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Adequacy study of a wind farm considering terrain and wake effect

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Abstract: When wind passes over a hilly land, the complex terrain feature will observably change wind speed. Wind speed distribution on a hill is much different from that on a flat area. The wake produced by an upstream wind turbine generator (WTG) may affect power output of a downstream WTG. When a WTG fails, wind speed distribution over a wind farm also changes, which leads to different wind power output. This study proposes a time sequential Monte Carlo simulation technique to evaluate adequacy of a wind farm with considering the combined effect of terrain, wake and WTG reliability. A quadratic interpolation method is used to study the optimal locations of WTGs in order to maximise power output of a wind farm. A test wind farm and the IEEE reliability test system (RTS) are analysed to illustrate the models and proposed technique.

1 Introduction

Wind is regarded as an important alternative to traditional fossil fuels owing to its clean and renewable nature. Recent public concerns on environment issues associated with conventional energy sources have led to a rapid growth of wind power penetration in power systems throughout the world. Power output from a wind turbine generator (WTG) depends on wind speed. It is hard to predict power output of a wind farm because of the intermittent and uncertain nature of wind speed. Wind speed models, which are very important for power system operation, have been studied [1-4]. A method using the Weibull distribution function and a lag-one auto-regressive (AR) model was developed to determine power output of WTGs [2]. Two time-series models generated using available wind data were proposed to simulate the hourly wind speed [3]. The authors in [4] and [5] proposed two wind farm models using simulation and analytical methods, respectively. In the late 70s and early 80s, as wind power generation appeared to have great technological and economic potential, the efforts were made to evaluate its impending impacts on generation reliability [1]. With continuously increasing wind power penetration in power systems, adequacy of a wind farm becomes an important issue for power system reliability evaluation. Many research works have been done on reliability assessment of power systems with wind power penetration. The authors in [6] studied adequacy of generation systems with WTG using Monte Carlo simulation method. The impact of adding WTG on reliability of distribution systems was investigated [7, 8]. Reliability of a power system with wind farms was analysed based on steady-state power flow [9]. Literature [10] summarised the factors affecting the reliability of wind power systems. Wake effect is an

important factor which affects the power output of a wind farm. The Jenson model, wind sheer and shade area model were developed in detail to determine wake effect [11-13]. A quadratic interpolation method was proposed to calculate the optimal distance of WTGs with considering wake effect on flat area [14]. However, the impact of the combined WTG reliability and wake effect on power output of a wind farm were not considered. One the other hand, most existing techniques consider only the wake effect of a wind farm in a flat area. When wind passes over a hilly land, complex terrain feature will change the wind speed. Wind speed distribution over a hill is much different from that on a flat area. Therefore wind speed data measured in the limited locations are not sufficient to accurately evaluate power outputs of WTGs at different parts of a wind farm. The terrain influence of a hilly land on wind speed has not been considered in adequacy study of a wind farm.

In this paper, both wake and terrain influence on wind speed model over a hilly land wind farm has been comprehensively studied based on the measured wind speeds at the limited locations. A time sequential Monte-Carlo simulation technique is used to evaluate adequacy of a wind farm with considering combined effect of WTG reliability and location, and wake and terrain. A quadratic interpolation method is used to determine the optimal locations of WTGs in a wind farm. A test wind farm and the IEEE RTS are analysed to illustrate the models and proposed technique.

2 Wind farm modelling

2.1 Reliability model of a WTG

A WTG is represented by a two-state reliability model [15]. In the two-state model, failure rate λ and repair rate μ of a WTG

are assumed to be constant. Therefore the time to failure τ_1 and the time to repair τ_2 are exponentially distributed and can be calculated by (1) and (2), respectively, based on uniform random number generated [9]

$$\tau_1 = -\frac{1}{\lambda} \ln \gamma_1 \tag{1}$$

$$\tau_2 = -\frac{1}{\mu} \ln \gamma_2 \tag{2}$$

where γ_1 and γ_2 are the uniform distribution stochastic variables between 0 and 1. An operating history for a WTG can be simulated based on τ_1 and τ_2 .

2.2 Wind speed model

An essential prerequisite in adequacy analysis of a wind farm is to realistically simulate the hourly wind speed. The Weibull parameters [13] of hourly wind speed are usually used to generate sequential wind speeds

$$v = c[-\ln \gamma_i]^{1/k} \tag{3}$$

where *c* is the size parameter reflecting the average wind speed; *k* is the shape parameter reflecting the skewness of Weibull distribution; and γ_i is a uniform stochastic variable between 0 and 1.

2.3 Power output of a WTG

The power output p(v) of a WTG is determined based on wind speed v using (4) [5].

$$p(v) = \begin{cases} 0 & (v < v_{ci}) \cup (v > v_{co}) \\ p_r(v^3 - v_{ci}^3)/(v_r^3 - v_{ci}^3) & v_{ci} \le v < v_r \\ p_r & v_r \le v \le v_{co} \end{cases}$$
(4)

where $p_{\rm r}$, $v_{\rm ci}$, $v_{\rm r}$ and $v_{\rm co}$ are the rated power, cut-in speed, rated speed and cut-off speed of a WTG, respectively.

3 Modelling terrain effect

3.1 Wind speeds on a flat area

Wind speed at any height on a flat area can be calculated based on the measured wind speed at a given height using the following exponent formula [11]

$$v_n = v_1 (h_n / h_1)^{\alpha} \tag{5}$$

where α is the coefficient of wind speed variation, which is generally 1/7 [13], v_n and v_1 are the wind speeds at height h_n and h_1 , respectively.

3.2 Hill models

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Many wind farms in China are located at mountain areas. Unlike an offshore wind farm, wind speed, direction and turbulence change with landscape when wind blows over a hill. Therefore wind speeds over a hill cannot be evaluated using (5). In order to consider the impact of terrain on wind speed, the shape model of a hill has to be developed. The mathematical models of a hill have been studied [16]. Fig. 1 is a general two dimensional representation of a hill.



Fig. 1 Model of a hill

Hill height y is a function of horizontal distance x. H is the maximum height of a hill, y = H/2 is the height at the two horizontal distances $x = \pm L_1$.

The two-dimensional (2D) models such as bell-shaped model, Gaussian model, cosine squared model and sinusoidal model have been proposed to represent different terrain features. The shape equations of those models are shown in Table 1.

The slope gradient s which represents the roughness of a hill is defined as [16]

$$s = H/2L_1 \tag{6}$$

Fig. 2 shows the 2D profiles of the four models for H = 60 m and s = 0.15. It can be seen from Fig. 2 that the models have different slopes. The bell-shaped model is smoother compared with the other three models. The model selection depends on hill shape of a wind farm. The bell-shaped model is used in the paper.

3.3 Wind speeds over a hill

Wind speed measured at height *h* over a flat land is defined as the natural wind speed v_0 . Based on dynamic air flow, natural wind speed v_0 will be amplified when it passes a hill. The wind speed v(x) at horizontal distance *x* and height *h* above hill surface is [17]

$$v(x) = (1 + 4as)v_0$$
(7)

 Table 1
 Equations of different hill models

Model	Equation
bell shaped	$y = H/[1 + (x/L_1)^2]$
Gaussian	$y = H \exp[-(x/L_1)^2 \ln 2]$
cosine squared	$y = H\cos^2(\pi x/4L_1)$
sinusoidal	$y = H[1 + \cos(\pi x/2L_1)]/2$



Fig. 2 Hill shapes of the four models

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Fig. 3 Wind speed curves over a hill

where location parameter a is defined as follows

$$a = \left(1 - \frac{|x|}{1.5L}\right) \exp\left(-\frac{2.5h}{L_1}\right), \quad s < 0.3$$
$$a = \left(1 - \frac{|x|}{1.5L}\right) \exp\left(-\frac{1.5h}{H}\right), \quad s > 0.3$$

where $L = L_1$ for the windward point and $L = L_2$ for the leeward point.

Fig. 3 shows wind speed curves over a hilly land at height *h* above hill surface based on the natural wind speed v_0 using (7) and (5), respectively. Wind speed is amplified by hill. The highest wind speed is at the peak of the hill when x = 0. Therefore the influence of terrain on the output power of a wind farm should be considered in adequacy study.

4 Modelling wake effect

When wind blows through a wind farm with multiple rows of WTGs, the wind speed on a downstream WTG is usually lower than that on an upstream WTG. This phenomenon is called wake effect. Wake effect can be represented using the Jensen model [14] and Lissanman model [11]. The Jensen model is usually used to represent the wake effect in a flat area and the Lissaman model is used for a complex terrain.

4.1 Jensen model

The Jensen model [14] can be explained using Fig. 4, where r is the radius of a WTG blade and r_w is the radius of the wake area caused by WTG1 at distance x.

In this model, wind speed v_x on WTG2 at distance x can be calculated based on wind speed v_0 on WTG1 at x = 0 using



Fig. 4 Wake effect representation of two WTGs in a flat land



Fig. 5 Lissaman model

the following equations [14]

$$v_x = v_0 \left[1 - (1 - \sqrt{1 - C_{\rm T}}) \left(\frac{r}{r + Kx} \right)^2 \right]$$
 (8)

$$d_{\rm w} = (1 - \sqrt{1 - C_{\rm T}}) \left(\frac{r}{r + Kx}\right)^2$$
 (9)

$$K = 0.01 \frac{1}{\ln(h/z)}$$
(10)

where d_w is the coefficient related to wind speed reduction in flat area; C_T is the thrust coefficient of a wind turbine; *K* is the declining coefficient of wake effect; *h* is the height of WTG and *z* is the surface roughness [11].

4.2 Lissaman model

For hill wind farms or wind farms with different hub height of WTGs, the Lissaman model [11] shown in Fig. 5 is used to describe the wake effect of a complex terrain, where WTG1 and WTG2 are installed in a hill.

The wind speed v'_x applied to WTG2 at distance x from WTG1 can be calculated by

$$v'_x = v'_0 [1 - d'_w] \tag{11}$$

where v'_0 is the wind speed at height *h* over hilly land at x = 0 and can be calculated using (7) and d'_w is the coefficient related to wind speed reduction.

Supposing that the energy loss caused by wake effect in Fig. 5 is the same with the loss in Fig. 4, d'_{w} can be obtained as [11]

$$d'_{\rm w} = d_{\rm w} \left[\frac{v_0}{v'_0} \right]^2 \tag{12}$$

5 Modelling power output of a wind farm

The power output of a wind farm can be determined based on the power outputs of WTGs with considering WTG reliability states and wind speed distribution in the farm when considering terrain and wake effect. Reliability state of a WTG at hour t can be seen from its operating history. Wake effect is considered only between two neighbouring WTGs. A failed WTG will have no wake effect on the downstream WTG owing to no movement of blade.

When the wind direction is given, the wind at the first row of WTGs is assumed to be natural wind speed v_0 . Wake effect will be considered for the downstream WTGs. The wind speed in the part of hilly terrain will be modified based on the amplification factor.

The procedure for calculating output power of a wind farm is as below:

Step 1: Input WTG parameters, WTG distribution and the hourly average natural wind speeds for the given simulation years.

Step 2: Create operating history of WTGs for the given simulation years.

Step 3: Calculate the wind speeds of WTGs for period *t* with considering terrain influence.

Step 4: Adjust the wind speeds of WTGs for period t with considering wake effect.

Step 5: Calculate the output power of each WTG for period t based on the wind speed and the total power output of the wind farm.

Step 6: Stop the procedure if all the simulation years are considered, otherwise go to step 3.

It should be noted that the first row of WTGs has to be changed properly if wind direction changes. Wind power output is calculated using (4), which means that all WTGs are controlled to operate in maximum power point tracking mode in order to harness maximum active power from wind.

6 Optimal distribution of WTGs

In order to minimise the influence of wake effect on WTG power output, WTGs in a wind farm should be properly allocated based on its terrain landscape and wind speed distribution.

Wind transmission through WTGs is like cone-shaped [12] as shown in Fig. 6. Fig. 6a shows that WTG2 is installed in the same line with WTG1 in the wind direction x. The blade of WTG2 is completely covered by the shadow area of the wake of WTG1. To minimise the wake effect, WTG2 should be deviated a distance d from x-axis as shown in Fig. 6b.



Fig. 6 Wake effect areas

a Two WTGs are in the same line as wind direction b Two WTGs are not in the same line as wind direction



Fig. 7 Incomplete shade area on WTG2 over a flat land



Fig. 8 Incomplete shade area on WTG2 over a hill

The radius r_w of WTG1 wake at distance x can be calculated based on the length r of the blade as [12]

$$r_{\rm w} = r + x \tan \alpha \tag{13}$$

where α is the cone vertex factor and $\tan \alpha$ is 0.04 for natural wind speed, otherwise, $\tan \alpha$ is 0.08 [11].

WTG2 can be completely, incompletely or not covered by WTG1 wake, which depends on the location of WTG2. The two incomplete cases for flat and hilly lands are shown in Figs. 7 and 8, respectively, where o_w is the centre of the shadow area of WTG1 wake, o_r is the centre of WTG2 blades and y is the vertical distance between the blade centres of the two WTGs.

 A_s in Fig. 7 can be calculated as [14]

$$A_{s} = \arccos\left(\frac{r_{w}^{2} + d^{2} - r^{2}}{2r_{w}d}\right)r_{w}^{2} + \arccos\left(\frac{r^{2} + d^{2} - r_{w}^{2}}{2rd}\right)r^{2} - \sin\left[\arccos\left(\frac{r_{w}^{2} + d^{2} - r^{2}}{2r_{w}d}\right)\right]r_{w}d$$
(14)

 A_s in Fig. 8 can be calculated as

$$A_{s} = \arccos\left(\frac{r_{w}^{2} + d^{2} + y^{2} - r^{2}}{2r_{w}\sqrt{d^{2} + y^{2}}}\right)r_{w}^{2}$$
$$+ \arccos\left(\frac{r^{2} + d^{2} + y^{2} - r_{w}^{2}}{2r\sqrt{d^{2} + y^{2}}}\right)r^{2}$$
$$- \sin\left[\arccos\left(\frac{r_{w}^{2} + d^{2} + y^{2} - r^{2}}{2r_{w}\sqrt{d^{2} + y^{2}}}\right)\right]r_{w}\sqrt{d^{2} + y^{2}} \quad (15)$$

When WTG_{*i*} is only affected by upstream WTG_{*i*-1}, the wind speed v_i of WTG_{*i*} can be calculated as [14]

$$v_i = \sqrt{v_0^2 + C(v_{i-1}^2 - v_0^2)}$$
(16)

where $C = A_s/A_r$ is the coefficient of wake effect, $A_r = \pi r_r^2$ and v_{i-1} can be calculated using (8) or (11).

The quadratic interpolation method [10] is used to calculate the optimal deviating distance d'. The objective is to provide

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the total maximum power output from a wind farm under d = d'. The objective function is

$$\max\left\{\sum_{i=1}^{n} p_i(d)\right\} \tag{17}$$

where $p_i(d)$ is the output power of WTG_{*i*}.

In order to solve (17), an interpolation function $f(d) = a_* + b_*d + c_*d^2$ is constructed to make $f(d) = \sum_{i=1}^{n} p_i(d)$ in different interpolation points d_1 , d_2 and d_3 . The maximum value of f(d) is

$$-\frac{b_*}{2a_*} = \frac{(d_2^2 - d_3^2)p(d_1) + (d_3^2 - d_1^2)p(d_2) + (d_1^2 - d_2^2)p(d_3)}{2[(d_2 - d_3)p(d_1) + (d_3 - d_1)p(d_2) + (d_1 - d_2)p(d_3)]}$$
(18)

It should be noted that the proposed method is based on single wind direction. In the system studies of this paper, two directions of west and north wind are separately considered. However, the wind direction may change with time in a practical wind farm. If there is wind direction data for a practical wind farm, the probability method can be used in the layout design, which means the probabilities of wind being in different directions have to be combined with the optimisation technique.

7 Reliability indices

Time sequential Monte-Carlo simulation technique is used to determine the expected wind farm output and reliability indices of power systems with wind farms. In this technique, the operating and repairing histories for WTGs and conventional generators (CGs) are generated using (1) and (2). Hourly time varying wind speed and load model is used in the analysis. The power output of a WTG for hour *t* is determined based on the states of WTGs and wind speed distribution over a wind farm.

7.1 Reliability indices

Several reliability indices have been proposed to represent adequacy of generating systems [15]. The following reliability indices are used in this paper to evaluate adequacy of power systems with wind farms.

Loss of load expectation (LOLE)

$$LOLE = \frac{1}{N} \sum_{t=1}^{N_t} LOLE_t \quad (hour/year)$$
(19)

$$\text{LOLE}_t = \begin{cases} 0, & P_{\text{G}t} \ge P_{\text{L}t} \\ 1 \text{ h}, & P_{\text{G}t} < P_{\text{L}t} \end{cases}$$
(20)

Loss of load probability (LOLP)

$$LOLP = \frac{LOLE}{N}$$
(21)

Expectation energy not supplied (EENS)

$$EENS = \frac{1}{N} \sum_{t=1}^{N_t} EENS_t \quad (MWh/year)$$
(22)

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$$\text{EENS}_{t} = \begin{cases} 0, & P_{\text{G}t} \ge P_{\text{L}t} \\ P_{\text{L}t} - P_{\text{G}t}, & P_{\text{G}t} < P_{\text{L}t} \end{cases}$$
(23)

A new index designated expected energy produced (EEP) is proposed to represent the annual energy output of a wind farm. The EEP can be calculated using the following equations

$$EEP = \frac{1}{N} \sum_{t=1}^{N_t} \sum_{i=1}^{N_w} P_{ti} \quad (MWh/year)$$
(24)

where *N* is the total number of the sampling years; N_t is the total number of simulation hours; LOLE_t is the loss of load expectation for hour t; EENS_t is the value of EENS for hour t; P_{Gt} is the total generating capacity in hour t; P_{Lt} is the total load in hour t and N_w is the number of available WTGs at time t.

7.2 Simulation procedure

The simulation procedure to assess the reliability of a generation system is as follow.

Step 1: Input data of CGs and WTGs.

Step 2: Input hourly natural wind speed data and load data. *Step 3*: Generate operating histories for CGs and WTGs using

(1) and (2) based on generated uniform number.

Step 4: Calculate the output of each WTG for hour *t* based on the WTG state with considering terrain and wake effect.

Step 5: Calculate the total output of WTGs for hour *t* and the total output of generators.

Step 6: Calculate the LOLE_{*t*} and EENS_{*t*} for hour *t* and update the total LOLE, EENS and EEP.

Step 7: Let t = t + 1. If $t < N_t$, go to step 4, otherwise go to next step.

Step 8: Output the reliability indices and energy output of wind farm.

Sinning reserve and its ramp rate have to be considered in power system real time reliability evaluation if wind penetration level is high and wind speed changes fast. As adequacy study of wind farms focuses on long-term planning problem, it is assumed that there is instantaneous support from the synchronous generators when wind speed is suddenly reduced.

8 System studies

8.1 Wind power output with terrain and wake effect

A wind farm which consists of 33×1.5 MW identical WTGs is shown in Fig. 9. The WTGs are originally distributed in 3 rows and 11 columns. The distance between two rows is 300 m and the distance between two columns is 200 m. The WTGs at columns 2, 3, 4, 9, 10 and 11 are located in the two hills with the height of 60 m. Assume that length of a WTG blade is 27 m. All WTGs have the same height of 60 m. The cut-in, cut-out and rated wind speeds are 3, 25 and 12 m/s, respectively. The wind speeds are the hourly average wind speeds measured at the flat area of a wind farm in the Shanxi province of China. Two major wind directions in this wind site are west wind (v1) taking up 60% of the year and north wind (v2) taking up 40% of the year. WTGs are sequentially numbered WTG1, WTG2,



Fig. 9 Original wind farm layout



Fig. 10 Average power outputs of WTGs for the four cases

WTG3, ..., WTG33 starting from row 1 and followed by rows 2 and 3.

Fig. 10 shows the average power output of each WTG for the following four cases:

- Case 1: Without considering wake and terrain effect.
- Case 2: With considering wake effect only.
- Case 3: With considering terrain influence only.

Case 4: With considering both wake and terrain influence.

The outputs of WTG5, WTG6, WTG7 and WTG8 for case 1 are the same without considering the wake effect because of

Table 2 Total energy of wind farm for four cases

Cases	Case 1	Case 2	Case 3	Case 4
EEP, MWh/year	158 710	85011	175 820	117 690

the same height. The power outputs of WTG5, WTG6, WTG7 and WTG8 for case 2 are sequentially reduced because of the wake effect. The lowest power output is from WTG11 for case 2. The power output of a WTG over the hill for case 3 is larger than that of the same WTG for case 1 owing to wind speed amplification effect of a hilly land. The power output of WTGs on the top of the hills such as WTG3 and WTG10 are the highest for case 3. The outputs of WTG23 are the same for the four cases because it is always the first unit, which faces the wind directions.

Table 2 shows the yearly total energy output of the wind farm for the four cases. The output is the highest when considering only terrain effect. The output is the lowest when considering only the wake effect. It can be concluded that the terrain and wake effect will significantly influence power output of a wind farm and has to be considered in adequacy studies.

8.2 Optimal WTG distribution

The optimal deviation distance d for a downstream WTG is determined by the quadratic interpolation method. Fig. 11 is



Fig. 11 Optimal wind farm layout



Fig. 12 Average power curves of WTGs for the three cases

Table 3 Expected energy produced of the wind farm

Cases	Case 1	Case 2	Case 3
EEP, MWh/year	117 690	150 010	155 430

the optimal layout of the wind farm with considering the terrain effect in west wind direction. The WTGs at columns 2, 4, 6, 8 and 10 are deviated by 60.3 m to south from the original location in Fig. 9. However, d is 58.1 m without considering the terrain effect using the technique in [14].

Fig. 12 shows the average power outputs of WTGs for the three cases. Table 3 shows the total expected energy produced of the wind farm for the three cases.

Case 1: The original distribution d = 0 m under west wind. *Case 2*: The optimal distribution d = 60.3 m under both west and north wind.

Case 3: The optimal distribution d = 60.3 m under west wind.

Fig. 12 and Table 3 shows that average power output of a WTG and the EEP of the wind farm in the optimal layout d = 60.3 m are increased significantly compared with that in the original layout. Therefore the optimal arrangement can reduce the wake effect and increase the output power of a wind farm. The EEP for case 2 is less than that for case 3 because the optimal deviation is calculated based on west wind. When considering north wind direction, the WTG in the second row of the wind farm should be deviated by 44.6 m from wind direction.

It is noted that wind in most wind farms in Shanxi are from northwest with range of 90°. Currently there is no detail wind direction data from the wind farms. If there is wind direction data, the probability method can be used to determine probability distribution of wind directions. In this case, both wind directions and their probabilities have to be considered in wind farm layout design, which means that the optimal deviation distance d for each wind direction

Table 4 Reliability indices

Indices	LOLE, hour/year	LOLP	EENS, MWh/year	EEP, MWh/year
case 1	7.72	0.000881	752.7	159 620
case 2	8.51	0.000974	840.2	58 507
case 3	8.19	0.000938	800.7	102 270
case 4	7.86	0.000898	760.6	148 600
case 5	7.79	0.000890	753.6	158 500

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should be weighted by its probability and the expected deviation distance should be calculated with considering all wind directions. More research has to be done for the optimal distribution of WTGs when considering wind directions.

8.3 Reliability analysis

The wind farm in Section 1 is added to the IEEE RTS [18]. The time varying hourly load model in [18] is used in reliability analysis. The adequacy studies of the modified IEEE RTS for the following cases have been studied using the proposed technique.

Case 1: The original WTG distribution without considering wake and terrain influence.

Case 2: The original WTG distribution with considering only wake effect.

Case 3: The original WTG distribution with considering both wake and terrain influence.

Case 4: The optimal WTG distribution with considering only wake effect.

Case 5: the optimal WTG distribution with considering wake and terrain influence.

Table 4 shows reliability indices for the five cases using the Monte-Carlo approach.

The results show that the LOLE and EENS for case 1 are the lowest because the EEP is the highest among all the cases when the wake effect is ignored. The LOLE and EENS for case 2 are the highest because the EEP from the wins farm is the lowest when the wake effect is considered. The LOLE is reduced from 8.51 of case 2 to 8.19 of case 3 because the power output from the wind farm is increased by wind speed amplification of the terrain effect. The EEPs from case 4 and case 2 shows that optimal distribution significantly reduces the wake effect and also increases the system reliability. The results for case 4 and case 5 show that the terrain effect can further increase the system output power.

It is noted that all WTGs are assumed to be controlled to operate in the maximum power point tracking mode to harness maximum active power from wind. Owing to uncertainty and intermittent characteristics of wind speed in most wind sites, wind power capacity credit (can be dispatched) of a WTG is very limited compared with conventional generator as indicated in many publications. Therefore a power system with wind farms should has an adequate spinning reserve and fast response speed from CGs to incorporate the random variation of wind power for system normal operation.

9 Conclusion

This paper proposes a time sequential Monte Carlo simulation technique to evaluate adequacy of a wind farm with considering the combined effect of terrain, wake and WTG reliability. A quadratic interpolation method is used to study the optimal locations of WTGs in order to maximise power output of a wind farm. A test wind farm and the IEEE RTS are analysed to illustrate the models and proposed technique. Reliability with and without the optimal WTG distribution is investigated using the proposed technique. The simulation results show that a hilly land can amplify wind speed and increase power output of a wind farm. The wake effect reduces the wind speed at a downstream WTG

and also the output power of a wind farm. The optimal layout can reduce the wake effect, increase the output power of WTGs and improve adequacy of wind farms.

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