Physics Letters A 376 (2012) 377–386

Contents lists available at SciVerse ScienceDirect

Physics Letters A

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A novel approach for multiple mobile objects path planning: Parametrization method and conflict resolution strategy

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ARTICLE INFO

Article history:
Received 28 April 2011
Received in revised form 11 August 2011
Accepted 29 August 2011
Available online 10 October 2011
Communicated by A.R. Bishop

Keywords:
Mobile object
The shortest path planning
Parametrization method
Time-varying nonlinear programming problem
Priority scheme
Conflict resolution strategy

ABSTRACT

We present a new approach containing two steps to determine conflict-free paths for mobile objects in two and three dimensions with moving obstacles. Firstly, the shortest path of each object is set as goal function which is subject to collision-avoidance criterion, path smoothness, and velocity and acceleration constraints. This problem is formulated as calculus of variation problem (CVP). Using parametrization method, CVP is converted to time-varying nonlinear programming problems (TNLPP) and then resolved. Secondly, move sequence of object is assigned by priority scheme; conflicts are resolved by multilevel conflict resolution strategy. Approach efficiency is confirmed by numerical examples.

1. Introduction

This Letter addresses path planning problems for multiple mobile objects, which is an important topic in the engineering application field specially in robot industry, military, surgery planning etc. [1,2], and has been studied extensively. The task is to plan collision-free paths for the mobile objects that bring each object from specified start configuration to goal configuration in environment with static or moving obstacles [1]. Multiple mobile objects path planning system is characterized by objects occupying different positions in the same work space and moving in parallel in most cases. Therefore, in this system, when planning paths for mobile objects, we should consider their collaboration as all mobile objects influence each other.

Many techniques have been proposed for the path planning of mobile objects. Approaches based on coordination are widely used in path planning problems. Coordinated approaches are usually divided into centralized planning and decoupled planning [3]. In centralized planning, all the paths of mobile objects are planned by a planner [4,5]. Whereas in decoupled planning, a path is computed for each object independently and a coordination diagram is used to plan a collision-free trajectory for each object along its path [6,7]. In terms of calculation speed and practicality, various methods of above two series show excellent or imperfect performance. Centralized approach is complete but calculatingly time-consuming, and then it only can be applied to path planning in simple environment. Decoupled approach is applicable to path planning under any complex environment, but the planned path is far from high quality. In order to improve calculation speed and obtain high-quality planned path, we propose parametrization method and conflict resolution strategy. Meanwhile, if one adds any constraints into our model, the main structure of our algorithm will not be changed. For example, as nonholonomic characters [8] of some vehicles are taken into account, the constrained velocities should belong to the given boundary. Then, in this Letter path planning approach could be extended not only to robots but also to all other objects.

Zamirian et al. in [9] have applied novel parametrization method and fuzzy aggregation for single object path planning. In [10], the Bernstein–Bezier curve method is adopted to obtain the optimal planned paths for multiple mobile objects.

In this Letter, we extend the parametrization method of [9] to path planning problems for multiple mobile objects. Then, we adopt conflict resolution strategy to achieve conflict-free path planning. Generally conflict resolution strategies include traffic rules method,

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0375-9601/$ – see front matter
doi:10.1016/j.physleta.2011.08.065

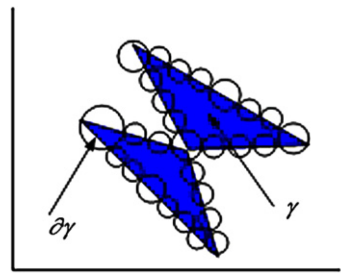


Fig. 1. Obstacle k and its boundary, that is covered with circles.

velocity adjustment method, coordination method, re-planning method, priority scheme, and so on. In priority scheme, priority of each object is assigned according to certain rules. If there existing conflict, the object with higher priority could have authority over object with lower priority and some coordination actions are taken by those objects, and finally the conflict avoidance is implemented. Earlier Erdmann and Lozano-Perez [11] proposed the priority scheme to cope with path planning problems. In [11], each object got its priority and the lower priority objects were treated as obstacles of the higher priority objects, and path of each object was computed in turn. Warren [12] extended the work of [11] and introduced the potential field method to plan path for objects with certain priority. Berg and Overmars [1] set rules that object with longer path length corresponded to higher priority were based on construction of the start and end configuration roadmaps for each mobile robot. However, it takes some time to build roadmaps for all objects and pays no attention to position distribution of objects in planned environment. Roadmap method and potential method demonstrate limitations when they are applied to path planning in dynamic environment, though they are excellent in static environment. Priority scheme has more advantages over the former methods. It is fairly extended to motion planning for multiple objects in dynamic environments with moving obstacles. Also, it is applicable to situations where multiple objects have different start times and continuously share a common environment with dynamic obstacles. Contrast to [1], in priority scheme, we take the location distribution of all objects and obstacles into account and assign corresponding priority to almost every object. Meanwhile, when priority scheme could not provide priority for some objects, multilevel conflict resolution strategy is used to achieve the overall conflict-free path planning for multiple mobile objects.

Multiple mobile objects path planning problems are not just a coarse sum of single mobile object path planning problem; they also involve collision and conflict problems. In this Letter, multiple mobile objects path planning problems are divided into two sequential phases. In the first phase, the goal is to minimize path length such that the path is smoothness and safe, for each mobile object in the same environment. This goal is formulated as the CVP whose variable is mobile object path $x(.)$, for each mobile object. Then by using parametrization method and some calculations, the CVP is converted to the sequential of TNLPPs whose variable is a polynomial function with unknown constant coefficients. With some calculation, the TNLPP is equivalent to a conventional nonlinear programming problem (NLPP). It is proved that the solution of this sequence of the NLPPs tends to the solution of the CVP similar to [9]. The solution of above NLPPs can be obtained by common software, such as Matlab, Lingo or others. After that, planned path for each mobile object is achieved. Planned paths of all mobile objects form the initial path set of multiple mobile objects *Path_initial*. In the second phase, based on *Path_initial*, conflict resolution strategy is used to achieve safety conflict-free planned paths.

Some advantages of our approach are: our planned path model is simpler than others, our optimal path is shorter than others, in contrast to [10] our optimal solution is feasible in every environment (because the fourth-order Bezier curve cannot be feasible in every environment, especially in present stationary or moving obstacles), and the polynomial of our approach is general which is more flexible than the polynomial of [10] since in [10] the points p_1 and p_{n-1} are achieved according to initial data and are constant.

The rest of this Letter is organized as follows. Section 2 describes the problem formulation. Section 3 introduces parametrization method for path planning of single mobile object and then based on the method we achieve initial paths set *Path_initial*. Section 4 details priority scheme and multilevel conflict resolution strategy. Section 5 presents illustrative path planning examples for multiple mobile objects using the proposed approach. Finally, Section 6 presents conclusions.

2. Problem formulation

We suppose object A_i which is a r_i-radius circle or sphere with center $x_i(t) = (x_{1i}(t), x_{2i}(t), x_{3i}(t))$, $t \in [0, t_{fi}]$, is a single rigid and free moving object in a two- or three-dimensional space in the presence of stationary or moving obstacles ($i = 1, 2, \ldots, m$, m is the number of mobile objects). Where $x_i(.)$ is a continuously unknown differentiable real vector-valued function which is the path of mobile object A_i, and t_{fi} is a given real number as final moving time of A_i. Also, we suppose obstacle k is a r_k-radius circle or sphere with center $\alpha_k(t) = (\alpha_{1k}(t), \alpha_{2k}(t), \alpha_{3k}(t))$, $k = 1, 2, \ldots, q$. $t \in [0, t_k]$, where $\alpha_k(.)$, $k = 1, 2, \ldots, q$ are known continuous real vector-valued functions which are the paths of motion obstacles, and t_k is a given real number as final moving time of obstacle k. We emphasize that all obstacles are considered as circles or spheres in plane or space, respectively. A non-circular with geometric shape γ_k of the k_{th} obstacle with compact boundary $\partial\gamma_k$, can be represented by a finite number of circles. Thus, we can substitute these circles with the obstacle γ_k (see Fig. 1).

Meanwhile, we suppose $x_i(.) \in X_i = \{x_i(t) \mid x_i(t) \in C^1(0, t_{fi}),\ a_i(t) \leqslant x_i(t) \leqslant b_i(t),\ c_i(t) \leqslant \dot{x}_i(t) \leqslant d_i(t),\ e_i(t) \leqslant \ddot{x}_i(t) \leqslant f_i(t),\ x_i(0) = x_{0i}, x_i(t_{fi}) = x_{fi},\ t \in [0, t_{fi}],\ 1 \leqslant i \leqslant m\}$, where $a_i(t) = (a_{1i}(t), a_{2i}(t), a_{3i}(t))$, $b_i(t) = (b_{1i}(t), b_{2i}(t), b_{3i}(t))$, $c_i(t) = (c_{1i}(t), c_{2i}(t), c_{3i}(t))$, $d_i(t) = (d_{1i}(t), d_{2i}(t), d_{3i}(t))$, $e_i(t) = (e_{1i}(t), e_{2i}(t), e_{3i}(t))$, and $f_i(t) = (f_{1i}(t), f_{2i}(t), f_{3i}(t))$, are known continuous real vector-valued functions as the boundaries of $x_i(t)$, $\dot{x}_i(t)$, and $\ddot{x}_i(t)$ for all $t \in [0, t_{fi}]$ respectively, also x_{0i} and x_{fi} are given constant vectors in $\Re^3$ as the initial and final points of $x_i(.)$, $i = 1, 2, \ldots, m$.

Three main criteria should be taken into account in the assessment of the planned path $x_i(.)$ of mobile object A_i: path length, collision-avoidance criterion which means distance between object A_i and obstacles is no less than safety distance, and the path smoothness.

The first criterion is the planned path length, which is defined as follows:

$$I_0\big(x_i(t_{fi})\big) = \int_0^{t_{fi}} \sqrt{\dot{x}_{1i}^2(t) + \dot{x}_{2i}^2(t) + \dot{x}_{3i}^2(t)}\,dt = \int_0^{t_{fi}} \big\|\dot{x}_i(t)\big\|_2\,dt.$$

For the second criterion, set

$$\varphi_k\big(x_i(t)\big) = \sqrt{x_{1i}(t) - \alpha_{1k}(t)^2 + x_{2i}(t) - \alpha_{2k}(t)^2 + x_{3i}(t) - \alpha_{3k}(t)^2} - (r_i + r_k) = \big\|x_i(t) - \alpha_k(t)\big\|_2 - (r_i + r_k),$$

where $\varphi_k(x_i(t))$, $k = 1, 2, \ldots, q$, is the distance between object A_i and obstacle k at the moment t.

The third criterion is the path smoothness. As we introduce the optimal path by a polynomial function that belongs to $C^\infty[0, t_{fi}]$ (the set of highly smooth functions), then this criterion is automatically satisfied.

Now, suppose d_{ki} is a given safety distance between A_i and obstacle k, which guarantees A_i and k are free of collision in the motion of A_i. Distances between A_i and obstacles can be denoted as $(d_{1i}, \ldots, d_{ki}, \ldots, d_{qi})$, where $k = 1, 2, \ldots, q$, $i = 1, \ldots, m$. Then for obtaining the length of the shortest path mobile object A_i in environment X_i, named I_{0i}, with constraints $\varphi_k(x_i(t)) \geqslant d_{ki}$, for every $t \in [0, t_{fi}]$, the following CVP is defined:

$$\min I_0\big(x_i(t_{fi})\big) = \int_0^{t_{fi}} \big\|\dot{x}_i(t)\big\|_2\,dt \quad \text{s.t.} \quad \begin{cases} \varphi_k(x_i(t)) \geqslant d_{ki} & k = 1, \ldots, q,\ t \in [0, t_{fi}], \\ x_i(.) \in X_i & i = 1, \ldots, m. \end{cases} \tag{1}$$

After solving above CVP for each mobile object (which is considered in Section 3), we can determine planned paths for all mobile objects. All planned paths of mobile objects constitute the initial paths set of multiple mobile objects, represented as $Path\_initial = \{path_1, \ldots, path_i, \ldots, path_m\}$, $1 \leqslant i \leqslant m$, $path_i = x_i(t)$, $t \in [0, t_{fi}]$. Following that, the *Path_initial* set should be processed through conflict resolution. In conflict resolution phase, priority scheme assigns corresponding priority to each object, and multilevel conflict resolution strategy deals with conflicts of planned paths. Through this phase, each object could move according to its own priority and achieve conflict-free and almost parallel motion.

3. Phase 1: Determine *Path_initial* set of multiple mobile objects based on parametrization method

In this phase, we should solve the CVP (1) for each mobile object. Generous methods have been developed to resolve CVP (1) [2,9,13–15]. In [2], Borzabadi et al. defined the artificial control function $u(t)$ as $u(t) = \dot{x}_i(t)$, and obtained an approximate solution by using measure theory that was established by Rubio [16]. But, we use a new approach for solving the problem (1). Our approach has some advantages. In this approach, the number of unknowns is lower than the methods used in [13,14], there is no error in final condition, $x(t_{fi}) = x_{fi}$, whereas the error can be found in [2,16]. In contrast with methods like successive approximation approach [17] and state parametrization using Chebyshev polynomials [18], which are restricted to quadratic objective function, our method is expressed for a general objective function.

Let $p_n^i(t) = (p_{1n}^i(t), p_{2n}^i(t), p_{3n}^i(t))$, for all $t \in [0, t_{fi}]$, where $p_{jn}^i(.)$, $j = 1, 2, 3$; $i = 1, 2, \ldots, m$ are polynomials of degree at most n with unknown constant coefficients. Then, by substituting $p_n^i(.)$ instead of $x_i(.)$ in the problem (1), the sequence of the TNLPPs is obtained as follows:

$$\min I_0\big(p_n^i(t_{fi})\big) = \int_0^{t_{fi}} \big\|\dot{x}_i(t)\big\|_2\,dt \quad \text{s.t.} \quad \begin{cases} \varphi_k(p_n^i(t)) \geqslant d_{ki} & k = 1, \ldots, q,\ t \in [0, t_{fi}], \\ p_n^i(.) \in X_i & n = 1, 2, \ldots,\ i = 1, \ldots, m. \end{cases} \tag{2}$$

Now, we suppose Q is the set of $x_i(.)$ such that the problem (1) is feasible and $Q(n)$ is the set of $p_{ni}(.)$ such that the problem (2) is feasible. Also, we suppose Q and $Q(n)$ are not empty. Then, by the following theorem is proved that the sequence of the solutions of problem (2) converges to the solution of problem (1).

Theorem 1. *If* $\eta = \inf_Q I_0(x_i(t_{fi}))$ *and* $\eta(n) = \inf_{Q_n} I_0(p_n^i(t_{fi}))$, *then* $\eta = \lim_{n\to\infty} \eta(n)$.

Proof. See [9]. □

According to the constraints: safety distance between mobile object A_i and obstacle k, initial point and final point of mobile object A_i, allowed velocity and acceleration boundary of mobile object A_i, problem (2) is transformed as follows:

$$\min \int_0^{t_{fi}} \big\|\dot{p}_n^i(t)\big\|_2\,dt \quad \text{s.t.} \quad \begin{cases} \varphi_k(p_n^i(t)) \geqslant d_{ki} & k = 1, \ldots, q, \\ p_n^i(t) \geqslant a_i(t) & t \in [0, t_{fi}], \\ p_n^i(t) \leqslant b_i(t) & n = 1, 2, \ldots, \\ \dot{p}_n^i(t) \geqslant c_i(t), \\ \dot{p}_n^i(t) \leqslant d_i(t), \\ \ddot{p}_n^i(t) \geqslant e_i(t), \\ \ddot{p}_n^i(t) \leqslant f_i(t), \\ p_n^i(0) = x_{0i}, \quad p_n^i(t_{fi}) = x_{fi}. \end{cases} \tag{3}$$

Now, we partition the interval $[0, t_{fi}]$ to S equal parts as $h = t_{fi}/S$, set $E_i(t) = \|\dot{p}_n^i(t)\|_2$. Thus, by using a numerical integration method such as trapezoidal rule, problem (3) is converted to the following problem:

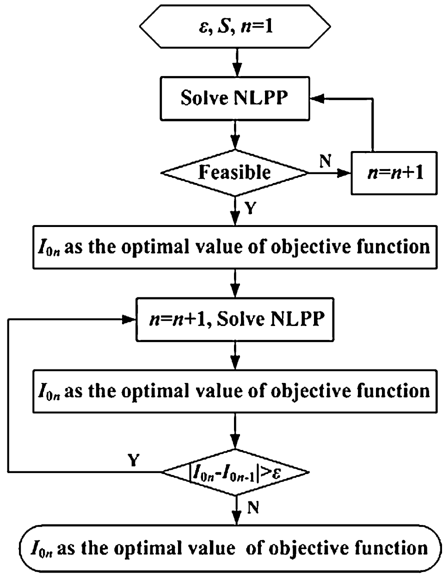


Fig. 2. Algorithm for solving the NLPPs.

$$\min h\big[E_i(0)+2E_i(h)+\cdots+2E_i\big((S-1)h\big)+E_i(t_{fi})\big]/2 \tag{4}$$

$$\text{s.t.}\quad \begin{cases} \varphi_k(p_n^i(sh)) \geqslant d_{ki} \quad k=1,\ldots,q, \\ p_n^i(sh) \geqslant a_i(sh) \quad s=0,1,\ldots,S, \\ p_n^i(sh) \leqslant b_i(sh) \quad n=1,2,\ldots, \\ \dot{p}_n^i(sh) \geqslant c_i(sh), \\ \dot{p}_n^i(sh) \leqslant d_i(sh), \\ \ddot{p}_n^i(sh) \geqslant e_i(sh), \\ \ddot{p}_n^i(sh) \leqslant f_i(sh), \\ p_n^i(0)=x_{0i}, \quad p_n^i(t_{fi})=x_{fi}. \end{cases} \tag{5}$$

Theorem 2. *The solutions of the problem* (4)–(5) *and* (3) *are the same, if in problem* (4)–(5) *S tends to infinity.*

Proof. See [19]. □

The problem (4)–(5) is a NLPP with $3n$ variables (the unknown constant coefficients of $p_{1n}^i(.)$, $p_{2n}^i(.)$, $p_{3n}^i(.)$, $i=1,2,\ldots,m$), which can be solved by using many softwares, such as Lingo, Matlab, etc.

The basics of attainment of *Path_initial* are single mobile object path planning based on parametrization. The polynomial coefficient n of the single mobile object optimal path is determined by the given path length error ε. Fig. 2 shows the specific process of solving NLPPs for each mobile object.

Optimal planned path of each mobile object is obtained by using algorithm of the NLPPs in turn. On the basis of above optimal planned path, *Path_initial* is achieved.

4. Phase 2: Determine the optimal path planning for multiple mobile objects based on conflict resolution

Conflict means as mobile objects parallel moving along the planned paths there existing collision or cross-line circumstances in paths of different mobile objects. Although initial multiple mobile objects paths set *Path_initial* has considered the safety distance between mobile object and obstacles and its velocity and acceleration constraints, *Path_initial* has not dealt with conflicts among mobile objects. Generally, in *Path_initial* there exist lots of conflicts. For such phenomena, conflict resolution approach based on priority scheme and multilevel conflict resolution is proposed to resolve conflict problems in *Path_initial*.

In our conflict resolution approach, firstly, we use priority scheme to assign priority to each mobile object; secondly, we adopt multi-level conflict resolution strategy to fulfill conflict-free path planning for mobile objects. Priority scheme is one conflict resolution strategy that has been frequently applied for robots path planning in [1,11,12,20]. In priority scheme, according to certain rules corresponding moving sequence is assigned to each mobile object. Based on the granted sequence, conflicts among mobile objects can be avoided. Mostly, moving sequence of every mobile object can be gained after priority scheme procession. However, sometimes moving sequence of some mobile objects cannot be acquired by priority scheme. Suppose the set of priority move sequence achieved by priority scheme named *PRI_1* and the set of remainder unspecified sequence named *PRI_2*. Then multilevel conflict resolution strategy is introduced to process *PRI_2*. After procession of priority scheme and multilevel conflict resolution strategy, conflict-free planned paths of all the mobile objects can be reached.

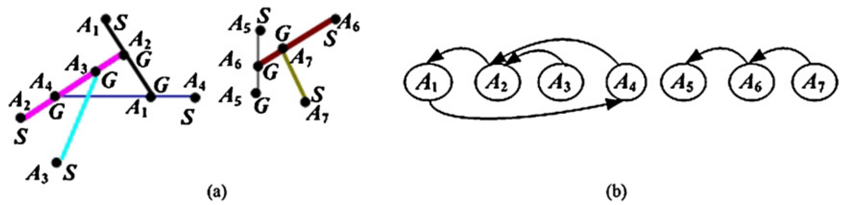


Fig. 3. Priority scheme attainment. (a) Planned path environment, (b) priority diagram.

4.1. PRI scheme

In priority scheme, priority of each mobile object influences the optimality of the resulting paths greatly [1]. Application of the appropriate priority scheme would achieve ideal planned path. In [20], Bennewitz performed a randomized search with hill-climbing method to find optimal schemes. As path planning problems for multiple mobile objects have been computed many times, using methods similar to [20] is very time-consuming. Contrast to [20], we propose straightforward rules to reach corresponding priority for each mobile object in the priority scheme. Priority of mobile object A_i can be denoted as PRI_i $(i = 1, \dots, m)$. PRI_h and PRI_l represent high priority and low priority respectively. We suppose if final point x_{fi} of A_i lies on the planned optimal path $Path_j$ of A_j, then PRI_i is lower than PRI_j and described as $PRI_i < PRI_j$, where $i \neq j$, $i, j = 1, \dots, m$.

In order to determine the set of priority move sequence *PRI*_1 and *PRI*_2, we propose sequential priority scheme algorithm called *PRI_Order* as follows. In *PRI_Order*, A, *A_init*, $relation_{ij}$, m stand for the set of mobile objects, the dynamic changing set of mobile objects, connection between A_i and A_j, the number of mobile objects, respectively. If $PRI_i > PRI_j$, then the arrow directs A_i from A_j in $relation_{ij}$.

Algorithm 1 *PRI_Order*(*relation*, m)

Input: All $relation_{ij}$ among mobile objects, Number of mobile objects m.
Output: *PRI*_1 given by *PRI_Order*, *PRI*_2 not processed by *PRI_Order*.

```
PRI_1, PRI_2 = NULL; A_init = A
foreach A_i in A_init do
for For all relation_ij, i ≠ j do
  if relation_ij is not null, the arrow directs to A_i then
    adds A_i into PRI_1.
    Delete A_i from A_init.
  end if
end for
end foreach
PRI_2 = A − PRI_1.
return PRI_1, PRI_2.
```

Fig. 3 gives the *PRI_Order* realization in environment with seven mobile objects. In Fig. 3(a), mobile objects $A_1, \dots, A_7$ are considered, the start and end points are presented as 'S' and 'G' respectively. Fig. 3(a) shows that final point x_{f2} of A_2 lies on the planned path $Path_1$ of A_1, final point x_{f3} of A_3 lies on planned path $Path_2$ of A_2, and so on. According to *PRI_Order*, as shown in Fig. 3(b), the arrows direct A_1 from A_2, A_2 from A_3, A_3 from A_4, A_4 from A_1, A_5 from A_6, A_6 from A_7, respectively. And *PRI*_1 and *PRI*_2 are depicted as *PRI*_1 $= \{A_5, A_6, A_7\}$, *PRI*_2 $= \{A_1, A_2, A_3, A_4\}$.

4.2. Multilevel conflict resolution strategy

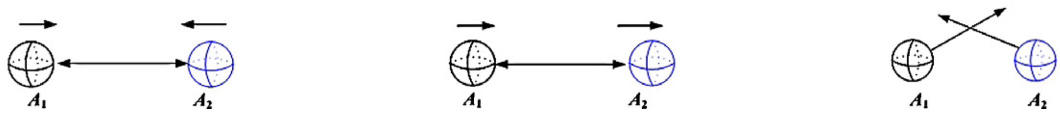
In Fig. 3(b), there exists no cyclic dependency in *PRI*_1, but exists a cyclic dependency in *PRI*_2. If multiple mobile objects in *PRI*_2 move, lots of collision and conflicts can be induced. Multilevel conflict resolution strategy is introduced to work out this troublesome problem. Optimal path length of A_i is denoted as $I_0(x_i)$ and the number of conflict points between A_i and A_j is presented as Con_{ij}, where $1 \leqslant i, j \leqslant m, i \neq j$. In multilevel conflict resolution strategy, the shorter the length $I_0(x_i)$ is, the smaller the Con_{ij} summation of A_i is, then the higher priority of A_i is. If the Con_{ij} summation of A_i is larger and A_i through simple move could lead other mobile objects to pass through without conflict, then priority of A_i is lower. As shown in Fig. 3(a), the Con_{ij} summation of A_4 equal to 3 which is larger than Con_{ij} summation of any other mobile object, and $I_0(x_4)$ is longer than any other $I_0(x_i)$ $(i = 1, 2, 3)$, then priority of A_4 is the lowest in *PRI*_2. In Fig. 3(a), A_4 could move to its final point without conflicts induced if A_4 takes several simple actions such as step aside, bypass nearby the conflict points. When the mobile object turning around or bypass the conflict points, it should take low-amplitude movement. We assume that if any mobile object moves to its final point and has no other movement, then path planning for this mobile object is completed.

4.3. Conflict resolution based on priority scheme and multilevel conflict resolution strategy

In this section we illustrate conflict resolution strategy for two and multiple mobile objects respectively.

Conflicts between two mobile objects are demonstrated from three different circumstances shown in Fig. 4. The three circumstances in Fig. 4(a), (b) and (c) are objects moving along the same line in the opposite direction and in the same direction, and not along the same line respectively.

In Fig. 4(a), final point of A_1 is in the planned path of A_2, and vice versa. According to priority scheme, we could not obtain priority sequence of A_1 and A_2 in this situation. Then, we adopt multilevel conflict resolution strategy to resolve this problem. In multilevel conflict resolution strategy, higher priority PRI_h corresponds to planned path length expressed as $\min(I_0(x_1), I_0(x_2))$. For the sake of simplify, we presume $PRI_1 > PRI_2$ in Fig. 4(a). Therefore, A_1 can move without intervention. Generally A_2 can take anyone of the following



(a) In the opposite direction along the same line. (b) In the same direction along the same line. (c) Not along the same line.

Fig. 4. Conflicts between two mobile objects under three different circumstances.

two measures. The first measure is A_2 should wait around the current motion location until A_1 passes through. The second measure is before moving to conflict point, A_2 should bypass near the conflict point for a while until the conflict is avoided, and then continue moving to final point x_{f2}.

In Figs. 4(b) and 4(c), if there exists no conflict between A_1 and A_2, then *Path_initial* of A_1,A_2 can be used directly. If there exist conflicts between A_1 and A_2, then we could adopt the same solution like conflict resolution measures applied in Fig. 4(a) to fulfill conflict-free path planning for A_1 and A_2.

Conflicts among three or more mobile objects can be shown in Fig. 3(a). To deal with conflicts among multiple mobile objects, we put forward basic decision processing as follows.

Firstly, if there is no conflict among multiple mobile objects, then *Path_initial* is used as the optimal planned paths.

When conflicts exist among mobile objects, priority scheme is used to acquire *PRI_1*, *PRI_2* as denoted in Section 4.1. As *PRI_2* is the set of unspecified sequence, we adopt multilevel conflict resolution strategy to achieve its priority allocation. Following that move sequence of each mobile object is granted according to its priority. In Fig. 3, above illustration can be presented as $PRI_5 > PRI_6 > PRI_7$ in *PRI_1* and $PRI_1 > PRI_2 > PRI_3 > PRI_4$ in *PRI_2*.

Thirdly, when comes to conflicts between two mobile objects, in order to fulfill conflict-free path planning, we could use anyone of the two conflict resolution measures applied in Fig. 4(a). Generally, conflicts can be resolved through this process.

Finally, if conflicts involved many mobile objects, objects should be grouped into three or two objects, and planning paths for every grouped mobile objects. In Fig. 3, we could partition *PRI_2* into two sets $\{A_1, A_2, A_3\}$ and $\{A_4\}$, and then using priority scheme and multilevel conflict resolution strategy to fulfill conflict-free path planning.

5. Experimental examples

Computer simulations are provided to validate the effectiveness of the proposed parametrization method and conflict resolution strategy, path planning for multiple mobile objects under different circumstances are discussed. We set $v_i(t) = (x_i(t))'$, $i = 1, 2, \ldots, m$, where $v_i(t)$ are interpreted as control functions which show the speed of object A_i in direct of $x_i(t)$ at the moment $t \in [0, t_{fi}]$. In the following figures, the scales are same and all quantities conform to a given unit system, for instance, meters, meters per second, etc. In this Letter, all the computations were run on a PC with CPU Intel Core2 Duo and 2 GB of RAM and all the codes are written in Matlab 7.0 software.

5.1. Case study for multiple mobile objects path planning with path conflict in two dimensions

Circumstance settings of 100×100 two dimensions are given as follows:

(1) Parameters of mobile objects: radius $r_1 = 1$, $r_2 = 3$, $r_3 = 2$. Initial points of center position $x_{01} = (37, 82)$, $x_{02} = (12, 57)$, $x_{03} = (42, 22)$, final points of center position $x_{f1} = (82, 11)$, $x_{f2} = (78, 90)$, $x_{f3} = (73, 25.2)$.
(2) Boundary conditions: velocity boundary $(0, -7.5) \leqslant (v_{11}, v_{21}) \leqslant (4.6, 0)$, $(0, 0) \leqslant (v_{12}, v_{22}) \leqslant (5.6, 3.2)$, $(0, 0) \leqslant (v_{13}, v_{23}) \leqslant (8, 0.6)$. Time boundary $T_{\max 1} = 10$, $T_{\max 2} = 12$, $T_{\max 3} = 6$.

Fig. 5 shows planned paths of multiple mobile objects in two dimensions, where 'S', 'G' represent start position and end position of mobile objects respectively. In the parallel moving procession of multiple mobile objects, distances between A_1, A_2 and A_3 are depicted in Fig. 6(a). Fig. 6(a) shows that A_2 and A_1, A_3 have no conflict (distance value is greater than 0), but A_1 and A_3 have collided at time nearby 8 s. Since final point of A_3 lies too close to the optimal path of A_1, according to priority scheme, we obtain the constraint $PRI_1 > PRI_3$ in this situation. According to priority of A_1 and A_3, A_1 should move firstly almost until it reaches to final point of A_3 and then movement of A_3 is allowed. Therefore, A_1, A_2 could move in parallel and A_3 should start to move after 4 s movement of A_1, A_2. Afterwards, through priority scheme distances r_{12}, r_{23}, r_{13} between mobile objects A_1, A_2 and A_3 are shown in Fig. 6(b). Velocities of A_1, A_2, A_3 are shown in Fig. 7(a), Fig. 7(b) and Fig. 7(c) respectively. All the velocity constraints are satisfied as all velocities are limited to velocity boundary. Results of this example demonstrate that in two dimensions our approach is fit for multiple mobile objects path planning problems.

To prove the validity of the proposed approach, we test it by using the case data in [10] and give the experimental results. Table 1 gives the raw data list and Fig. 8(a) shows the planned paths in [10]. In [10], the start and end points x_0, x_f, start and end velocities v_0, v_f, safety distance d_s, maximum allowed velocities $v_{\max}$ and accelerations $a_{\max}$ of mobile objects A_i, $i = 1, 2, 3$ are listed in Table 1. And all the allowed time boundary $T_{\max}$ of A_i, $i = 1, 2, 3$ are 5. The planned paths of A_i, $i = 1, 2, 3$ in [10] are given in Table 2. Their approach was based on fourth-order Bezier curve and the highest indexes of all their polynomials are restricted to four. Their approach was based on fourth-order Bezier curve.

Before using conflict resolution strategy, planned paths of A_1, A_2 and A_3 are shown in Fig. 8(b). In Fig. 8(b), the planned paths of A_1 and A_2 are almost overlapped. Fig. 8(c) shows planned paths of A_1, A_2 and A_3 handled by conflict resolution strategy.

Fig. 9(a) shows distances between A_1, A_2 and A_3 before treated with conflict resolution strategy when mobile objects moving in parallel. About 0.9 s and 1.3 s, distances between A_1 and A_3, A_1 and A_2 are less than 0.35 m respectively. About 1.2 s, 1.8 s, A_1 collided with A_3 and A_2 respectively. As start and final points of A_1 and A_2 are reversed, corresponding priority cannot be given according to priority scheme. Therefore, according to multilevel conflict solution strategy we could come to the following solution. In the proposed solution, A_1 begins to bypass at 1.8 s and turns to diagonal movement at 4 s, A_2 maintains its trajectory without change, from 0 to 0.8 s

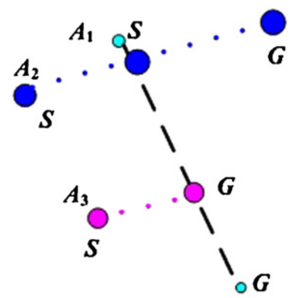


Fig. 5. Planned paths of the mobile objects A_1, A_2 and A_3 with conflicts.

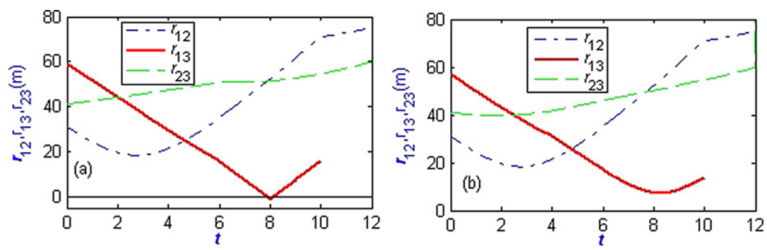


Fig. 6. Distances between any two mobile objects A_1, A_2 and A_3. (a) and (b) without or with priority scheme and conflict resolution respectively.

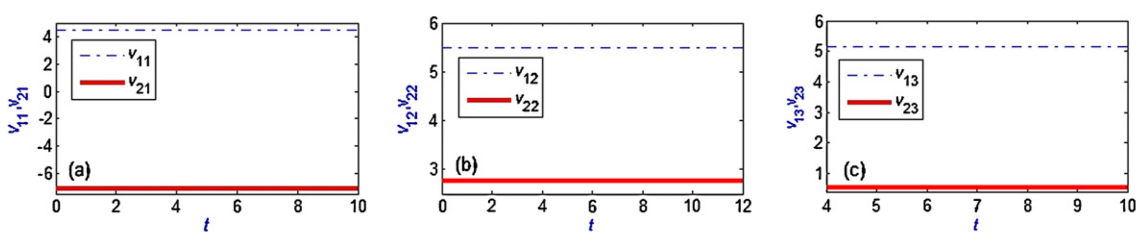


Fig. 7. (a), (b) and (c) are velocities of A_1, A_2 and A_3 with conflict resolution respectively.

Table 1
Initial information of mobile objects A_i.

A_i	x_0	x_f	v_0	v_f	a_{max}	d_s	v_{max}
A_1	$[0.2, 1.4, -\pi/4]^T$	$[1.4, 0.2, -\pi/4]^T$	0.4	0.4	0.5	0.35	0.8
A_2	$[1.4, 0.2, 3\pi/4]^T$	$[0.2, 1.4, 3\pi/4]^T$	0.4	0.5	0.5	0.35	0.8
A_3	$[0.2, 0.2, \pi/4]^T$	$[1.4, 1.4, \pi/4]^T$	0.4	0.4	0.5	0.35	0.8

Table 2
Planned paths of multiple mobile objects A_i given by [10] based on predefined data of [10].

A_i	$Path_i$ in [10]
A_1	$\begin{cases} x_{11}(t) = 0.2 + 0.2828t + 0.216t^2 - 0.098t^3 + 0.011t^4 \\ x_{21}(t) = 1.4 - 0.2828t - 0.0142t^2 + 0.0103t^3 - 0.00134t^4 \quad t \in [0, 4.5974] \end{cases}$
A_2	$\begin{cases} x_{12}(t) = 1.4 - 0.28285t + 0.063t^2 - 0.02t^3 + 0.00156t^4 \\ x_{22}(t) = 0.2 + 0.28284t - 0.236t^2 + 0.095t^3 - 0.0098t^4 \quad t \in [0, 4.5973] \end{cases}$
A_3	$\begin{cases} x_{13}(t) = 0.2 + 0.2828t - 0.063t^2 + 0.0231t^3 - 0.0023t^4 \\ x_{23}(t) = 0.2 + 0.28284t + 0.071t^2 - 0.035t^3 + 0.00403t^4 \quad t \in [0, 4.5973] \end{cases}$

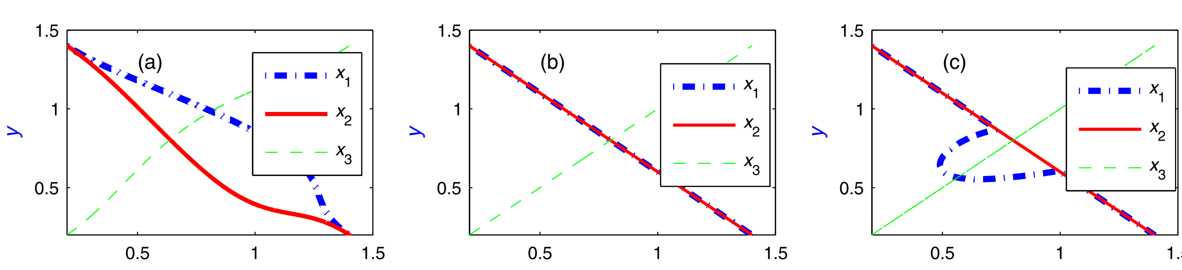


Fig. 8. Planned paths of A_1, A_2 and A_3 under environment in [10]. (a) are paths given in [10]. (b) and (c) are paths without or with conflict resolution, respectively.

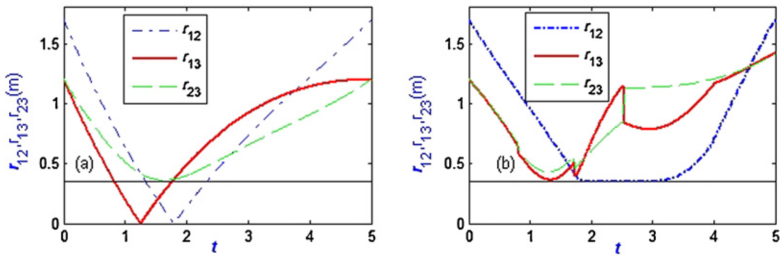


Fig. 9. Distances between any two mobile objects A_1, A_2 and A_3. (a) and (b) without or with priority scheme and conflict resolution respectively.

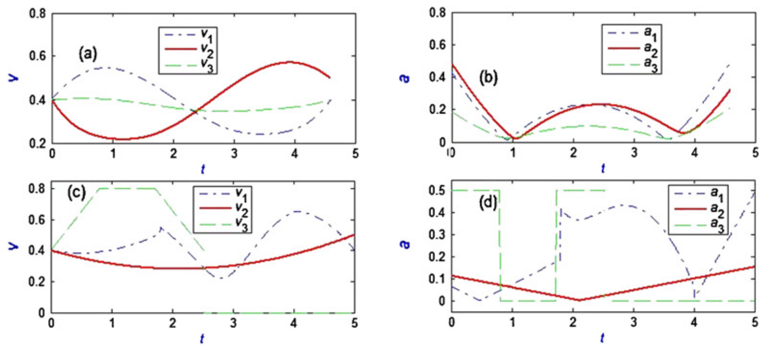


Fig. 10. Velocity and acceleration comparison of A_1, A_2 and A_3 in [10].

Table 3
Planned paths of multiple mobile objects based on parametrization and conflict solution and comparison with results of [10].

A_i	$Path_i$ in this Letter	Path length in [10]	Path length in this Letter
A_1	$\begin{cases} x_{11}(t) = 0.2 + 0.283t - 0.023t^2 + 0.017t^3 \\ x_{21}(t) = 1.4 - 0.283t + 0.023t^2 - 0.017t^3 \quad t \in [0, 1.8] \\ x_{11}(t) = 1.69 + 0.01t - 0.69t^2 + 0.26t^3 - 0.025t^4 \\ x_{21}(t) = -1.33 + 3.76t - 2.17t^2 + 0.49t^3 - 0.04t^4 \quad t \in (1.8, 4] \\ x_{11}(t) = 2.811 - 2.286t + 0.688t^2 - 0.058t^3 \\ x_{21}(t) = -1.211 + 2.286t - 0.688t^2 + 0.058t^3 \quad t \in (4, 5] \end{cases}$	> 1.83	1.829
A_2	$\begin{cases} x_{12}(t) = 1.4 - 0.2828t + 0.0398t^2 - 0.0063t^3 \\ x_{22}(t) = 1.4 + 0.2828t - 0.0398t^2 + 0.0063t^3 \quad t \in [0, 5] \end{cases}$	> 1.7	1.697
A_3	$\begin{cases} x_{13}(t) = 0.2 + 0.283t + 0.177t^2 \\ x_{23}(t) = 0.2 + 0.283t + 0.177t^2 \quad t \in [0, 0.8] \\ x_{13}(t) = 0.172 + 0.566t \\ x_{23}(t) = 0.172 + 0.566t \quad t \in (0.8, 1.721] \\ x_{13}(t) = -0.437 + 1.174t - 0.177t^2 \\ x_{23}(t) = -0.437 + 1.174t - 0.177t^2 \quad t \in (1.721, 2.521] \end{cases}$	> 1.70	1.697

and 1.721 s to 2.521 s, A_3 keeps to accelerated motion, from 0.8 s to 1.721 s, A_3 keeps to linear movement. In Fig. 8(c), at time 1.8 s there is a circuitous action of A_1. After application of conflict resolution strategy, Fig. 9(b) shows distances between any two are positive, which demonstrates that conflict-free path planning has been achieved.

Table 3 lists the planned path of A_1, A_2 and A_3 based on parametrization method and conflict resolution strategy and comparative results with [10]. Fig. 10 gives the velocity and acceleration curves of [10] and this Letter. Results of [10] are shown in Fig. 10(a) and (b). With our proposed approach, as are shown in Fig. 10(c) and (d), the constraints of velocity and acceleration are satisfied, respectively.

In our planned path model, the index of polynomials is not restricted to one certain number, and the planned path length is shorter than [10]. Then our approach can be applied to any environment, especially with stationary or moving obstacles. The polynomial of our approach is general which is more flexible than the polynomial of [10] since in [10] the points p_1 and p_{n-1} are achieved according to initial data and are constant. But, our optimal paths with conflict resolution are piecewise smooth which is a disadvantage in our approach. Then, we overcome this disadvantage by adding of the low-amplitude movement requirement.

5.2. Case study for multiple mobile objects path planning with path conflicts in three dimensions

Circumstance settings of $100 \times 100 \times 100$ three dimensions with static and moving obstacles are given as follows:

(1) Parameters of mobile objects: radius $r_1 = 6$, $r_2 = 5$, $r_3 = 7$. Initial points of center position $x_{01} = (41, 20, 75)$, $x_{02} = (75, 12, 65)$, $x_{03} = (80, 70, 91)$, final points of center position $x_{f1} = (52, 31, 15)$, $x_{f2} = (12, 75, 15)$, $x_{f3} = (50, 27, 48)$.
(2) Boundary conditions: velocity boundary $(0, 0, -9) \leqslant (v_{11}, v_{21}, v_{31}) \leqslant (5, 2, 0)$, $(-10, 0, -8) \leqslant (v_{12}, v_{22}, v_{32}) \leqslant (0, 10, 0)$, $(-7, -8, -8) \leqslant (v_{13}, v_{23}, v_{33}) \leqslant (0, 0, 0)$. Time boundary $T_{\max 1} = 8$, $T_{\max 2} = 8$, $T_{\max 3} = 6$. Safety distance $d_{ki} = 1$, $k = 1, 2, 3, 4$, $i = 1, 2, 3$. $d_s = 5$.

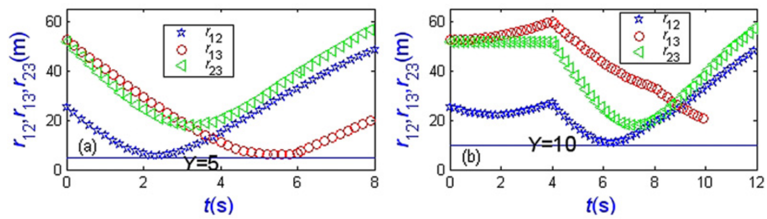


Fig. 11. Distances between any two mobile objects A_1, A_2 and A_3. (a) and (b) without or with priority scheme and conflict resolution respectively.

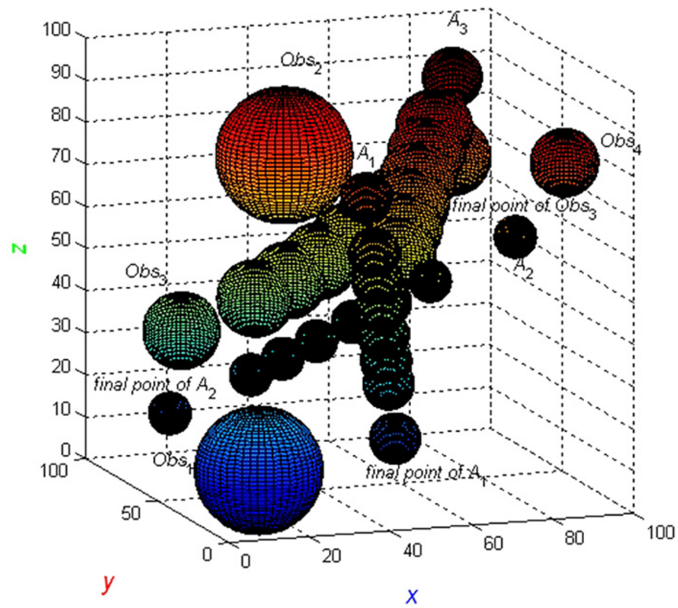


Fig. 12. Planned paths of A_1, A_2, A_3 and movement of obstacle 3 in three dimensions with obstacles.

(3) Parameters of obstacles: radius $r\_obs_1 = 15$, $r\_obs_2 = 16$, $r\_obs_3 = 9$, $r\_obs_4 = 8$. Center position $\alpha_1 = (18, 30, 10)$, $\alpha_2 = (36, 63, 77)$, $\alpha_{30} = (6, 50, 40)$, $\alpha_4 = (90, 20, 80)$. Path of obstacle 3 $\alpha_3(t) = (\alpha_{13}(t), \alpha_{23}(t), \alpha_{33}(t))$, $t \in [0, 15]$ is given as follows:

$$\begin{cases} \alpha_{13}(t) = 6 + 7.1087733t - 0.14502925t^2, \\ \alpha_{23}(t) = 50 + 1.743633t - 0.018315431t^2 - 0.00060253t^3 \quad t \in [0, 15], \\ \alpha_{33}(t) = 40 + 1.4318847t + 0.12392547t^2 - 0.00514415t^3. \end{cases}$$

After all motion constraints of static and moving obstacles are taken into account, *Path_initial* can be obtained by solving NLPPs (4)–(5), where $Path\_initial = \{path_1, path_2, path_3\}$, $path_i = x_i(t)$, $t \in [0, t_{fi}]$. And path lengths of A_1, A_2 and A_3 are $l_{01} = 61.78$, $l_{02} = 101.374$, $l_{03} = 67.457$ respectively. The specific paths expression are listed as follows:

$$\begin{cases} x_{11}(t) = 41 + 1.6492375t - 0.03427973t^2, \\ x_{21}(t) = 20 + 1.48335125t + 0.01288319t^2 - 0.00330338t^3 \quad t \in [0, 8], \\ x_{31}(t) = 75 - 7.86432125t - 0.08008794t^2 + 0.01570352t^3, \end{cases}$$

$$\begin{cases} x_{12}(t) = 75 - 10.1118425t + 0.27960531t^2, \\ x_{22}(t) = 12 + 10.0440725t - 0.103909625t^2 - 0.02090305t^3 \quad t \in [0, 8], \\ x_{32}(t) = 65 - 8.04250375t + 0.10166953t^2 + 0.01529918t^3, \end{cases}$$

$$\begin{cases} x_{13}(t) = 80 - 6.76658833t + 0.183320306t^2, \\ x_{23}(t) = 70 - 7.9033225t - 0.08640553t^2 + 0.025602t^3 \quad t \in [0, 6], \\ x_{33}(t) = 91 - 7.9095483t - 0.07118928t^2 + 0.01861158t^3. \end{cases}$$

Fig. 11(a) shows distances between any two mobile objects A_1, A_2 and A_3 without conflict resolution in three dimensions with static and moving obstacles. Nearby 2.5 s and 5.5 s distances between A_1 and A_2, A_1 and A_3 are less than the value of d_s. Thus, we need to deal with conflicts among planned paths in *Path_initial*. When conflict resolution strategy is applied, we should keep trajectory of A_1 without any change and let A_2, A_3 start to move at $t = 4$ s. The adopted strategy means that initial motion of A_1 is maintained and A_2, A_3 are stopped for 4 s. After treated with priority scheme and multilevel conflict solution strategy, distances between any two mobile objects are shown in Fig. 11(b). It illustrates that after conflict resolution procession, all the distances are greater than 10 and satisfy the safety distance requirement.

After treatment of conflict resolution strategy, planned path of A_1 remains unchanged, and planned paths of A_2, A_3 are given as follows:

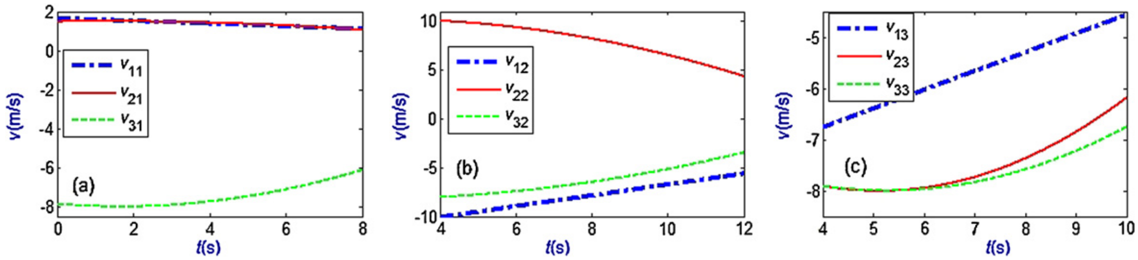


Fig. 13. (a), (b) and (c) are velocities of A_1, A_2 and A_3 in 3D with conflict resolution respectively.

$$\begin{cases} x_{12}(t) = 119.92 - 12.349t + 0.27961t^2, \\ x_{22}(t) = -28.501 + 9.8720t + 0.14693t^2 - 0.0209t^3 \quad t \in [4, 12], \\ x_{32}(t) = 97.818 - 8.1215t - 0.08192t^2 + 0.0153t^3, \end{cases}$$

$$\begin{cases} x_{13}(t) = 110.00 - 8.2332t + 0.18332t^2, \\ x_{23}(t) = 98.592 - 5.9832t - 0.39363t^2 + 0.02560t^3 \quad t \in [4, 10], \\ x_{33}(t) = 120.31 - 6.4467t - 0.29453t^2 + 0.01861t^3. \end{cases}$$

Fig. 12 shows planned paths of A_1, A_2 and A_3 after procession of conflict resolution strategy in three-dimensional environments with static and moving obstacles, and also shows the movement path of obstacle 3. Velocities of A_1, A_2, A_3 are shown in Fig. 13(a), Fig. 13(b) and Fig. 13(c) respectively. As all velocities are limited to velocity boundary, consequently all the velocity constraints are satisfied. Results of this example indicate that the proposed approach is valid for mobile objects path planning with static or moving obstacles in three dimensions.

6. Conclusion

This Letter extends parametrization method to path planning problems for multiple mobile objects. Meanwhile, priority scheme and multilevel conflict resolution strategy are combined to actualize mobile objects conflict-free movement. Results of several numerical examples have verified the effectiveness of the proposed approach. Compared with other methods, the presented approach has some virtues: the model is simpler and its variables are fewer; planned path is shorter and more flexible; no conflicts exist in planned paths; path planning method is suitable for two and three dimensions with static and moving obstacles. Based on the work of this Letter, in future works, we will develop a moderate scale path planning system for multiple mobile objects to meet the requirement of large-scale production.

Acknowledgements

The authors wish to sincerely thank the Editor-in-Chief Prof. A.R. Bishop and anonymous reviewers for their constructive and valuable comments which lead to the better presentation of this Letter. And the first two authors are partially supported by National Science Foundation of China (90924301 and 70971046).

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