Fatigue & Fracture of Engineering Materials & Structures

doi: 10.1111/j.1460-2695.2011.01572.x

Multiaxial fatigue life prediction method based on path-dependent cycle counting under tension/torsion random loading

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Received in final form: 4 February 2011

ABSTRACT A path-dependent cycle counting method is proposed by applying the distance formula between two points on the tension-shear equivalent strain plane for the identified half-cycles first. The Shang–Wang multiaxial fatigue damage model for an identified half-cycle and Miner's linear accumulation damage rule are used to calculate cumulative fatigue damage. Therefore, a multiaxial fatigue life prediction procedure is presented to predict conveniently fatigue life under a given tension and torsion random loading time history. The proposed method is evaluated by experimental data from tests on cylindrical thin-walled tubes specimens of En15R steel subjected to combined tension/torsion random loading, and the prediction results of the proposed method are compared with those of the Wang–Brown method. The results showed that both methods provided satisfactory prediction.

Keywords cycle counting; fatigue damage; life prediction; multiaxial fatigue.

INTRODUCTION

Fatigue failure of mechanical components and structures under multiaxial loading conditions is a common concern, because most engineering components, such as pressure vessels, automobile suspension and transmission parts, gas turbines, crankshafts, and so on, are subjected to multiaxial random loading in service. Multiaxial fatigue life prediction is an extremely intractable issue due to the inherent complexities of fatigue crack initiation and growth under multiaxial random loading.

For fatigue life prediction under general multiaxial random loading, three main steps are included: multiaxial cycle counting, damage evaluation for a cycle and damage accumulation. The accuracy of fatigue life prediction depends on all the above steps. Firstly, an efficient cycle counting method is needed to make a conversion of random loading history into some different constant amplitudes. Secondly, a good multiaxial fatigue damage parameter is needed, which can be based on stress,[1] strain,[2] energy,[3–6] and the critical plane.[7–9] Finally, a multiaxial fatigue damage accumulation model is needed to predict fatigue life.

To predict accurately fatigue life under general multiaxial random loading, some researcher have proposed some different approaches, such as, energy approach,[3] Bannantine–Soice's method,[10] Wang–Brown's method,[11] Jiang's method[12] and EVICD method.[13,14] Although there have been some research efforts towards the fatigue life prediction method under multiaxial random loading, unfortunately, there is no perfect approach.

In this paper, a path-dependent cycle counting method and a fatigue life prediction procedure are presented under tension/torsion random loading. The path-dependent cycle counting method can be described as a procedure seeking the maximum distance from a starting point along loading path on the $\varepsilon - \gamma/\sqrt{3}$ plane. For the fatigue life prediction procedure, the path-dependent cycle counting method is used to count half-cycles first. The Shang–Wang multiaxial fatigue damage model for an identified half-cycle and Miner's linear accumulation damage rule are used to calculate cumulative fatigue damage.

BACKGROUND THEORY

A brief review for the path-dependent maximum range (PDMR) method and the Wang–Brown method is introduced. Some comments are performed for the two methods.

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PDMR method

Dong *et al.*[15] presented a PDMR cycle counting method and defined the path-dependent effective stress range to evaluate the fatigue life. First, map $\sigma_s(t)$ and $\tau_s(t)$ time histories to the $\sigma_s - \sqrt{\beta}\tau_s$ plane. Search the maximum possible distance between any two data points within the entire loading history. Next, the load path traversed in the process of identifying the maximum distance consists of some load path segments, and the two time points with the maximum distance are defined as the half-cycle. Repeat the previous steps for the remaining loading path segments until all paths have been counted and counted only once.

Wang–Brown's method

Wang and Brown[11] proposed a multiaxial fatigue life prediction method, which uses a multiaxial reversal counting method based on the equivalent relative strain, Wang–Brown's damage parameter and Miner's linear accumulation damage rule. The principle of this method is that the maximum equivalent strain over entire history is found and defined as the first major turning point A, which is used as a starting point to rearrange the equivalent strain history. Then, equivalent relative strain is calculated corresponding to the first maximum reference point. A reversal is identified from zero to the point of maximum increase of equivalent relative strain, and then a new major turning is found and used to determine a new equivalent relative strain for further cycle counting. The above steps are repeated for the residual multiaxial loading history.

A strain-based multiaxial fatigue damage model proposed by Wang and Brown[16] is used here:

$$\frac{(\Delta\gamma_{\max}/2) + S\left(\varepsilon_n^*\right)}{1 + v' + (1 - v')S} = \frac{\sigma_f' - 2\sigma_{n,\mathrm{mean}}}{E}\left(2N_f\right)^b + \varepsilon_f'\left(2N_f\right)^c \tag{1}$$

where $\Delta\gamma_{\max}$, ε_n^* are the shear strain range and the normal strain excursion between two turning points of shear strain on the maximum shear plane, respectively. S is a material parameter that represents the influence of the normal strain on fatigue crack growth and it can be determined by correlating uniaxial and pure torsion fatigue data. v' is the effective Poisson's ratio, and σ_{mean} is the mean normal stress on the maximum shear strain plane. The other parameters E, σ_f', ε_f', b and c are the uniaxial strain lifetime equation constants. More details can be found in Refs.[16–18].

Both the PDMR method and the Wang–Brown method can identify half-cycles (or reversals) for a known multiaxial loading history. However, both methods also have some shortcomings in the cycle counting procedure.

In the PDMR method, it will spend a lot of computing time because the distance between any two data points needs to be calculated repeatedly within the entire loading history. It may need an advanced algorithm to solve the problem. In addition, parameter β is a material constant in the $\sigma_s - \sqrt{\beta}\tau_s$ plane, and it needs to be determined in advance for different materials. For stress loading history, Dong *et al.*[15] describe a complete cycle counting procedure, and the effectiveness of the PDMR method is also demonstrated in correlating with a large amount of multiaxial fatigue data. However, for strain loading history, Dong *et al.*[15] describe only qualitatively the entire cycle counting process on the $\varepsilon - \gamma$ plane.

In the Wang–Brown method, the first major turning point relies on the identification of peak equivalent strain (always positive). Dong *et al.*[15] took a loading path as an example to point out the disadvantage of the Wang–Brown method. Wang and Brown[17] also proposed two approaches to solve the problem. One solution would be to identify the attained peak value for the equivalent relative strain range between the two data points. However, they also noted that this method required a double-maximization procedure and may involve more computation. An alternative method is to identify the first turning point from a relative strain graph, rather than the absolute equivalent strain, and the mean level of strain for the whole loading block is used as a reference value. But the expression of the mean strain is not formulated in Ref. [17]. Wang–Brown's method fails to note that the multiaxial cycle count procedure does not produce a unique set of reversals for some loading cases. For example, some loading cases may include a number of peak points with the same equivalent strain. In this case, a number of the first major turning points will be identified. In addition, the iterative process of effective Poisson's ratio is quite complex, which will require accurate stress and strain.

THE PROPOSED FATIGUE LIFE PREDICTION METHOD

The proposed cycle counting procedure

In order to solve effectively the disadvantages of Wang–Brown's method and PDMR method, a simple path-dependent cycle counting method is proposed by applying the distance formula between two points on the $\varepsilon - \gamma/\sqrt{3}$ plane. The method can be summarized as follows.

1. Map $\varepsilon(t)$ and $\gamma(t)$ time histories on the $\varepsilon - \gamma/\sqrt{3}$ plane. Here the distance between two points on the $\varepsilon - \gamma/\sqrt{3}$

plane is expressed with following formula:

$$\varepsilon_{\mathrm{dis}}(t) = \sqrt{(\varepsilon(t) - \varepsilon(t_R))^2 + \frac{1}{3}(\gamma(t) - \gamma(t_R))^2} \quad (2)$$

where t_R is the reference time point.

2 Search the maximum distance between every point and any point within the whole loading history by Eq. (3):

$$\varepsilon_{\mathrm{dis}}(t_c) = \max_{\substack{0 \le t_c \le t_n \\ 0 \le t \le t_n}} \sqrt{(\varepsilon(t) - \varepsilon(t_c))^2 + \frac{1}{3}(\gamma(t) - \gamma(t_c))^2} \quad (3)$$

where the time period $0 < t \le t_n$ represents the loading block, and t_c represents every time point. $\varepsilon_{\mathrm{dis}}(t_c)$ is the maximum distance time history between every point and any point.

3 Find the maximum value point A of $\varepsilon_{\mathrm{dis}}(t_c)$ within the whole loading history by Eq. (4), and denote the corresponding time as t_A :

$$\varepsilon_{\mathrm{dis}}(t = t_A) = \max_{0 < t_c \le t_n} \varepsilon_{\mathrm{dis}}(t_c). \quad (4)$$

4 Rearrange the data so that a new data block starts from the point A. Move the data before the point A to the end of the data file to form a new time history.

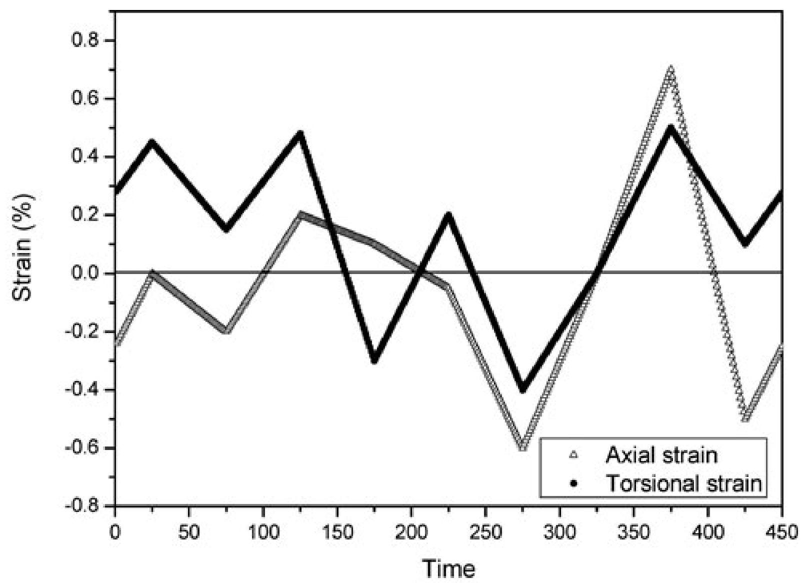
5 Find the maximum value of distance $\varepsilon_{\mathrm{dis}}(t)$ by Eq. (2) with respect to the first point in the block. A half-cycle is counted by combining a sequence of data points where $\varepsilon_{\mathrm{dis}}(t)$ is increasing up to the maximum value to complete one half-cycle. A fragment of the strain history occurs when ever $\varepsilon_{\mathrm{dis}}(t)$ experiences a decrease.

6 With the first point of each fragmented data block being the initial point defining $\left(\varepsilon(t_R), \frac{\gamma(t_R)}{\sqrt{3}}\right)$ in the distance formula (in Eq. (2)), repeat step (5) for each block. Further fragmented blocks skipped over during each pass of cycle counting are to be treated in a similar way, until all half-cycles are counted.

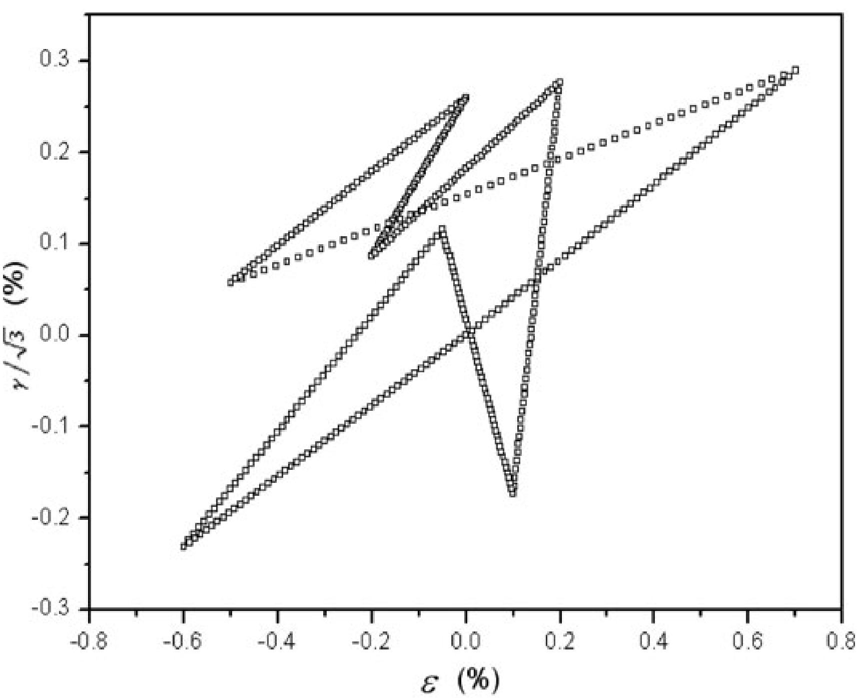
Let us take an axial-torsion strain loading history shown in Fig. 1 as an example to explain the proposed method. The counting procedure of this complex multiaxial loading path is depicted in detail as follows.

a. Search the maximum distance between every point and any point within the whole loading history by Eq. (3), and the maximum distance time history is shown in Fig. 2.
b. Find the maximum value (point A) of the maximum distance time history within the whole loading history by Eq. (4), and denote the corresponding time as t_A, shown in Fig. 2.
c. Rearrange the data so that a new data block starts from the point A. Move the data before the point A to the end of the data file to form a new time history, shown in Fig. 3. The whole cycle counting procedure will start from the point A.
d. The proposed cycle counting procedure is described qualitatively in Fig. 4a for the loading path on the $\varepsilon - \gamma/\sqrt{3}$ plane. Point A is a starting point, and search the maximum distance along the direction of the arrow on whole $\varepsilon - \gamma/\sqrt{3}$ plane. Point B will be identified as point of the maximum distance from point A. So one half-cycle (A–B) is identified. Then the data will be deleted between points A and B at the same time. Point B is defined as a staring point, and search maximum distance point between point B and any point within the remaining loading history. One half-cycle (B–C–C′–D) can be obviously identified from the Fig. 4a. The procedure will be repeatedly implemented, until all half-cycles are counted. For the complex loading history, the counted eight half-cycles include A–B, B–C–C′–D (A), C–E–E′–F, E–G, G–E′, F–H–H′–C′, H–I, I–H′.
e. The distance formula (Eq. (2)) between two points on the $\varepsilon - \gamma/\sqrt{3}$ plane is used to implement quantitatively the cycle counting. The whole procedure is shown in Fig. 4b. The distance is calculated with respect to the point A, which is shown by a curve starting from the point A. It can be seen that the peak point of this curve is point B. Therefore one half-cycle (A–B) is identified. Subsequently the distances with respect to points B are plotted (the curve starts from the point B). The peak point of this curve is point D. Because the distance decreased in point C, the curve is fragmented at point C. So one half-cycle (B–C–C′–D) is identified. Now plot the distance with respect to point C, resulting in curve starting from the point C. One half-cycle (C–E–E′–F) is identified, which is fragmented at points E and F, respectively. Now plot the curves of the distance with respect to these two points. For the data between E and E′, two half-cycles (E–G and G–E′) are identified. The curve starting from point F is fragmented at point H, one half-cycle (F–H–H′–C′) is also identified. Finally, for the data between H and H′, two half-cycles (H–I and I–H′) are identified. Altogether eight half-cycles are counted for this loading history.

The proposed cycle counting method is described in detail above under axial and torsion strain loading. For uniaxial loading or pure torsion loading, the proposed method is equivalent to the simplified rain-flow counting for a repeating history in uniaxial random loading in ASTM.[19] The difference between the proposed cycle counting procedure and the Wang–Brown cycle counting process appears to be the definition of equivalent strain and starting point in the history. Comparison of the two methods will be shown in the discussion section. The proposed cycle counting procedure is quite simple, and it is convenient to develop a computer code. In the study, a FORTRAN computer code with the cycle counting procedure was compiled to carry out the proposed path-dependent cycle counting method.



(a)



(b)

Fig. 1 An axial-torsional strain random loading; (a) An axial-torsional strain random loading history; (b) An axial-torsional strain loading path on the $\varepsilon - \gamma/\sqrt{3}$ plane.

Fatigue life prediction procedure

Once half-cycles are identified through the proposed method above, fatigue damage can be calculated. In order to estimate conveniently fatigue damage, an axial strain-based multiaxial fatigue damage parameter, which does not include the weight constants, proposed by Shang and Wang[9] is used here.

$$\sqrt{\varepsilon_n^{*2} + \frac{1}{3}\left(\frac{\Delta\gamma_{\max}}{2}\right)^2} = \frac{\sigma_f' - 2\sigma_{n,\mathrm{mean}}}{E}\left(2N_f\right)^b + \varepsilon_f'\left(2N_f\right)^c \tag{5}$$

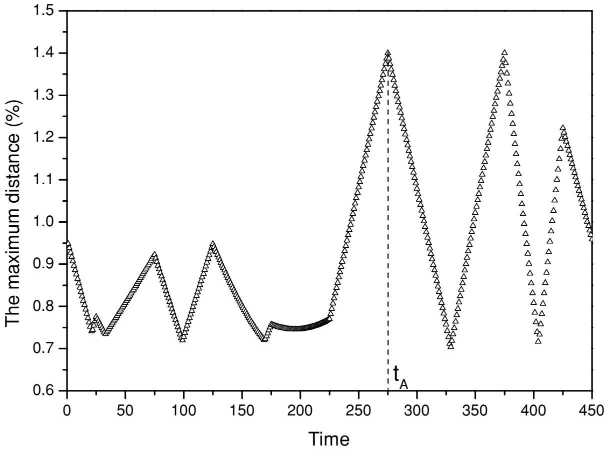


Fig. 2 The maximum distance between every point and any point in strain loading history.

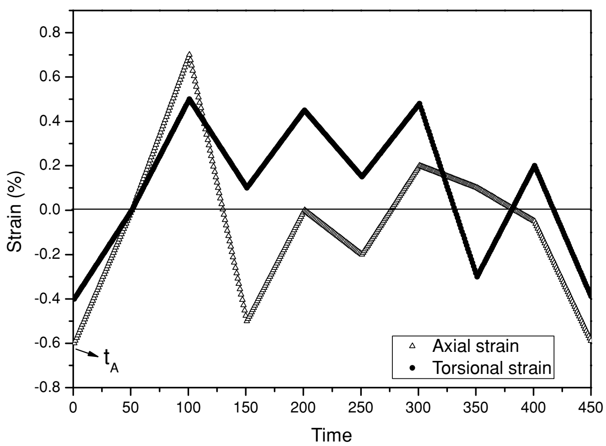


Fig. 3 Rearranged strain loading history with the starting point A (identified in Fig. 2).

where $\Delta\gamma_{\max}$, ε_n^* are the shear strain range and the normal strain excursion between two turning points of shear strain on the maximum shear plane, respectively, σ_{mean} is the mean normal stress on the maximum shear strain plane. The other parameters E, σ_f', ε_f', b and c are the constants of uniaxial strain lifetime equation.

Once the fatigue damage is calculated for each half-cycle identified, total fatigue damage for the known loading is accumulated with the Miner linear rule:

$$D = \sum_{i=1}^{n} \frac{1}{2N_{\text{fi}}} \tag{6}$$

where n is the number of half-cycles and N_{fi} associated fatigue life with amplitude being equal to the ith half-cycle. The total fatigue damage for the loading block is given as D.

Determination of fatigue damage parameter on the critical plane in the counted half-cycle

In this study, only a plane stress state will be considered, because fatigue cracks often occur on the surface of a component. The $\Delta\gamma_{\max}$ plane can be determined by checking the shear strain history on all planes inclined from the axis of the specimen. For an identified half-cycle loading history, the actual search for the maximum shear plane, the normal strain excursion, the mean normal stress on the maximum shear plane and computation of the damage are executed by a computer code developed in this study. In the algorithm of the program, the angle measured counter-clockwise from the specimen axis to the normal vector on an incline plane is denoted byθ. For each increment of θ by 1° the stress and strain profiles, the fatigue parameter and the damage are evaluated. The determination procedure of fatigue damage parameter $\Delta\gamma_{\max}/2$, ε_n^* and $\sigma_{n,\text{mean}}$ on the critical plane in the counted half-cycles are depicted in detail as follows.

For the tubular specimen under tension/torsion loading, the strains on the plane that makes an angle θ with the X axis defined the direction of tensile loading, are expressed as

$$\varepsilon_\theta = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2\theta) + \frac{\gamma_{xy}}{2}\sin(2\theta) \tag{7a}$$

$$\frac{\gamma_\theta}{2} = \frac{\varepsilon_x - \varepsilon_y}{2}\sin(2\theta) - \frac{\gamma_{xy}}{2}\cos(2\theta) \tag{7b}$$

where $\varepsilon_y = -v\varepsilon_x$.

Equation (7) can be rewritten as

$$\varepsilon_\theta = \frac{1-v}{2}\varepsilon_x + \frac{1+v}{2}\varepsilon_x\cos(2\theta) + \frac{\gamma_{xy}}{2}\sin(2\theta) \tag{8a}$$

$$\frac{\gamma_\theta}{2} = \frac{(1+v)\,\varepsilon_x}{2}\sin(2\theta) - \frac{\gamma_{xy}}{2}\cos(2\theta). \tag{8b}$$

Here, the critical plane is defined as the plane of maximum range of shear strain with the maximum normal strain range. If the identified half-cycle by the proposed multiaxial cycle counting method is used to calculate multiaxial fatigue damage by the critical plane approach, the critical plane in an half-cycle needs to be determined during a multiaxial strain loading. The procedure for finding the orientation of the critical plane and relevant stress and strain parameters on it is as follows:

1. Read the data of a multiaxial stress–strain history in a half-cycle counted strain cycle time period.
2. Calculate the shear strain range $\Delta\gamma$ and normal strain range $\Delta\varepsilon$ in a determined interval of angle from 0° to 180°. In this paper, it was found that, when the interval

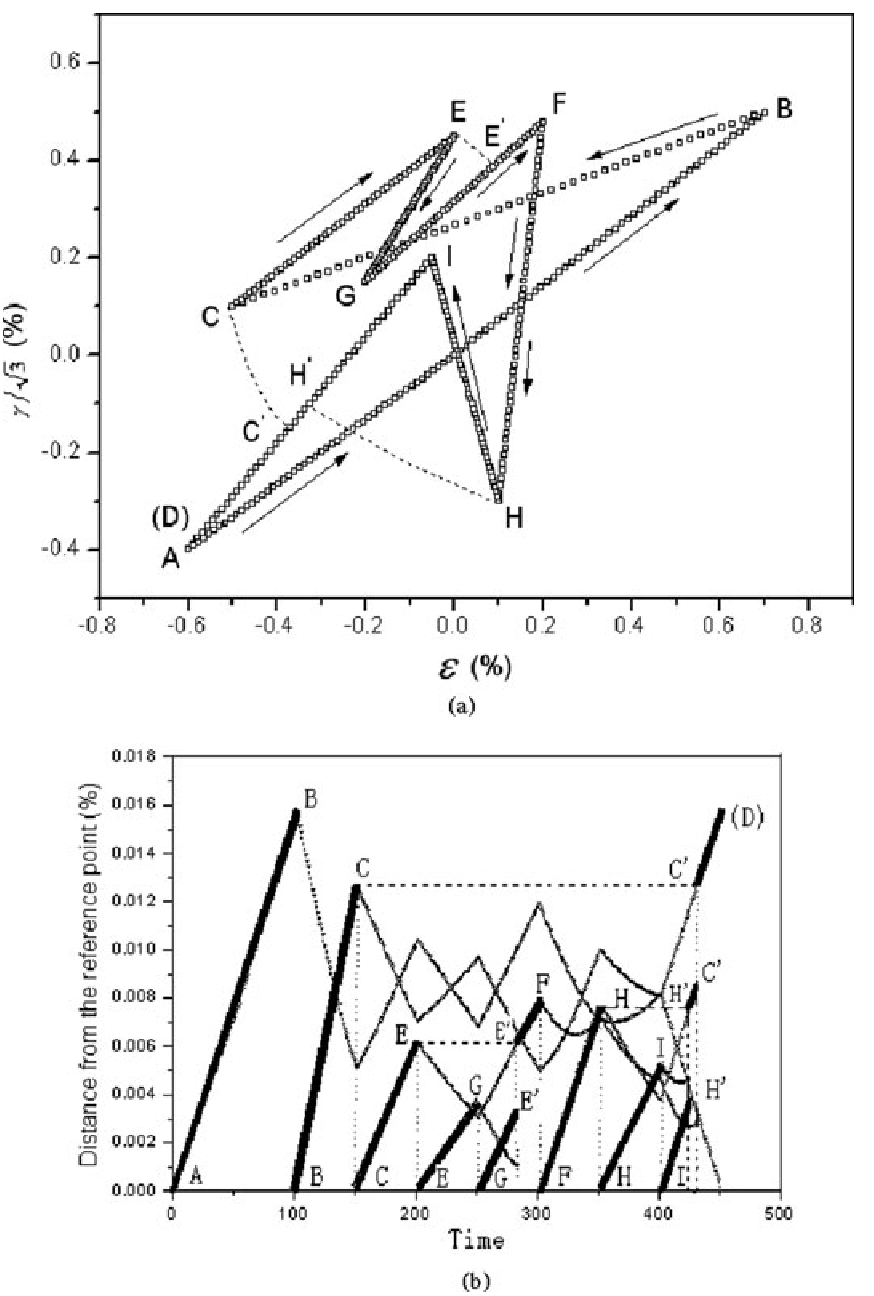


Fig. 4 The proposed cycle counting quantitatively procedure for the loading path.

angle was taken 1°, a more accurate result can be obtained for calculating the multiaxial fatigue damage.

3 Search the angle of the maximum shear plane with the maximum normal strain range as the orientation of the critical plane θ_c .

4 Denote the time instant of a pair of $\gamma_{\max}$ turning points on the critical plane as t_1 and t_2 .

5 Seek the maximum excursion of the normal strain over two turning points of shear strain on the critical plane $[t_1, t_2]$:

$$\varepsilon_n^* = \max_{t1 \le t < t2} \varepsilon_{\theta_c}(t) - \min_{t1 \le t < t2} \varepsilon_{\theta_c}(t)\,. \tag{9}$$

6 Determine the maximum and minimum normal stresses by the following relationship:

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta_c) + \frac{\tau_{xy}}{2}\sin(2\theta_c) \tag{10}$$

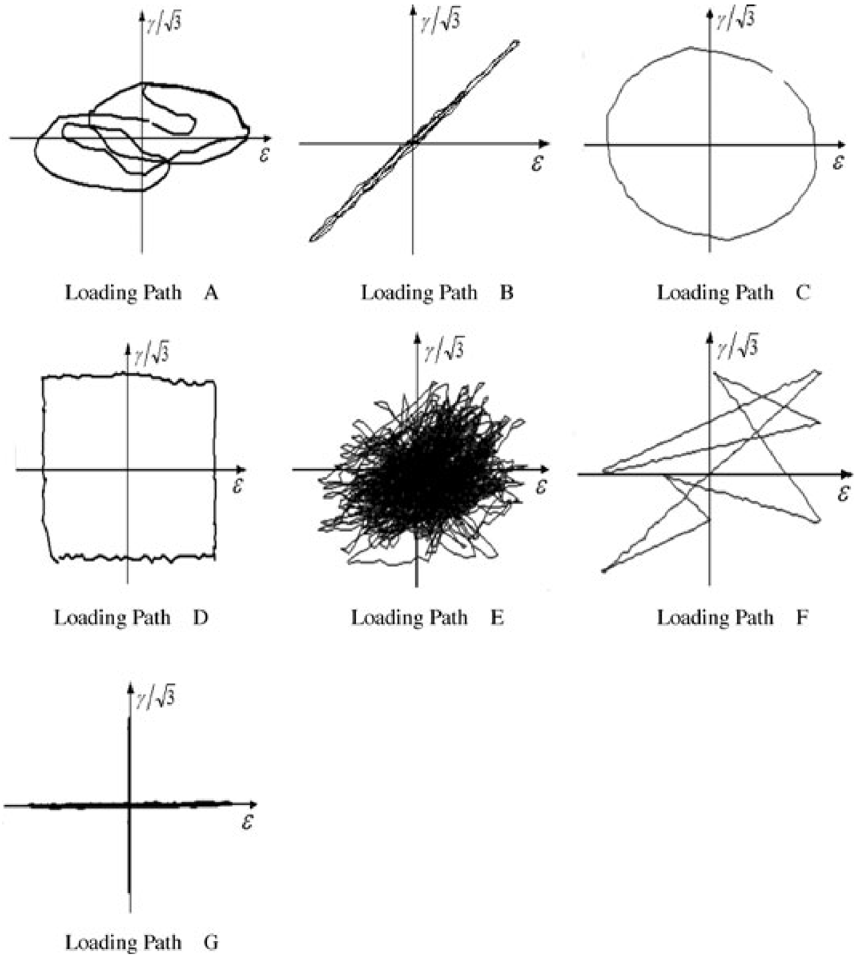


Fig. 5 Variable amplitude loading paths of En15R steel.[18]

where σ_n is the normal stress on the critical plane, θ_c is the angle of the critical plane orientation, σ_x, σ_y, τ_{xy} are the multiaxial stress components that should be obtained from a multiaxial incremental plasticity computational model in an half-cycle period, in which their values take the experimental data in this paper.

7 Search the values of the maximum and minimum normal stresses on the critical plane for all time points during the half-cycle identified, so the mean normal stress can be obtained on the critical plane.

COMPARISON BETWEEN EXPERIMENTAL RESULTS AND PREDICTIONS

The experimental data were generated at the University of Sheffield, and was used in Refs. [11,18]. The specimens are thin-walled tubes made of Enl5R steel alloy. A total of 41 specimens were tested under various loading conditions. In this study, data from all single-block loading tests are used to verify the proposed method, including 20 specimens and a loading history measured from an automotive vehicle. The variable amplitude loading paths are shown in Fig. 5. The details of materials, specimen geometry and fatigue tests performed in this investigation were reported in Ref. [18]. The fatigue properties of En15R steel are listed in Table 1.

The comparison between predicted and experimental lives is shown in Fig. 6. The Wang–Brown method, including Wang–Brown's cycle count method and fatigue damage model (Eq. (1)), is also related with the same experimental data. The comparison between predicted and experimental lives is also shown in Fig. 6. It can be seen that most predictions are within a factor of 2. The results showed that the Wang–Brown method and the proposed

Table 1 Fatigue properties of En15R steel[18]

E(GPa)	v	σ_f' (MPa)	ε_f'	b	c
205	0.28	1114	0.259	−0.097	−0.515

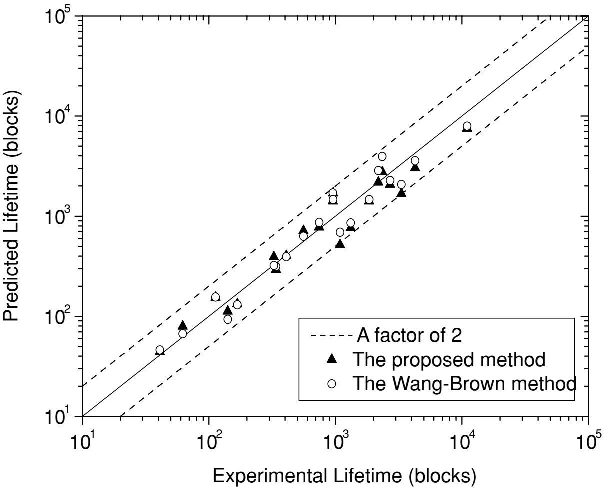


Fig. 6 Comparison between experimental and predicted lives for En15R steel.

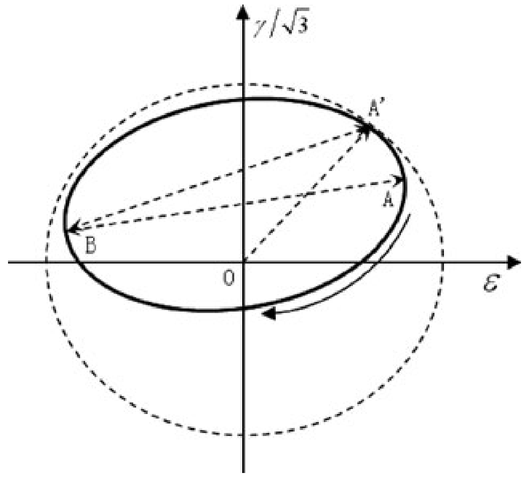


Fig. 7 Comparison of two methods to determine the first major turning point.

multiaxial fatigue life prediction method provided satisfactory predictions.

DISCUSSION

In the Wang–Brown method, the equivalent strain is expressed as

$$\varepsilon_{eq}(t) = \frac{1}{\sqrt{2}(1+v')}\sqrt{(\varepsilon_x(t)-\varepsilon_y(t))^2 + (\varepsilon_y(t)-\varepsilon_z(t))^2 + (\varepsilon_z(t)-\varepsilon_x(t))^2 + \frac{3}{2}(\gamma_{xy}^2(t)+\gamma_{yz}^2(t)+\gamma_{zx}^2(t))}.$$

The effective Poisson ratio can be evaluated using deformation theory, $v' = 0.5 - (0.5 - v)\Delta\sigma_{eq}/E\left(\Delta\varepsilon_{eq}\right)$. It is noted that the iterative process of effective Poisson's ratio is quite complex procedure. In addition, the first major turning point is a crucial point, which determines the number of half-cycles counted for whole given loading history. In the Wang–Brown method, the maximum equivalent strain over entire history is only defined as the first major turning point. If there are many equal maximum equivalent strain over entire history, how to choose the first major turning point, a clear solution has not been given.

In this paper, a path-dependent cycle counting method is proposed in the multiaxial fatigue life prediction method. The entire cycle counting procedure is quit simple, which is similar to the Wang–Brown method, because the distance formula on the $\varepsilon - \gamma/\sqrt{3}$ plane equivalent to von Mises relative strain when the effective Poisson's ratio takes 0.5. It avoids the iterative process of effective Poisson's ratio in the Wang and Brown method. For the determination of the first major turning point, the method is adopted, which is similar to the first solution of Wang and Brown's suggestion. The first initial point is identified through double-maximum distance procedure to avoid the major turning point relying on peak equivalent strain (always positive) in the Wang–Brown method. Although the double-maximum procedure involves more computation, it seems to be acceptable. This is mainly because the distance formula is simple, and that this process is only used once. Once the first major turning point is identified, a unique set of reversals will be counted. In present study, the first major turning point is defined as the point, which obtains the maximum value point A of the maximum distance time history at the first time.

To compare the two methods which determine the first major turning point, let us take the ellipse loading path on the $\varepsilon - \gamma/\sqrt{3}$ plane shown in Fig. 7 as an example. If the peak distance is defined as the first major turning point (similar to the Wang–Brown method), point A′ will be used as the first major turning point. The path will be counted two half-cycles (A′–B, and B–A′). If the double-maximum procedure proposed is used to determine the first major turning point, point A will be used as the first major turning point. The path will be counted two half-cycles (A–B, and B–A). Obviously, the latter approach is better reasonable under the ellipse loading path.

The entire cycle counting process on the $\varepsilon - \gamma/\sqrt{3}$ plane is also similar to the PDMR method, because both methods borrow the distance formula on different planes. However, the distance between any two data points needs to be calculated repeatedly within the entire loading history in the PDMR method. It is too time consuming. To solve the problem, an advanced algorithm was developed by Zhigang Wei *et al*.[20] In present study, the proposed method does not require special algorithms, and the implementation process is simple. However, it is only suitable for tension and torsion strain random loading condition, because the cycle counting procedure depend on loading path on the $\varepsilon - \gamma/\sqrt{3}$ plane. It has been known that fatigue cracks often occur on the surface of a component, where a plane stress state will be considered. The proposed method can be applied to this condition such as plane stress state.

For the fatigue damage calculated in the identified half-cycles, the Wang–Brown damage parameter (Eq. (1)) is used as the equivalent strain in the Wang–Brown variable amplitude multiaxial fatigue life prediction method. In the PDMR method, Dong *et al*.[15] proposed that the path length was defined as the effective stress or strain range in the same stress or strain space as that of cycle counting procedure. In this paper, Shang–Wang damage parameter (Eq. (5)) is applied to evaluate the fatigue damage in the half-cycles identified with the cycle counting method proposed. Comparing the three damage parameters, the Wang–Brown parameter includes a weight constant S that tend to increase as the fatigue life increases,[21] and the constant S is determined by fitting the uniaxial fatigue data and the torsion fatigue data. In the PDMR method, the path length is computed by using the integral method. For strain loading history, Kitade *et al*.[22] defined the length of strain path as a damage parameter on the $\varepsilon - \gamma/\sqrt{3}$ plane,

$$\Delta\varepsilon_{\text{path}} = \frac{1}{2}\int_{\text{cycle}} d\bar{\varepsilon} \quad \bar{\varepsilon} = \left(\varepsilon^2 + \frac{\gamma^2}{3}\right)^{\frac{1}{2}}. \tag{11}$$

However, Itoh *et al*.[23] verified that scatter of the results by using the damage parameter with the length of strain path was large. For Shang–Wang damage parameter, the parameter does not include any weight constants and the constants of uniaxial strain lifetime equation are only required. The applicability of the Shang–Wang parameter has been verified for constant amplitude proportional and nonproportional low-cycle fatigue data in Ref. [9] In this study, Shang–Wang parameter is expanded to predict variable amplitude multiaxial fatigue life. The predicted results showed that the proposed multiaxial fatigue life prediction method provided satisfactory predictions. Therefore, the proposed multiaxial fatigue life prediction procedure is a promising technique for engineer application. In addition, the proposed method must be verified by more tests, and future work is needed to examine the accuracy and efficiency of the method in dealing with real engineering components under service loading.

CONCLUSIONS

A procedure is presented to predict conveniently fatigue life under tension and torsion random loading. The fatigue life prediction method is verified and compared between predicted and experimental results on cylindrical thin-walled tubes specimens of En15R steel subjected to combined tension/torsion random loading. The following conclusions can be drawn from the present study.

1. For a given loading-time history under tension and torsion random loading, the proposed path-dependent cycle counting method can identify conveniently half-cycles, and reduce to the simplified rain-flow counting for a repeating history in uniaxial random loading in ASTM.
2. The Wang–Brown method and the proposed method predicted fatigue life for tension and torsion random loading with most prediction results falling within a factor of 2. Both methods can be used under tension and torsion random loading.
3. From the view of the implementation of both fatigue life prediction procedures, the distance formula is simpler than the equivalent strain formula in the Wang–Brown method, and Shang-Wang fatigue damage model does not include the weight constants. The proposed method appeared to be a better choice for engineer application.

Acknowledgements

Authors would also like to thank Prof. Drew Nelson of Stanford University provide the multiaxial fatigue experimental data from Prof. M.W Brown of the University of Sheffield, Dr. T. E. Langlais of Seagate Technology. The authors gratefully acknowledge the financial support of the National Natural Science Foundation of China (No. 50975005, 50575004), the international cooperative research project of the National Natural Science Foundation of China (No. 50710105077, 51010006), the Beijing Municipal Natural Science Foundation (No. 3092003) and the Science and technology plan Key Project of Beijing Education Committee (No. KZ201010005003).

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