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Alpha-reliable combined mean traffic equilibrium model with stochastic travel times

ZHANG Wen-yi(张文义), GUAN Wei(关伟), SONG Li-ying(宋丽英), SUN Hui-jun(孙会君)

Key Laboratory for Urban Transportation Complex Systems Theory and Technology of Ministry of Education, Beijing Jiaotong University, Beijing 100044, China

Abstract: Based on the reliability budget and percentile travel time (PTT) concept, a new travel time index named combined mean travel time (CMTT) under stochastic traffic network was proposed. CMTT here was defined as the convex combination of the conditional expectations of PTT-below and PTT-excess travel times. The former was designed as a risk-optimistic travel time index, and the latter was a risk-pessimistic one. Hence, CMTT was able to describe various routing risk-attitudes. The central idea of CMTT was comprehensively illustrated and the difference among the existing travel time indices was analyzed. The Wardropian combined mean traffic equilibrium (CMTE) model was formulated as a variational inequality and solved via an alternating direction algorithm nesting extra-gradient projection process. Some mathematical properties of CMTT and CMTE model were rigorously proved. Finally, a numerical example was performed to characterize the CMTE network. It is founded that that risk-pessimism is of more benefit to a modest (or low) congestion and risk network, however, it changes to be risk-optimism for a high congestion and risk network.

Key words: travel behavior; risk attitude; travel time reliability; combined mean travel time; wardropian user equilibrium

1 Introduction

The reliability-based methodology has been widely used to investigate the traveller's routing behaviours in stochastic traffic network. Within this framework, a traveller aims not only to minimize the travel time but also to improve the punctual arrival probability (or arrival reliability). FRANK [1] employed the reliability-based percentile travel time (PTT) index to measure the quality of a route, and the shortest routes possessed the least PTT. FAN et al [2] applied PTT into an adaptive routing problem. Note that, when travellers were insatiable, the PTT-minimum routes corresponded to the first-order non-dominated ones according to the stochastic dominance theory [3]. Inspired from this, NIE and WU [4] explored the PTT-shortest routing problem. Later, the correlations between adjacent links were incorporated [5]. Recently, NIE [6] proposed a percentile user equilibrium model for traffic network analysis.

Another measure to model travel time reliability is the effective travel time (ETT) index. Different from PTT, ETT argues that a traveller reserves a positive safety margin to hedge against the travel time variation in daily trips [7]. Mathematically, ETT comprises the mean travel time (MTT) index and a safety margin which is the product of standard deviation and a scalar called punctuality factor. When the travel time distributes normally, the punctuality factor has a one-to-one correspondence with reliability. However, such relationship does not hold in other cases [8]. LO and TUNG [9] proposed a probabilistic user equilibrium model, where the travellers merely selected the routes experiencing the minimum MTT and satisfying the reliability restrictions. LO et al [10] extended the probabilistic user equilibrium model to the travel time budget (TTB) model with degradable link capacities, and the central limit theorem guaranteed the normal distributions of route travel times. When travel time distributed normally, TTB was equivalent with PTT, value at risk (VaR) [11] was applied widely in finance area, and alpha-reliable travel time index [12]. Based on TTB, many further studies, considering endogenous [13−14] or exogenous random sources [15−16], were explored. CHEN and ZHOU [17] argued the importance of unreliable aspect in a traveller's routing process. They introduced a mean-excess travel time (METT) index which was defined as the conditional expectation of the TTB-excess travel times. METT was analogous to the conational value at risk (CVaR) [18] used in financial engineering. Compared with TTB, METT actually represented a kind of more pessimistic routing

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Corresponding author: GUAN Wei, Professor, PhD; Tel: +86−10−51687142; E-mail: weig@bjtu.edu.cn

behaviour. FOSGERAU and KARLSTRÖM [19] analysed the value of reliability, measured by the derivative of expected utility with respect to standard deviation, in scheduling activities. They verified that the maximal expected utility was the linear combination of mean and standard deviation for all travel time distributions. Later, FOSGERAU and ENGELSON [20] further reported their study on the value of travel time variance.

Due to the randomness of travel time, someone would like to select the robust routes to avoid the worst possibility. This kind of routing behaviour can be described by the dominant-strategy-based self-routing games. At the equilibrium states, travellers only distribute on the routes with the minimum upper bounds of travel time. In order to obtain these upper bounds, two kinds of methods were developed, i.e., the robust optimization [21] and distribution-free-based method [22].

With the development of ITS technology, more and more (link/route) travel time samples are available. However, the TTB-based methodology is initially developed to describe those risk-pessimistic routing behaviours. METT is a more risk-pessimistic travel time index since METT individuals care merely about the unreliable events with travel times exceeding the budgeted travel time. Analogous pessimistic behaviours may be common in financial investments where investors have to be enough supersensitive and scrupulous to avoid potential investment-ruins because of frequent and huge transactions. In daily trips, travellers also regard the performance of reliable domain, with travel time less than budget, as an important aspect. In order to capture this kind of routing behaviour, a combined mean travel time (CMTT) index is introduced and explored comprehensively.

2 Combined mean travel time

2.1 Notations

Considering a strongly connected transportation network $G=(N, A)$, where N and A ($a\in A$) denote the sets of nodes and links, respectively. Let R and S denote a subset of N for which travel demand q^{rs} is generated from origin $r\in R$ to destination $s\in S$. The main notations are listed below, and the unlisted ones will be explained when they are firstly encountered.

K	Route index
K^{rs}	Set of routes from node r to s
f_k^{rs}	Flow on route k from node r to s, $\boldsymbol{f}=(\cdots,f_k^{rs},\cdots)^{\mathrm{T}}$
v_a	Flow on link a
α	Confidence level or budgeted reliability
T_k^{rs}	Random travel time on route k from node r to node s
t_k^{rs}	Sample value with respect to T_k^{rs}
T_a	Random travel time on link a
ξ	Artificial constant larger than T_k^{rs}
λ	Combined weight of MBTT to estimate CMTT
$\xi_k^{rs}(\alpha)$	Alpha-reliable PTT/TTB on route k from r to s
$\eta_k^{rs}(\alpha)$	Alpha-reliable METT on route k from r to s
$\varphi_k^{rs}(\alpha)$	Alpha-reliable MBTT on route k from r to s, $\boldsymbol{\varphi}(\boldsymbol{f})=(\cdots,\varphi_k^{rs}(\alpha),\cdots)^{\mathrm{T}}$
$\psi_k^{rs}(\alpha';\lambda)$	Alpha-reliable CMTT on route k from r to s, $\boldsymbol{\psi}(\boldsymbol{f})=(\cdots,\psi_k^{rs}(\alpha),\cdots)^{\mathrm{T}}$
π_α^{rs}	Minimum alpha-reliable CMTT for the routes in K^{rs}, $\boldsymbol{\pi}=(\cdots,\pi_k^{rs}(\alpha),\cdots)^{\mathrm{T}}$
μ_k^{rs}	Expectation/mean of T_k^{rs}
σ_k^{rs}	Standard deviation of T_k^{rs}
δ_{ak}^{rs}	0−1 indicator variable, and $\delta_{ak}^{rs}=1$ if route k uses link a, and 0, otherwise
$f(\cdot)$	Operator of probability density function
$E(\cdot)$	Expectation operator of random variable
$\mathrm{Var}(\cdot)$	Variance operator of random variable
$\mathrm{Pr}(\cdot)$	Operator of cumulative density function
$N(\mu,\sigma)$	Normal distribution indicator with mean μ and standard deviation σ
$\Phi(\cdot)$	Cumulative density function for the standard normal distribution
$\phi(\cdot)$	Probability density function for the standard normal distribution
$\Phi^{-1}(\cdot)$	Inverse of $\Phi(\cdot)$

2.2 Definition of mean-below travel time

Due to the huge gap in the transaction amount and frequency between everyday travels and normal investments, the travellers can not behave the same sensitivity towards unreliability as the financial investors do. Therefore, the reliability indices (e.g., CVaR and VaR) used in finance area may not be well suitable to travel decision-makings. In daily life, one may occasionally suffer from very long duration on an often-used route, yet next time he/she still chooses it. A possible reason is that he/she habitually believes analogous events will emerge on the other routes; it may also attribute to his/her optimistic risk-attitude towards these habitual routes. In other words, he/she believes that the occasional events would not happen to him/her in the limited daily trips. Such kind of overconfidence behaviour was also reported in the studies on the cognitive psychology [23], i.e., people tended to overestimate their own subjective cognitive capacities. Subsequently, a new travel time index named mean-below travel time (MBTT) will be introduced to capture such a kind of risk-optimistic routing behaviour.

Definition 1: The alpha-reliable MBTT of a route is the conditional expectation of the corresponding PTT-below travel times, i.e.,

$$\varphi_k^{rs}(\alpha)=E\left(T_k^{rs}:T_k^{rs}\le\xi_k^{rs}(\alpha)\right),\ \forall k\in K^{rs},\ r\in R,\ s\in S \tag{1}$$

where $\xi_k^{rs}(\alpha)=\min\left\{\xi:\Pr(T_k^{rs}\le\xi)\ge\alpha\right\}$ can be further rewritten as $\xi_k^{rs}(\alpha)=\left\{\xi:\Pr(T_k^{rs}\le\xi)=\alpha\right\}$. Note that $\xi_k^{rs}(\alpha)$ is equivalent to the alpha-reliable TTB when T_k^{rs} distributes normally. So, in the remaining context TTB and PTT will be used interchangeably. Accordingly, Eq. (1) can be formulated as

$$\varphi_k^{rs}(\alpha)=\int_{-\infty}^{\xi_k^{rs}(\alpha)}\frac{t_k^{rs}f(t_k^{rs})}{\Pr(T_k^{rs}\le\xi_k^{rs}(\alpha))}\mathrm{d}t_k^{rs},\ \forall k\in K^{rs},\ r\in R,s\in S \tag{2}$$

MBTT is regarded as a risk-optimistic travel time index since the MBTT-conducted individuals merely care about the performance of PTT-below reliable domain and believe the PTT-excess unreliable cases will not happen to them in limited trips.

Define $\eta_k^{rs}(\alpha)=E\left(T_k^{rs}:T_k^{rs}\ge\xi_k^{rs}(\alpha)\right)$, which is the alpha-reliable METT explored by CHEN and ZHOU [17], the following proposition can be obtained.

Proposition 1: Equation $\varphi_k^{rs}(\alpha)=\frac{1}{\alpha}E(T_k^{rs})-\frac{1-\alpha}{\alpha}\eta_k^{rs}(\alpha)$ holds in general for all routes.

Proof:

Since

$$E(T_k^{rs})=\int_{-\infty}^{+\infty}t_k^{rs}f(t_k^{rs})\mathrm{d}t_k^{rs}=\int_{-\infty}^{\xi_k^{rs}(\alpha)}t_k^{rs}f(t_k^{rs})\mathrm{d}t_k^{rs}+\int_{\xi_k^{rs}(\alpha)}^{+\infty}t_k^{rs}f(t_k^{rs})\mathrm{d}t_k^{rs}=\alpha\int_{-\infty}^{\xi_k^{rs}(\alpha)}\frac{t_k^{rs}f(t_k^{rs})}{\Pr(T_k^{rs}\le\xi_k^{rs}(\alpha))}\mathrm{d}t_k^{rs}+(1-\alpha)\int_{\xi_k^{rs}(\alpha)}^{+\infty}\frac{t_k^{rs}f(t_k^{rs})}{\Pr(T_k^{rs}\ge\xi_k^{rs}(\alpha))}\mathrm{d}t_k^{rs}=\alpha\varphi_k^{rs}(\alpha)+(1-\alpha)\eta_k^{rs}(\alpha)\Rightarrow\varphi_k^{rs}(\alpha)=\frac{1}{\alpha}E(T_k^{rs})-\frac{1-\alpha}{\alpha}\eta_k^{rs}(\alpha),$$
$$\forall k\in K^{rs},r\in R,s\in S$$

According to **Proposition 1**, $\varphi_k^{rs}(1)=E(T_k^{rs})$ is derived, i.e., MBTT changes to be MTT when α=1.

2.3 Estimation of mean-below travel time

In order to estimate the alpha-reliable MBTT, an essential step is to obtain the expectation and variation of route travel time. To focus on the essence of MBTT, the link travel time $T_a(a\in A)$ is assumed to be mutually independent. This assumption is also adopted in the other studies (see Refs. [15−16]). Accordingly, $\mu_k^{rs}=\sum_{a\in A}E(T_a)\delta_{ak}^{rs}$ and $\sigma_k^{rs}=\left[\sum_{a\in A}\mathrm{Var}(T_a)\delta_{ak}^{rs}\right]^{\frac{1}{2}}$ for all routes are promised. In addition, assume the link travel time is bounded. Regardless of the probability distributions of link travel time, applying the Lyapunov condition [24] for the central limit theorem, $T_k^{rs}\sim N\left(\mu_k^{rs},\sigma_k^{rs}\right)$ can be concluded. Accordingly, the alpha-reliable MBTT, defined by Eq. (1), can be estimated as

$$\varphi_k^{rs}(\alpha)=\mu_k^{rs}-\frac{\sigma_k^{rs}}{\sqrt{2\pi}\alpha}\exp\left(-\frac{\left[\Phi^{-1}(\alpha)\right]^2}{2}\right),\ \forall k\in K^{rs},\ r\in R,s\in S \tag{3}$$

The detailed deduction is offered in **Appendix A**. In Eq. (3), the variability of travel time may result from both exogenous and endogenous disturbances, such as stochastic supply, stochastic demand, and etc. The negative scalar before the standard deviation implies MBTT is universally smaller than MTT, making MBTT into a risk-optimistic travel time index.

2.4 Combined mean travel time

As mentioned above, MBTT is a risk-optimistic travel time index reflecting the performance of reliable cases, whereas METT is just the opposite. Hence, MBTT and METT can be regarded as two boundaries (i.e. risk-optimistic and risk-pessimistic) of routing risk-attitudes. In reality, the extreme behaviours conducted by MBTT and METT may be not common. Most travellers consider both reliable aspect and unreliable aspect to make rational and overall decisions. In addition, due to heterogeneous personalities, travellers' routing risk-attitudes should also be various. Accordingly, we suggest using a combined mean travel time (CMTT) index to describe the heterogeneous routing risk-attitudes. Mathematically, the alpha-reliable CMTT here is formulated as the convex combination of alpha-reliable MBTT and METT, i.e.,

$$\psi_k^{rs}(\alpha;\lambda)=\lambda\varphi_k^{rs}(\alpha)+(1-\lambda)\eta_k^{rs}(\alpha)=\mu_k^{rs}+\frac{(\alpha-\lambda)\sigma_k^{rs}}{\sqrt{2\pi}\alpha(1-\alpha)}\exp\left(-\frac{\left[\Phi^{-1}(\alpha)\right]^2}{2}\right),\ \forall k\in K^{rs},r\in R,s\in S \tag{4}$$

where the non-negative scalar $\lambda\in[0,1]$ reflects the degree of risk-optimistic, and a larger value represents a more optimistic risk-attitude. According to Eq. (4), when λ takes 0, α, and 1, $\psi_k^{rs}(\alpha;\lambda)$ returns to the alpha-reliable METT, MTT, and MBTT, respectively. Therefore, CMTT is able to describe various routing risk-attitudes. The formulations of alpha-reliable METT $\eta_k^{rs}(\alpha)$ are listed in Table 1.

Table 1 Routing risk-attitudes and formulations of c for four travel time indices

Alpha-reliable travel time index	Formulation of c in Eq. (5)	Value of coefficient c/Routing risk-attitude		
		α<0.5	α=0.5	α>0.5
MBTT	$-\dfrac{1}{\sqrt{2\pi}\alpha}\exp\left(-\dfrac{\left[\Phi^{-1}(\alpha)\right]^2}{2}\right)$	Negative/ Risk-optimistic	$-\sqrt{\dfrac{2}{\pi}}$/Risk-optimistic	Negative/ Risk-optimistic
PTT [1]/TTB [10]	$\Phi{-}1(\alpha)$	Negative/ Risk-optimistic	0/Risk-neutral	Positive/ Risk-pessimistic
METT [17]	$\dfrac{1}{\sqrt{2\pi}(1-\alpha)}\exp\left(-\dfrac{\left[\Phi^{-1}(\alpha)\right]^2}{2}\right)$	Positive/ Risk-pessimistic	$\sqrt{\dfrac{2}{\pi}}$/Risk-pessimistic	Positive/ Risk-pessimistic
CMTT	$\dfrac{(\alpha-\lambda)}{\sqrt{2\pi}\alpha(1-\alpha)}\exp\left(-\dfrac{\left[\Phi^{-1}(\alpha)\right]^2}{2}\right)$	Negative/ Risk-optimistic if λ>α, 0/Risk-neutral if λ=α, Positive/ Risk-pessimistic if λ<α		

2.5 Comparison analyses

In this sub-section, the routing risk-attitudes conducted by above mentioned four travel time indices (i.e. MBTT, TTB/PTT, METT and CMTT) within a general framework will be compared. For this, a general alpha-reliable travel time index is defined, i.e.,

$$G_k^{rs}(\alpha)=\mu_k^{rs}+c\sigma_k^{rs},\ \forall k\in K^{rs}, r\in R, s\in S \tag{5}$$

where c is an underdetermined coefficient, which reflects a traveller's risk-attitude towards variability. Risk-optimism, risk-neutralism and risk-pessimism are specified when c takes negative value, zero, and positive value, respectively. Table 1 lists the formulations of c for diverse travel time indices.

Table 1 indicates that MBTT and METT are purely risk-optimistic and risk-pessimistic travel time indices, respectively. TTB seems to be able to express both risk-optimism and risk-pessimism. However, TTB needs to impose an unreliable reliability (e.g., smaller than 0.5) to formulate the risk-pessimism. This doesn't coincide with the reality since everyone usually preserves a relatively high reliability (e.g., larger than 0.8). Hence, TTB is actually a risk-pessimistic travel time index. By contrast, CMTT is capable of capturing various risk-attitudes within rational reliability levels. Therefore, CMTT is a complement and effective travel time index for modeling routing behaviours and implementing sensitivity analysis for risk-attitudes.

3 Combined mean traffic equilibrium model and solution algorithm

3.1 Combined mean traffic equilibrium model

Let $\boldsymbol{\delta}=\left[\delta_{ak}^{rs}\right]$ be the route-link incidence matrix. The feasible region $\boldsymbol{\Omega}_1$ described in terms of route flow can be described as follows:

$$\sum_k f_k^{rs}=q^{rs},\ \forall r\in R, s\in S \tag{6}$$

$$v_a=\sum_r\sum_s\sum_k f_k^{rs}\delta_{ak}^{rs},\ \forall a\in A \tag{7}$$

$$f_k^{rs}\ge 0,\ \forall k\in K^{rs}, r\in R, s\in S \tag{8}$$

where Eq. (6) is the conservation constraint of travel demand; Eq. (7) is a definitional conservation constraint between link flow and route flows; Eq. (8) promises route flow to be non-negative.

As mentioned above, CMTT individuals aim to minimize the alpha-reliable CMTTs. These collective self-routing actions will finally reach a long-term habitual traffic equilibrium, which can be formulated as the following complementary expression.

$$f_k^{rs*}\left(\psi_k^{rs*}(\alpha)-\pi_\alpha^{rs}\right)=0,\ \psi_k^{rs*}(\alpha)-\pi_\alpha^{rs}\ge 0,$$
$$f_k^{rs*}\ge 0, \forall k\in K^{rs}, r\in R, s\in S \tag{9}$$

The above expression is the alpha-reliable combined mean traffic equilibrium (CMTE) model. It implies $\psi_k^{rs*}(\alpha)=\pi_\alpha^{rs}$ if $f_k^{rs*}>0$ and $\psi_k^{rs*}(\alpha)\ge\pi_\alpha^{rs}$ if $f_k^{rs*}=0$, i.e., the used routes possess equal and minimum CMTT, while the unused ones possess the higher CMTTs. This equilibrium condition formulated in Eq. (9) is a Wardropian CMTE.

Proposition 2: The Wardropian CMTE expressed by Eq. (9) is equivalent to the following variational inequality problem (VIP).

$$\sum_{r\in R}\sum_{s\in S}\sum_{k\in K^{rs}}\psi_k^{rs*}(\alpha)\left(f_k^{rs}-f_k^{rs*}\right)\ge 0\ \forall \boldsymbol{f}\in\boldsymbol{\Omega}_1 \tag{10}$$

with vector version expressed as

$$\boldsymbol{\psi}(\boldsymbol{f}^*)^{\mathrm{T}}\left(\boldsymbol{f}-\boldsymbol{f}^*\right)\ge 0,\ \forall \boldsymbol{f}\in\boldsymbol{\Omega}_1 \tag{11}$$

Proof: Proposition 2 can be guaranteed by the equivalent condition between nonlinear complementary problem (NCP) and VIP (see FACCHINEI and PANG [25]).

Proposition 3: Assume CMTT is positive and continuous, and then the above CMTE model has at least one optimal solution.

Proof: According to **Proposition 2**, we only need to consider its equivalent VIP (11). Since $\boldsymbol{\Omega}_1$ is nonempty and convex, and the mapping $\psi(\boldsymbol{f})$ is continuous. Hence, the VIP (Eq. (11)) has at least one solution (see NAGURNEY [26]).

Consider the following link-route relationship

$$T_k^{rs} = \sum_{a \in A} T_a \delta_{ak}^{rs},\ \forall k \in K^{rs}, r \in R, s \in S \tag{12}$$

and the physical meaning of CMTT, it is reasonable to make the positive and continuous assumptions for CMTT in the above propositions. Therefore, the validity of VIP (Eq. (11)) and the existence of optimal solution are ensured.

3.2 Solution algorithm

CHEN and ZHOU [17] applied the modified alternating direction (MAD) algorithm [27] to solve the mean-excess traffic equilibrium (METE) model. The essence of MAD algorithm is transforming the linear constrained VIP into a uniform pattern, which helps to make the subsequent projection process much easier. Due to the economic meaning of Lagrangian multiplier vector $\boldsymbol{\pi}$ (i.e., the realized minimum cost vector between the O-D pairs) for the demand conservation constraints [25], VIP (Eq. (11)) can be reformulated as an equivalent VIP ($\boldsymbol{F}$, $\boldsymbol{\Omega}$) with closed and unified pattern as follows:

Finding $\boldsymbol{u}^* \in \boldsymbol{\Omega}$, such that

$$\boldsymbol{F}(\boldsymbol{u}^*)^{\mathrm{T}}(\boldsymbol{u}-\boldsymbol{u}^*) \geq 0,\ \forall \boldsymbol{u} \in \boldsymbol{\Omega} \tag{13}$$

where

$$\boldsymbol{u} = \begin{pmatrix} \boldsymbol{f} \\ \boldsymbol{\pi} \end{pmatrix}, \boldsymbol{F}(\boldsymbol{u}) = \begin{pmatrix} \psi(\boldsymbol{f}) - \boldsymbol{\Lambda}^{\mathrm{T}}\boldsymbol{\pi} \\ \boldsymbol{\Lambda}^{\mathrm{T}}\boldsymbol{f} - \boldsymbol{Q} \end{pmatrix}, \boldsymbol{\Omega} = \boldsymbol{R}_+^m \times \boldsymbol{R}_+^w \tag{14}$$

where $\boldsymbol{\Lambda}$ is the route-OD incidence matrix; $\boldsymbol{R}_+^m$ and $\boldsymbol{R}_+^w$ represent the m-dimensional and w-dimensional non-negative Euclidean spaces, respectively, m and w are the total numbers of routes and OD pairs, respectively. $\boldsymbol{Q}$ is a vector grouping travel demand q^{rs}.

The extra-gradient method [28], rather than the MAD algorithm [27], is employed here for two reasons. Firstly, the convergence condition for extra-gradient method is less strict [29]. Secondly, this work is focused on the illustration of the central idea of CMTT, and the extra-gradient method is adequate.

4 Numerical example

4.1 Network characteristics

The following example was conducted to perform the algorithm and characterize the CMTE model. For this reason, a hypothetical transportation network (see Fig. 1) with six paths from origin 1 to destination 10 was applied.

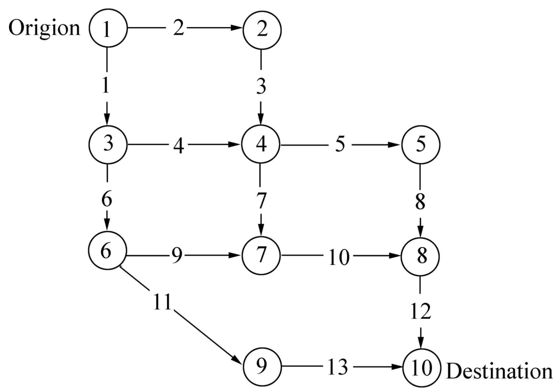

Fig. 1 Test network

BPR link cost function was adopted in the subsequent numerical simulation.

$$T_a(v_a) = t_a^0 \left[1 + \beta \left(\frac{v_a}{C_a} \right)^n \right],\ \forall a \in A \tag{15}$$

where t_a^0 was the free-flow travel time on link a, C_a was the link capacity, and β and n are the predefined parameters. Recently, based on the BPR formulation, many investigations were explored to study the stochastic traffic equilibrium under various disturbances, such as variation for link capacity [9−10] and travel demand [30]. Later, the variations considering both supply aspect and demand aspect were explored [15−16]. For simplicity and without loss of generality, only the link capacity degradation was considered here. Assumed the link capacities distributed uniformly, i.e., $C_a \sim U(\theta_a \overline{C}_a,\ \overline{C}_a)$, $\forall a \in A$, where $\overline{C}_a$ and θ_a (vector $\boldsymbol{\Theta} = (\cdots, \theta_a, \cdots)$) are the design capacity and its degradable degree of link a, respectively. Then, the expectations and variations of link travel times could be deduced (see Ref. [9] for detail).

$$E(T_a) = t_a^0 + \beta t_a^0 v_a^n \frac{(1-\theta_a^{1-n})}{\overline{C}_a^n (1-\theta_a)(1-n)},\ \forall a \in A \tag{16}$$

$$\mathrm{Var}(T_a) = \beta^2 (t_a^0)^2 v_a^{2n} \left\{ \frac{(1-\theta_a^{1-2n})}{\overline{C}_a^{2n}(1-\theta_a)(1-2n)} - \left[\frac{(1-\theta_a^{1-n})}{\overline{C}_a^n (1-\theta_a)(1-n)} \right]^2 \right\},\ \forall a \in A \tag{17}$$

where β=0.15 and n=4. The standard values of other parameters are listed in Table 2.

Table 2 Network parameters and information

Link	t_a^0 /min	$\overline{C}_a$ /(pcu·h^{-1})	Link	t_a^0 /min	$\overline{C}_a$ /(pcu·h^{-1})
1	10	1 000	8	10	1 000
2	10	1 000	9	4	1 500
3	10	1 000	10	10	2 000
4	5	1 600	11	30	1 000
5	10	1 000	12	10	1 000
6	5	1 000	13	10	1 000
7	10	1 000	—	—	—

4.2 Numerical results

Based on Fig. 1, three experimental scenarios were designed.

Scenario 1: Convergence analysis of the algorithm.

Scenario 2: Sensitivity analysis (SA) for risk-attitudes under different demand levels.

Scenario 3: SA for the risk-attitudes under different capacity degradation levels.

The numerical parametric values of the above three scenarios are listed in Table 3, where Q denotes the travel demand and vector $\boldsymbol{\Theta}$ denotes the capacity degradable degree. For simplicity, $\boldsymbol{\Theta}$=0.8 meant $\theta_a = 0.8,\ \forall a \in A$.

Table 3 Numerical scenarios

Scenario No.	α	Q/(pcu·h^{-1})	$\boldsymbol{\Theta}$	λ
1	0.9	4 000	0.8	0.5
2	0.9	[3 000:1 000:4 000]	0.8	[0.0:0.1:1.0]
3	0.9	4 000	[0.6:0.1: 0.9]	[0.0:0.1:1.0]

The numerical results for Scenarios 1, 2 and 3 are displayed in Figs. 2, 3 and 4, respectively. In Figs. 2 and 3, the indicator λ_0 denotes that the travelers across the

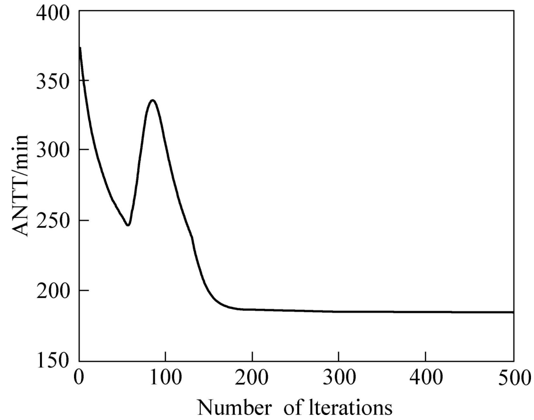

Fig. 2 Convergence curve

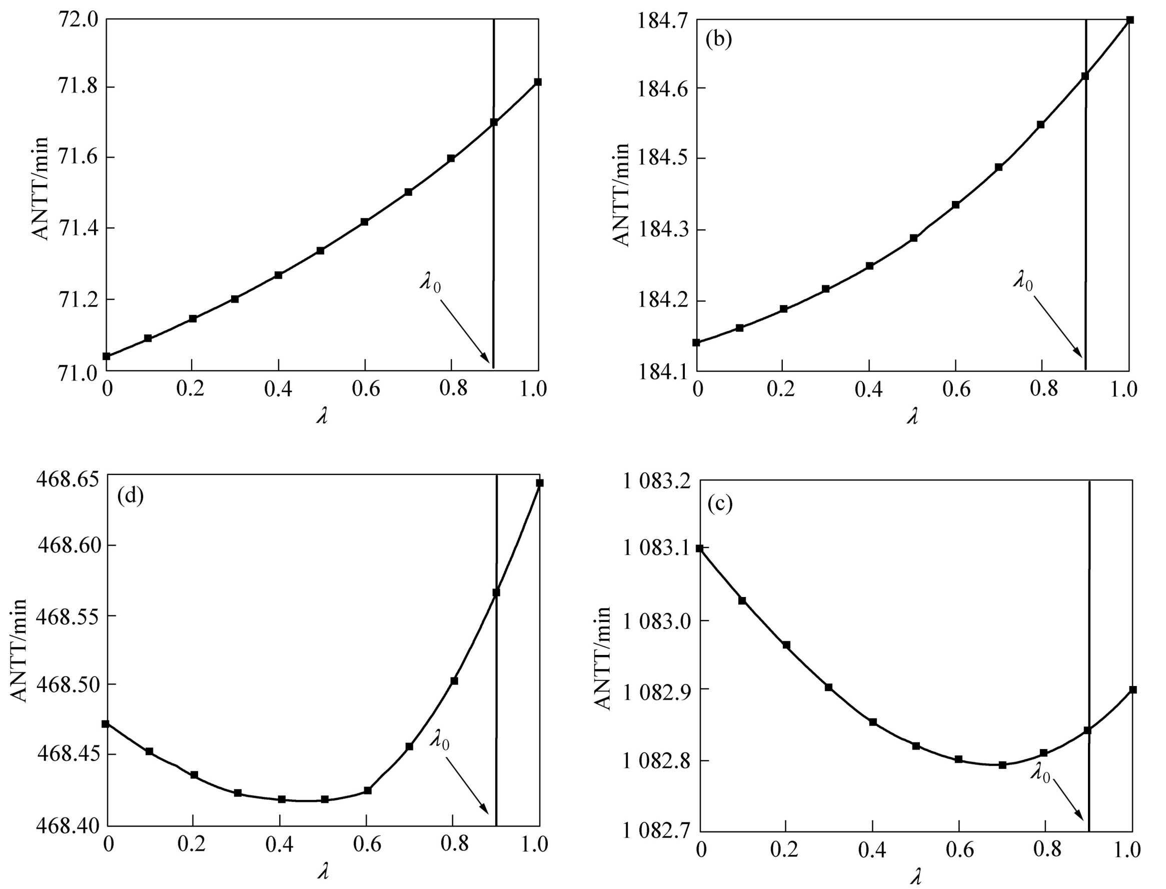

Fig. 3 SA for travelers' risk-attitudes under different demand levels: (a) Q=3 000 pcu/h; (b) Q=4 000 pcu/h; (c) Q=5 000 pcu/h; (d) Q=6 000 pcu/h

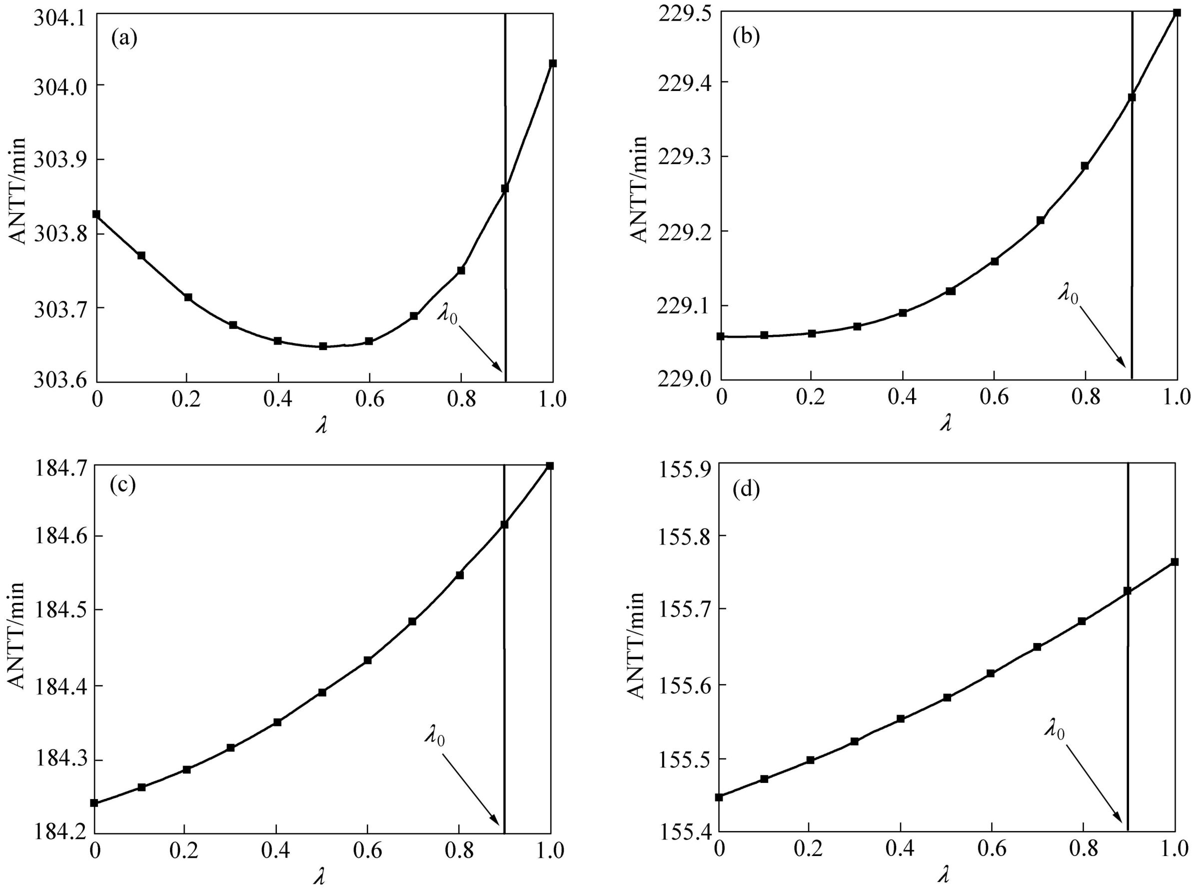

Fig. 4 SA for travelers' risk-attitudes under different degradable levels of capacity: (a) Θ=0.6; (b) Θ=0.7; (c) Θ=0.8; (d) Θ=0.9

network are risk-neutral at point $\lambda=\lambda_0$. Figure 2 performs the evolution of the average network travel time (ANTT). Note that, in the following context, all units will be omitted to simplify the presentation.

Figure 2 shows that ANTT fluctuates obviously in the beginning 100 iterations, and decreases steadily until it converges to the required accuracy (equal to 1.0×10^{-4}) as iteration proceeds. The whole computation process, executed on a microcomputer with CPU frequency of 1.5 GHz and 512 MB RAM, consumes about 2 s. Consequently, the current solution algorithm is adequate to the following simulations.

Figure 3 displays the effects of travellers' risk-attitudes on the network performance (measured by ANTT) under different demand levels.

Figure 3 indicates that: 1) ANTT surges to about 1 083 from about 72 as Q increases from 3 000 to 6 000; 2) Under a given demand level, the variation of ANTT is not obvious as λ increases, which provides the evidence for the existence of risk-optimistic routing behaviours, because risk-optimists do not feel apparent loss from their optimistic routing behaviour in the long run; 3) Overall, modest risk-pessimist helps to alleviate the network congestion and save personal MTT. In addition, when the demand level is low (e.g., $Q\leq4$ 000 pch/h), METT travellers enjoy less personal MTT than MBTT ones, whereas it is just the opposite for high demand level cases (e.g., $Q\geq5$ 000 pch/h). This demonstrates that the optimists do not always suffer from the most loss in daily travels, and this phenomenon is especially obvious in the high demand cases.

Figure 4 displays the effects of travellers' risk-attitudes on the network performance (measured by ANTT) under different capacity degradation levels.

Figure 4 shows some analogous phenomena as Fig. 3 does. 1) ANTT decreases to about 156 from about 304 as $\boldsymbol{\Theta}$ increases from 0.6 to 0.9; 2) Under a specific capacity degradation level, unobvious changes happen on ANTT as λ increases, which further verifies the existence of risk-optimistic routing behaviours; 3) Generally, modest risk-pessimist contributes to alleviating the network congestion and thrift of private MTT. Moreover, when the capacity degradation is relatively low (e.g., $\boldsymbol{\Theta}\geq0.8$), METT travellers enjoy less private MTT than MBTT travellers, while the superiority decreases in the higher degradation level cases (e.g., $\boldsymbol{\Theta}\leq0.7$).

5 Conclusions and future research

1) Compared with the existing travel time indices (e.g., PTT/TTB and METT), CMTT is capable of capturing various routing risk-attitudes, spreading from

risk- optimism to risk-pessimism.

2) Numerical study demostrates the efficiency of algorithm. It can also be found that, for low and modest demand and capacity degradation levels, the risk-pessimistic routing behaviour is more beneficial to both network and individuals. However, as the demand and capacity degradation levels increase, the superiority of risk-pessimism becomes more and more obvious.

3) Many further works are worthy of exploring based on the proposed CMTE model. More efficient algorithms for generating the CMTT routes and solving the proposed model need to be developed and tested on a large-scale network. Empirical studies need to be performed to obtain a better understanding of the travellers' routing risk-attitudes under the traffic information service. To incorporate multiple risk-attitudes into the current CMTE model is also a valuable extension. Finally, by factoring various sources of travel time variation and general cost into the proposed modelling approach, the CMTE model will be more realistic.

Appendix A: Estimating MBTT

Note that some super-scripts and sub-scripts will be omitted to facilitate the following estimation. Since $T \sim N(\mu, \sigma)$ and $\alpha = \Pr\left(T \le \xi(\alpha)\right)$, the moment generating function $M_{T:T\le\xi(\alpha)}(z)$ can be formulated as follows:

$$M_{T:T\le\xi(\alpha)}(z) = \int_{-\infty}^{\xi(\alpha)} \mathrm{e}^{tz} \frac{1}{\Pr\left(T \le \xi(\alpha)\right)} \frac{1}{\sqrt{2\pi}\sigma} \mathrm{e}^{-\frac{(t-\mu)^2}{2\sigma^2}} \mathrm{d}t \tag{A1}$$

For brevity, we rewrite $M_{T:T\le\xi(\alpha)}(z)$ as $M(z)$ in the remaining deduction. Let $T = \mu + \sigma S$, where $S \sim N(0, 1)$, then Eq. (A1) can be further deduced as

$$\begin{aligned} M(z) = {} & \frac{1}{\alpha}\mathrm{e}^{\mu z}\int_{-\infty}^{\frac{\xi(\alpha)-\mu}{\sigma}} \mathrm{e}^{\sigma s z} \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{s^2}{2}} \mathrm{d}s = \\ & \frac{1}{\alpha}\mathrm{e}^{\mu z}\int_{-\infty}^{\frac{\xi(\alpha)-\mu}{\sigma}} \mathrm{e}^{\frac{\sigma^2 z^2}{2}} \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{(s-\sigma z)^2}{2}} \mathrm{d}s = \\ & \frac{1}{\alpha}\mathrm{e}\left(\mu z + \frac{\sigma^2 z^2}{2}\right)\int_{-\infty}^{\frac{\xi(\alpha)-\mu}{\sigma}} \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{(s-\sigma z)^2}{2}} \mathrm{d}s. \end{aligned}$$

Let $Y = S - \sigma z$, then

$$\begin{aligned} M(z) = {} & \frac{1}{\alpha}\mathrm{e}\left(\mu z + \frac{\sigma^2 z^2}{2}\right)\int_{-\infty}^{\frac{\xi(\alpha)-\mu}{\sigma}-\sigma z} \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{y^2}{2}} \mathrm{d}y = \\ & \frac{1}{\alpha}\mathrm{e}\left(\mu z + \frac{\sigma^2 z^2}{2}\right)\Phi\left(\frac{\xi(\alpha)-\mu}{\sigma} - \sigma z\right) \end{aligned} \tag{A2}$$

Taking the first-order derivative of Eq. (A2) with respect to z, we obtain

$$\begin{aligned} M'(z) = {} & \frac{1}{\alpha}(\mu + \sigma^2 z)\exp\left(\mu z + \frac{\sigma^2 z^2}{2}\right)\Phi\left(\frac{\xi(\alpha)-\mu}{\sigma} - \right. \\ & \left. \sigma z\right) - \frac{\sigma}{\alpha}\exp\left(\mu z + \frac{\sigma^2 z^2}{2}\right)\phi\left(\frac{\xi(\alpha)-\mu}{\sigma} - \sigma z\right) \end{aligned} \tag{A3}$$

where $\Phi(y) = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{x^2}{2}} \mathrm{d}x$, $\phi(y) = \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{y^2}{2}}$. Let z=0 in Eq. (A3) and define $T = \mu + \sigma S$, because $\alpha \equiv \int_{-\infty}^{\xi(\alpha)} f(t)\mathrm{d}t$, we have $\alpha \equiv \int_{-\infty}^{\frac{\xi(\alpha)-\mu}{\sigma}} \phi(s)\mathrm{d}s \equiv \Phi \cdot \left(\frac{\xi(\alpha)-\mu}{\sigma}\right)$ and

$$M'(0) = \mu - \frac{\sigma}{\alpha}\phi\left(\frac{\xi-\mu}{\sigma}\right) \tag{A4}$$

Let $\Phi^{-1}(\alpha) = \frac{\xi-\mu}{\sigma}$, we finally derive the alpha-reliable mean-below travel time (MBTT) index as follows.

$$\varphi(\alpha) = M'(0) = \mu - \frac{\sigma}{\sqrt{2\pi}\alpha}\exp\left(\frac{\left[\Phi^{-1}(\alpha)\right]^2}{2}\right) \tag{A5}$$

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