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Finite-time combination-combination synchronization of four different chaotic systems with unknown parameters via sliding mode control

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Abstract In this paper, we apply the nonsingular terminal sliding mode control technique to realize the novel combination-combination synchronization between combination of two chaotic systems as drive system and combination of two chaotic systems as response system with unknown parameters in a finite time. On the basic of the adaptive laws and finite-time stability theory, an adaptive combination sliding mode controller is proposed to ensure the occurrence of the sliding motion in a given finite time for four different chaotic systems. In theory, it is proved that the sliding mode technique can realize fast convergence for four different chaotic systems in the finite time. Some criteria and corollaries are derived for finite-time combination-combination synchronization of four different chaotic systems. Numerical simulation results are shown to verify the effectiveness and correctness of the combination-combination synchronization.

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1 Introduction

Extremely sensitivity to initial conditions and system parameter changes, fractal properties of the motion in phase plane, broad Fourier transform spectra, strange attractors, are a lot of special characteristics for chaotic systems. As a result of these characters and many available practical applications in actual lives, synchronization of chaotic systems has attracted many scientists in the field of sociology, biology, mathematics, physics, and engineering sciences. A essential model for chaos synchronization is the drive-response structure, where the response system trajectories should keep pace with the drive system trajectories. As time goes on, all kinds of synchronization have been intensively studied and a lot of theoretical results have been obtained, such as complete synchronization [1], antisynchronization [2], generalized synchronization [3–7], phase synchronization [8], antiphase synchronization [9], lag synchronization [10], partial synchronization [11], projective synchronization [12–17], time scale synchronization [18], and combination synchronization [19–21], compound synchronization [17], etc. In this period, several theoretical methods have been developed to realize chaos synchronization such as the Ott, Grebogi, and Yorke (OGY) method [22], feedback control method [23, 24], active control method [25], passive control method [26], backstepping method [27], adaptive control method [5, 28], sliding mode control method [29], impulsive control method [30, 31], coupling control method [32–34], etc.

For these above approaches, it has been shown that the sliding mode control is a more popular method, which is relatively higher efficient and easily realized. Sliding mode control is a special case of variable structure systems. By designing a switching surface and applying a discontinuous control law, the trajectories of dynamic systems can be forced to slide along the desired sliding surface. Then the dynamical systems can be driven to realize the desired performance. Generally speaking, the two main advantages of sliding-mode control are the simple dynamic behaviors of the system with the designed switching functions and strong robustness to system uncertainties. It is also a potential robust approach for controlling the nonlinear dynamic systems [35], especially systems with uncertain factors [36]. It has been studied extensively and received many applications due to its insensitivity to system parameter variations, external disturbance rejection, and fast dynamic response [37]. Due to its superiority, this technique has been used to cope with the control problem of chaotic system [38].

These works have guaranteed that the trajectories of the response system can convergence to the trajectories of the drive system with infinite settling time. In a practical engineering process, it has more potential to synchronize the chaotic systems in the given finite time. The finite-time control approaches are powerful techniques to realize faster convergence to control systems. Finite-time synchronization implies the optimality in the given time, and what is more, the finite-time control techniques have shown better robustness and disturbance rejection properties [39]. More and more researchers have applied the sliding-mode control to finite-time synchronization. The terminal sliding mode concept has been proposed to address the finite-time control issue [40]. In [41], the nonsingular terminal sliding mode control for chaotic systems with uncertain parameters or disturbance has been discussed. A new nonsingular terminal sliding surface has been introduced to deal with the problem of finite-time chaos synchronization between two different chaotic systems with fully unknown parameters [36].

However, the above chaos synchronization have been paid a lot of attention by many researchers, and so far the studies of chaos synchronization have mostly been limited to within one drive system and one response system. Recently, an active backstepping design has proposed to achieve combination synchronization between two drive systems and one response system [19, 20]. Although the combination synchronization of chaotic systems has more advantages and potential than synchronization between one drive system and one response system, the study of combination synchronization has mostly been limited to within one response system. To extend combination synchronization, regarding the finite-time combination-combination synchronization between combination of two chaotic systems as drive system and combination of two chaotic systems as response system with model uncertainties and unknown parameters, there are few results in the literature so far, which still remains as an open and challenging problem to be solved.

Motivated by the above discussions, we utilize the general benefits of the combination synchronization scheme [19, 20] for combination of two drive systems and one response system, and extend it to a combination of two drive systems and combination of two response systems. We apply the nonsingular terminal sliding mode control technique to realize the novel combination-combination synchronization in a finite time. Based on the adaptive laws and finite-time control idea, an adaptive sliding mode controller is proposed to ensure the occurrence of the sliding motion in the given finite time for combination-combination synchronization.

Our method is advantageous and has potential. The combination of all the chaotic systems is considered as the drive systems or response systems, such that a universal combination of the drive systems, a universal combination of response systems, and a universal adaptive sliding mode controller for a nonsingular terminal sliding surface will be constructed. According to our actual requirements, we chose the corresponding system or systems combination, the corresponding parameter values are given to the drive systems, and response systems to realize synchronization. We need not redesign the controller for two systems or systems combination for every application. Too much time and energy are saved for our future practical application. If synchronization can be controlled intelligently at will, then it may be possible to attain vastly better performance for secure communication and information processing.

This work is presented as follows: In Sect. 2, problem formulation and preliminary lemmas are described, followed by Sect. 3 which suggests the sliding mode controller design procedure and stability analysis. The illustrative example is provided to validate the effectiveness and the feasibility of the proposed dynamical sliding mode control strategy in Sect. 4. Some conclusions are finally presented in Sect. 5.

2 Problem formulation

In this section, we give the formulation of the system description and synchronization problem, which are necessary for our further discussion.

The first drive system is given as follows:

$$\begin{cases} \dot{x}_{11}(t) = f_{11}\big(x_1(t)\big) + F_{11}\big(x_1(t)\big)\theta_1, \\ \dot{x}_{12}(t) = f_{12}\big(x_1(t)\big) + F_{12}\big(x_1(t)\big)\theta_1, \\ \vdots \\ \dot{x}_{1n}(t) = f_{1n}\big(x_1(t)\big) + F_{1n}\big(x_1(t)\big)\theta_1, \end{cases} \tag{1}$$

where $x_1(t) = [x_{11}, x_{12}, \ldots, x_{1n}]^T$ is the state vector of the first drive system (1), f_{1i} $(i = 1, 2, \ldots, n)$ is a continuous nonlinear function, F_{1i} $(i = 1, 2, \ldots, n)$ is the ith row of $n \times m_1$ matrix $(F_1(x_1(t)))$ whose elements are continuous nonlinear functions, $\theta_1 = [\theta_{11}, \theta_{12}, \ldots, \theta_{1m_1}]^T$ is a $m_1 \times 1$ unknown parameter vector of the first drive system (1), and the second drive system is described as follows:

$$\begin{cases} \dot{x}_{21}(t) = f_{21}\big(x_2(t)\big) + F_{21}\big(x_2(t)\big)\theta_2, \\ \dot{x}_{22}(t) = f_{22}\big(x_2(t)\big) + F_{22}\big(x_2(t)\big)\theta_2, \\ \vdots \\ \dot{x}_{2n}(t) = f_{2n}\big(x_2(t)\big) + F_{2n}\big(x_2(t)\big)\theta_2, \end{cases} \tag{2}$$

where $x_2(t) = [x_{21}, x_{22}, \ldots, x_{2n}]^T$ is the state vector of the second drive system (2), f_{2i} $(i = 1, 2, \ldots, n)$ is a continuous nonlinear function, F_{2i} $(i = 1, 2, \ldots, n)$ is the ith row of $n \times m_2$ matrix $(F_2(x_2(t)))$ whose elements are continuous nonlinear functions, $\theta_2 = [\theta_{21}, \theta_{22}, \cdots, \theta_{2m_2}]^T$ is a $m_2 \times 1$ unknown parameter vector of the second drive system (2).

The first response system is given as follows:

$$\begin{cases} \dot{y}_{11}(t) = g_{11}\big(y_1(t)\big) + G_{11}\big(y_1(t)\big)\delta_1 + u_1(t), \\ \dot{y}_{12}(t) = g_{12}\big(y_1(t)\big) + G_{12}\big(y_1(t)\big)\delta_1 + u_2(t), \\ \vdots \\ \dot{y}_{1n}(t) = g_{1n}\big(y_1(t)\big) + G_{1n}\big(y_1(t)\big)\delta_1 + u_n(t), \end{cases} \tag{3}$$

where $y_1(t) = [y_{11}, y_{12}, \ldots, y_{1n}]^T$ is the state vector of the first response system (3), g_{1i} $(i = 1, 2, \ldots, n)$ is a continuous nonlinear function, G_{1i} $(i = 1, 2, \ldots, n)$ is the ith row of $n \times m_3$ matrix $(G_1(y_1(t)))$ whose elements are continuous nonlinear functions, $\delta_1 = [\delta_{11}, \delta_{12}, \cdots, \delta_{1m_3}]^T$ is a $m_3 \times 1$ unknown parameter vector of the first response system (3), $u = [u_1, u_2, \ldots, u_n]^T$ is the vector of control input, and the second response system is described as follows:

$$\begin{cases} \dot{y}_{21}(t) = g_{21}\big(y_2(t)\big) + G_{21}\big(y_2(t)\big)\delta_2 + u_1^*(t), \\ \dot{y}_{22}(t) = g_{22}\big(y_2(t)\big) + G_{22}\big(y_2(t)\big)\delta_2 + u_2^*(t), \\ \vdots \\ \dot{y}_{2n}(t) = g_{2n}\big(y_2(t)\big) + G_{2n}\big(y_2(t)\big)\delta_2 + u_n^*(t), \end{cases} \tag{4}$$

where $y_2(t) = [y_{21}, y_{22}, \ldots, y_{2n}]^T$ is the state vector of the second response system (4), g_{2i} $(i = 1, 2, \ldots, n)$ is a continuous nonlinear function, G_{2i} $(i = 1, 2, \ldots, n)$ is the ith row of $n \times m_4$ matrix $(G_2(y_2(t)))$ whose elements are continuous nonlinear functions, $\delta_2 = [\delta_{21}, \delta_{22}, \ldots, \delta_{2m_4}]^T$ is a $m_4 \times 1$ unknown parameter vector of the second response system (4), and $u^* = [u_1^*, u_2^*, \ldots, u_n^*]^T$ is the vector of control input.

To solve the finite-time synchronization problem, the error between combination of drive systems and combination of response systems can be defined as $e(t) = Ax_1(t) + Bx_2(t) - Cy_1(t) - Dy_2(t)$. Therefore, the following Definition 1 is given.

Definition 1 Consider the combination of drive systems (1), (2), and the combination of response systems (3), (4), respectively. If there exist four constant matrices A, B, C, $D \in R^{n\times n}$ and $C \neq 0$ or $D \neq 0$ a constant $T = T(e(0)) > 0$, such that

$$\begin{aligned} &\lim_{t\to T}\|e(t)\| \\ &\quad = \lim_{t\to T}\|Ax_1(t) + Bx_2(t) - Cy_1(t) - Dy_2(t)\| \\ &\quad = 0, \end{aligned} \tag{5}$$

and $\|e(t)\| \equiv 0$, if $t \geq T$, then the combination of drive systems (1) and (2) are realized combination-combination synchronization with the combination of response systems (3) and (4) in a finite time, where $\|\cdot\|$ represents the matrix norm.

Remark 1 The constant matrices A, B, C, D are called the scaling matrices. In addition, A, B, C, D can be extended to matrices functions of state variables x_1, x_2, y_1, and y_2.

Remark 2 If $C = 0$ or $D = 0$, then the finite-time combination-combination synchronization problem will be reduced to the finite-time combination synchronization problem.

Remark 3 If $A = 0, C = I$, $D = 0$ or $A = C = 0$, $D = I$ or $B = 0$, $C = I$, $D = 0$ or $B = C = 0$, $D = I$, then the finite-time combination synchronization problem will be reduced to finite-time projective synchronization, where I is a $n \times n$ identity matrix.

Remark 4 If $A = 0$, $C = -I$, $D = 0$ or $A = C = 0$, $D = -I$ or $B = 0$, $C = -I$, $D = 0$ or $B = C = 0$, $D = -I$, then finite-time combination synchronization problem will be reduced to finite-time projective anti-synchronization.

Remark 5 If the scaling matrices $A = B = C = 0$ or $A = B = D = 0$, then finite-time combination synchronization will be turned into a chaos control problem in the finite time.

Remark 6 Definition 1 shows that the combination of drive systems and combination of response systems can be extended to three or more chaotic systems. In addition, drive systems and response systems of the combination can be identical or different.

Remark 7 Most of the previous proposed synchronization are included as combination-combination synchronization special cases in our model. The combination of all the chaotic systems is regarded as the drive systems or response systems, such that we can design a universal combination of the drive systems, a universal combination of response systems, and a universal adaptive sliding mode controller for a nonsingular terminal sliding surface. According to our actual requirements, we chose the corresponding system or systems combination, the corresponding parameter values are given to the drive systems, and response systems to realize synchronization. For example, if we want to make the drive system (1) and the response system (3) proceed synchronization, we only set the scaling matrices $B = D = 0$; if we want to make the combination of drive systems (1) and (2) synchronize with the response system (3), we only set the scaling matrices $D = 0$. We need not redesign the controller for two systems or systems combination every application. Too much time and energy are saved for our future practical application.

For the convenience of our discussions, we assume $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n}$, $C = (c_{ij})_{n\times n}$, $D = (d_{ij})_{n\times n}$ in our synchronization scheme. The error dynamics is obtained as follows:

$$
\begin{cases}
\dot{e}_1(t) = \sum_{j=1}^{n} \Big[a_{1j}\big(f_{1j}(x_1) + F_{1j}(x_1)\theta_1\big) \\
\qquad + b_{1j}\big(f_{2j}(x_2) + F_{2j}(x_2)\theta_2\big) \\
\qquad - c_{1j}\big(g_{1j}(y_1) + G_{1j}(y_1)\delta_1 + u_j(t)\big) \\
\qquad - d_{1j}\big(g_{2j}(y_2) + G_{2j}(y_2)\delta_2 + u_j^*(t)\big)\Big], \\
\dot{e}_2(t) = \sum_{j=1}^{n} \Big[a_{2j}\big(f_{1j}(x_1) + F_{1j}(x_1)\theta_1\big) \\
\qquad + b_{2j}\big(f_{2j}(x_2) + F_{2j}(x_2)\theta_2\big) \\
\qquad - c_{2j}\big(g_{1i}(y_1) + G_{1j}(y_1)\delta_1 + u_j(t)\big) \\
\qquad - d_{2j}\big(g_{2j}(y_2) + G_{2j}(y_2)\delta_2 + u_j^*(t)\big)\Big], \\
\vdots \\
\dot{e}_n(t) = \sum_{j=1}^{n} \Big[a_{nj}\big(f_{1j}(x_1) + F_{1j}(x_1)\theta_1\big) \\
\qquad + b_{nj}\big(f_{2j}(x_2) + F_{2j}(x_2)\theta_2\big) \\
\qquad - c_{nj}\big(g_{1j}(y_1) + G_{1j}(y_1)\delta_1 + u_j(t)\big) \\
\qquad - d_{nj}\big(g_{2j}(y_2) + G_{2j}(y_2)\delta_2 + u_j^*(t)\big)\Big].
\end{cases}
\tag{6}
$$

It is clear that the finite-time synchronization problem can be transformed to the equivalent problem about the finite-time stabilization of the error system (6). The objective of this paper is to design a suitable feedback control law $u(t)$ such that for any given the combination of drive systems (1), (2), and the combination of response systems (3), (4), with unknown parameters, the finite-time stability of the resulting error system (6) can be achieved according to Definition 1.

Assumption 1 Assume the unknown vector parameters $\theta_1, \theta_2, \delta_1$, and δ_2 are bounded by

$$\begin{aligned} &\|\theta_1\| \le \xi_{\theta_1}, \qquad \|\theta_2\| \le \xi_{\theta_2}, \\ &\|\delta_1\| \le \xi_{\delta_1}, \qquad \|\delta_2\| \le \xi_{\delta_2}, \end{aligned} \tag{7}$$

where ξ_{θ_1}, ξ_{θ_2}, ξ_{δ_1}, and ξ_{δ_2} are four known positive constants.

Lemma 1 *The system is described by*

$$\dot{x} = f(x), \quad f(0) = 0, \quad x \in R^n, \tag{8}$$

where $f : D \to R^n$ is a continuous function. Assume there exists a continuous differential positive-definite function $V(x) : D \to R$, constants $p > 0$, $0 < \eta < 1$, so that

$$\dot{V} + pV^{\eta}(x) \le 0, \quad \forall x \in D. \tag{9}$$

Then the origin of system (8) is a locally finite-time stable equilibrium, and the settling time related to the initial state $x(0) = x_0$, one obtains

$$T(x_0) \le \frac{V^{1-\eta}(x_0)}{p(1-\eta)}. \tag{10}$$

What is more, if $D = R^n$ and $V(x)$ are also unbounded (i.e., $V(x) \to +\infty$ as $\|x\| \to +\infty$), then the origin is a globally finite-time stable equilibrium for system (8).

Lemma 2 (see [36]) *Suppose $a_1, a_2, \ldots, a_n$, and $0 < q < 2$ are all real constants, then the following inequality holds true*:

$$\begin{aligned} &\left(a_1^2 + a_2^2 + \cdots + a_n^2\right)^{q/2} \\ &\quad \le |a_1|^q + |a_2|^q + \cdots + |a_n|^q. \end{aligned} \tag{11}$$

According to Lemma 2, we can get the following Lemma 3.

Lemma 3 *For the real constants $a_1, a_2, \ldots, a_n \in R$, the following inequality holds true*:

$$\begin{aligned} &\left(a_1^2 + a_2^2 + \cdots + a_n^2\right)^{1/2} \\ &\quad \le |a_1| + |a_2| + \cdots + |a_n|. \end{aligned} \tag{12}$$

3 Design of sliding mode controller

In this section, we will discuss chaos synchronization for four different chaotic systems with unknown parameters via a finite-time sliding mode controller. There are two main steps for the design scheme of the proposed finite-time controller. First, we should define a nonsingular terminal sliding surface for the desired sliding motion, and second, determine the control law to guarantee the existence of the sliding motion in the given finite time. The sliding mode surface with integral operator is defined as follows:

$$\begin{aligned} &s_i(t) = h_i\big(e_i(t) - e_i(0)\big) + \int_0^t \operatorname{sgn}\big(e_i(\tau)\big)\big|e_i(\tau)\big|^{\sigma} d\tau, \\ &\quad i = 1, 2, \ldots, n, \end{aligned} \tag{13}$$

where $s_i(t) \in R$, $h_i > 0$ and $0 < \sigma < 1$ are constants, $\operatorname{sgn}(\cdot)$ stands for the signum function.

Differentiating both sides of Eq. (13), it yields

$$\begin{aligned} &\dot{s}_i(t) = h_i\dot{e}_i(t) + \operatorname{sgn}\big(e_i(t)\big)\big|e_i(t)\big|^{\sigma} = 0, \\ &\quad i = 1, 2, \ldots, n. \end{aligned} \tag{14}$$

Therefore, the dynamics of the nonsingular terminal sliding mode can be obtained as follows:

$$\begin{aligned} &\dot{e}_i(t) = -\frac{1}{h_i}\operatorname{sgn}\big(e_i(t)\big)\big|e_i(t)\big|^{\sigma}, \\ &\quad i = 1, 2, \ldots, n. \end{aligned} \tag{15}$$

Theorem 1 *This system (8) is finite-time stable and its trajectories converge to the equilibrium $e(t) = 0$ in the given finite time for the sliding mode dynamics (15), T_1, described by*

$$T_1 \le \frac{[1/2\sum_{i=1}^n e_i^2(0)]^{\frac{1-\sigma}{2}}}{(1-\sigma)\rho 2^{\frac{1-\sigma}{2}}}, \tag{16}$$

where $\rho = \min\{1/h_i (i = 1, 2, \ldots, n)\}$.

Proof Selecting a positive definite function as a Lyapunov function candidate as follows:

$$V(t) = \frac{1}{2}\sum_{i=1}^n e_i^2. \tag{17}$$

Its derivative with respect to time t along the trajectory of (15) is

$$\dot{V}(t) = \sum_{i=1}^{n} e_i \dot{e}_i. \tag{18}$$

Substituting $\dot{e}_i$ from (15) into the above equation (18), one has

$$\dot{V}(t) = \sum_{i=1}^{n} e_i \left(-\frac{1}{h_i} \operatorname{sgn}\bigl(e_i(t)\bigr)\bigl|e_i(t)\bigr|^{\sigma} \right). \tag{19}$$

Since $\operatorname{sgn}(e_i) = |e_i|/e_i$, one obtains

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^{n} -\frac{1}{h_i}\bigl|e_i(t)\bigr|^{(\sigma+1)} \\ &\le -\rho \sum_{i=1}^{n} \bigl|e_i(t)\bigr|^{(\sigma+1)}. \end{aligned} \tag{20}$$

Using Lemma 2, one can obtain

$$\begin{aligned} \dot{V}(t) &= -2^{\frac{\sigma+1}{2}} \rho \left(\frac{1}{2} \sum_{i=1}^{n} e_i^2 \right)^{\frac{\sigma+1}{2}} \\ &= -2^{\frac{\sigma+1}{2}} \rho V^{\frac{\sigma+1}{2}}. \end{aligned} \tag{21}$$

Therefore, from Lemma 1, the error e_i $(i = 1, 2, \dots, n)$ will converge to zero in the finite time

$$T_1 \le \frac{[1/2 \sum_{i=1}^{n} e_i^2(0)]^{\frac{1-\sigma}{2}}}{(1-\sigma)\rho 2^{\frac{1-\sigma}{2}}}.$$

Hence, the proof is completed. □

After choosing the corresponding sliding surface, we design the control law to drive the error system trajectories go onto the sliding surface in the finite time. Therefore, to ensure the existence of the sliding motion in the finite time (i.e., to ensure that the error trajectories $e_i(t)$ converge to the sliding surface $s_i(t) = 0$), the corresponding finite-time combination control law is designed as follows:

$$\begin{aligned} &\sum_{j=1}^{n} \bigl(c_{ij} u_j(t) + d_{ij} u_j^*(t)\bigr) \\ &\quad = \sum_{j=1}^{n} \bigl[a_{ij}\bigl(f_{1j}(x_1) + F_{1j}(x_1)\hat{\theta}_1\bigr) \\ &\quad + b_{ij}\bigl(f_{2j}(x_2) + F_{2j}(x_2)\hat{\theta}_2\bigr) \\ &\quad - c_{ij}\bigl(g_{1j}(y_1) + G_{1j}(y_1)\hat{\delta}_1\bigr) \\ &\quad - d_{ij}\bigl(g_{2j}(y_2) + G_{2j}(y_2)\hat{\delta}_2\bigr)\bigr] + \frac{1}{h_i} \operatorname{sgn}(e_i)|e_i|^{\sigma} \\ &\quad + \mu\bigl(\|\hat{\theta}_1\| + \|\hat{\theta}_2\| + \|\hat{\delta}_1\| + \|\hat{\delta}_2\| \\ &\quad + \xi_{\theta_1} + \xi_{\theta_2} + \xi_{\delta_1} + \xi_{\delta_2}\bigr)\left(\frac{s_i}{h_i\|S\|^2}\right) + k_i \operatorname{sgn}(s_i), \\ &i = 1, 2, \dots, n, \end{aligned} \tag{22}$$

where $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\delta}_1$, and $\hat{\delta}_2$ are estimations for θ_1, θ_2, δ_1, and δ_2, respectively; $\mu = \min\{k_i h_i (i = 1, 2, \dots, n)\} > 0$, $S(t) = [s_1, s_2, \dots, s_n]$, k_i is a positive constant.

To deal with the unknown parameters, appropriate adaptive laws are given as follows:

$$\begin{aligned} \dot{\hat{\theta}}_1 &= \bigl[F_1(x_1)\bigr]^T \lambda_1, & \hat{\theta}_1(0) &= \hat{\theta}_{10}, \\ \dot{\hat{\theta}}_2 &= \bigl[F_2(x_2)\bigr]^T \lambda_2, & \hat{\theta}_2(0) &= \hat{\theta}_{20}, \\ \dot{\hat{\delta}}_1 &= -\bigl[G_1(y_1)\bigr]^T \lambda_3, & \hat{\delta}_1(0) &= \hat{\delta}_{10}, \\ \dot{\hat{\delta}}_2 &= -\bigl[G_2(y_2)\bigr]^T \lambda_4, & \hat{\delta}_2(0) &= \hat{\delta}_{20}, \end{aligned} \tag{23}$$

where

$$\begin{aligned} \lambda_1 &= \left[\sum_{j=1}^{n} h_1 s_1 a_{1j}, \sum_{j=1}^{n} h_2 s_2 a_{2j}, \dots, \sum_{j=1}^{n} h_n s_n a_{nj} \right], \\ \lambda_2 &= \left[\sum_{j=1}^{n} h_1 s_1 b_{1j}, \sum_{j=1}^{n} h_2 s_2 b_{2j}, \dots, \sum_{j=1}^{n} h_n s_n b_{nj} \right], \\ \lambda_3 &= \left[\sum_{j=1}^{n} h_1 s_1 c_{1j}, \sum_{j=1}^{n} h_2 s_2 c_{2j}, \dots, \sum_{j=1}^{n} h_n s_n c_{nj} \right], \\ \lambda_4 &= \left[\sum_{j=1}^{n} h_1 s_1 d_{1j}, \sum_{j=1}^{n} h_2 s_2 d_{2j}, \dots, \sum_{j=1}^{n} h_n s_n d_{nj} \right], \end{aligned} \tag{24}$$

$\hat{\theta}_{10}$, $\hat{\theta}_{20}$, $\hat{\delta}_{10}$, and $\hat{\delta}_{20}$ are the initial values and the adaptive parameters $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\delta}_1$, and $\hat{\delta}_2$, respectively.

The proposed combination control law in (22) and adaptive laws in (23) will guarantee the finite-time occurrence of the sliding motion, which is proved in the following Theorem 2.

Theorem 2 *Suppose the error system* (6) *is controlled with the combination controller* (22) *and adaptive laws in* (23). *The states of the system* (6) *will go toward the sliding surface and will reach the sliding surface* $s_i(t)=0$ *in the finite time*, T_2, *determined by*

$$T_2 \le \frac{\sqrt{2}}{\mu}\left(\frac{1}{2}\sum_{i=1}^{n}\left[s_i^2(0)\right]+\frac{1}{2}\left\|\hat{\theta}_1(0)-\theta_1\right\|^2 + \frac{1}{2}\left\|\hat{\theta}_2(0)-\theta_2\right\|^2+\frac{1}{2}\left\|\hat{\delta}_1(0)-\delta_1\right\|^2 + \frac{1}{2}\left\|\hat{\delta}_2(0)-\delta_2\right\|^2\right)^{\frac{1}{2}}. \tag{25}$$

Proof Choosing a positive definite function as a Lyapunov function candidate as follows:

$$\dot{V}(t)=\frac{1}{2}\sum_{i=1}^{n}\left[s_i^2\right]+\frac{1}{2}\|\tilde{\theta}_1\|^2+\frac{1}{2}\|\tilde{\theta}_2\|^2 + \frac{1}{2}\|\tilde{\delta}_1\|^2+\frac{1}{2}\|\tilde{\delta}_2\|^2, \tag{26}$$

where $\tilde{\theta}_1=\hat{\theta}_1-\theta_1$, $\tilde{\theta}_2=\hat{\theta}_2-\theta_2$, $\tilde{\delta}_1=\hat{\delta}_1-\delta_1$, and $\tilde{\delta}_2=\hat{\delta}_2-\delta_2$ are the parameters errors (it is clear that $\dot{\tilde{\theta}}_1=\dot{\hat{\theta}}_1$, $\dot{\tilde{\theta}}_2=\dot{\hat{\theta}}_2$, $\dot{\tilde{\delta}}_1=\dot{\hat{\delta}}_1$, and $\dot{\tilde{\delta}}_2=\dot{\hat{\delta}}_2$).

Taking the time t derivative of $V(t)$ along the trajectory of (14), one has

$$\dot{V}(t)=\sum_{i=1}^{n}[s_i\dot{s}_i]+\tilde{\theta}_1^T\dot{\hat{\theta}}_1+\tilde{\theta}_2^T\dot{\hat{\theta}}_2+\tilde{\delta}_1^T\dot{\hat{\delta}}_1+\tilde{\delta}_2^T\dot{\hat{\delta}}_2. \tag{27}$$

Substituting (14) into the above equation (27), one has

$$\dot{V}(t)=\sum_{i=1}^{n}\left[s_i\left(h_i\dot{e}_i(t)+\operatorname{sgn}\left(e_i(t)\right)\left|e_i(t)\right|^{\sigma}\right)\right] +\tilde{\theta}_1^T\dot{\hat{\theta}}_1+\tilde{\theta}_2^T\dot{\hat{\theta}}_2+\tilde{\delta}_1^T\dot{\hat{\delta}}_1+\tilde{\delta}_2^T\dot{\hat{\delta}}_2. \tag{28}$$

Inserting $\dot{e}_i$ from (6) and adaptive laws (23) into the above equation (28), one obtains

$$\begin{aligned}\dot{V}(t)=\sum_{i=1}^{n}\Bigg\{s_i\Bigg[h_i\sum_{j=1}^{n}&\Big(a_{ij}\big(f_{1j}(x_1)+F_{1j}(x_1)\hat{\theta}_1\big)\\ &+b_{ij}\big(f_{2j}(x_2)+F_{2j}(x_2)\hat{\theta}_2\big)\\ &-c_{ij}\big(g_{1j}(y_1)+G_{1j}(y_1)\hat{\delta}_1\big)\\ &-d_{ij}\big(g_{2j}(y_2)+G_{2j}(y_2)\hat{\delta}_2\big)\\ &-\big(c_{ij}u_j(t)+d_{ij}u_j^*(t)\big)\Big)\\ &+\operatorname{sgn}\big(e_i(t)\big)\big|e_i(t)\big|^{\sigma}\Bigg]\Bigg\}\\ &+(\hat{\theta}_1-\theta_1)^T\big[F_1(x_1)\big]^T\lambda_1\\ &+(\hat{\theta}_2-\theta_2)^T\big[F_2(x_2)\big]^T\lambda_2\\ &-(\hat{\delta}_1-\delta_1)^T\big[G_1(y_1)\big]^T\lambda_3\\ &-(\hat{\delta}_2-\delta_2)^T\big[G_2(y_2)\big]^T\lambda_4.\end{aligned} \tag{29}$$

Since

$$\begin{aligned}&\sum_{i=1}^{n}\sum_{j=1}^{n}s_ih_ia_{ij}F_{1j}(x_1)\theta_1=\theta_1^T\big[F_1(x_1)\big]^T\lambda_1,\\ &\sum_{i=1}^{n}\sum_{j=1}^{n}s_ih_ib_{ij}F_{2j}(x_2)\theta_2=\theta_2^T\big[F_2(x_2)\big]^T\lambda_2,\\ &\sum_{i=1}^{n}\sum_{j=1}^{n}s_ih_ic_{ij}G_{1j}(y_1)\delta_1=\delta_1^T\big[G_1(y_1)\big]^T\lambda_3,\\ &\sum_{i=1}^{n}\sum_{j=1}^{n}s_ih_id_{ij}G_{2j}(y_2)\delta_2=\delta_2^T\big[G_2(y_2)\big]^T\lambda_4,\end{aligned} \tag{30}$$

the above equation (29) can be written as

$$\begin{aligned}\dot{V}(t)=\sum_{i=1}^{n}\sum_{j=1}^{n}\Big\{s_i\Big[h_i\big(&a_{ij}f_{1j}(x_1)+b_{ij}f_{2j}(x_2)\\ &-c_{ij}g_{1j}(y_1)-d_{ij}g_{2j}(y_2)\big)\\ &-\big(c_{ij}u_j(t)+d_{ij}u_j^*(t)\big)\\ &+\operatorname{sgn}\big(e_i(t)\big)\big|e_i(t)\big|^{\sigma}\Big]\Big\}+\hat{\theta}_1^T\big[F_1(x_1)\big]^T\lambda_1\\ &+\hat{\theta}_2^T\big[F_2(x_2)\big]^T\lambda_2-\hat{\delta}_1^T\big[G_1(y_1)\big]^T\lambda_3\\ &-\hat{\delta}_2^T\big[G_2(y_2)\big]^T\lambda_4.\end{aligned} \tag{31}$$

Substituting (22) into the above equation (31) and simplifying it, we have

$$\begin{aligned}\dot{V}(t)=\sum_{i=1}^{n}\sum_{j=1}^{n}\Big[s_ih_i\Big(&-a_{ij}F_{1j}(x_1)\hat{\theta}_1-b_{ij}F_{2j}(x_2)\hat{\theta}_2\\ &+c_{ij}G_{1j}(y_1)\hat{\delta}_1+d_{ij}G_{2j}(y_2)\hat{\delta}_2\\ &-\mu\big(\|\hat{\theta}_1\|+\|\hat{\theta}_2\|+\|\hat{\delta}_1\|\\ &+\|\hat{\delta}_2\|+\xi_{\theta_1}+\xi_{\theta_2}+\xi_{\delta_1}+\xi_{\delta_2}\big)\end{aligned}$$

$$\times\left(\frac{s_i}{h_i\|S\|^2}\right)+k_i\,\mathrm{sgn}(s_i)\bigg)\bigg] + \hat{\theta}_1^T\big[F_1(x_1)\big]^T\lambda_1+\hat{\theta}_2^T\big[F_2(x_2)\big]^T\lambda_2 - \hat{\delta}_1^T\big[G_1(y_1)\big]^T\lambda_3-\hat{\delta}_2^T\big[G_2(y_2)\big]^T\lambda_4. \tag{32}$$

Since

$$\begin{aligned}
&\sum_{i=1}^n\sum_{j=1}^n s_ih_ia_{ij}F_{1j}(x_1)\hat{\theta}_1=\hat{\theta}_1^T\big[F_1(x_1)\big]^T\lambda_1,\\
&\sum_{i=1}^n\sum_{j=1}^n s_ih_ib_{ij}F_{2j}(x_2)\hat{\theta}_2=\hat{\theta}_2^T\big[F_2(x_2)\big]^T\lambda_2,\\
&\sum_{i=1}^n\sum_{j=1}^n s_ih_ic_{ij}G_{1j}(y_1)\hat{\delta}_1=\hat{\delta}_1^T\big[G_1(y_1)\big]^T\lambda_3,\\
&\sum_{i=1}^n\sum_{j=1}^n s_ih_id_{ij}G_{2j}(y_2)\hat{\delta}_2=\hat{\delta}_2^T\big[G_2(y_2)\big]^T\lambda_4,
\end{aligned} \tag{33}$$

the above equation (33) can be further rewritten as

$$\begin{aligned}
\dot{V}(t)=\sum_{i=1}^n\bigg[&s_ih_ik_i\,\mathrm{sgn}(s_i)-s_ih_i\mu\big(\|\hat{\theta}_1\|+\|\hat{\theta}_2\|\\
&+\|\hat{\delta}_1\|+\|\hat{\delta}_2\|+\xi_{\theta_1}+\xi_{\theta_2}+\xi_{\delta_1}+\xi_{\theta_2}\big)\\
&\times\left(\frac{s_i}{h_i\|S\|^2}\right)\bigg].
\end{aligned} \tag{34}$$

Using the fact $\sum_{i=1}^n s_i(\frac{s_i}{\|S\|^2})=1$, we have

$$\begin{aligned}
\dot{V}(t)=\sum_{i=1}^n\big[s_ih_ik_i\,\mathrm{sgn}(s_i)\big]&-\mu\big(\|\hat{\theta}_1\|+\|\hat{\theta}_2\|+\|\hat{\delta}_1\|\\
&+\|\hat{\delta}_2\|+\xi_{\theta_1}+\xi_{\theta_2}+\xi_{\delta_1}+\xi_{\theta_2}\big).
\end{aligned} \tag{35}$$

According to Assumption 1, we have

$$\begin{aligned}
&\|\hat{\theta}_1-\theta_1\|\le\|\hat{\theta}_1\|+\|\theta_1\|\le\|\hat{\theta}_1\|+\xi_{\theta_1},\\
&\|\hat{\theta}_2-\theta_2\|\le\|\hat{\theta}_2\|+\|\theta_2\|\le\|\hat{\theta}_2\|+\xi_{\theta_2},\\
&\|\hat{\delta}_1-\delta_1\|\le\|\hat{\delta}_1\|+\|\delta_1\|\le\|\hat{\delta}_1\|+\xi_{\delta_1},\\
&\|\hat{\delta}_2-\delta_2\|\le\|\hat{\delta}_2\|+\|\delta_2\|\le\|\hat{\delta}_2\|+\xi_{\delta_2}.
\end{aligned} \tag{36}$$

It can be concluded that

$$\begin{aligned}
&-\big(\|\hat{\theta}_1\|+\xi_{\theta_1}\big)\le-\|\hat{\theta}_1-\theta_1\|,\\
&-\big(\|\hat{\theta}_2\|+\xi_{\theta_2}\big)\le-\|\hat{\theta}_2-\theta_2\|,\\
&-\big(\|\hat{\delta}_1\|+\xi_{\delta_1}\big)\le-\|\hat{\delta}_1-\delta_1\|,\\
&-\big(\|\hat{\delta}_2\|+\xi_{\delta_2}\big)\le-\|\hat{\delta}_2-\delta_2\|.
\end{aligned} \tag{37}$$

From (35), we have

$$\begin{aligned}
\dot{V}(t)=\sum_{i=1}^n\big[s_ih_ik_i\,\mathrm{sgn}(s_i)\big]&-\mu\|\hat{\theta}_1-\theta_1\|\\
&-\mu\|\hat{\theta}_2-\theta_2\|-\mu\|\hat{\delta}_1-\delta_1\|\\
&-\mu\|\hat{\delta}_2-\delta_2\|.
\end{aligned} \tag{38}$$

By the fact $\mathrm{sgn}(s_i)=|s_i|/s_i$, we gain

$$\begin{aligned}
\dot{V}(t)=\sum_{i=1}^n\big[h_ik_i|s_i|\big]&-\mu\|\hat{\theta}_1-\theta_1\|-\mu\|\hat{\theta}_2-\theta_2\|\\
&-\mu\|\hat{\delta}_1-\delta_1\|-\mu\|\hat{\delta}_2-\delta_2\|.
\end{aligned} \tag{39}$$

According to Lemma 3, we get

$$\begin{aligned}
\dot{V}(t)&=-\mu\Bigg(\sum_{i=1}^n\big[|s_i|\big]-\|\hat{\theta}_1-\theta_1\|-\|\hat{\theta}_2-\theta_2\|\\
&\qquad-\|\hat{\delta}_1-\delta_1\|-\|\hat{\delta}_2-\delta_2\|\Bigg)\\
&\le-\sqrt{2}\mu\Bigg(\frac{1}{2}\sum_{i=1}^n\big[s_i^2\big]+\frac{1}{2}\|\hat{\theta}_1-\theta_1\|^2\\
&\qquad+\frac{1}{2}\|\hat{\theta}_2-\theta_2\|^2\\
&\qquad+\frac{1}{2}\|\hat{\delta}_1-\delta_1\|^2+\frac{1}{2}\|\hat{\delta}_2-\delta_2\|^2\Bigg)\\
&=-\sqrt{2}\mu V^{1/2}.
\end{aligned} \tag{40}$$

Therefore, from Lemma 2, the error trajectories $e_i(t)$ will be driven to the sliding surface $s_i(t)=0$ in the finite time $T_2\le\frac{\sqrt{2}}{\mu}(\frac{1}{2}\sum_{i=1}^n[s_i^2(0)]+\frac{1}{2}\|\hat{\theta}_1(0)-\theta_1\|^2+\frac{1}{2}\|\hat{\theta}_2(0)-\theta_2\|^2+\frac{1}{2}\|\hat{\delta}_1(0)-\delta_1\|^2+\frac{1}{2}\|\hat{\delta}_2(0)-\delta_2\|^2)^{\frac{1}{2}}$. Hence, the proof is achieved completely. □

Remark 8 According to the Theorems 1 and 2, the sliding mode combination control law (22) and adaptive laws (23) and the sliding surface (13), we can complete synchronization between combination of the drive systems (1) and (2) reach combination of the response systems (3) and (4) in the finite time $T \leq T_1 + T_2$.

Remark 9 Assume systems (1), (2), (3), and (4) satisfy $f_{1i}(\cdot) = f_{2i}(\cdot) = g_{1i}(\cdot) = g_{2i}(\cdot)$ and $F_{1i}(\cdot) = F_{2i}(\cdot) = G_{1i}(\cdot) = G_{2i}(\cdot)$, $i = 1, 2, \dots, n$, the designed controller is also appropriate for the chaos synchronization of four identical chaotic systems with different initial values and unknown parameters.

Remark 10 According to Eqs. (16) and (25), the convergence times T_1 and T_2 are proportional to the value of parameter h_i. Therefore, the smaller h_i results in shorter convergence times T_1 and T_2. In other words, according to the control input in Eq. (22), it is clear that the control input $u_i(t)$ is proportional to the inverse of h_i. This means that a smaller h_i results in a larger $u_i(t)$. Therefore, the parameter h_i should be selected in accordance with the convergence times T_1 and T_2 to be short and the control input $u_i(t)$ not to be very large, considering the designer requirements.

Remark 11 Since Eq. (22) has been designed for the combination controller, the control inputs u_j and u_j* are not specified. We may have different choices for inputs u_j and u_j* to complete the combination synchronization. How to design the better controller in real applications to optimize factors such as robustness to errors in the parameters and immunity to noise and disturbance, is the research direction in future.

Assume

$$
\begin{aligned}
A &= \operatorname{diag}(a_{11}, a_{22}, \dots, a_{nn}),\\
B &= \operatorname{diag}(b_{11}, b_{22}, \dots, b_{nn}),\\
C &= \operatorname{diag}(c_{11}, c_{22}, \dots, c_{nn}),\\
D &= \operatorname{diag}(d_{11}, d_{22}, \dots, d_{nn}),
\end{aligned}
$$

the following corollaries are easily obtained from Theorem 1, and their proofs are similar with Theorem 1, so the processes will be omitted here.

Corollary 1 (i) *Assume $d_{ii} = 0$ $(i = 1, 2, \dots, n)$, if the control law is chosen as follows*:

$$
\begin{aligned}
u_i(t) = \frac{1}{c_{ii}}\Big[&a_{ii} f_{1i}(x_1) + a_{ii} F_{1i}(x_1)\hat{\theta}_1 + b_{ii} f_{2i}(x_2)\\
&+ b_{ii} F_{2i}(x_2)\hat{\theta}_2 - c_{ii} g_{1i}(y_1) - c_{ii} G_{1i}(y_1)\hat{\delta}_1\\
&+ \frac{1}{h_i}\operatorname{sgn}(e_i)|e_i|^{\sigma} + \mu\big(\|\hat{\theta}_1\| + \|\hat{\theta}_2\| + \|\hat{\delta}_1\|\\
&+ \xi_{\theta_1} + \xi_{\theta_2} + \xi_{\delta_1}\big)\left(\frac{s_i}{h_i\|S\|^2}\right) + k_i \operatorname{sgn}(s_i)\Big],\\
i = 1, 2, \dots, n. &
\end{aligned}
\tag{41}
$$

Meanwhile, appropriate adaptive laws are given as follows:

$$
\begin{aligned}
\dot{\hat{\theta}}_1 &= \big[F_1(x_1)\big]^T \lambda_1, & \hat{\theta}_1(0) &= \hat{\theta}_{10},\\
\dot{\hat{\theta}}_2 &= \big[F_2(x_2)\big]^T \lambda_2, & \hat{\theta}_2(0) &= \hat{\theta}_{20},\\
\dot{\hat{\delta}}_1 &= \big[G_1(y_1)\big]^T \lambda_3, & \hat{\delta}_1(0) &= \hat{\delta}_{10},
\end{aligned}
\tag{42}
$$

where $\lambda_1 = [h_1 s_1 a_{11}, h_2 s_2 a_{22}, \dots, h_n s_n a_{nn}]$, $\lambda_2 = [h_1 s_1 b_{11}, h_2 s_2 b_{22}, \dots, h_n s_n b_{nn}]$, $\lambda_3 = [h_1 s_1 c_{11}, h_2 s_2 c_{22}, \dots, h_n s_n c_{nn}]$, then the drive systems (1) and (2) will achieve combination synchronization with the response system (3) in the finite time via the sliding mode control.

(ii) *Assume $c_{ii} = 0$ $(i = 1, 2, \dots, n)$, if the control law is chosen as follows*:

$$
\begin{aligned}
u_i^*(t) = \frac{1}{d_{ii}}\Big[&a_{ii} f_{1i}(x_1) + a_{ii} F_{1i}(x_1)\hat{\theta}_1 + b_{ii} f_{2i}(x_2)\\
&+ b_{ii} F_{2i}(x_2)\hat{\theta}_2 - d_{ii} g_{2i}(y_2) - d_{ii} G_{2i}(y_2)\hat{\delta}_2\\
&+ \frac{1}{h_i}\operatorname{sgn}(e_i)|e_i|^{\sigma} + \mu\big(\|\hat{\theta}_1\| + \|\hat{\theta}_2\| + \|\hat{\delta}_2\|\\
&+ \xi_{\theta_1} + \xi_{\theta_2} + \xi_{\delta_2}\big)\left(\frac{s_i}{h_i\|S\|^2}\right) + k_i \operatorname{sgn}(s_i)\Big],\\
i = 1, 2, \dots, n. &
\end{aligned}
\tag{43}
$$

Meanwhile, appropriate adaptive laws are given as follows:

$$
\begin{aligned}
\dot{\hat{\theta}}_1 &= \big[F_1(x_1)\big]^T \lambda_1, & \hat{\theta}_1(0) &= \hat{\theta}_{10},\\
\dot{\hat{\theta}}_2 &= \big[F_2(x_2)\big]^T \lambda_2, & \hat{\theta}_2(0) &= \hat{\theta}_{20},\\
\dot{\hat{\delta}}_2 &= \big[G_2(y_2)\big]^T \lambda_4, & \hat{\delta}_2(0) &= \hat{\delta}_{20},
\end{aligned}
\tag{44}
$$

where $\lambda_1 = [h_1 s_1 a_{11}, h_2 s_2 a_{22}, \dots, h_n s_n a_{nn}]$, $\lambda_2 = [h_1 s_1 b_{11}, h_2 s_2 b_{22}, \dots, h_n s_n b_{nn}]$, $\lambda_4 = [h_1 s_1 d_{11}, h_2 s_2 d_{22}, \dots, h_n s_n d_{nn}]$, then the drive systems (1) and (2) will achieve combination synchronization with the response system (4) in the finite time via the sliding mode control.

Corollary 2 (i) *Assume* $b_{ii}=d_{ii}=0$, $c_{ii}=1$, $(i=1,2,\dots,n)$, *if the control law is chosen as follows*:

$$\begin{aligned} u_i(t) = {} & a_{ii} f_{1i}(x_1) + a_{ii} F_{1i}(x_1)\hat{\theta}_1 - g_{1i}(y_1) \\ & - G_{1i}(y_1)\hat{\delta}_1 + \frac{1}{h_i}\operatorname{sgn}(e_i)|e_i|^{\sigma} \\ & + \mu\big(\|\hat{\theta}_1\| + \|\hat{\delta}_1\| + \xi_{\theta_1} + \xi_{\delta_1}\big) \\ & \times \left(\frac{s_i}{h_i\|S\|^2}\right) + k_i \operatorname{sgn}(s_i), \\ & i=1,2,\dots,n. \end{aligned} \tag{45}$$

Meanwhile, appropriate adaptive laws are given as follows:

$$\begin{aligned} \dot{\hat{\theta}}_1 &= \big[F_1(x_1)\big]^T \lambda_1, \qquad \hat{\theta}_1(0) = \hat{\theta}_{10}, \\ \dot{\hat{\delta}}_1 &= \big[G_1(y_1)\big]^T \lambda_3, \qquad \hat{\delta}_1(0) = \hat{\delta}_{10}, \end{aligned} \tag{46}$$

where $\lambda_1=[h_1s_1a_{11},h_2s_2a_{22},\dots,h_ns_na_{nn}]$, $\lambda_3=[h_1s_1,h_2s_2,\dots,h_ns_n]$, *then the drive system* (1) *will achieve the projective synchronization with the response system* (3) *in the finite time via the sliding mode control*.

(ii) *Assume* $a_{ii}=d_{ii}=0$, $c_{ii}=1$, $(i=1,2,\dots,n)$, *if the control law is chosen as follows*:

$$\begin{aligned} u_i(t) = {} & b_{ii} f_{2i}(x_2) + b_{ii} F_{2i}(x_2)\hat{\theta}_2 - g_{1i}(y_1) \\ & - G_{1i}(y_1)\hat{\delta}_1 + \frac{1}{h_i}\operatorname{sgn}(e_i)|e_i|^{\sigma} \\ & + \mu\big(\|\hat{\theta}_2\| + \|\hat{\delta}_1\| + \xi_{\theta_2} + \xi_{\delta_1}\big) \\ & \times \left(\frac{s_i}{h_i\|S\|^2}\right) + k_i \operatorname{sgn}(s_i), \\ & i=1,2,\dots,n. \end{aligned} \tag{47}$$

Meanwhile, appropriate adaptive laws are given as follows:

$$\begin{aligned} \dot{\hat{\theta}}_2 &= \big[F_2(x_2)\big]^T \lambda_2, \qquad \hat{\theta}_2(0) = \hat{\theta}_{20}, \\ \dot{\hat{\delta}}_1 &= \big[G_1(y_1)\big]^T \lambda_3, \qquad \hat{\delta}_1(0) = \hat{\delta}_{10}, \end{aligned} \tag{48}$$

where $\lambda_2=[h_1s_1b_{11},h_2s_2b_{22},\dots,h_ns_nb_{nn}]$, $\lambda_3=[h_1s_1,h_2s_2,\dots,h_ns_n]$, *then the drive system* (2) *will achieve the projective synchronization with the response system* (3) *in the finite time via the sliding mode control*.

(iii) *Assume* $b_{ii}=c_{ii}=0$, $d_{ii}=1$ $(i=1,2,\dots,n)$, *if the control law is chosen as follows*:

$$\begin{aligned} u_i^*(t) = {} & a_{ii} f_{1i}(x_1) + a_{ii} F_{1i}(x_1)\hat{\theta}_1 - g_{2i}(y_2) \\ & - G_{2i}(y_2)\hat{\delta}_2 + \frac{1}{h_i}\operatorname{sgn}(e_i)|e_i|^{\sigma} \\ & + \mu\big(\|\hat{\theta}_1\| + \|\hat{\delta}_2\| + \xi_{\theta_1} + \xi_{\delta_2}\big) \\ & \times \left(\frac{s_i}{h_i\|S\|^2}\right) + k_i \operatorname{sgn}(s_i), \\ & i=1,2,\dots,n. \end{aligned} \tag{49}$$

Meanwhile, appropriate adaptive laws are given as follows:

$$\begin{aligned} \dot{\hat{\theta}}_1 &= \big[F_1(x_1)\big]^T \lambda_1, \qquad \hat{\theta}_1(0) = \hat{\theta}_{10}, \\ \dot{\hat{\delta}}_2 &= \big[G_2(y_2)\big]^T \lambda_4, \qquad \hat{\delta}_2(0) = \hat{\delta}_{20}, \end{aligned} \tag{50}$$

where $\lambda_1=[h_1s_1a_{11},h_2s_2a_{22},\dots,h_ns_na_{nn}]$, $\lambda_4=[h_1s_1,h_2s_2,\dots,h_ns_n]$, *then the drive system* (1) *will achieve the projective synchronization with the response system* (4) *in the finite time via the sliding mode control*.

(iv) *Assume* $a_{ii}=c_{ii}=0$, $d_{ii}=1$ $(i=1,2,\dots,n)$, *if the control law is chosen as follows*:

$$\begin{aligned} u_i^*(t) = {} & b_{ii} f_{2i}(x_2) + b_{ii} F_{2i}(x_2)\hat{\theta}_2 - g_{2i}(y_2) \\ & - G_{2i}(y_2)\hat{\delta}_2 + \frac{1}{h_i}\operatorname{sgn}(e_i)|e_i|^{\sigma} \\ & + \mu\big(\|\hat{\theta}_2\| + \|\hat{\delta}_2\| + \xi_{\theta_2} + \xi_{\delta_2}\big) \\ & \times \left(\frac{s_i}{h_i\|S\|^2}\right) + k_i \operatorname{sgn}(s_i), \\ & i=1,2,\dots,n. \end{aligned} \tag{51}$$

Meanwhile, appropriate adaptive laws are given as follows:

$$\begin{aligned} \dot{\hat{\theta}}_2 &= \big[F_2(x_2)\big]^T \lambda_2, \qquad \hat{\theta}_2(0) = \hat{\theta}_{20}, \\ \dot{\hat{\delta}}_2 &= \big[G_2(y_2)\big]^T \lambda_4, \qquad \hat{\delta}_2(0) = \hat{\delta}_{20}, \end{aligned} \tag{52}$$

where $\lambda_2=[h_1s_1b_{11},h_2s_2b_{22},\dots,h_ns_nb_{nn}]$, $\lambda_4=[h_1s_1,h_2s_2,\dots,h_ns_n]$, *then the drive system* (2) *will achieve the projective synchronization with the response system* (4) *in the finite time via the sliding mode control*.

Corollary 3 (i) *Suppose* $a_{ii} = b_{ii} = d_{ii} = 0$, $c_{ii} = 1$ $(i = 1, 2, \dots, n)$, *if the control law is chosen as follows*:

$$u_i(t) = -g_{1i}(y_1) - G_{1i}(y_1)\hat{\delta}_1 + \frac{1}{h_i}\operatorname{sgn}(e_i)|e_i|^\sigma + \mu\big(\|\hat{\delta}_1\| + \xi_{\delta_1}\big)\left(\frac{s_i}{h_i\|S\|^2}\right) + k_i \operatorname{sgn}(s_i), \quad i = 1, 2, \dots, n. \tag{53}$$

Meanwhile, appropriate adaptive law is given as follows:

$$\dot{\hat{\delta}}_1 = \big[G_1(y_1)\big]^T \lambda_3, \qquad \hat{\delta}_1(0) = \hat{\delta}_{10}, \tag{54}$$

where $\lambda_3 = [h_1 s_1, h_2 s_2, \dots, h_n s_n]$, *then the equilibrium point* $(0, 0, \dots, 0)$ *of the response system* (3) *is asymptotically stable in the finite time via the sliding mode control.*

(ii) *Suppose* $a_{ii} = b_{ii} = c_{ii} = 0$, $d_{ii} = 1$ $(i = 1, 2, \dots, n)$, *if the control law is chosen as follows*:

$$u_i^*(t) = -g_{2i}(y_2) - G_{2i}(y_2)\hat{\delta}_2 + \frac{1}{h_i}\operatorname{sgn}(e_i)|e_i|^\sigma + \mu\big(\|\hat{\delta}_2\| + \xi_{\delta_2}\big)\left(\frac{s_i}{h_i\|S\|^2}\right) + k_i \operatorname{sgn}(s_i), \quad i = 1, 2, \dots, n. \tag{55}$$

Meanwhile, appropriate adaptive law is given as follows:

$$\dot{\hat{\delta}}_2 = \big[G_2(y_2)\big]^T \lambda_4, \qquad \hat{\delta}_2(0) = \hat{\delta}_{20}, \tag{56}$$

where $\lambda_4 = [h_1 s_1, h_2 s_2, \dots, h_n s_n]$, *then the equilibrium point* $(0, 0, \dots, 0)$ *of the response system* (4) *is asymptotically stable in the finite time via the sliding mode control.*

4 Numerical simulations

In the following section, the Lorenz system, Lü system, Chen system, and Rössler system are taken as an example to validate the effectiveness of the proposed approaches. It is assumed that the Lorenz system and Lü system are two drive systems, Chen system and Rössler system are two response systems. The dynamic equations of four systems are, respectively, given by

$$\dot{x}_1 = \underbrace{\begin{pmatrix} 0 \\ -x_{11}x_{13} \\ x_{11}x_{12} \end{pmatrix}}_{f_1(x_1)} + \underbrace{\begin{pmatrix} x_{12} - x_{11} & 0 & 0 \\ 0 & x_{11} & 0 \\ 0 & 0 & -x_{13} \end{pmatrix}}_{F_1(x_1)} \underbrace{\begin{pmatrix} 10 \\ 28 \\ 8/3 \end{pmatrix}}_{\theta_1}, \tag{57}$$

$$\dot{x}_2 = \underbrace{\begin{pmatrix} 0 \\ -x_{21}x_{23} \\ x_{21}x_{22} \end{pmatrix}}_{f_2(x_2)} + \underbrace{\begin{pmatrix} x_{22} - x_{21} & 0 & 0 \\ 0 & x_{21} & 0 \\ 0 & 0 & -x_{23} \end{pmatrix}}_{F_2(x_2)} \underbrace{\begin{pmatrix} 36 \\ 20 \\ 3 \end{pmatrix}}_{\theta_2}, \tag{58}$$

$$\dot{y}_1 = \underbrace{\begin{pmatrix} 0 \\ -y_{11}y_{13} \\ y_{11}y_{12} \end{pmatrix}}_{g_1(y_1)} + \underbrace{\begin{pmatrix} y_{12} - y_{11} & 0 & 0 \\ -y_{11} & y_{11} + y_{12} & 0 \\ 0 & 0 & -y_{13} \end{pmatrix}}_{G_1(y_1)} \underbrace{\begin{pmatrix} 35 \\ 28 \\ 3 \end{pmatrix}}_{\delta_1} + \underbrace{\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}}_{u}, \tag{59}$$

$$\dot{y}_2 = \underbrace{\begin{pmatrix} 0 \\ 0 \\ y_{23}y_{22} + 0.2 \end{pmatrix}}_{g_2(y_2)} + \underbrace{\begin{pmatrix} -y_{22} - y_{23} & 0 & 0 \\ y_{21} & y_{22} & 0 \\ 0 & 0 & -y_{23} \end{pmatrix}}_{G_2(y_2)} \underbrace{\begin{pmatrix} 1 \\ 0.2 \\ 5.7 \end{pmatrix}}_{\delta_2} + \underbrace{\begin{pmatrix} u_1^* \\ u_2^* \\ u_3^* \end{pmatrix}}_{u^*}. \tag{60}$$

Assume $\sigma = 1/2$, $h_1 = 1$, $h_2 = 2$, $h_3 = 3$, according to Eq. (13), three sliding surfaces are defined as

$$\begin{cases} s_1 = e_1(t) - e_1(0) + \int_0^t \operatorname{sgn}\big(e_1(\tau)\big)|e_1(\tau)|^{1/2} d\tau, \\ s_2 = 2\big(e_2(t) - e_2(0)\big) + \int_0^t \operatorname{sgn}\big(e_2(\tau)\big)|e_2(\tau)|^{1/2} d\tau, \\ s_3 = 3\big(e_3(t) - e_3(0)\big) + \int_0^t \operatorname{sgn}\big(e_3(\tau)\big)|e_3(\tau)|^{1/2} d\tau. \end{cases} \tag{61}$$

Furthermore, $a_{11} = a_{22} = a_{33} = 1$, $b_{11} = b_{22} = b_{33} = 1$, $c_{11} = c_{22} = c_{33} = 1$ and $d_{11} = d_{22} = d_{33} = 1$, are arbitrarily given. ξ_{θ_1}, ξ_{θ_2}, ξ_{δ_1}, and ξ_{δ_2} are chosen equal to 100, the vector $K = [k_1, k_2, k_3] = [1, 1, 1]$ is selected as the switching gain, then $\mu = \min\{1, 2, 3\} = 1$. The error dynamics are obtained as follows:

$$\begin{cases} \dot{e}_1(t) = (x_{12} - x_{11})(10 - \hat{\theta}_{11}) \\ \qquad + (x_{22} - x_{21})(36 - \hat{\theta}_{21}) \\ \qquad - (y_{12} - y_{11})(35 - \hat{\delta}_{11}) \\ \qquad - (-y_{22} - y_{23})(1 - \hat{\delta}_{21}) + \operatorname{sgn}(e_1)|e_1|^{1/2} \\ \qquad + \big(\|\hat{\theta}_1\| + \|\hat{\theta}_2\| + \|\hat{\delta}_1\| + \|\hat{\delta}_2\| + 400\big) \\ \qquad \times \left(\frac{s_1}{\|S\|^2}\right) + \operatorname{sgn}(s_1), \\ \dot{e}_2(t) = x_{11}(28 - \hat{\theta}_{12}) + x_{21}(20 - \hat{\theta}_{22}) \\ \qquad + y_{11}(35 - \hat{\delta}_{11}) \\ \qquad - (y_{11} + y_{12})(28 - \hat{\delta}_{12}) - y_{21}(1 - \hat{\delta}_{21}) \\ \qquad - y_{22}(0.2 - \hat{\delta}_{22}) + \frac{1}{2}\operatorname{sgn}(e_2)|e_2|^{1/2} \\ \qquad + \big(\|\hat{\theta}_1\| + \|\hat{\theta}_2\| + \|\hat{\delta}_1\| + \|\hat{\delta}_2\| + 400\big) \\ \qquad \times \left(\frac{s_2}{2\|S\|^2}\right) + \operatorname{sgn}(s_2), \\ \dot{e}_3(t) = x_{13}\left(\frac{8}{3} + \hat{\theta}_{13}\right) - x_{23}(3 - \hat{\theta}_{23}) \\ \qquad + y_{13}(3 - \hat{\delta}_{13}) \\ \qquad + y_{23}(5.7 - \hat{\delta}_{23}) + \frac{1}{3}\operatorname{sgn}(e_3)|e_3|^{1/2} \\ \qquad + \big(\|\hat{\theta}_1\| + \|\hat{\theta}_2\| + \|\hat{\delta}_1\| + \|\hat{\delta}_2\| + 400\big) \\ \qquad \times \left(\frac{s_3}{3\|S\|^2}\right) + \operatorname{sgn}(s_3). \end{cases} \tag{62}$$

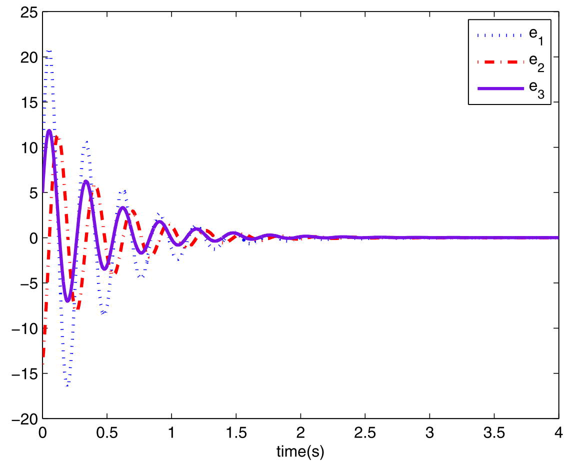

Fig. 1 Synchronization errors e_1, e_2, e_3 between drive systems (57), (58), and response systems (59), (60)

To tackle the unknown parameters, appropriate adaptive laws are given as:

$$\dot{\hat{\theta}}_1 = \begin{pmatrix} \hat{\theta}_{11} \\ \hat{\theta}_{12} \\ \hat{\theta}_{13} \end{pmatrix} = \begin{pmatrix} (x_{12} - x_{11})s_1 \\ 2x_{11}s_2 \\ -3x_{13}s_3 \end{pmatrix}, \tag{63}$$

$$\dot{\hat{\theta}}_2 = \begin{pmatrix} \hat{\theta}_{21} \\ \hat{\theta}_{22} \\ \hat{\theta}_{23} \end{pmatrix} = \begin{pmatrix} (x_{22} - x_{21})s_1 \\ 2x_{21}s_2 \\ -3x_{23}s_3 \end{pmatrix}, \tag{64}$$

$$\dot{\hat{\delta}}_1 = \begin{pmatrix} \hat{\delta}_{11} \\ \hat{\delta}_{12} \\ \hat{\delta}_{13} \end{pmatrix} = \begin{pmatrix} -(y_{12} - y_{11})s_1 \\ y_{11}s_1 - 2(y_{11} + y_{12})s_2 \\ -3y_{13}s_3 \end{pmatrix}, \tag{65}$$

$$\dot{\hat{\delta}}_2 = \begin{pmatrix} \hat{\delta}_{21} \\ \hat{\delta}_{22} \\ \hat{\delta}_{23} \end{pmatrix} = \begin{pmatrix} (y_{22} + y_{23})s_1 \\ -y_{21}s_1 - 2y_{22}s_2 \\ 3y_{23}s_3 \end{pmatrix}. \tag{66}$$

The initial states for two drive systems and for two response systems are arbitrarily given by $(x_{11}, x_{12}, x_{13}) = (-2, 1, 1.5)$, $(x_{21}, x_{22}, x_{23}) = (1, 1, 2)$, $(y_{11}, y_{12}, y_{13}) = (-4, 5, -6)$, and $(y_{21}, y_{22}, y_{23}) = (-5, 11, 4.5)$; thus, we have $(e_{10}, e_{20}, x_{30}) = (8, -14, 5)$. The initial values of estimated parameters are chosen as $\hat{\theta}_1 = (15, 15, 8)$, $\hat{\theta}_2 = (36, 20, 8)$, $\hat{\delta}_1 = (68, 40, 12)$, and $\hat{\delta}_2 = (3, 1, 10)$. The error time response and adaptation parameters θ_{1i}, θ_{2i} δ_{1i}, δ_{2i}, $(i = 1, 2, 3)$ are shown in Figs. 1–5, respectively. As it can be seen, the synchronization error $e(t)$ converges to zero in Fig. 1, which means that the combination-combination synchronization between two drive systems for Lorenz system and Lü system and two response systems for

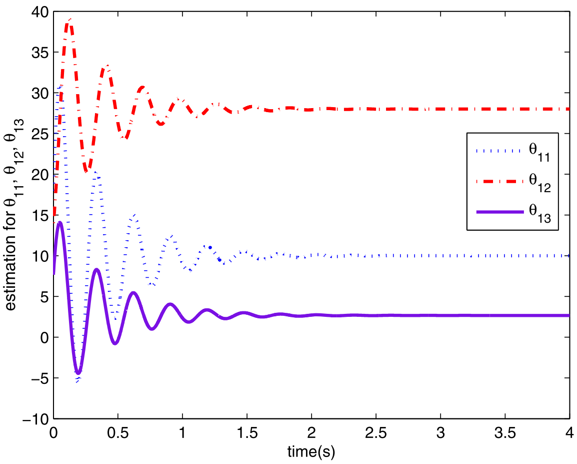

Fig. 2 Time response of the adaptive vector parameter $\hat{\theta}_1$

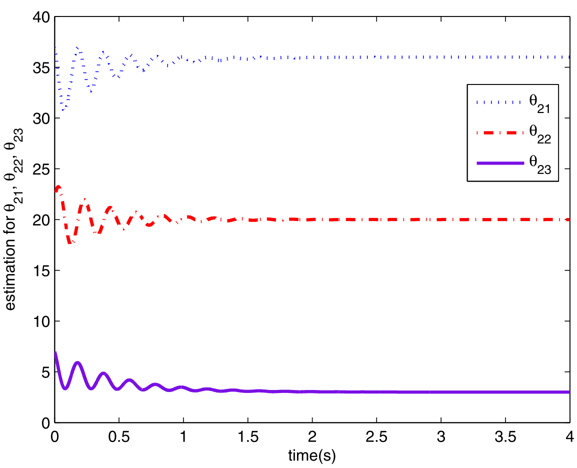

Fig. 3 Time response of the adaptive vector parameter $\hat{\theta}_2$

Chen system and Rössler system is realized in the finite time via the sliding mode control. Furthermore, the $\hat{\theta}_{1i}$, $\hat{\theta}_{2i}$ $\hat{\delta}_{1i}$, $\hat{\delta}_{2i}$ $(i = 1, 2, 3)$ are tended to the expected values θ_{1i}, θ_{2i} δ_{1i}, δ_{2i} $(i = 1, 2, 3)$ in Figs. 2, 3, 4 and 5, respectively.

Remark 12 According to the simulation parameters, $T_1 \le 16.0508$, then we compute $T_2 \le 39.8459$, the adaptive laws (63)–(66) and the sliding surface (61) can complete synchronization between combination of the drive systems (57) and (58) reach combination of the response systems (59) and (60) in the finite time $T \le T_1 + T_2 = 55.8967$, the simulation results have good agreement with Remark 8.

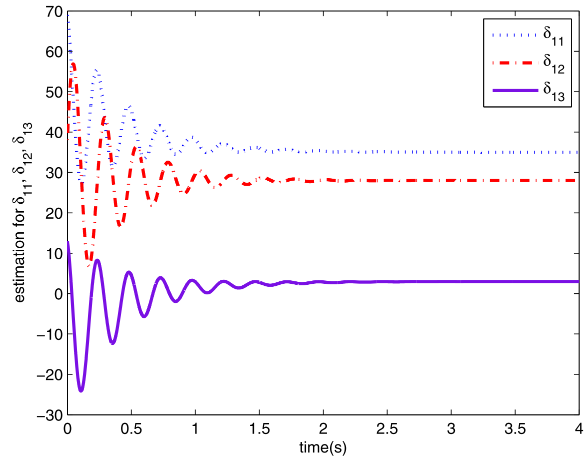

Fig. 4 Time response of the adaptive vector parameter $\hat{\delta}_1$

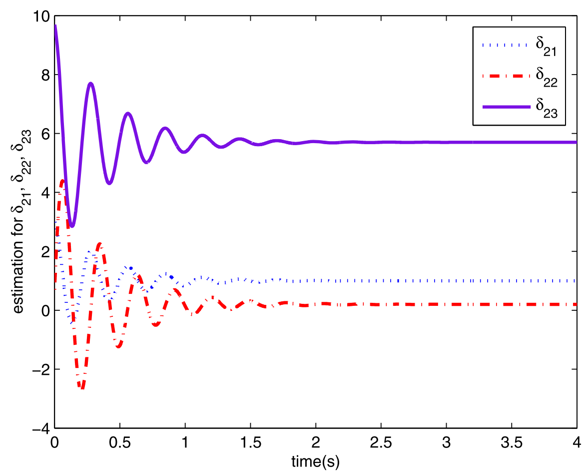

Fig. 5 Time response of the adaptive vector parameter $\hat{\delta}_2$

5 Conclusion

In this paper, we have introduced the scheme of finite-time combination-combination chaos synchronization between combination of two chaotic systems as the drive system and combination of two chaotic systems as the response system with unknown parameters. A nonsingular terminal sliding mode manifold has been chosen and its finite-time convergence has been proved analytically. Combined to the adaptive laws and finite-time control method, an adaptive combination finite-time sliding mode controller has been designed. The proposed technique has had finite-time convergence and stability in both the reaching and sliding mode surface. The simulation results have shown that the proposed combination controller works well for synchronizing the combination of drive systems and combination of response systems in the finite time; even the parameters of both the combination of drive systems and combination of response systems are fully unknown. However, most of existing works only study the synchronization of the master-slave type. We consider the synchronization of combination for drive systems and combination for response systems. Our method may be more advantageous and have more potential than the traditional method to complete intelligent synchronization. How to realize combination-combination chaos synchronization in actual practice is our next research topic.

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