Multipeaked fundamental and vortex solitons in azimuthally modulated Bessel lattices

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We demonstrate the existence of multipeaked fundamental and vortex solitons in defocusing Kerr media with an imprinted azimuthally modulated Bessel lattice. Multipeaked solitons emanating from the fundamental linear lattice modes are stable in their entire existence domains. The number of soliton peaks is determined by the azimuthal index. Multipeaked vortex solitons with high topological charges in lattices exhibit special amplitude and phase distributions that resemble those of azimuthons. We reveal that the "stability rule" for vortex solitons in defocusing Kerr media is exactly opposite to that in focusing media. Multipeaked vortex solitons we obtained may provide a missing link between the radially symmetric vortices and nonrotating soliton clusters.

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1. INTRODUCTION

Optical solitons are spatially localized, nondiffracting modes existing in nonlinear optical media. Recent progress in theoretical and experimental study opens up many possibilities in the study of novel types of solitons [1,2]. Special attention is paid to solitons in optically induced lattices these days because the lattice can behave as an effective and tunable potential to confine and capture optical radiation [1–3]. In photorefractive materials, harmonic lattices are usually created by interfering several plane waves whose intensity and intersection angles determine the lattice depth and period [3]. Stable fundamental [4], dipole [5], vortex [4,6,7], necklace-like [8], higher-band solitons [9], etc., can be supported by two-dimensional harmonic lattices.

Besides the harmonic lattice, there is another important optical lattice with unique symmetry, the Bessel lattice, which can be created by nondiffracting Bessel beams with cylindrical symmetry. Kartashov and co-workers systematically investigated the dynamics of various types of solitons supported by Bessel lattices, including multipole-mode solitons [10], ring-profile vortex solitons, and [11], spatiotemporal solitons [12]. Necklace [13] and broken ring solitons [14] can also be trapped stably in different order Bessel lattices. Solitons trapped at different lattice rings can be set into controlled rotation inside each ring [15,16]. Discrete solitons and soliton rotation in Bessel-like lattices were observed [17]. For a review of the early works, see [18].

Interestingly, Bessel lattices with azimuthal modulation are possible [19,20]. Such lattices resemble highly nonlinear microstructured fibers [21], and may be realized in experiment by several incoherent Bessel beams with different intensities and orders [19]. The complex lattices can also be created in photorefractive crystals by the phase-imprinting technique [21,22]. The azimuthally modulated lattices exhibit several discrete guiding channels of linear refractive index. Stable soliton complexes and azimuthal switching in focusing cubic media with modulated Bessel lattices were reported in [19]. Neighboring components in a soliton complex are out of phase. Ring-shaped and single-site solitons were observed in azimuthally modulated lattices [21,22]. Especially, by using group-theory techniques, Kartashov *et al.* derived a general "charge/stability rule" for vortex solitons supported by the azimuthal Bessel lattice [20].

In [23], Desyathikov and his co-workers introduced a novel class of spatially localized self-trapped ringlike singular optical beams in focusing cubic and saturable media, the so-called "azimuthons." The amplitude of such a state is a spatially localized ring modulated azimuthally, and the phase of the azimuthon is a staircase function of the polar angle. This concept provided an important missing link between the radially symmetric vortices and rotating soliton clusters [24]. Following this work, stable azimuthons in nonlocal nonlinear media were found when the nonlocality parameter exceeds a certain threshold value [25,26]. Families of azimuthons can be found by considering internal modes of classical vortex solitons [27]. Two-dimensional azimuthons and vector azimuthons were also predicted in Bose–Einstein condensates confined by a parabolic trap [28,29].

Thus far, stable azimuthons or azimuthonlike solitons in media with local nonlinear responses, have not been reported, to our knowledge. In this paper, we show that azimuthally modulated Bessel lattices in defocusing cubic media can support two types of multipeaked solitons whose amplitude distribution are similar to those of azimuthons. The existence and stability properties of vortex solitons with special amplitude and phase distributions are discussed in detail. When the lattice is not modulated, multipeaked vortex solitons will degenerate to the conventional radially symmetric vortex solitons [11]. On the other hand, if the phase of multipeaked vortex solitons is removed, it will exhibit as a discrete soliton cluster. It is the combination of nontrivial phase and lattice confinement that affords the existence of multipeaked vortex solitons. Thus, the multipeaked vortex solitons we obtained provide a missing link between the radially symmetric vortices and nonrotating soliton clusters, although they break the radial symmetry due to the potential we used. Similar to the "azimuthons" stated in [23], the nonlinear localized modes we discuss can also be attributed to the two contributions induced by the internal energy flow and the modulated beam. In sharp contrast to the cases in focusing cubic media [20], we reveal that the "stability rule" in defocusing cubic media is quite the reverse. The result is in good agreement with the conclusion given by [30,31], where the stability of discrete vortex solitons supported by hexagonal photonic lattices in focusing media is opposite to the stability in the defocusing one, although the discussions were limited to the single-charged and double-charged vortex solitons.

2. MODEL

We consider beam propagation along the z axis in defocusing cubic media with an imprinted transverse refractive index modulation. The dynamics of the nonlinear modes supported by such a scheme can be described by the nonlinear Schrödinger equation for the normalized complex field q:

$$i\frac{\partial q}{\partial z}+\frac{1}{2}\left(\frac{\partial^2 q}{\partial x^2}+\frac{\partial^2 q}{\partial y^2}\right)-|q|^2q+pR(x,y)q=0. \quad (1)$$

Here the longitudinal z and transverse x, y coordinates are scaled to the diffraction length and input beam width, respectively. The parameter p describes the lattice depth. The profile of the modulated lattice is given by $R(x,y)=J_{n_J}^2[(2b_{\mathrm{lin}})^{1/2}r]\cos^2(n\phi)$, where n_J denotes the order of the Bessel function, $r=(x^2+y^2)^{1/2}$ is the radius, ϕ is the azimuthal angle, n stands for the azimuthal index, and b_{lin} defines the transverse lattice scale. Typical transverse linear refractive index modulation induced by the first-order Bessel lattices with azimuthal index $n=2$ and 5 are shown in Figs. 1(a) and 1(b), respectively. The local lattice maxima situated closer to the lattice center are more pronounced than others. The number of guiding channels in the main ring is given by $2n$.

Such lattices were proposed by Kartashov *et al.* in [19,20] and created by Fischer *et al.* [21,22]. Experimentally, Eq. (1) can be realized by launching a modulated Bessel beam into a photorefractive crystal in the ordinary polarization direction and a soliton beam in the extraordinary polarization direction [10]. In the particular case of optical lattice induction in strontium-barium niobate crystal biased with dc electric field of $\sim 10^5\,\mathrm{V/m}$, for laser beams with $10\,\mu\mathrm{m}$, the propagation distance $z\sim 1$ corresponds to 1 mm of actual crystal length, while amplitude $q\sim 1$ corresponds to peak intensity of about $50\,\mathrm{mW/cm^2}$ [20].

Note that Eq. (1) can also be treated as Gross–Pitaevskii equation for a two-dimensional Bose–Einstein condensate with repulsive interatomic interactions trapped in an optical lattice created by an azimuthally modulated Bessel beam.

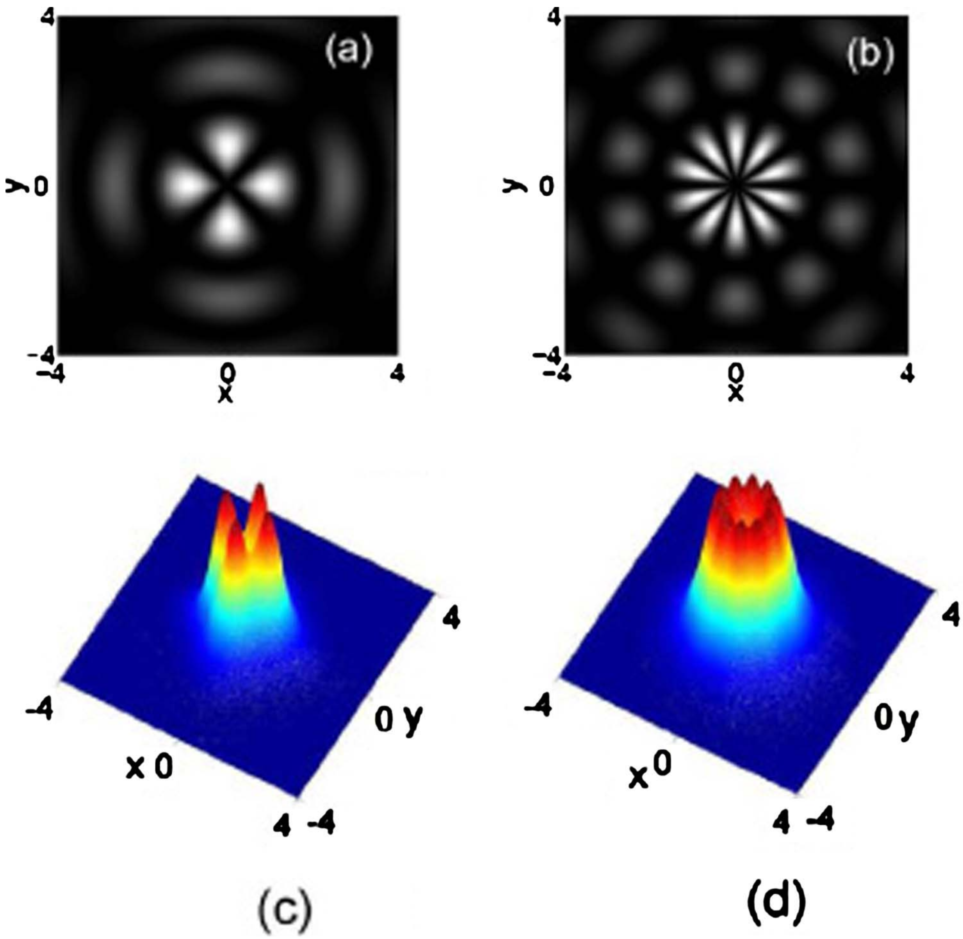

Fig. 1. (Color online) First-order Bessel lattices with azimuthal index (a) $n=2$ and (b) $n=5$. Profiles of the first linear modes with (c) $n=2$ and (d) $n=5$. All quantities are plotted in dimensionless units.

Equation (1) conserves several quantities, including the power: $U = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|q|^2 dxdy$.

We search for stationary solutions of Eq. (1) in the form of $q(x,y,z) = [w_r(x,y) + iw_i(x,y)]\exp(ibz)$, where w_r and w_i are real and imaginary parts of the solution profiles and b is a nonlinear propagation constant. The twisted phase structure of the stationary solutions can be defined by $m = \oint \arctan[w_i(x,y)/w_r(x,y)]/2\pi$, where m is the so-called "topological charge" of vortex solitons. Obviously, the multipeaked fundamental solitons only have nonzero w_r. Substituting the expression into Eq. (1), we obtain

$$\frac{1}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)w_{r,i} - bw_{r,i} - (w_r^2 + w_i^2)w_{r,i} + pRw_{r,i} = 0. \quad (2)$$

The soliton profiles are found numerically by a two-dimensional relaxation algorithm. In numerical calculations, because one can use scaling transformation $q(x,y,z) \to \chi q(\chi x, \chi y, \chi^2 z, \chi^2 p)$ to obtain various families of lattice solitons from a given family [15], we fix $b_{\mathrm{lin}} \equiv 2$ and vary b, p, and n without loss of generality.

To elucidate the stability properties of solitons, we search for perturbed solutions of Eq. (1) in the form $q(x,y,z) = [w_r + iw_i + (u_r + iu_i)\exp(\lambda z)]\exp(ibz)$, where u_r and u_i are the real and imaginary parts of the perturbations, respectively. Substituting the perturbed solution into Eq. (1) and linearizing $u_{r,i}$ around $w_{r,i}$ yield a system of coupled Schrödinger-type equations for perturbation components $u_{r,i}$:

$$\begin{aligned} \pm\lambda u_{i,r} = {} & \frac{1}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)u_{r,i} - bu_{r,i} + pRu_{r,i} \\ & - (3w_{r,i}^2 + w_{i,r}^2)u_{r,i} - 2w_r w_i u_{i,r}, \end{aligned} \quad (3)$$

where $u_{r,i}$ may grow with a complex rate λ during the propagation of solitons. The eigenfunctions $u_{r,i}$ and eigenvalues λ can be solved numerically. The solitons are stable only when all real parts of λ equal zero.

3. MULTIPEAKED FUNDAMENTAL SOLITONS

Before we discuss the dynamics of localized nonlinear modes, it is important to understand the origin of such nonlinear modes. After removing the nonlinear term in Eq. (2), the linear equation has infinite eigenvalues and the corresponding linear eigenmodes. Nonlinear modes bifurcate from these linear modes when the nonlinearity cannot be ignored. Fundamental solitons always bifurcate from the first linear modes and higher-order solitons associate with the other linear modes. Corresponding to the azimuthal Bessel lattices shown in Figs. 1(a) and 1(b), we plot the first eigenmodes of the linearized equation of Eq. (2) in Figs. 1(c) and 1(d). The profiles of linear modes possess several amplitude peaks covering on a constitutive ringlike substrate; thus, they look like azimuthons. Such linear modes intuitively reveal the possible profiles of nonlinear modes in a nonlinear system.

Now, we address the properties of the multipeaked fundamental solitons in the modulated Bessel lattice. For the sake of simplicity, we select the first-order Bessel lattice with different azimuthal index as a linear guide for the laser beam. For azimuthal index $n = 2$, the power of multipeaked solitons is a monotonically decreasing function of the propagation constant [Fig. 2(a)]. It approaches to infinity at $b \to 0$ and vanishes at $b \to b_{\mathrm{co}}$, where b_{co} stands for the upper cutoff of the propagation constant. Comparing the power curves at $p = 10$ and 20, one finds that, for a fixed propagation constant, the deeper the lattice is modulated, the higher the power will be. The existence domain of multipeaked solitons expands with the growth of lattice depth [Fig. 2(b)]. We did not find an increasing b_{co} branch corresponding to the decreasing b_{co} branch, which occurs in focusing media for soliton complex when the lattice is shallow [19].

An example of multipeaked fundamental solitons marked in Fig. 2(a) is shown in Fig. 2(c). The complex configuration resembles the amplitude distribution of azimuthons mentioned in [23]. The four bright peaks of solitons reside only in the four guiding channels of the azimuthal Bessel lattices. Multipeaked solitons expand to the outer lattice rings at small propagation constants and reside on the main guiding lattice ring at large propagation constants. The profiles of multipeaked solitons do not cross the transverse plane. That is to say, multipeaked solitons here are fundamental solitons that bifurcate from the first linear modes of the linearized equation of Eq. (2). The local minima of amplitude profile on the lattice ring does not equal zero, which differs from the lattice distribution.

To shed more light on the multipeaked solitons in the modulated Bessel lattices with different azimuthal indices, we also studied the existence of multipeaked solitons in the first-order lattices with azimuthal index $n = 3\ldots 8$. The basic properties, such as power or existence domain, are similar to those of $n_J = 1$ and $n = 2$. The difference between the local maxima and minima of amplitude decreases with the growth of azimuthal index n. A multipeaked soliton approaches to a ring-profile distribution when we further increase azimuthal index n. However, the profiles of multipeaked solitons still exhibit special amplitude distributions when the lattice order is increased.

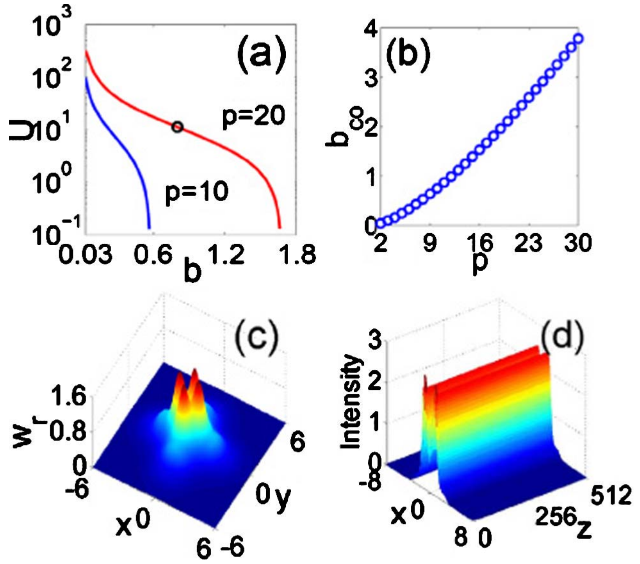

Fig. 2. (Color online) Properties of multipeaked fundamental solitons in the first-order Bessel lattices with azimuthal index $n = 2$. (a) Power versus propagation constant. (b) Propagation constant cutoff b_{co} versus lattice depth p. (c) Profile of soliton marked by circle in (a). (d) Stable propagation of (c), cut of intensity distribution at $y = 0$ is shown. White noise $\sigma_{\mathrm{noise}}^2 = 0.01$ was added into the initial input.

In focusing cubic media, one- and two-dimensional lattice soliton configurations can be stable only when the field changes sign between neighboring channels. The main guiding ring of an nth Bessel lattice can support stable soliton complexes formed by $2n$ out-of-phase bright spots [19]. On the contrary, multipeaked solitons carrying $2n$ in-phase bright peaks, which are found in defocusing cubic media modulated by azimuthal Bessel lattices, are stable in their entire existence domain. According to the Vakhitov–Kolokolov stability criterion, which is valid for fundamental solitons, the multipeaked solitons in defocusing media are all stable since $dU/db < 0$. To verify this prediction, we solved the coupled eigenequations [Eqs. (3)] for varying lattice depth, lattice order, and azimuthal index, and found that all real parts of λ equal zero. We also conduct extensive numerical propagation simulations of multipeaked solitons with different parameters by the split-step Fourier algorithm. The perturbed input condition of Eq. (1) is $q|_{z=0} = [w_r(x,y) + iw_i(x,y)][1 + \sigma_{\mathrm{noise}}\rho(x,y)]$, where $w(x,y)$ describes the stationary solution, $\rho(x,y)$ is a random function with Gaussian distribution, and variance $\sigma_{\mathrm{noise}} = 0.1$. A stable propagation of multipeaked solitons is displayed in Fig. 2(d). The white noise added into the initial input radiates away soon and the soliton reemerges in its unperturbed shape after a short propagation distance.

To further understand the characteristic properties of multipeaked solitons, we studied the existence of multipeaked solitons in different order Bessel lattices modulated azimuthally. Various families of multipeaked solitons in lattices with different order n_J and different azimuthal index n were found. It is worth mentioning that multipeaked solitons with odd number peaks can be supported by lattices with azimuthal index $n = 1.5$, 2.5, etc. Such solitons consisting of $2n$ peaks have been proved to be stable by various ways mentioned above and propagation simulation results at $z = 512$ are plotted in Fig. 3. We should note that stable solitons with odd number peaks are very rare, with the only exception found in nonlocal nonlinear media [26]. Deeper lattices are needed for supporting multipeaked solitons with more peaks. The difference between the local maxima and minima around the lattice ring decreases with the growth of peak number $2n$.

4. MULTIPEAKED VORTEX SOLITONS

Mathematically, the refractive index modulation contributed by the modulated Bessel lattice increases linearly with the growth of lattice depth. However, this relationship cannot hold for a practical crystal when the lattice is modulated very deep. Thus, the practical realization of stable vortex solitons with higher topological charges becomes infeasible by solely increasing the lattice depth of the first-order lattice to a very large value. Fortunately, the higher-order Bessel lattice can

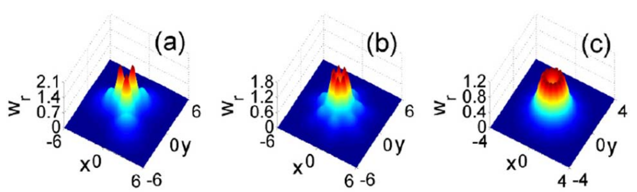

Fig. 3. (Color online) Stable propagation results of multipeaked solitons in the first-order Bessel lattices with odd azimuthal index. (a) $n = 1.5$, $p = 25$, $b = 0.9$. (b) $n = 3.5$, $p = 25$, $b = 0.9$. (c) $n = 7.5$, $p = 35$, $b = 2$. The propagation distance is 512 and white noise was added into the initial inputs.

suppress the azimuthal instability of vortex solitons effectively [32]. To study the properties of vortex solitons with higher charges, one must consider the higher-order modulated Bessel lattices.

The following discussion will focus on multipeaked vortex solitons carrying different topological charges supported by azimuthally modulated Bessel lattices imprinted in defocusing cubic media. For the convenience of comparing with the results of [20], we assume $R(x,y) = J_n^2[(2b_{\mathrm{lin}})^{1/2}r]\cos^2(n\phi)$, where the order of lattice equals the azimuthal index. We also search for stationary solutions of vortex solitons by relaxation methods. A Gauss beam multiplying a phase dislocation with charge m was selected as an initial iterative guess solution.

Figure 4 displays some instances of multipeaked vortex solitons in lattices with $n = 4$ and 6. The vortex solitons exhibit spatially modulated patterns that are in contrast to the vortices in unmodulated Bessel lattices [11], where the vortices are ring-shaped. Note that the amplitude and phase distributions of multipeaked vortex solitons we found are very similar to those of azimuthons [23]. The profiles of vortex solitons possess several amplitude peaks covering on a constitutive ringlike substrate. The number of amplitude peaks is also determined by the azimuthal index n. Similar to the vortices in focusing media [20], vortices with similar amplitude distributions allow different topological charges. In the fourth-order Bessel lattices with azimuthal index $n = 4$, vortex solitons can be found only for $m = 1$, 2, and 3. For fixed b and p, the discreteness of vortex solitons increases with the growth of the topological charge m, while the "radii" of the vortices are almost the same. For fixed p and n, the vortex solitons will expand to the outer lattice rings at small b and shrink to the main guiding lattice ring at larger b. The local minima of multipeaked vortex solitons around the lattice ring approaches to zero when $b \to b_{\mathrm{co}}$.

We also find multipeaked vortex solitons in the lattices with different azimuthal indices. Numerical study reveals that vortex solutions can be found only when the relation $0 < m < n$ is satisfied. The relation also holds for the vortex solitons in focusing cubic media [11]. The reason is that all nonlinear modes originate from the linear modes. Because linear modes do not exist when $m \geq n$, there are no nonlinear modes for $m \geq n$. The phase difference between the neighboring components is $m\pi/n$, which differs from the vortex solitons in harmonic lattices [6] or necklace solitons in Bessel lattices [13]. For $m > n$, a vortex soliton should have a phase change greater than π between the neighboring peaks. However, the defocusing nonlinearity does not allow such phase difference, which explains the existence condition of vortex solitons ($0 < m < n$) qualitatively.

The properties of multipeaked vortex solitons in the azimuthally modulated Bessel lattices are summarized in Fig. 5. Similar to the power curves of multipeaked solitons in Fig. 2(a), the power of vortex solitons is a descending curve due to the defocusing nonlinearity [Fig. 5(a)]. Vortex solutions cannot be found when the propagation constant exceeds a certain value that corresponds to an eigenvalue of the linearized equation of Eq. (2). The upper propagation constant cutoffs of vortex solutions with $m = 1$ and 3 are displayed in Figs. 5(b) and 5(c). The existence areas expand with the growth of lattice depth for a fixed topological charge and shrink with the growth of topological charges for a fixed

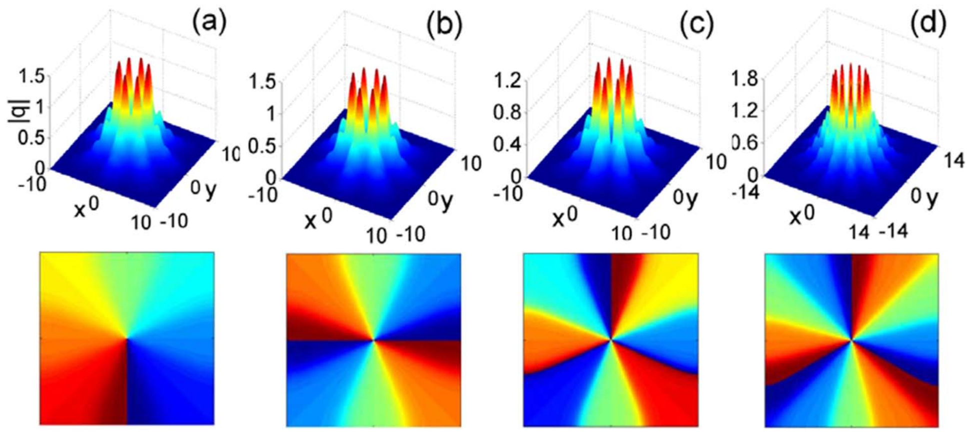

Fig. 4. (Color online) Amplitude distributions of multipeaked vortex solitons with (a) $m = 1$, (b) 2, and (c), (d) 3. Parameters $n = 4$, $p = 30$, and $b = 0.5$ in (a)–(c) and $n = 6$, $p = 45$, and $b = 0.5$ in (d). Bottom row: the corresponding phase structures.

lattice depth. There is a lower threshold lattice depth for supporting multipeaked vortex solitons. Comparing the points of $b \to 0$ in Figs. 5(b) and 5(c), we find that the threshold lattice depth grows with the increase of topological charge m.

To comprehensively understand the stability properties of multipeaked vortex solitons in the lattices with different depths and azimuthal indices, we performed the linear stability analysis on vortex solitons in lattices with order (azimuthal index) n up to 10 and lattice depths $p \le 80$. We numerically derived an important "stability rule" for vortex solitons in the azimuthally modulated Bessel lattices imprinted in defocusing media. That is, vortex solitons might be stable only when the topological charge satisfies the condition

$$0 < m \le n/2, \tag{4}$$

where $n > 2$. Vortex solitons with $m = n/2$ for even n may be stable or unstable depending on the lattice parameters. There exists a narrow instability area near $b \to 0$ when the lattice is modulated shallow (near its lower threshold value). For deeper lattices similar to the stable ring-profile vortex solitons in Bessel lattices [11], completely stable vortex solitons are possible. A summary of "stability rule" is presented in Table 1. The table shows the stability status of vortex solitons for different lattice orders. It is exactly opposite to Table 1 in [11], which was derived by the group theory and is valid in the focusing cubic media. This finding also verifies the very recent reports [30,31] in which the stability of discrete vortex solitons supported by hexagonal photonic lattices in focusing media is proved to be opposite to the stability in the defocusing ones. We note that our conclusion is more general because the above two studies are restricted to the single- and double-charge discrete vortex solitons.

Linear instability analysis results of some unstable vortex solitons in the fourth-, fifth- and sixth-order lattices are shown in Figs. 5(d)–5(f). Note the relation between the azimuthal index and topological charge does not satisfy the condition Eq. (4). The instability domain vanishes only when the propagation constant approaches to its upper cutoff.

The vortex solitons aforementioned are restricted to the particular cases of $n = n_J$. In fact, vortex solutions can also

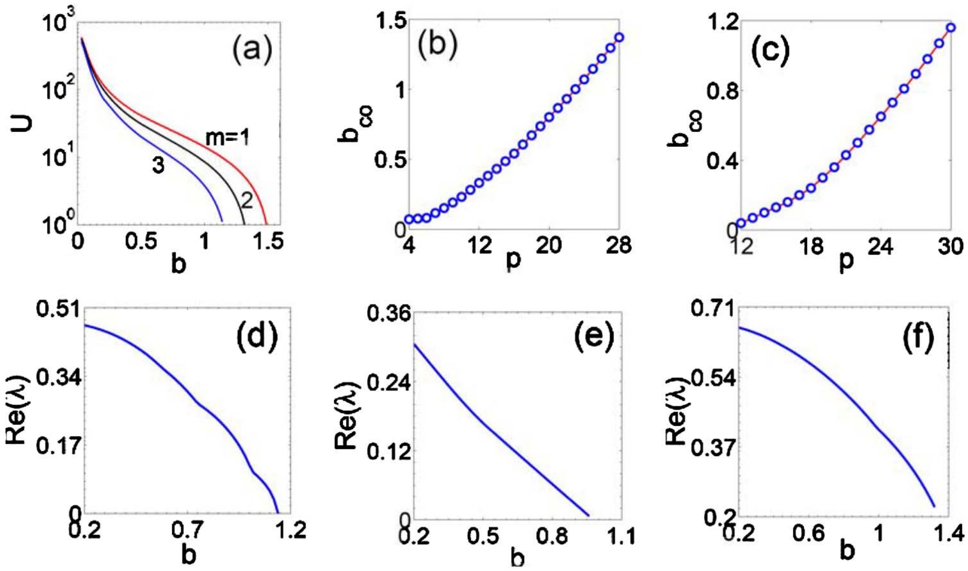

Fig. 5. (Color online) (a) Power of multipeaked vortex solitons with $m = 1$, 2, and 3 versus propagation constant, $n = 4$, $p = 30$. Propagation constant cutoff b_{co} versus lattice depth p for vortex solitons with (b) $m = 1$ and (c) 3. Real parts of instability growth rate λ versus propagation constant for vortex solitons supported by the (d) fourth-, (e) fifth-, and (f) sixth-order lattices with $p = 30$, $m = 3$; $p = 35$, $m = 3$; and $p = 35$, $m = 5$, respectively. Azimuthal index $n = 4$ in (a)–(d), 5 in (e), and 6 in (f).

Table 1. Stability Status of Vortex Solitons for Different Lattice Orders

Lattice Order	Available Charges and Stability Status				
$n=2$	$m=1$ unstable	—	—	—	—
$n=3$	$m=1$ stable	$m=2$ unstable	—	—	—
$n=4$	$m=1$ stable	$m=2$ stable	$m=3$ unstable	—	—
$n=5$	$m=1$ stable	$m=2$ stable	$m=3$ unstable	$m=4$ unstable	—
$n=6$	$m=1$ stable	$m=2$ stable	$m=3$ stable	$m=4$ unstable	$m=5$ unstable

be found when $n \neq n_J$. Contrary to intuition and the cases in nonlocal media [26], the charge m of available vortex solutions is independent of the lattice order n_J but less than the azimuthal index n. The initial input guess solutions with $m \geq n$ may converge to the nonlinear modes of the following three different categories: 1. a vortex with charge $m' < n$; 2. a multipole-mode or necklace soliton with neighboring components out- of phase; or 3. a multipole-mode or necklace soliton embedded into a global skew phase whose charge $m'' = m - n$. Thus, we conclude that vortex solutions can be found only for $m < n$. The reason may be attributed to the Kerr media with a local nonlinear response in our model [23].

Finally, we stress that the defocusing nonlinearity is a necessary ingredient for the existence of the stable multipeaked fundamental and vortex solitons. The linear modes guided by the purely modulated Bessel lattices occupy only the first lattice ring [Figs. 1(c) and 1(d)], while the multipeaked (vortex) solitons exhibit modulated multiring structures, e.g., see Figs. 4 and 5 (also see Fig. 3 in [11]). One the other hand, it is the defocusing nonlinearity that affords the nonzero local intensity minima of vortex solitons around the rings of lattices. The nonlinear modes converge to the linear modes when the propagation constant approaches to its upper cutoff value.

5. CONCLUSIONS

To summarize, we investigate the existence, stability, and propagation dynamics of azimuthonlike solitons supported by the azimuthally modulated Bessel lattice in a defocusing medium. We reveal that the scheme supports completely stable multipeaked fundamental solitons. Specially, stable solitons with odd numbers of intensity peaks, which are very rare, can exist in the azimuthally modulated Bessel lattices. Another interesting result we uncovered is that the "stability rule" for vortex solitons in defocusing cubic media is opposite to that in focusing media. We show that stable azimuthonlike (vortex) solitons can exist in local nonlinear media with an appropriate optical potential. Our results reported here can be easily generated into the Bose–Einstein condensates trapped in azimuthally modulated Bessel lattices.

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