A Dual-Frequency Transformer for Complex Impedances With Two Unequal Sections

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***Abstract*—In this letter, a small dual-frequency transformer with two unequal sections for complex impedances is proposed. To design this transformer, two different groups of nonlinear equations and the corresponding solutions processes are obtained. The results of numerical examples show that two complex impedances can be matched at two different frequencies simultaneously. This proposed transformer can be regarded as the extension of small dual-frequency transformer in two sections for two resistances.**

***Index Terms*—Complex impedances, dual-frequency, impedance match.**

I. INTRODUCTION

DEVICES in modern communication systems are required to work at two different frequencies in many cases, such as GSM and TD-SCDMA. To fulfill this dual-frequency operation requirement, a novel transformer of one-third wavelength in two sections for a frequency and its first harmonic has been proposed in [1]. Later on, comprehensive analysis and exact solutions on dual-frequency transformer with conventional ideal transmission line have been presented in [2], [3]. And this novel transformer has been used in dual-frequency power dividers [4], [5]. However, the dual-frequency transformers in [1]–[5] are only suitable to match between two resistances, namely, load resistance R_L and real characteristic impedance Z_0. Considering that it is usual to match two complex impedances (for example, the input or output impedances of RF chips and power amplifiers) at dual-frequency, this letter extends the application scope of dual-frequency transformer from two resistances to two complex impedances and deduces two groups of parameter design equations for the transformer in dual-frequency matching conditions. Furthermore, by applying the optimization toolbox [6] based on the proposed process, numerical examples are given to present the matching characteristics and prove the validity of the proposed matching structure.

II. DESIGN EQUATIONS

A dual-frequency transformer of two sections of lossless transmission lines (physical lengths are l_1 and l_2 while the corresponding electrical lengths are ψ_1 and ψ_2) between two complex impedances (Z_0 and Z_L) is illustrated in Fig. 1. Z_{in} and Z_{L2} are the respective input impedances from the front-end of two sections, while Z_L corresponds to the load impedance.

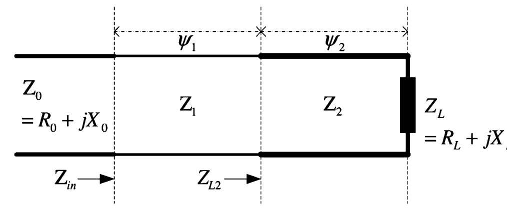

Fig. 1. Dual-frequency transformer for complex impedances.

The expressions of these input impedances are as follows:

$$Z_{in} = Z_1\frac{Z_{L2}+jZ_1\tan(\psi_1)}{Z_1+jZ_{L2}\tan(\psi_1)}, \tag{1}$$

$$Z_{L2} = Z_2\frac{Z_L+jZ_2\tan(\psi_2)}{Z_2+jZ_L\tan(\psi_2)}. \tag{2}$$

In order to match to Z_0, the following equation is necessary:

$$Z_{in} = Z_0^* = R_0 - jX_0. \tag{3}$$

And (1) can be rewritten as

$$Z_{L2} = Z_1\frac{R_0-jX_0-jZ_1\tan(\psi_1)}{Z_1-X_0\tan(\psi_1)-jR_0\tan(\psi_1)}. \tag{4}$$

Combination of the equation $Z_L = R_L + jX_L$ and equating (2) and (4), the following equation can be obtained as:

$$Z_2\frac{R_L+jX_L+jZ_2\tan(\psi_2)}{Z_2+j(R_L+jX_L)\tan(\psi_2)} = Z_1\frac{R_0-jX_0-jZ_1\tan(\psi_1)}{Z_1-j(R_0-jX_0)\tan(\psi_1)}. \tag{5}$$

Rearranging (5) and separating the real and imaginary parts, we can obtain the following equations:

$$\left(R_LZ_1^2-R_0Z_2^2\right)\tan(\psi_1)\tan(\psi_2)=Z_1Z_2(R_L-R_0)$$
$$+[Z_1\tan(\psi_2)+Z_2\tan(\psi_1)]\times(R_0X_L-R_LX_0), \tag{6a}$$
$$(R_0R_L+X_0X_L)[Z_1\tan(\psi_2)+Z_2\tan(\psi_1)]-Z_2Z_1^2\tan(\psi_1)-Z_1Z_2^2\tan(\psi_2)$$
$$=Z_1Z_2(X_L+X_0)-\left(X_LZ_1^2+X_0Z_2^2\right)\tan(\psi_1)\tan(\psi_2). \tag{6b}$$

In order to satisfy (6a) and (6b) at two frequencies ($f_1, f_2 = mf_1, m \geq 1, \beta = 2\pi/\lambda_1$), the following equations can be obtained as:

$$\left(R_LZ_1^2-R_0Z_2^2\right)\tan(\beta l_1)\tan(\beta l_2)=Z_1Z_2(R_L-R_0)$$
$$+[Z_1\tan(\beta l_2)+Z_2\tan(\beta l_1)](R_0X_L-R_LX_0), \tag{7a}$$
$$(R_0R_L+X_0X_L)[Z_1\tan(\beta l_2)+Z_2\tan(\beta l_1)]$$
$$-Z_2Z_1^2\tan(\beta l_1)-Z_1Z_2^2\tan(\beta l_2)$$
$$=Z_1Z_2(X_L+X_0)-\left(X_LZ_1^2+X_0Z_2^2\right)\tan(\beta l_1)\tan(\beta l_2). \tag{7b}$$
$$\left(R_LZ_1^2-R_0Z_2^2\right)\tan(m\beta l_1)\tan(m\beta l_2)=Z_1Z_2(R_L-R_0)$$
$$+[Z_1\tan(m\beta l_2)+Z_2\tan(m\beta l_1)](R_0X_L-R_LX_0), \tag{7c}$$

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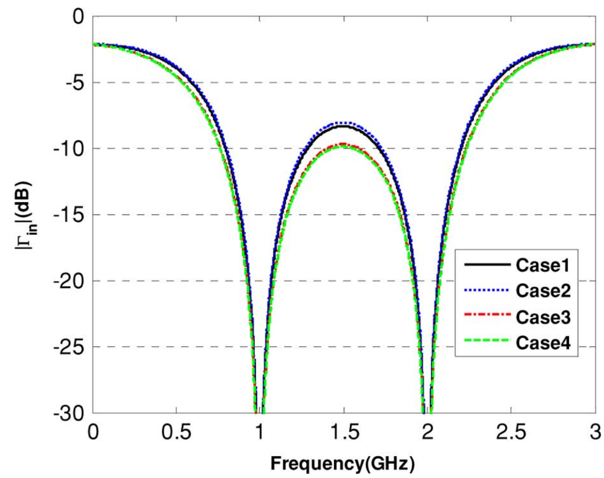

Fig. 2. Comparison of frequency characteristics of dual-frequency transformers (the frequency ratio $m = 2$) with different reactances.

$$\begin{aligned}&(R_0R_L + X_0X_L)[Z_1\tan(m\beta l_2) + Z_2\tan(m\beta l_1)] - Z_2Z_1^2\tan(m\beta l_1)\\&- Z_1Z_2^2\tan(m\beta l_2)\\&= Z_1Z_2(X_L + X_0) - (X_LZ_1^2 + X_0Z_2^2)\tan(m\beta l_1)\tan(m\beta l_2). \quad (7d)\end{aligned}$$

Seen from (7a)–(7d), there are four variables within four equations. The values of R_0, R_L, X_0, X_L and m are known, and the goal is to find the values of $Z_1, Z_2, \beta l_1$ and βl_2. In order to assure desired parameter values are significant for compact microwave implementation, constraint conditions should be given as

$$\begin{aligned}&Z_1 > 0, \quad Z_2 > 0,\\&0 < \beta l_{i,i=1,2} < \pi/2. \end{aligned} \quad (8)$$

Unfortunately, the nonlinear (7) can not be solved analytically unless $X_0 = X_L = 0$ (This case has been analyzed in [1]–[3]). There are many mathematical methods to solve these kinds of nonlinear equations. Here, we propose a simple and direct method to solve (7) as follows:

1) Let $X_0 = X_L = 0$ and apply the analytical solution in Monzon's theory [3] to obtain the initial values, given by

$$l_{1s} = l_{2s} = \frac{\lambda_1}{2(1+m)}, \quad (9a)$$

$$Z_{1s} = \sqrt{\frac{R_0(R_L - R_0)}{2\tan^2(\frac{\pi}{1+m})} + \sqrt{\left[\frac{R_0(R_L - R_0)}{2\tan^2(\frac{\pi}{1+m})}\right]^2 + R_0^3R_L}}, \quad (9b)$$

$$Z_{2s} = \frac{R_LR_0}{Z_{1s}}. \quad (9c)$$

2) Optimization algorithms are then used to obtain the final desired values which satisfy (7) around the initial values; note that the desired values are close to the initial values. It is necessary to point out that either the immune algorithm, the genetic algorithm, or some optimization toolbox [6] can be applied in this step.

Through numerical solutions, it can be observed that there is a relationship between Z_{L2} values at the two frequencies, given by

$$Z_{L2}|_{f_1} = \mathrm{conj}(Z_{L2}|_{f_2}) = R_{L2} + jX_{L2}, \quad (10)$$

where $\mathrm{conj}(*)$ represents the conjugate function.

The relationship (10) can be contributed to deduce other nonlinear equations instead of (7) to decrease the number of variables. Based on single transmission line transformations analysis between complex impedances [7], [8], we can obtain

$$Z_1 = \sqrt{R_0R_{L2} + \frac{X_0^2R_{L2} - X_{L2}^2R_0}{R_0 - R_{L2}}}, Z_2 = \sqrt{R_LR_{L2} + \frac{X_{L2}^2R_L - X_L^2R_{L2}}{R_{L2} - R_L}}, \quad (11a)$$

$$\tan(\beta l_1) = \frac{Z_1(R_0 - R_{L2})}{R_0X_{L2} - R_{L2}X_0}, \tan(\beta l_2) = \frac{Z_2(R_{L2} - R_L)}{R_{L2}X_L + R_LX_{L2}}, \quad (11b)$$

$$\tan(m\beta l_1) = \frac{Z_1(R_{L2} - R_0)}{R_0X_{L2} + R_{L2}X_0}, \tan(m\beta l_2) = \frac{Z_2(R_{L2} - R_L)}{R_{L2}X_L - R_LX_{L2}}. \quad (11c)$$

Considering (8), (11) can be simplified to two nonlinear equations including two unknown parameters R_{L2} and X_{L2}, given by

$$m\arctan\left[\frac{Z_1(R_0 - R_{L2})}{R_0X_{L2} - R_{L2}X_0}\right] = \arctan\left[\frac{Z_1(R_{L2} - R_0)}{R_0X_{L2} + R_{L2}X_0}\right] + \pi, \quad (12a)$$

$$m\arctan\left[\frac{Z_2(R_{L2} - R_L)}{R_{L2}X_L + R_LX_{L2}}\right] = \arctan\left[\frac{Z_2(R_{L2} - R_L)}{R_{L2}X_L - R_LX_{L2}}\right] + \pi \quad (12b)$$

where Z_1 and Z_2 are in terms of R_{L2} and X_{L2} from (11a).

The solutions process of (12) is similar with that of (7). The initial value can be obtained by (9) and (2). Obviously, (12) is convenient because there are just two variables. Once the values of (10) are deduced from (12), the values of $Z_1, Z_2, \beta l_1$ and βl_2 can be obtained directly from (11). In addition, a small dual-frequency transformer may not exist in some special cases or that the solutions may not satisfy (8). These rigorous situations of special cases are relevant with the values of impedances and frequency ratios m, which need further research.

III. Numerical Examples

In this section, some numerical examples are presented to verify the design methods. It can be found that the final results of (7) and (12) are the same. To simulate the frequency characteristics of this two-section transformer based on the lossless transmission line model, the reflection coefficient is defined as [9]

$$\Gamma_{\text{in}} = \frac{Z_{\text{in}} - Z_0^*}{Z_{\text{in}} + Z_0}. \quad (13)$$

For convenience we choose the first frequency $f_1 = 1$ GHz in these examples. When the frequency ratio $m = 2$ is fixed, four cases with different reactances are simulated. The parameters obtained from (7) or (12) are listed in Table I and the corresponding frequency characteristics are shown in Fig. 2. Another example with the frequency ratio $m = 3.5$ is simulated and the design parameters are listed in Table II while the corresponding frequency characteristics are shown in Fig. 3. In order to compare the characteristic of this transformer with different frequency ratios, the final example is illustrated in Fig. 4 and the design parameters are listed in Table III.

Obviously, observed from Fig. 2–4, the design parameters in Tables I–III satisfy the matching conditions at two different frequencies simultaneously. It can be observed from Tables I–III that the physical lengths of two sections are not equal when reactances are non-zero, which is different from

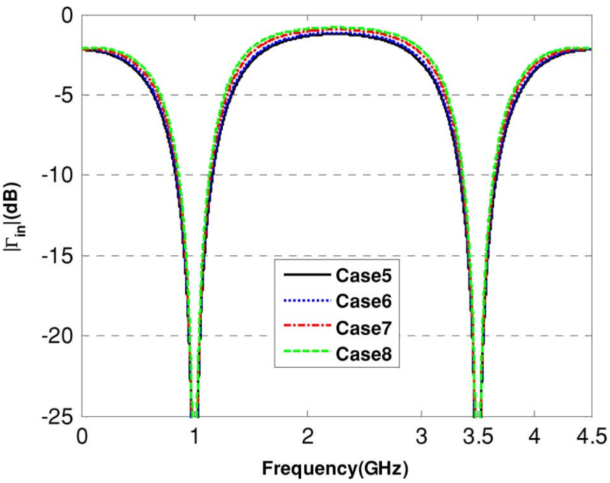

Fig. 3. Comparison of frequency characteristics of dual-frequency transformers (the frequency ratio $m = 3.5$) with different reactances.

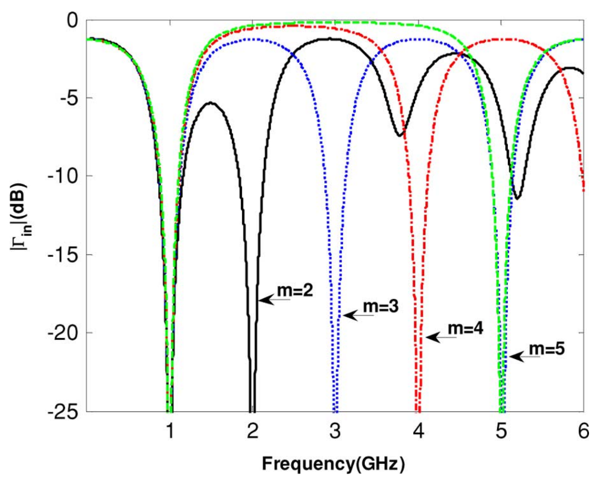

Fig. 4. Comparison of frequency characteristics of dual-frequency transformer with different frequency ratios.

TABLE I
PARAMETERS OF THE DUAL-FREQUENCY TRANSFORMER
($m = 2, R_0 = 50, R_L = 400$)

Type	X_0	X_L	Z_1	Z_2	l_1/λ_1	l_2/λ_1
Case 1	0	0	102.789	194.573	0.1667	0.1667
Case 2	0	30	104.372	195.167	0.1667	0.1717
Case 3	20	0	99.378	205.595	0.1392	0.1667
Case 4	15	-60	99.142	207.466	0.1456	0.1559

the traditional dual-frequency transformer between two resistances. It is interesting that the physical lengths in Case 3 and Case 4 ($X_0 > 0$ and $X_L \leq 0$) are smaller than the one in Case1, which means that there may be smaller dual-frequency transformers between two special complex impedances than the Monzon's transformer. From Table I–III, the physical lengths of two sections in all cases are still very small. Actually, the bandwidth of the matching will become smaller as the ratio $r = \mathrm{Max}(R_L, R_0)/\mathrm{Min}(R_L, R_0) > 1$ increases [3].

Fig. 4 shows that the transformer between two complex impedances can be designed at two arbitrary frequencies. The matching characteristics at the fixed frequency $f_1 = 1$ GHz are very similar, although the frequency ratios are different. The second matching frequencies are changed along with the frequency ratios.

TABLE II
PARAMETERS OF THE DUAL-FREQUENCY TRANSFORMER
($m = 3.5, R_0 = 50, R_L = 400$)

Type	X_0	X_L	Z_1	Z_2	l_1/λ_1	l_2/λ_1
Case 5	0	0	163.480	122.339	0.1111	0.1111
Case 6	0	40	166.622	120.633	0.1111	0.1134
Case 7	-20	0	175.400	115.734	0.1198	0.1111
Case 8	-20	80	183.293	114.165	0.1194	0.1153

TABLE III
PARAMETERS OF THE DUAL-FREQUENCY TRANSFORMER WITH DIFFERENT FREQUENCY RATIOS ($R_0 = 50, X_0 = 40, R_L = 400, X_L = 300$)

m	Z_1	Z_2	l_1/λ_1	l_2/λ_1
2	136.373	296.592	0.1324	0.2108
3	175.250	177.667	0.1057	0.1431
4	218.979	125.571	0.0879	0.1099
5	262.573	97.281	0.0750	0.0896

IV. CONCLUSION

The nonlinear equations for dual-frequency matching between two complex impedances are obtained by a strict derivation. Two simple numerical solution processes are proposed to solve two groups of nonlinear equations. Through simulating some examples, the proposed matching structure and numerical methods are verified. It is believed that this small dual-frequency transformer can be used widely in dual band microwave circuits.

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