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# A novel four-dimensional autonomous hyperchaotic system

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A novel four-dimensional autonomous hyperchaotic system is reported in this paper. Some basic dynamical properties of the new hyperchaotic system are investigated in detail by means of a continuous spectrum, Lyapunov exponents, fractional dimensions, a strange attractor and Poincaré mapping. The dynamical behaviours of the new hyperchaotic system are proved by not only performing numerical simulation and brief theoretical analysis but also by conducting an electronic circuit experiment.

**Keywords:** hyperchaos, dynamical behaviors, circuit experiment
**PACC:** 0545

## 1. Introduction

Hyperchaos with more than one positive Lyapunov exponent has been studied with increasing interest in the last few years. As a higher-dimensional chaotic system, the hyperchaotic system has more complex and richer dynamical behaviours than a low-dimensional chaotic system. Historically, the first hyperchaotic system was reported by Rössler in 1979.[1] Due to its great potential applications in technology, more researchers paid attention to constructing and controlling hyperchaos.[2−5] Recently, based on an originally chaotic system some hyperchaos has been designed. Particularly, using a basic electronic circuit to generate the hyperchaos has become a hot topic in the nonlinear research field.[6−12]

The present paper reports on a new four-dimensional hyperchaotic system, which also exhibits complex and abundant hyperchaotic dynamic behaviours. More precisely, some complicated dynamics has been analysed and explored in detail by using Poincaré maps, bifurcation diagrams and Lyapunov exponents. Finally, a novel oscillation circuit is designed for physically realizing this hyperchaotic system. The results of the circuit experiment are shown to be in good agreement with computer numerical simulations. It is believed that this hyperchaotic system can enhance the security of the communication scheme in information processing and image manipulation.

## 2. System description and analysis

This novel hyperchaotic system can be described by the following four-dimensional autonomous differential equation:

$$\begin{aligned} \dot{x} &= a(y - x + y^2) + ew, \\ \dot{y} &= by - kxz + mw, \\ \dot{z} &= -cz + hy - lw, \\ \dot{w} &= -dz, \end{aligned} \tag{1}$$

where $x, y, z$ and $w$ are state variables, and $a, b, c, k, h, e, m, l$ and $d$ are all positive real parameters. When the values of the parameters in system (1) are selected as $a = 0.5, b = 2.5, c = 4, k = 1, h = 1, e = 1, m = 1, l = 0.25$ and $d = 0.25$, this new chaotic system becomes hyperchaotic.

In order to reveal the hyperchaotic dynamical properties of this nonlinear system, first, we discuss the equilibria of the four-dimensional autonomous system (1). Let

$$\begin{aligned} &a(y - x + y^2) + ew = 0, \\ &by - kxz + mw = 0, \\ &hy - cz - lw = 0, \\ &-dz = 0, \end{aligned} \tag{2}$$

then system (1) will have only a real equilibrium, which is described as $O(0, 0, 0, 0)$. For equilibrium

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$O(0,0,0,0)$, system (1) is linearized, and the relevant Jacobian matrix is

$$J_0 = \begin{bmatrix} -a & a(1+2y) & 0 & e \\ -kz & b & -kx & m \\ 0 & h & -c & -l \\ 0 & 0 & -d & 0 \end{bmatrix} = \begin{bmatrix} -0.5 & 0.5 & 0 & 1 \\ 0 & 2.5 & 0 & 1 \\ 0 & 1 & -4 & -0.25 \\ 0 & 0 & -0.25 & 0 \end{bmatrix}. \tag{3}$$

To gain its eigenvalues, let

$$|\lambda I - J_0| = 0,$$

then these eigenvalues corresponding to equilibrium $O(0,0,0,0)$ will be obtained as follows:

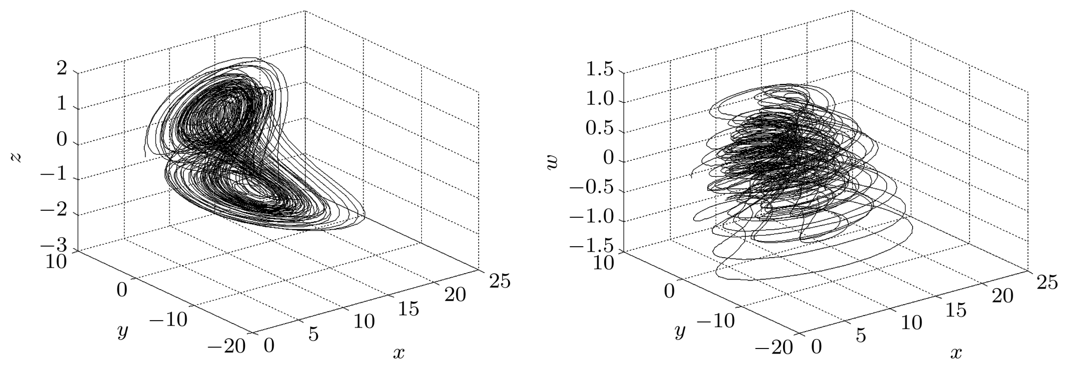
$$\lambda_1 = -0.5, \quad \lambda_2 = -4.025,$$

$$\lambda_3 = 2.4844, \quad \text{and} \quad \lambda_4 = 0.0406,$$

where $\lambda_1$ and $\lambda_2$ both are negative real roots, while $\lambda_3$ and $\lambda_4$ both are positive real roots. Therefore, equilibrium $O(0,0,0,0)$ is an unstable saddle in this nonlinear four-dimensional autonomous system.

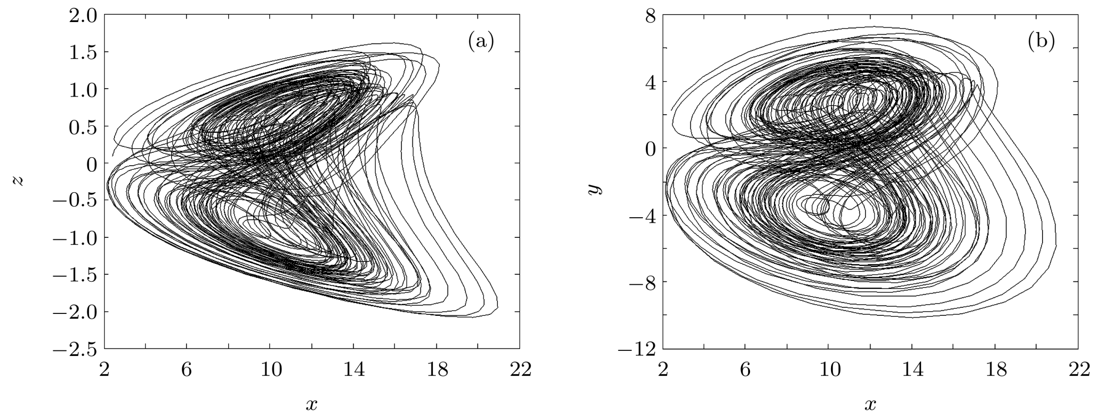
The initial values of state variables $x, y, z$ and $w$ of system (1) are taken as 2.4, 2.2, 0.08 and 0, respectively. According to both numerical and theoretical analyses, it has been confirmed that new system (1) is possessed of sophisticated and abundant hyperchaotic dynamical behaviours.

The hyperchaotic strange attractors are shown in Figs.1 and 2, which are also new butterfly-shaped attractors.




(a) (b)

**Fig.1.** $x$–$y$–$z$ (a) and $x$–$y$–$w$ (b) three-dimensional view of system (1).



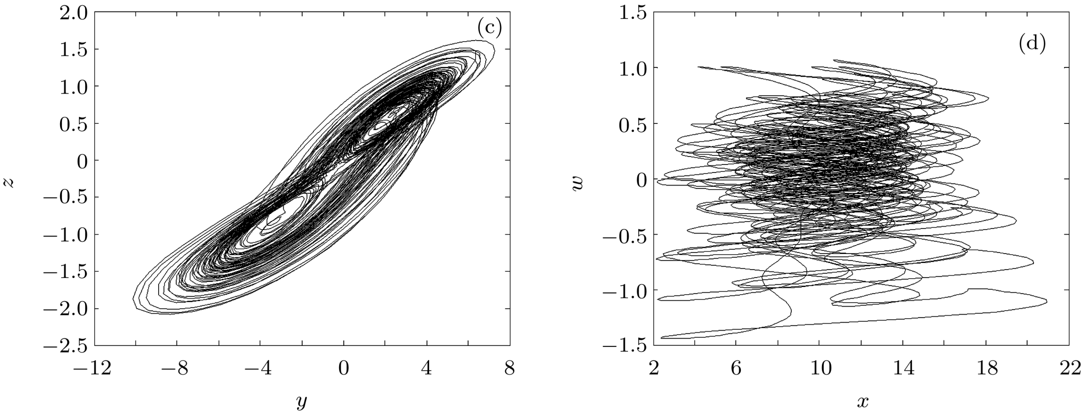



**Fig.2.** Strange attractors of system (1) in $x$–$z$ (a), $x$–$y$ (b), $y$–$z$ (c), and $x$–$w$ (d) phase planes.

The aforementioned analyses show that it is apparently a new chaotic attractor, and its sensitivity to the initial condition is a prominent characteristic of chaotic behaviour: when the assigned initial values change, the chaotic dynamical behaviour of this system will disappear quickly.

The waveforms of $x(t)$ in time domain are shown in Fig.3, and apparently, they are non-periodic in the continuous-time four-dimensional autonomous chaotic system (1), which shows basic chaotic dynamical properties.

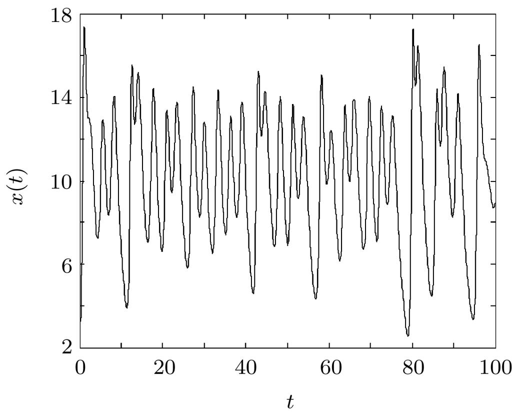



**Fig.3.** $x(t)$ waveform of system (1).

The spectrum of non-linear system (1) is also studied, and it is continuous as shown in Fig.4.

The Poincaré mapping of this non-linear autonomous system is also analysed. It can be seen that the Poincaé mapping is composed of these points as shown in Fig.5.

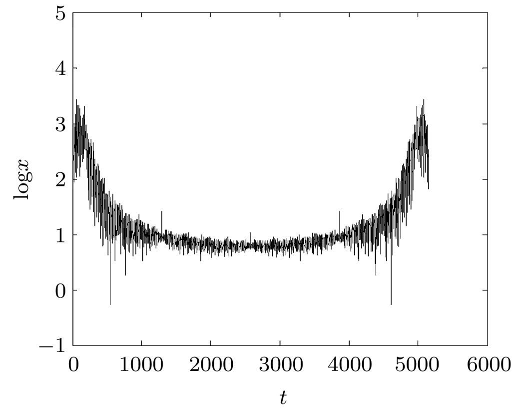



**Fig.4.** Spectrum of $|x|$ in system (1).

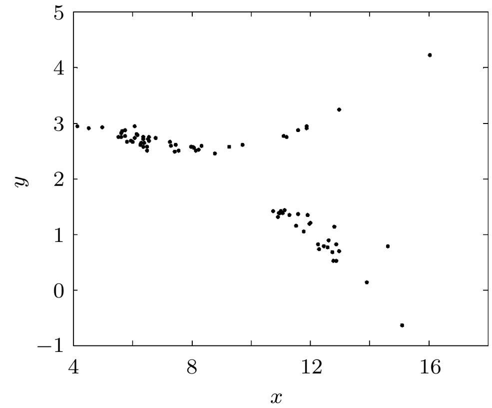



**Fig.5.** Poincaré map of the $x$–$y$ plane of system (1).

The bifurcation diagram of $x$ versus $a$ is given in Fig.6, which shows richer and complex dynamical behaviours.

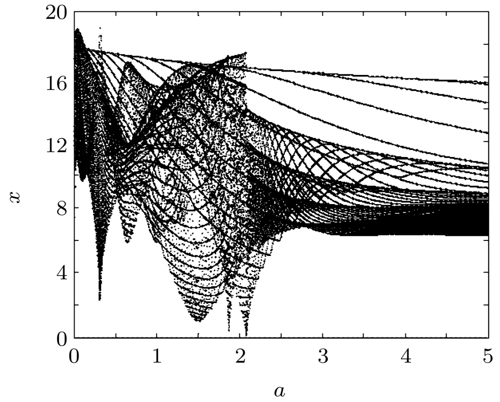



**Fig.6.** Bifurcation diagram versus $a$ with $b = 2.5, h = 1, k = 1, e = 1, m = 1, l = 0.25$, and $d = 0.25$ of system (1).

The divergence of system (1) is given by

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{w}}{\partial w} = -a + b - c - d = -\delta. \quad (4)$$

Since $\nabla V = -\delta = -0.5 + 2.5 - 4 - 0.25 = -2.25$ and its value is a negative constant, system (1) is dissipative. Therefore, this shows that each volume contained in the trajectory of this system shrinks to zero as $t \to \infty$ at an exponential rate $V(0)\mathrm{e}^{-\delta t}$. Thus, all orbits of system (1) ultimately are confined to a subset of zero volume, and it is apparent that the asymptotic motion for the non-linear system settles down to an attractor finally.

According to chaos theory, the Lyapunov exponent is a measure of exponential rates of divergence and convergence of nearby trajectories in phase space of system (1). As is well known, there is more than one positive exponent in a hyperchaotic four-dimensional autonomous system.

The computation results of Lyapunov exponents are shown in Fig.7. The two maximum values of positive Lyapunov exponents of the nonlinear autonomous system (1) are calculated to be $L_1 = 0.1277$ and $L_2 = 0.0444$, which show the expanding nature in different directions in phase space. Another Lyapunov exponent is

$$L_3 = 0,$$

which shows the critical nature between the expanding and the contracting nature in different directions in phase space.

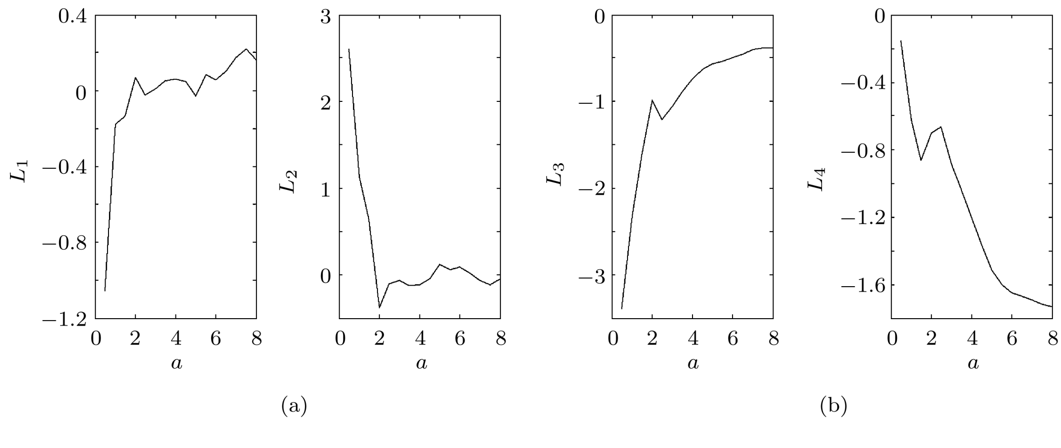



**Fig.7.** Lyapunov exponents of system (1).

While the negative Lyapunov exponent of the nonlinear autonomous system (1) is

$$L_4 = -2.1673,$$

which shows the contracting nature in different directions in phase space. Therefore, by theoretical analysis and numerical simulation, the Lyapunov dimension of this hyperchaotic system is obtained as follows:

$$\begin{aligned} D_L &= j + \frac{1}{|L_{j+1}|} \sum_{i=1}^{j} L_i \\ &= 3 + \frac{(L_1 + L_2 + 0)}{|L_4|} \\ &= 3 + \frac{0.1277 + 0.0444 + 0}{|-2.1673|} = 3.079. \end{aligned} \quad (5)$$

The Lyapunov dimension of this new hyperchaotic system is also fractional dimension.

The above theoretical analysis and numerical simulation both show that system (1) is a new hyperchaotic system and possesses a more sophisticated topological structure and abundant hyperchaotic dynamical properties.[13−20]

## 3. Circuitry experimental confirmation of the hyperchaotic system

In this section, an electronic oscillation circuit is designed to realize the new hyperchaotic system (1) based on Ref.[21]. The designed hyperchaotic oscillation circuit is shown in Fig.8, and it comprises linear resistors, linear capacitors, operational amplifiers and analogue multipliers. The four state variables $x, y$, $z$ and $w$ are respectively obtained from the terminal outputs of $v_{C_1}, v_{C_2}$, $v_{C_3}$ and $v_{C_4}$ in this electronic circuit. The experimental phase portraits of the new transverse butterfly-shaped attractor (1) are shown in Fig.9. The results of the circuit experiment are shown to be in agreement with numerical simulations. The designed chaos oscillator shown in Fig.8 is very useful in the electronic technique and communication engineering.[21−26]

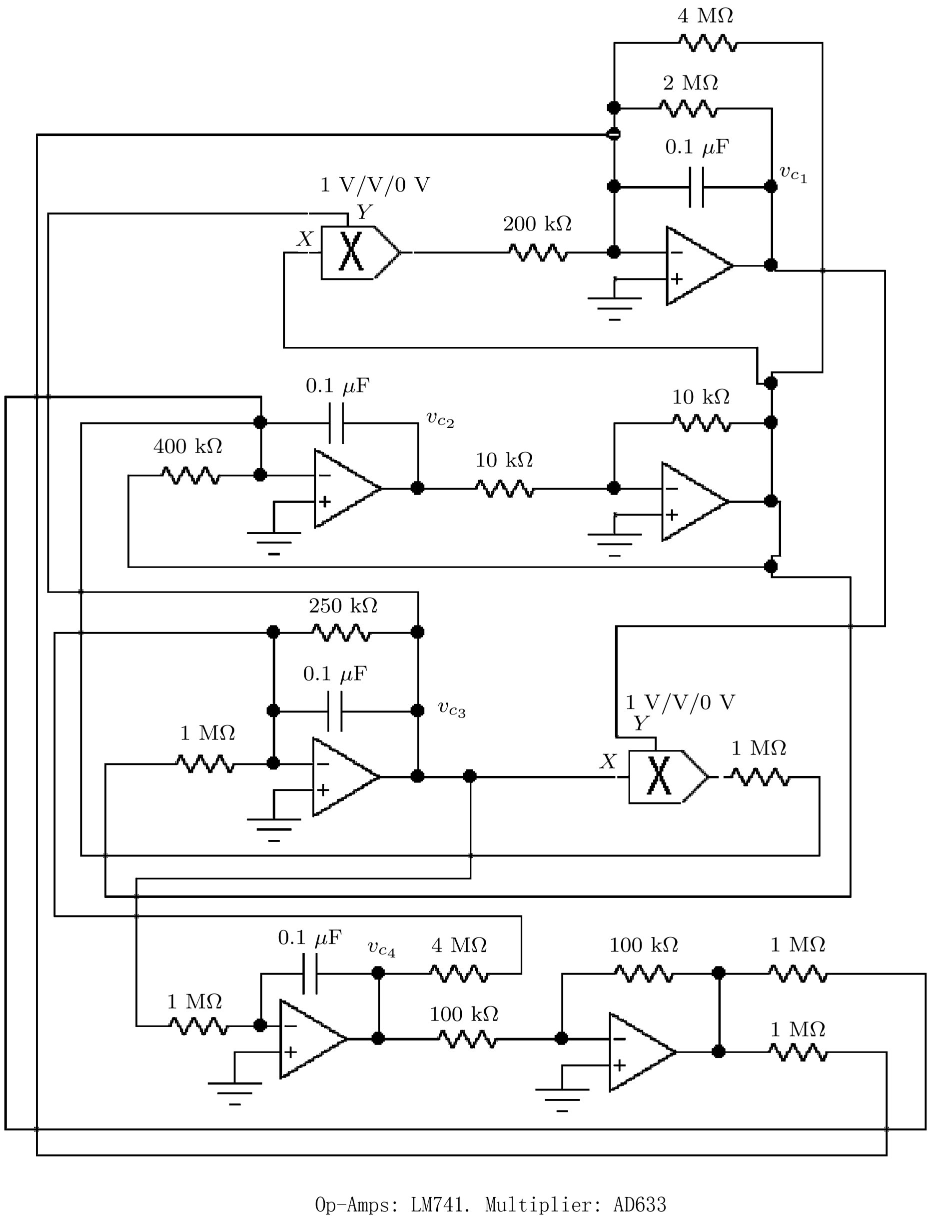



**Fig.8.** Circuitry realization of the hyperchaotic system (1).

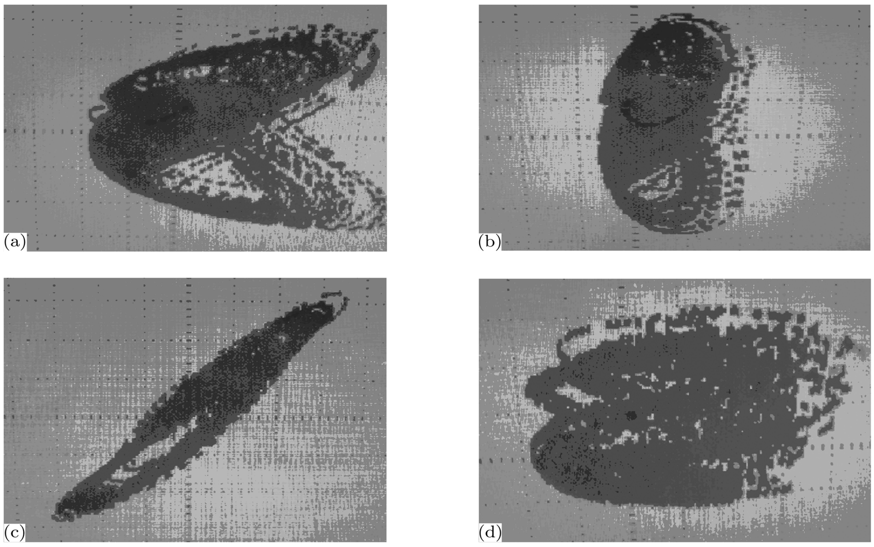



**Fig.9.** Experimental phase portraits of system (1) in (a) $x$–$z$ plane (2 V/div, 0.5 V/div); (b) $x$–$y$ plane (5 V/div, 2 V/div); (c) $y$–$z$ plane (2 V/div, 0.5 V/div); (d) $x$–$w$ plane (2 V/div, 0.2 V/div).

# 4. Conclusion

There are abundant and complex dynamical behaviours in the new hyperchaotic system (1). This new hyperchaotic attractor is different from the hyperchaotic Lorenz attractor, the hyperchaotic Chen attractor, the hyperchaotic system proposed by Chen *et al.*[8] It is a new transverse butterfly-shaped hyperchaotic attractor of a Lorenz-like system. This new hyperchaotic attractor can be realized with an electronic oscillation circuit and has great potential applications in the electronic technique and communication engineering. These new hyperchaotic attractors and their forming mechanism need further study. Their topological structure should be completely and thoroughly investigated. More detailed theoretical analyses and simulations are expected to occur elsewhere in the near future.

# References

[1] Rossler O E 1979 *Phys. Lett.* A **71** 155
[2] Chen G R and Ueta T 1999 *Int. J. Bifurrc. Chaos* **9** 1465
[3] Lü J H, Chen G R and Zhang S C 2002 *Chaos, Solitions and Fractals* **14** 669
[4] Jia Q 2007 *Phys. Lett.* A **366** 217
[5] Sun M, Jia Q and Tian L X 2007 *Chaos, Solitions and Fractals* **32** 168
[6] Wang F Q and Liu C X 2006 *Chin. Phys.* **15** 963
[7] Sun M, Tian L X and Zeng C Y 2008 *Nonlinear Analysis: Real World Applications* **04** 019
[8] Chen Z Q, Yang Y, Qi G Y and Yuan Z Z 2006 *Phys. Lett.* A **360** 696
[9] Liu L, Su Y C and Liu C X 2007 *Chin. Phys.* **16** 1897
[10] Liu L, Liu C X and Zhang Y B 2007 *Chin. Phys. Lett.* **24** 2756
[11] Liu C X 2007 *Chin. Phys.* **16** 3279
[12] Liu C X and Liu L 2008 *Chin. Phys.* B **17** 2829
[13] Wolf A, Swift J B, Swinney H L and Vastano J A 1985 *Physica* D **16** 285
[14] Lu J H, Zhou T S and Zhang S C 2002 *Chin. Phys.* **11** 12
[15] Guan X P, Peng H P, Li L X and Wang Y Q 2001 *Acta Phys. Sin.* **50** 26 (in Chinese)
[16] Yang L B and Yang T 2000 *Acta Phys. Sin.* **49** 1039 (in Chinese)
[17] Lou X S, Kong Lj and Qu W L 1998 *Acta Phys. Sin.* **47** 1078 (in Chinese)
[18] Tang G N, Luo X S and Kong L J 2000 *Acta Phys. Sin.* **49** 30 (in Chinese)
[19] Lü J H, Chen G R and Zhang S C 2002 *Int. J. Bifurc. Chaos* **12** 1001
[20] Zhang S H and Shen K 2003 *Chin. Phys.* **13** 149
[21] http://www.electronics workbench.com/
[22] Wang J Z, Chen Z Q and Yuan Z Z 2006 *Chin. Phys.* **15** 1216
[23] Yu S M, Ma Z G, Q S S, Peng S G and Lin Q H 2004 *Chin. Phys.* **13** 317
[24] Wang F Q and Liu C X 2006 *Chin. Phys.* **15** 1971
[25] Wang G Y, Qiu S S, Li H W, Li G F and Zhang Y 2006 *Chin. Phys.* **15** 2872
[26] Lu J G 2006 *Chin. Phys.* **15** 83