218 2 −0.211 2	0.014 6 0.309 3]	12
0.5	0.100	[0.008 4 −0.003 9	−2.412 8 −0.261 7	0.018 3 0.370 3]	10
0.75	0.100	[0.008 4 −0.003 9	−2.409 4 −0.261 2	0.018 3 0.371 8]	10
1	0.100	[0.003 1 −0.002 2	0.605 6 −0.141 9	0.009 3 0.208 2]	18

In the process of computation, Eqs. (30) and (31) are correspondingly modified below:

$$\boldsymbol{\Xi}_1' = \begin{bmatrix} \boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{12} & \boldsymbol{\Theta}_1 \\ \boldsymbol{\Phi}_{12}^{\mathrm{T}} & \boldsymbol{\Phi}_{22} & \boldsymbol{\Theta}_2 \\ \boldsymbol{\Theta}_1^{\mathrm{T}} & \boldsymbol{\Theta}_2^{\mathrm{T}} & \varepsilon^{-1}\boldsymbol{\Pi}_1 \end{bmatrix} \geq 0 \tag{33}$$

$$\boldsymbol{\Xi}_2' = \begin{bmatrix} \boldsymbol{\Xi}_{11} & \boldsymbol{\Xi}_{12} & \tau\boldsymbol{\Pi}_1\boldsymbol{A}^{\mathrm{T}} \\ \boldsymbol{\Xi}_{12}^{\mathrm{T}} & \boldsymbol{\Xi}_{22} & \tau(\boldsymbol{\Pi}_1\boldsymbol{A}_d^{\mathrm{T}} + \boldsymbol{\Theta}_3^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}) \\ \tau\boldsymbol{A}\boldsymbol{\Pi}_1 & \tau(\boldsymbol{A}_d\boldsymbol{\Pi}_1 + \boldsymbol{B}\boldsymbol{\Theta}_3) & -\tau\varepsilon\boldsymbol{\Pi}_1 \end{bmatrix} < 0 \tag{34}$$

Matrices to be determined in the design of the controller $\boldsymbol{K}$ when τ=0.1 and μ=0.5 are given below:

$$\boldsymbol{\Pi}_1 = \begin{bmatrix} 0.0663 & -0.0001 & -0.1071 & -0.0780 & -0.0016 & 0.0007 \\ -0.0001 & 0.0000 & 0.0001 & -0.0009 & 0.0000 & 0.0000 \\ -0.1071 & 0.0001 & 0.1987 & 0.1159 & 0.0031 & -0.0013 \\ -0.0780 & -0.0009 & 0.1159 & 1.3030 & 0.0020 & -0.0024 \\ -0.0016 & 0.0000 & 0.0031 & 0.0020 & 0.0001 & -0.0000 \\ 0.0007 & 0.0000 & -0.0013 & -0.0024 & -0.0000 & 0.0000 \end{bmatrix} \times 10^3$$

$$\boldsymbol{\Theta}_3 = [-0.238\,0 \quad -0.008\,2 \quad 0.648\,2 \quad -2.793\,9 \quad 0.006\,8 \quad 0.005\,4]$$

The curves for rotor angle deviation and rotor angular speed deviation when τ=0.1 s and μ=0.5 are shown in Fig. 1 and Fig. 2, respectively. "delay=0.1s" represents that input signal of the stabilization controller is delayed for 0.1 s. It is found that deviations of rotor angle and rotor angular speed can return to be zero within several seconds by the designed time delay controller with the given gain $\boldsymbol{K}$=[0.008 4 −2.412 8 0.018 3 −0.003 9 −0.261 7 0.370 3] even if the input signal is delayed for 0.1 s. That coincidences with τ= 0.1 s when μ=0.5 and $\boldsymbol{K}$=[0.008 4 −2.412 8 0.018 3 −0.003 9 −0.261 7 0.370 3] in Table 1. It is clear that delay margin is enlarged up to 0.1 s when the stabilization controller is added to the power system. In another word, power system can return to be stable when it is subjected to small signal disturbance even if the input signal is delayed for 0.1 s by using the stabilization controller properly designed in Table 1.

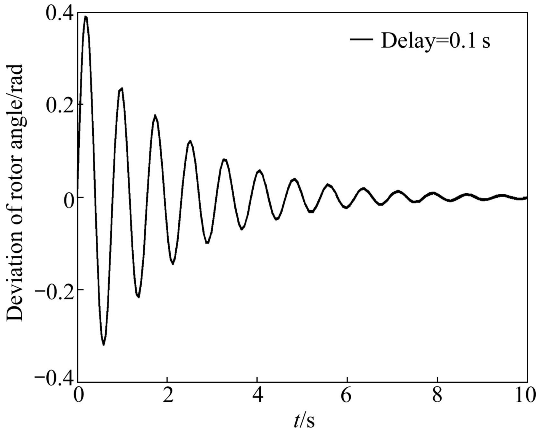

Fig. 1 Deviation of rotor angle (τ=0.1 s and μ=0.5)

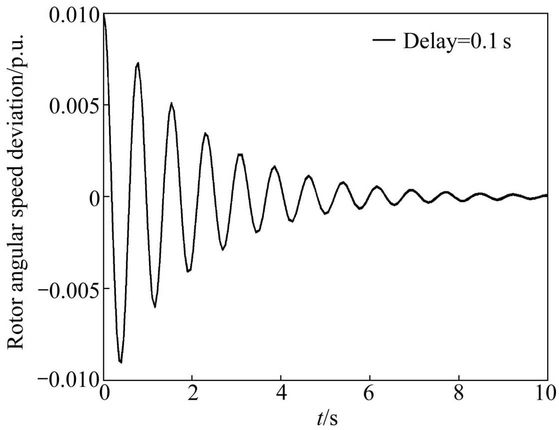

Fig. 2 Deviation of rotor angular speed (τ=0.1 s and μ=0.5)

The curve for control input $\boldsymbol{u}(t)$ with respect to time when τ=0.1 and μ=0.5 is shown in Fig. 3. It is noted that the control input $\boldsymbol{u}(t)$ is zero between 0 to 0.1 s after disturbance because the system state signals required for stabilization control have not transmitted to the controller. It is also found that the amplitude of control input is relatively larger at early stabilization process, while at later stabilization process, it decreases dramatically up to zero. That means that when the power system gradually returns to its original stable status, control input decreases correspondingly to zero.

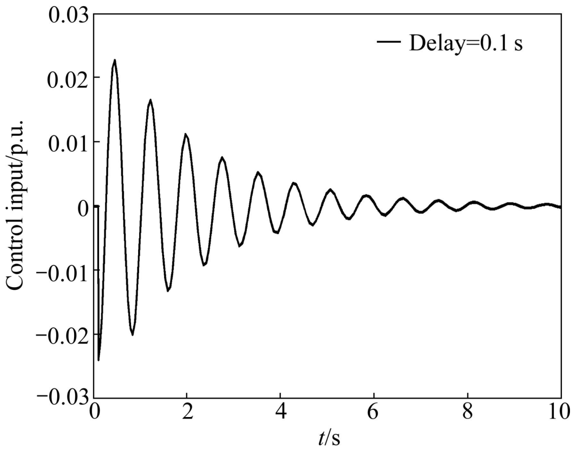

Fig. 3 Control input (τ=0.1 s and μ=0.5)

5 Conclusions

1) A modified power system small signal stability model considering signal transmission delay is constructed. The new model overcomes the disadvantage that the conventional small signal stability model can not deal with delay influence.

2) A power system delay-dependent stability criterion, which is based on Lyapunov-Krasovskii functional and has less conservativeness, is proposed. The novel stability criterion takes the place of the conventional eigenvalue analysis method and can be used to analyze small signal stability of time delay power system.

3) A stabilization controller considering delay influence is proposed to ensure power system stability under disturbances. The new controller can be easily realized because its design approach is in the form of LMIs. Simulation tests show that it is effective in power system stability control. The application of the proposed stabilization controller in large-scale pratical power system is needed to be studied further.

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