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Materials

An analytical model for the prediction of cross-section profile and mean roll radius in alloy bar rolling

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Abstract: In a round-oval-round pass rolling sequence, the cross-section profile of an outgoing workpiece was predicted first after getting the maximum spread. The concept “critical point on the contact boundary” was proposed and the coordinates of the critical point were solved. The equivalent contact section area was represented and the mean roll radius was determined. The validity of this model was examined by alloy bar rolling experiment and rigid-plastic FEM simulation. Compared with the existing models, the mean roll radius obtained by this model is similar to experiment data.

Key words: alloy steel; critical point; cross-section profile; mean roll radius; round-oval-round

1. Introduction

In strip (or plate) rolling process, the calculation of rolling speed by the multiplication of roll rpm and roll radius is very simple. However, in rod (or bar) rolling process, the roll surface is not smooth for the groove on the roll, so the determination of rolling speed becomes difficult, as the roll radius is not constant along the direction of roll axis. Consequently, for calculating the rolling speed of the workpiece in the grooved roll, the “mean roll radius” has been used as equivalent radius to replace the varying roll radius along the roll groove profile.

For calculating the mean roll radius, the cross-section profile of the outgoing workpiece should be predicted. Shinokura and Takai [1-3] presented an experimentally based model for the prediction of cross-section profile in oval pass rolling. Kemp [4] proposed a model for the prediction of cross-section profile in oval and round groove rolling, but did not represent the equation for the cross-section profile. Kim [5] represented a free surface scheme for the analysis of plastic deformation in shape rolling.

For determining the mean radius of the grooved roll, some calculating models were proposed by scholars.

This article studied the deformation law of the alloy bar (or rod) in the groove, and the cross-section profile of an outgoing workpiece has been predicted, and then a novel model was proposed to calculate the mean roll radius. The mean roll radius calculated by this was compared with the existing models.

2. Prediction of cross-section profile of the outgoing workpiece

According to the research of Lee [6-9], the free surface profile at the exit cross-section can be expressed as a circular arc. As can be seen in Figs. 1 and 2, the radius of the circular arc is shown as R_s, and the intersection between the free surface and the groove curve is defined as “critical point on the contact boundary”. So the cross-section profile can be predicted when R_s and the critical point (C_Y, C_Z) are known. The coordinate of the critical point (C_Y, C_Z) must be solved for the exact determination of cross-section profile. Consequently, the model for solving the critical point (C_Y, C_Z) should also be built.

2.1. Critical point in round-oval pass rolling

Once the groove profile and roll gap are known, the position of the critical point (C_Y, C_Z) can be just determined by the maximum spread Δb or maximum

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width $W_{\max}$.

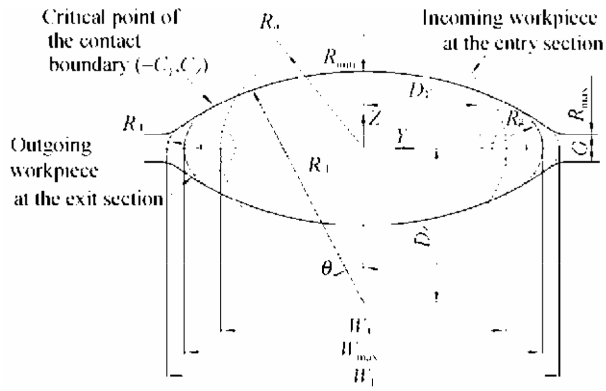

Fig. 1. Parameter designation of round-oval pass for solving the critical point (C_Y, C_Z).

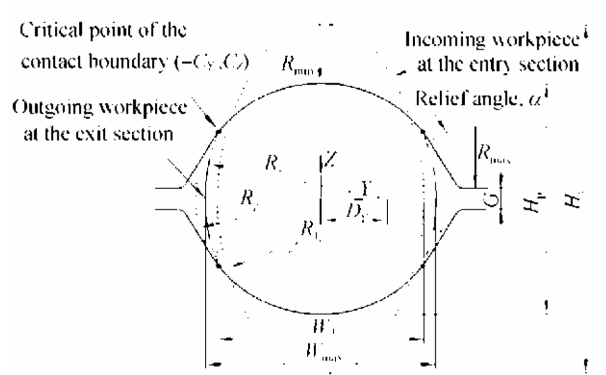

Fig. 2. Parameter designation of oval-round pass for solving the critical point (C_Y, C_Z).

R_s can be predicted as the linear interpolation of R_a and R_f (Fig. 1) or R_1 and R_g (Fig. 2), once W_{max} is known. W_{max} is the maximum width of the outgoing workpiece, which can be calculated by Shinokura and Takai's equation [3].

In round-oval pass rolling, R_s and R_f may be given by

$$R_s = R_a \cdot W_t + R_f \cdot (1 - W_t) \tag{1}$$

$$R_f = \frac{R_1 \cdot H_p - (W_f^2 + H_p^2)/4}{2R_1 - W_f} \tag{2}$$

$$W_t = \frac{W_f - W_{max}}{W_f - W_i} \tag{3}$$

$$W_{max} = W_i + \Delta b \tag{4}$$

where R_a is the radius of curvature of the incoming cross-section. W_t a weighting function, W_i the width of the inlet cross-section, W_f the width of the roll groove area, Δb the maximum spread of the outgoing workpiece.

The critical point (C_Y, C_Z), at which the oval groove intersects the surface profile of the outgoing workpiece, can be formulated as two simultaneous circular equations:

$$(C_Y - D_Y)^2 + C_Z^2 = R_s^2 \tag{5}$$

$$(D_Z + C_Z)^2 + C_Y^2 = R_1^2 \tag{6}$$

where D_Y, the distance along the direction of Y-axis between the origin coordinate and the center of the arc R_s, is shown as

$$D_Y = (W_{max} - 2R_s)/2 \tag{7}$$

where D_z, the distance along the direction of Z-axis between the origin coordinate and the center of the arc R_1, is shown as

$$D_Z = R_1 - H_p / 2 \tag{8}$$

where R_1 is the radius of the oval groove , H_p the thickness of the roll groove area.

A new Eq. (9) can be obtained by Eqs. (5) and (6).

$$a \cdot C_Z^2 + b \cdot C_Z + c = 0 \tag{9}$$

where a, b, c are shown , respectively, as

$$a = 1 + \frac{D_Z^2}{D_Y^2} \tag{10}$$

$$b = 2D_Z - \frac{k_0 \cdot D_Z}{D_Y^2} \tag{11}$$

$$c = D_Z^2 - R_1^2 + \frac{k_0^2}{4D_Y^2} \tag{12}$$

where k_0 is shown as

$$k_0 = R_1^2 + D_Y^2 - R_s^2 - D_Z^2 \tag{13}$$

Solving the simple binomial Eq. (9), C_Z can be expressed explicitly in terms of a, b, c as

$$C_Z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{14}$$

According to Eq. (14), two solutions of C_Z will be obtained. However, in these two solutions, only one is effective or available. In other words, one of two solutions has to be excluded.

The reference point (C_{Y0}, C_{Z0}), at which the oval groove intersects the surface profile of incoming workpiece, can be formulated as

$$(D_Z + C_{Z0})^2 + C_{Y0}^2 = R_1^2 \tag{15}$$

$$C_{Z0}{}^2 + C_{Y0}^2 = R_a^2 \tag{16}$$

Solving Eqs. (15) and (16), yields

$$C_{Z0} = \frac{R_1^2 - R_a^2 - D_Z^2}{2D_Z} \tag{17}$$

$$C_{Y0} = \sqrt{R_a^2 - C_{Z0}^2} \quad (18)$$

For the spread of the workpiece in rod rolling, C_{Z0} should be bigger than C_Z. So, the criterion for the determination of the only effective value C_Z can be shown as

$$C_Z < C_{Z0} \quad (19)$$

Once C_Z is determined, C_Y can be obtained by

$$C_Y = \sqrt{R_1^2 - (C_Z + D_Z)^2} \quad (20)$$

2.2. Critical point in oval-round pass rolling

In oval-round pass rolling, R_s may be given by

$$R_s = R_1 \cdot W_t + R_g \cdot (1 - W_t) \quad (21)$$

$$W_t = \frac{2R_g - W_{max}}{2R_g - W_i} \quad (22)$$

where R_1 is the radius of curvature of the incoming cross-section, W_t a weighting function, R_g the radius of the round groove.

The critical point (C_Y, C_Z), at which the round groove intersects the surface profile of an outgoing workpiece, can be formulated as

$$(C_Y + D_Y)^2 + C_Z^2 = R_s^2 \quad (23)$$

$$C_Z^2 + C_Y^2 = R_g^2 \quad (24)$$

where D_Y, the distance along the direction of Y-axis between the origin coordinate and the center of the arc R_s, is shown as

$$D_Y = R_s - W_{max} / 2 \quad (25)$$

Solving Eqs. (23) and (24), yields

$$C_Y = \frac{R_s^2 - R_g^2 - D_Y^2}{2D_Y} \quad (26)$$

$$C_Z = \sqrt{R_g^2 - C_Y^2} \quad (27)$$

The reference point (C_{Y0}, C_{Z0}), at which the round groove intersects the surface profile of an incoming workpiece, can be formulated as

$$(D_0 + C_{Y0})^2 + C_{Z0}^2 = R_1^2 \quad (28)$$

$$C_{Z0}^2 + C_{Y0}^2 = R_g^2 \quad (29)$$

$$D_0 = R_1 - W_i / 2 \quad (30)$$

Solving Eqs. (28) and (29), yields

$$C_{Y0} = \frac{R_1{}^2 - R_g^2 - D_0^2}{2D_0} \quad (31)$$

$$C_{Z0} = \sqrt{R_g^2 - C_{Y0}^2} \quad (32)$$

3. Calculating models of mean roll radius

In this section, the existing models for the calculation of mean roll radius are reviewed and the derivation procedure of the proposed analytical model is described.

3.1. Existing models for mean roll radius

(1) Wusatowski model [10].

Fig. 3 illustrates the physical meaning of mean roll radius in an oval and a round pass. It is a single value, which replaces the varying roll radius along the groove profile. Wusatowski model can be used to calculate mean roll radius when one knows information regarding the deformation of a workpiece, *i.e.*, the maximum spread and cross sectional area of the deformed workpiece.

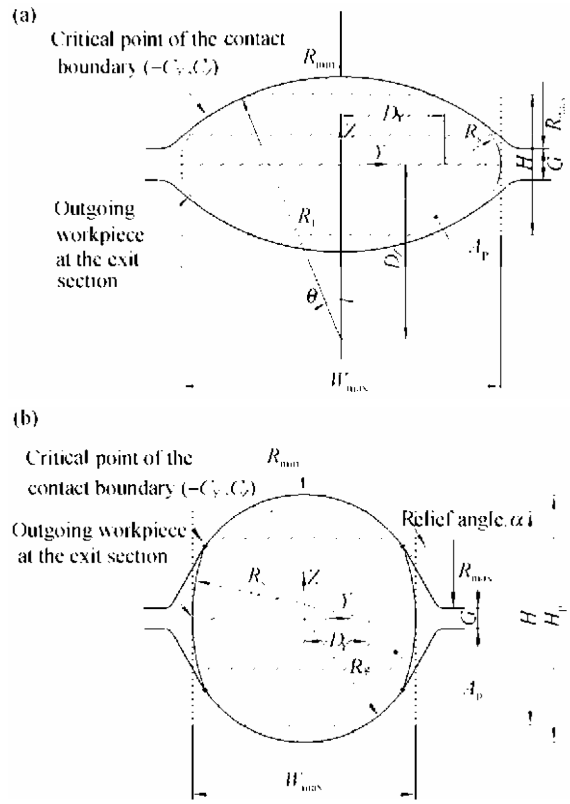

Fig. 3. Parameter signification for the calculation of mean roll radius by Wusatowski model [1]: (a) oval pass; (b) round pass.

The mean roll radius is

$$R_{mean} = R_{max} + G/2 - \overline{H}/2 \quad (33)$$

$$\overline{H} = A_p / W_{max} \quad (34)$$

where A_p is the cross sectional area of deformed workpiece at a given pass. The effective height of outgoing workpiece, $\overline{H}$, is calculated by using the equivalent rectangle approximation method which transforms a curved cross section into a rectilinear one while the net cross sectional area is maintained.

(2) Saito model [11].

Saito *et al.* [11] proposed that the effective height, $\bar{H}$, could be obtained by knowing the cross points of a roll groove profile and an incoming workpiece at a pass when they are overlapped (Fig. 4). Then the mean roll radius is

$$R_{\text{mean}} = R_{\max} + G/2 - \bar{H}/2 \tag{35}$$

$$\bar{H} = A_{\text{h}}/B_{\text{c}} \tag{36}$$

It should be noted that the mean roll radius obtained by Saito model depends on the cross sectional shape of the incoming (undeformed) workpiece, whereas that obtained by Wusatowski model depends on the outgoing (deformed) workpiece.

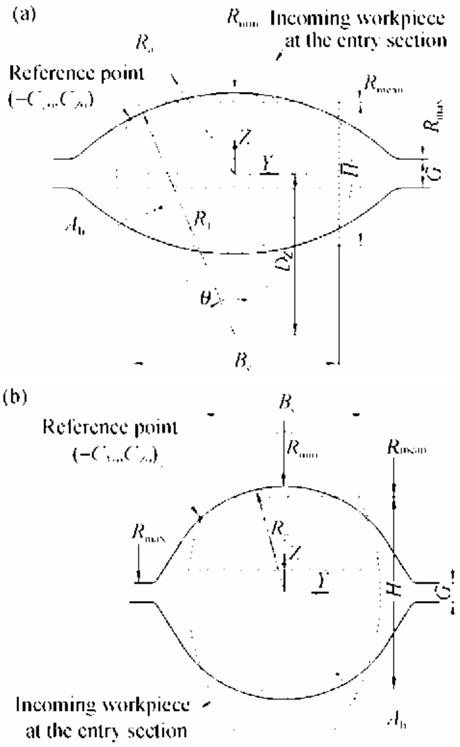

Fig. 4. Parameter signification for the calculation of mean roll radius by Satio *et al.*'s model [2]: (a) oval pass; (b) round pass.

(3) Lee model [9].

Lee's analytical model has been developed based on the mapping which transforms the definition of rolling speed in strip (or plate) rolling into that in rod (or bar) rolling. The mean roll radius proposed by Lee is represented by

$$R_{\text{mean}} = (R_1 + R_2 + \cdots + R_{N-1} + R_N)/N = \left(\sum_{i=1}^{N} R_N\right) \cdot \frac{1}{N} \tag{37}$$

$$N = \left|\frac{2C_Y}{\Delta l}\right| \tag{38}$$

$$\Delta l = C_Y \cdot \left(\frac{C_Z}{W_{\text{f}}}\right)^{\gamma} \tag{39}$$

$$\gamma = \left|\frac{W_{\text{f}}}{G}\right| \tag{40}$$

where N is the number of the point-wise roll radii along the periphery, γ an integer and a nondimensional constant, Δl the interval between the point-wise radius $R_1, R_2, R_3, \cdots, R_N$ (Fig. 5).

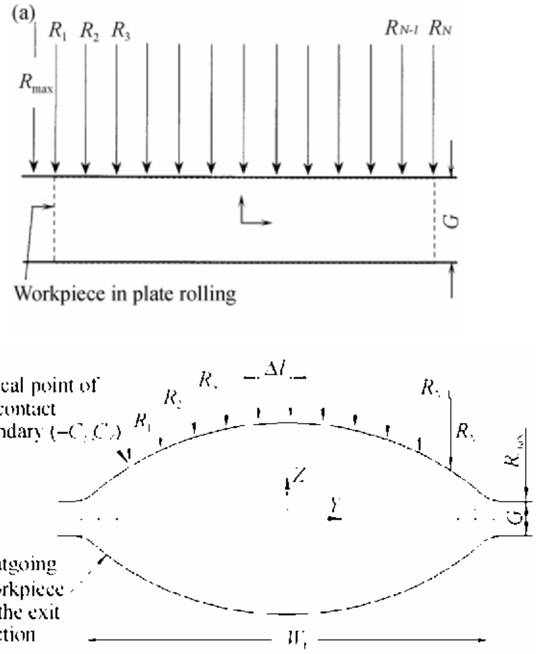

Fig. 5. Discrete distribution of radius at the exit section: (a) in strip or plate rolling; (b) in rod or bar rolling

3.2. Development of a novel model for mean roll radius

On the basis of summarizing and improving Wusatowski model and Lee model, a novel analytical model was put forward.

According to Saito model, the mean roll radius is determined purely from the geometry of the incoming workpiece and the roll groove profile at a pass when they are overlapped. Whereas, according to Wusatowski model, Lee model, and the novel model describes in this article, the mean roll radius is determined purely from the geometry of the outgoing workpiece and the roll groove profile. Moreover, in this article, the lateral flow of the deformed workpiece (along roll-axis direction) and contact boundary has been considered as the two pivotal influencing factors on the mean roll radius. So the concept of critical point and available contact section is proposed (Figs. 6 and 7). The concept "critical point" is defined as the point on the contact boundary at the exit section.

It affects the contact status and the shape of the outgoing workpiece directly, so it is indispensable for determining the mean roll radius. Once this point is known, the available contact zone (hatched zone *abcd*) at the exit section can be determined and furthermore, the equivalent contact zone (rectangular zone *ABCD*) can be obtained (Figs. 6 and 7).

First, the model of solving the critical point (C_Y, C_Z) should be built.

(1) The mean roll radius in round-oval pass rolling.

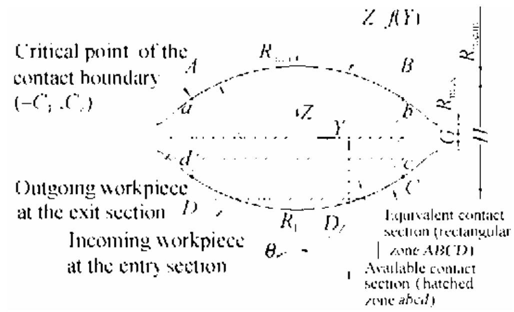

Fig. 6. Parameter designation of the round-oval pass.

The curve equation of the oval groove is expressed as

$$Z = f(Y) = \sqrt{R_1^2 - Y^2} - D_Z \tag{41}$$

where R_1 is the radius of the oval groove.

The area of the available contact section for the oval pass is expressed as

$$A_{abcd} = 2\int_{-C_Y}^{C_Y} \left(\sqrt{R_1^2 - Y^2} - D_Z\right) \mathrm{d}Y \tag{42}$$

The area of the equivalent contact section for the oval pass and round pass is

$$A_{ABCD} = \bar{H} \cdot 2C_Y \tag{43}$$

The mean height of the workpiece in the oval pass is shown as

$$\bar{H} = \frac{A_{ABCD}}{2C_Y} = \frac{A_{abcd}}{2C_Y} = \frac{2\int_{-C_Y}^{C_Y}\left(\sqrt{R_1^2 - Y^2} - D_Z\right)\mathrm{d}Y}{2C_Y} = \frac{2\int_0^{C_Y}\left(\sqrt{R_1^2 - Y^2}\right)\mathrm{d}Y - 2C_Y \cdot D_Z}{C_Y} \tag{44}$$

Integrating the Eq. (44) yields

$$\bar{H}\big/2 = \frac{R_1^2(\sin 2\theta_0 + 2\theta_0)}{4C_Y} - D_Z \tag{45}$$

where θ_0 is shown as

$$\theta_0 = \arcsin(C_Y / R_1) \tag{46}$$

The mean roll radius of the round pass may be given by

$$R_{\mathrm{mean}} = R_{\mathrm{max}} + G\big/2 - \bar{H}\big/2 \tag{47}$$

(2) The mean roll radius in oval-round pass rolling.

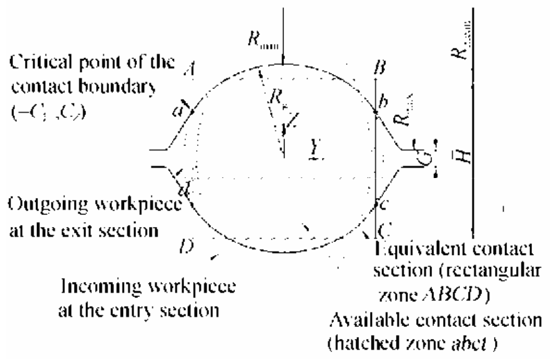

Fig. 7. Parameter designation of the oval-round pass.

The curve equation of the round groove is expressed as

$$Z = f(Y) = \sqrt{R_{\mathrm{g}}^2 - Y^2} \tag{48}$$

The area of the available contact section for the round pass is expressed as

$$A_{abcd} = 2\int_{-C_Y}^{C_Y}\left(\sqrt{R_{\mathrm{g}}^2 - Y^2}\right)\mathrm{d}Y \tag{49}$$

The area of the equivalent contact section for the oval pass and round pass is

$$A_{ABCD} = \bar{H} \cdot 2C_Y \tag{50}$$

The mean height of the workpiece in the round pass is shown as

$$\bar{H} = \frac{A_{ABCD}}{2C_Y} = \frac{A_{abcd}}{2C_Y} = \frac{2\int_{-C_Y}^{C_Y}\left(\sqrt{R_{\mathrm{g}}^2 - Y^2}\right)\mathrm{d}Y}{2C_Y} \tag{51}$$

Integrating Eq. (51) yields

$$\bar{H}\big/2 = \frac{2C_Y \cdot C_Z + \left(2\arctan(C_Y / C_Z\right) \cdot R_{\mathrm{g}}^2}{4C_Y} \tag{52}$$

The mean roll radius of the oval pass and round pass may be given by

$$R_{\mathrm{mean}} = R_{\mathrm{max}} + G\big/2 - \bar{H}\big/2 \tag{53}$$

3. Results and discussions

The calculation has been carried out according to the rolling schedule (Table 1) and the size of round-oval-round pass schedule (Table 2), which is always applied to Pomini rolling mills that belongs to Beiman Special Steel Co. Ltd. The material of the workpiece is structural alloy steel (40Cr).

The rolling process has been simulated by rigid-plastic FEM, the contact boundary and critical point is shown in Fig. 8 and the surface profile is shown in Fig. 9. The surface profile obtained by the prediction model, FEM simulation, and experimental data are shown as Fig. 10.

Table 1. Rolling schedule of the Pomini rolling mills

Pass	Roll diameter/ mm	Bite angle/ (°)	Motor speed/ $(r \cdot min^{-1})$	Roll speed/ $(r \cdot min^{-1})$	Input speed/ $(m \cdot s^{-1})$	Output speed/ $(m \cdot s^{-1})$	Neutral diameter/ mm	Height of the incoming workpiece/ mm
Oval pass	730	24.8	822	8.1	0.22	0.28	658.2	171.0
Round pass	730	28.0	887	10.5	0.28	0.35	627.1	195.4

Table 2. Size of the oval pass and round pass mm

Oval pass					Round pass			
H_p	W_f	R_1	D_Z	G	H_p	W_f	R_g	G
112	230	156.6	100.6	20	136	150.11	68	12

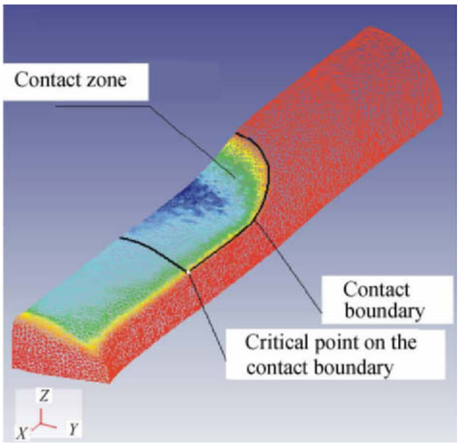

Fig. 8. Contact boundary and contact zone in oval pass rolling.

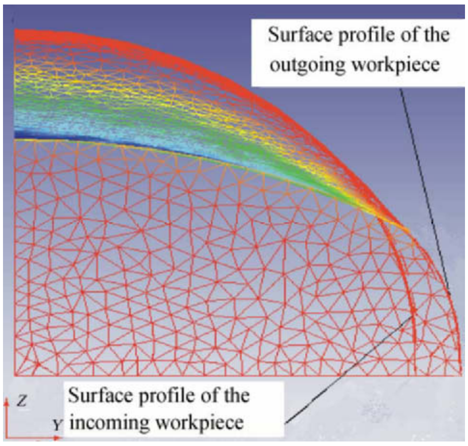

Fig. 9. Surface profile of the incoming workpiece and outgoing workpiece.

The mean roll radius can be obtained by the following steps:

Step 1: Calculate Δb and W_{max} by Shinokura and Takai's equation [7] and simulate the rolling process by the rigid-plastic FEM according to the rolling schedule from Pomini rolling mills.

Step 2: Predict the surface profile of the outgoing workpiece by Lee's model and determine R_s and D_Y in oval pass and round pass, respectively.

Step 3: Calculate the reference point (C_{Y0}, C_{Z0}) and solve the critical point (C_Y, C_Z).

Step 4: Calculate the bar section area A_p, A_h, A_{abcd} and $\bar{H}$ in oval pass and round pass, respectively.

Step 5: Calculate the mean roll radius R_{mean}.

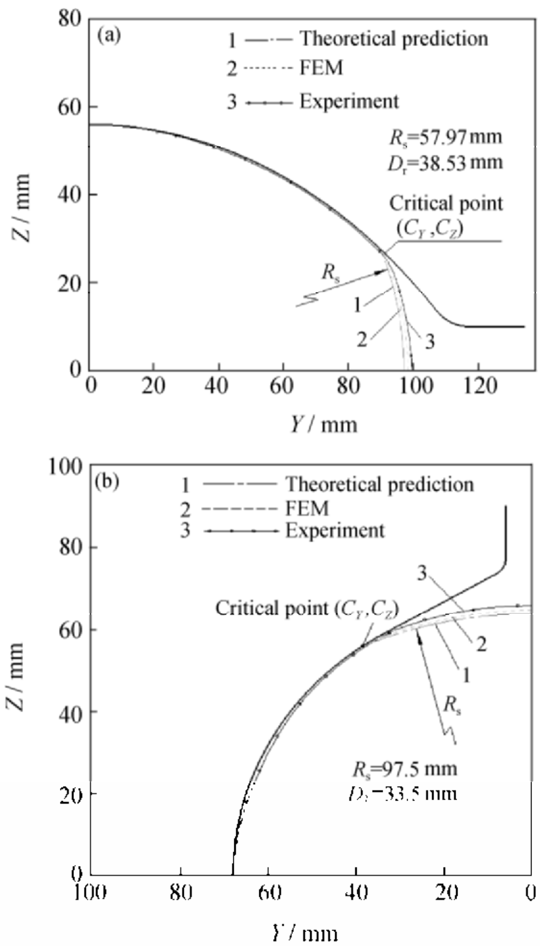

Fig. 10. Predicted and experimental surface profiles of the outgoing workpiece: (a) oval pass; (b) round pass.

The result is shown as Table 3 and Table 4.

The effective section areas $A_{\rm h}$, $A_{\rm p}$ and A_{abcd}, which are used to calculate the effective height of the outgoing workpiece, are expressed in Table 3. As can be seen in table 3, in the three section areas calculated by different models, the biggest and the smallest one are $A_{\rm h}$ and $A_{\rm p}$ respectively. The values of mean height $\overline{H}$ and mean roll radius $R_{\rm mean}$ calculated by different models are listed in Table 4. As can be seen in Table 4, the values of $\overline{H}$ and $R_{\rm mean}$ calculated from the new analytical model approach the practical data mostly.

Table 3. Critical point and section area of different models

Pass	Method	Max. spread, Δb/mm	Reference point, (C_{Y0},C_{Z0})/mm	Critical point, (C_Y,C_Z)/mm	Bar section area/ mm^2		
					Wusatowski model, $A_{\rm p}$	Saito model, $A_{\rm h}$	New model, A_{abcd}
Oval pass	Theoretical result	22.04	(77.95, 35.25)	(89.95, 27.94)	16711.28	15247.02	16476.69
	Experiment	24.40	(77.95, 35.25)	(90.79, 26.78)	16770.62	15247.02	16536.15
	Simulating result by FEM	25.25	(77.95, 35.25)	(91.56, 25.93)	16820.34	15247.02	16586.79
Round pass	Theoretical result	15.68	(48.6, 47.56)	(56.09, 38.45)	14079.75	11982.53	13341.14
	Experiment	16.87	(48.6, 47.56)	(56.67, 37.58)	141281.14	11982.53	13390.37
	Simulating result by FEM	18.49	(48.6, 47.56)	(57.92, 36.41)	14196.58	11982.53	13455.92

Table 4. Mean height and mean roll radius

Pass	Method	Effective height, $\bar{H}$ /mm				Mean roll diameter, $R_{\rm mean}$ /mm			
Oval pass	Practical data	91.8				329.1			
	Theoretical result	Wusatowski model	Saito model	Lee model	New model	Wusatowski model	Saito model	Lee model	New model
		86.59	97.8	97.16	91.59	331.71	326.1	326.42	328.95
	Absolute error/mm	−5.21	+6.00	+5.36	+0.21	+2.61	−3.00	−2.72	−0.15
	Relative error/%	−5.68	+6.54	+5.84	+0.23	+0.79	−0.91	−0.83	−0.05
Round pass	Practical data	117.7				316.15			
	Theoretical result	Wusatowski model	Saito model	Lee model	New model	Wusatowski model	Saito model	Lee model	New model
		125.51	123.28	122.47	118.93	312.25	313.36	314.76	315.54
	Absolute error/mm	−7.81	−5.58	−4.77	−1.23	−3.90	−2.79	−1.39	−0.61
	Relative error/%	−6.64	−4.74	−4.05	−1.11	−1.23	−0.88	−0.44	−0.19

In oval pass rolling, the mean roll radius $R_{\rm mean}$ computed by Saito model, which is very similar with that by Lee model, is less than that by the novel model. On the other hand, the mean roll radius computed by Wusatowski model overestimate those by Lee model and the novel model. In round pass rolling, the mean roll radius $R_{\rm mean}$ calculated by the novel model is bigger compared with all the other models.

4. Conclusions

(1) Whether the position of critical point is accurate enough or not depends on the precision of the spread model, in other words, when the maximum width is more close to the true spread, the critical point will be more correct. So, the spread model should be chosen carefully to express the law of lateral flow exactly.

(2) The surface profile of the outgoing workpiece is influenced by two pivotal parameters of the critical point (C_Y, C_Z) and $R_{\rm s}$. Compared with the experimental data and simulation results, the surface profile prediction model is proved to be correct.

(3) Wusatowski model can be used for rough estimation for its simplicity in calculation; Saito model can be applied for approximate research when the maximum spread Δb is small enough and can be ignored; Lee model has a good mathematical rationale, but in practice, the application of this model is not easy as it involves tremendous calculations.

(4) Compared with the existing models, the novel analytical model is more rational because the influence of maximum spread and contact status upon the mean roll radius has been considered in this model. The effective contact section area is an intermediate value between those of the Wusatowski model and Saito model.

(5) The mean roll radius calculated by the novel analytical model is more close to experiment data. So, it can be used as an available reference in alloy bar rolling.

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