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The statistical properties of two-dimensional turbulent wake of a heated cylinder

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Abstract

By analyzing the experimental data, the statistical properties of 2D turbulent wake of a heated cylinder are studied. The coherent structures may have the stronger effect upon the transverse velocity and temperature than the longitudinal velocity. In the near wake, the transverse velocity v and the temperature T have the bimodal distribution and the χ^2 distribution, respectively. But the distribution of the longitudinal velocity u is near the Gaussian even in the near wake. The statistical distributions for the velocity differences and the temperature differences are studied. In the near wake, the pdfs of the transverse velocity differences are concave functions only for the time separations $n\Delta t$: $n_{c_1} < n < n_{c_2}$. Otherwise, they will be the convex functions. The pdf of the temperature difference will change from the nearly exponential distribution to the Gaussian distribution as the time separation $n\Delta t$ increasing in the near wake. Using the detrended fluctuation analysis, it is found out that the longitudinal velocity and temperature have the long-range correlation with the exponent $\alpha \approx 0.7$ in the near and intermediate wake and the transverse velocity has the weakly anti-correlation with the exponent $\alpha \approx 0.44$ in the intermediate wake.

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Keywords: Turbulence; Long-range correlation; Detrended fluctuation analysis

Chicago convective experiment revealed that the probability density function (pdf) of the temperature is Gaussian for high Rayleigh numbers and nearly exponential for low Rayleigh numbers [1]. This experiment has inspired a lot of theoretical [2–5], numerical [6–9] and experimental [10–12] investigations for the turbulent statistics. For the inhomogeneous turbulence, the coherent structures, which are the large-scale

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self-organized structures, may enhance the intermittency of turbulence [13]. However, a complete understanding is still lack [13–15].

Since turbulent wake flow can be relatively easily obtained in a laboratory, it has been extensively studied [13,16,17]. The coherent structures in the wake of a cylinder are induced by that the vortices shed from the cylinder with varying degrees of intensity and regularity. The effect of the coherent structures will be weaker in the far wake. As a result, the intermittency properties have been found to agree with the Gaussian distribution in the far wake [17]. It is expected that the near wake will be dominated by the non-Gaussian statistics due to the effect of the coherent structures. Studying the turbulent wake is very helpful for understanding the inhomogeneous turbulence.

In this paper, we analyze the experimental data for the 2D turbulent wake of a heated cylinder. The data analysis shows that the coherent structures have the more stronger effect upon the transverse velocity and temperature than the longitudinal velocity. In the near wake, the statistical distribution of temperature is asymmetric and can be fitted very well by the χ^2 distribution. In order to filter the large-scale effect [9], we study the temperature differences. It is found that the temperature differences have the stretched exponential distributions for the small time separations and have the Gaussian distributions for the large time separations. The statistics for the longitudinal velocity differences and transverse velocity differences are also studied.

The problem whether the velocity and temperature fluctuations exhibit the long-range correlation have the theoretical and practical importance. Detrended fluctuation analysis (DFA) method has made a lot of achievements for detecting the long-range correlations in the DNA sequences [18,19], weather records [20] and heartbeat fluctuations [21]. We use the DFA method to study the time sequences for the velocity and temperature fluctuations in the wake. It is found out that the longitudinal velocity and temperature have the power-law long-range correlation in the wake that we checked and the transverse velocity exhibits the weakly anti-correlation in the intermediate wake.

Experiment to investigate the behavior of cylinder wake was carried out in a closed circuit wind tunnel with a square cross-section ($0.6\,\mathrm{m} \times 0.6\,\mathrm{m}$) of 2.0 m long [1]. A brass cylinder ($d = 12.7\,\mathrm{mm}$) generated the wake. The cylinder was installed horizontally in the mid-plane and spanned the full width of the working section. It was located at 20 cm downstream of the exit plane of the contraction. This resulted in a maximum blockage of about 2.1% and an aspect ratio of 47. Cylinder was electrically heated. Measurements were made at a free-stream velocity U_∞ of 7 m/s, or $Re = 5800$. A three-wire probe (an X-wire plus a cold wire, the latter placed about 1 mm upstream of the X-wire crossing point and orthogonal to the X-wire plane) was used to measure the velocity in the stream-wise and lateral directions u and v, respectively, and the temperature T. The three-wire probe was traversed across the flow. One X-wire was used in conjunction with the three-wire probe in order to provide a phase reference for the signals from the three-wire probe. The X-wire was fixed at $4d$ below the center of the cylinder. The hot wires were etched from a 5 μm diameter Wollaston (Pt–10% Rh) wire to a length of about 1 mm. As for the cold wire, a 1.27 μm diameter Wollaston (Pt–10% Rh) wire was etched to a length of about 1.2 mm and a temperature

[1] This experiment was performed in the Fluid Mechanics Laboratory of Hong Kong Polytechnic University.

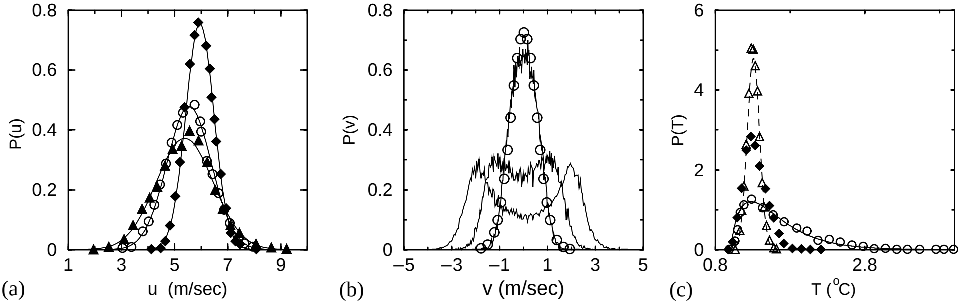

Fig. 1. (a) The pdfs of the longitudinal velocity $P(u)$ where filled triangle up, circle and filled diamond correspond to $x/d = 10, 20, 40$, respectively. Solid lines are the Gaussian distributions. (b) The pdfs of the transverse velocity $P(v)$ for $x/d = 10, 20, 40$ (from bottom to top) where circle corresponds to the Gaussian distribution. (c) The pdfs of temperature $P(T)$ for $x/d = 10$ (circle), $x/d = 20$ (filled diamond) and $x/d = 40$ (triangle up). The dashed line and solid line are the Gaussian distribution and the χ^2 distribution, respectively.

coefficient of 1.69×10^{-3}°C was used [22]. Constant-temperature and constant-current circuits were used for the operation of the hot wires and the cold wire, respectively. An overheat ratio of 1.8 was adopted for the X-wire, while a current of 0.1 mA was used in the cold wire. The sensitivity of the cold wire to the velocity fluctuations was negligible since the length-to-diameter ratio was about 1000, sufficiently large to allow the neglect of any low-wave-number attenuation of the temperature variance. Based upon the arguments of Antonia et al. [23], the frequency response of the wire, as indicated by -3 dB frequency, was estimated to be 2.2 kHz at the wind speed investigated. This was sufficient to avoid any high-frequency attenuation of the main quantities of interest. Signals from the circuits were offset, amplified and then digitized using a 16 channel (12 bit) A/D board and a personal computer at a sampling frequency of 3.5 kHz per channel. The duration of each record was 10 s. Data was recorded at the control points placed downstream distances from the cylinder on the axis of the flow.

We first study the one-point statistics for velocity and temperature. The statistical variance of the longitudinal velocity u will decrease as increasing x/d (where x is the distance from the cylinder). This means that the turbulent energy will decay in the far wake of cylinder. Fig. 1(a) shows that the probability density functions (pdfs) of the longitudinal velocity can be fitted very well by the Gaussian distribution although there is a slight departure for $x/d = 10$. In contrast to the longitudinal velocity, the transverse velocity v has the bimodal distribution for $x/d = 10$. It can be explained as the result of that the inverse revolving vortices shed from the cylinder alternately. The pdf $P(v)$ will approach the Gaussian in the intermediate wake $x/d = 40$ (see Fig. 1(b)). It is very interesting that the temperature has the asymmetrical distribution in the near wake. At the location of $x/d = 10$, we can fit it with the χ^2 distribution: $P(T) = 7.82(T - 1.06)^{0.81}e^{-3(T-1.06)}$ (see Fig. 1(c)). It will approach the Gaussian as x/d increases to $x/d = 40$, eventually.

Initiated by the celebrated Kolmogrov's work in 1941 and 1962 [24,25], much effort has been devoted to study the anomalous multiscaling in the inertial range of fluid

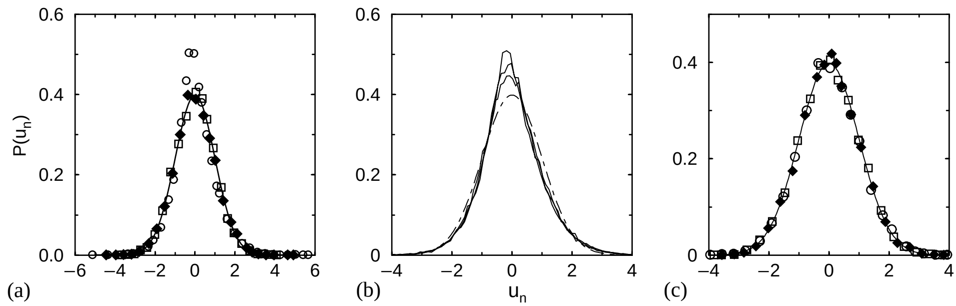

Fig. 2. The pdfs of the normalized longitudinal velocity differences $P(u_n)$: (a) for $x/d=10$ with the different time separations: $n=1$ (circle), $n=10$ (square), $n=100$ (filled diamond). Solid line is the Gaussian distribution; (b) for $x/d=10,20,40$ with the time separation $n=1$ and the Gaussian distribution(dot–dashed line) (from top to bottom); (c) for $x/d=10$ (circle), $x/d=20$ (square) and $x/d=40$ (filled diamond) with the time separation $n=100$. Solid line is the Gaussian distribution.

turbulence in terms of the structure function $S_N(r) \equiv \langle [(A(\mathbf{x}+\mathbf{r},t)-A(\mathbf{x},t)]^N \rangle$, where A is velocity or temperature [26–29]. In the recent, some studies [13,16,30] have been done for the anomalous multiscaling in turbulent wake flows. To study whether turbulent velocity and turbulent temperature are intermittency, we consider the pdfs for the velocity difference and temperature difference and investigate whether there is a change in its shape as the time separation changes. In order to give a clear picture, we investigate the normalized differences: $A_n=(A(t+n\Delta t)-A(t))/\langle [(A(t+n\Delta t)-A(t))^2] \rangle^{1/2}$, where $\Delta t=1/3.5\times 10^{-3}$ s, i.e., u_n, v_n and T_n are the normalized differences for longitudinal velocity u, transverse velocity v and temperature T, respectively. Fig. 2 shows that the pdfs of the normalized longitudinal velocity differences $P(u_n)$ are departure from the Gaussian for the small n and eventually approach the Gaussian for the large n at all the locations $x/d=10,20,40$. The statistical distribution for the normalized transverse velocity difference $P(v_n)$ is more interesting. In the near wake ($x/d=10$), the pdfs of the normalized transverse velocity differences $P(v_n)$ are the concave functions for the time separations $n_{c_1} < n < n_{c_2}$, in which $n_{c_1} \approx 8$ and $n_{c_2} \approx 185$. They are the convex functions for the small or large time separations (see Fig. 3). For $n=1, P(v_n)$ is very near the exponential distribution and can be approximated by the stretched exponential distribution $P(v_n)=0.7891\exp(1.58|v_n|^{0.92})$ at the location $x/d=10$. In the intermediate wake ($x/d=40$), $P(v_n)$ can be fitted by the Gaussian distribution for the large time separation $n=200$ (see Fig. 4). For the small time separation $n=1$, the pdf of the normalized temperature difference $P(T_n)$ is near the exponential distribution and can also be fitted by the stretched exponential distribution $P(T_n)=0.7891\exp(1.58|T_n|^{0.92})$. The pdf $P(T_n)$ changes from the stretched exponential distribution to the Gaussian as n increasing (see Fig. 5). The change in shape of the pdf with n indicates that the statistics are different at different time scales and the turbulent temperature is intermittent. This phenomenon has been also found out in the numerical simulations for the passive scalar advected by the quasifrozen velocity [8] and the Rayleigh–Bénard convection experiments [31]. It is also in agreement with the proposition of

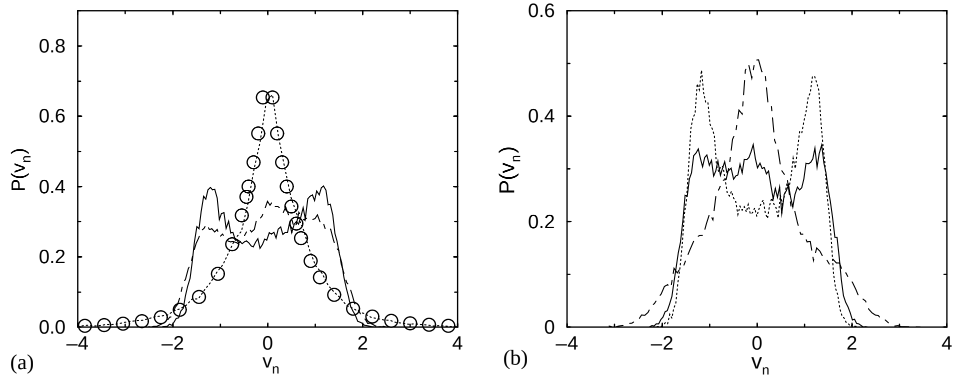

Fig. 3. The pdfs of the normalized transverse velocity differences $P(v_n)$ for $x/d = 10$ with the different time separations: (a) $n = 1$ (dotted line), $n = 8$ (dot–dashed line), $n = 10$ (solid line) and the stretched exponential distribution (circle); (b) $n = 180$ (dotted line), $n = 185$ (solid line) and $n = 190$ (dotted–dashed line).

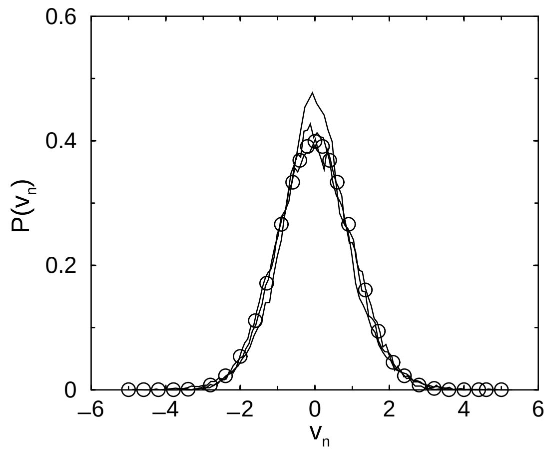

Fig. 4. The pdfs of the normalized transverse velocity differences $P(v_n)$ for $x/d = 40$ with the different time separations: $n = 1, 5, 200$ (from top to bottom). Circle corresponds to the Gaussian distribution.

Ching et al. [9] that the shape of pdfs of the scalar difference is not affected by the presence of the large-scale flow. We also note that both the transverse velocity difference v_n and temperature difference T_n have the nearly exponential distributions in the near wake $x/d = 10$ for the small time separation $n = 1$ although they are different for the large time separations.

In order to investigate the long-range correlations for turbulent velocity and turbulent temperature, we use the DFA method which is shown to be an effective tool for detecting the long-range correlation in random sequences [18–21]. For the random sequence I_i, the correlation function $C(n) = \langle I_i I_{i+n} \rangle$ is not calculated directly, but instead

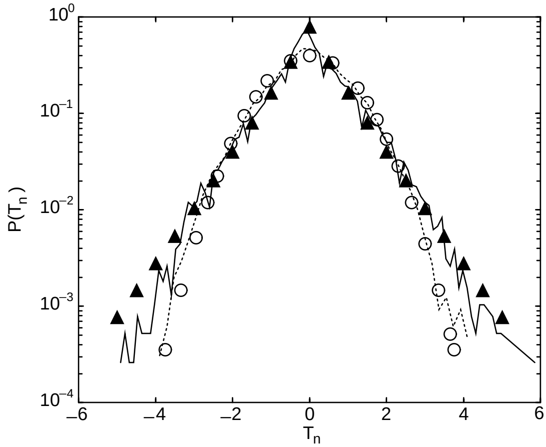

Fig. 5. The pdfs of the normalized temperature differences $P(T_n)$ for the different time separations: $n = 1$ (solid line) and $n = 100$ (dotted line) at the location $x/d = 10$. Filled triangle up and circle correspond to the stretched exponential distribution and Gaussian distribution, respectively.

study the profile for the random sequence: $Y_n = \sum_{i=1}^{n} I_i$. The fluctuations for the profile Y_n are calculated in the specified way proposed in Refs. [18,19]. If only short-range correlation (or no correlations) exists in the random sequence, the detrended walk must have the statistical properties of a random walk so $F(n) \sim n^{1/2}$. If the correlations are long range such that $C(n) \sim n^{-\gamma}$, the fluctuations will be described by the power law: $F(n) \sim n^{\alpha}$ where $\alpha = 1 - \gamma/2$. The exponent α can serve as an indicator of the presence and type of correlations:

1. If $\alpha > 1/2$, I_i is correlated such that positive values of I_i are likely to be close to each other, and the same is true for negative I_i values, i.e., the random sequence I_i is persistence.
2. If $\alpha < 1/2$, I_i is anti-correlated. The values I_i are organized such that positive and negative values are more like to alternate in time.

Using the original DFA method with a sliding box [18,19], we calculate the fluctuation functions $F(n)$ for the velocity and temperature in the wake. Fig. 6(a) and (b) show that the longitudinal velocity and temperature are always persistence in the near or intermediate wake with the exponent $\alpha \approx 0.7$. This exponent α is very near that obtained from the records of the maximum daily temperature of 14 meteorological stations around the globe [20], where $\alpha \approx 0.65$. We note that the slope 0.7 in Fig. 6(b) seems to fit better for small x/d while there seems to be a cross-over to stronger correlations on small scales in the intermediate wake. In the turbulent wake, the inverse revolving vortices shed from the cylinder alternately. The shedding frequency is measured to be in the range of 109–115 Hz. So, the periodic trend has the strong effect upon the transverse velocity in small time scale ($n < 31$) (see Fig. 6(c)). For the time

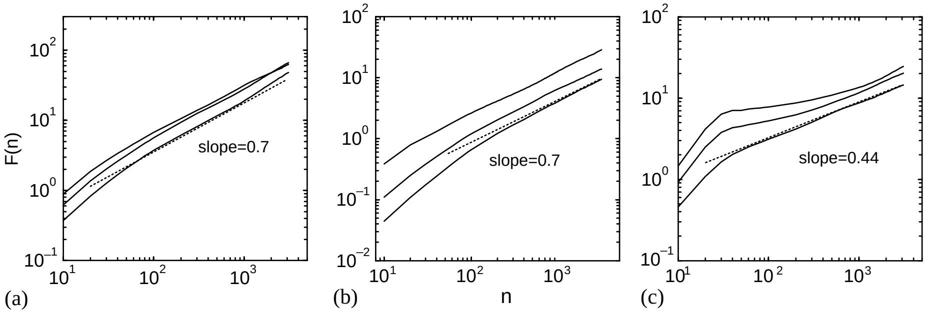

Fig. 6. The fluctuation functions $F(n)$ versus n for the different locations: $x/d = 10, 20, 40$ (from top to bottom) of (a) the longitudinal velocity, (b) the temperature and (c) the transverse velocity.

scale $n > 31$ (corresponding to about 110 Hz), the transverse velocity exhibits weakly anti-correlation with the exponent $\alpha \approx 0.44$ in the intermediate wake. However, the transverse velocity does not have the power-law long-range correlations in the near wake.

In this paper, we investigate the intermittency and the long-range correlation of the turbulent wake of a heated cylinder. The relations between the intermittency and the long-range correlation is also of interest. This reserves the future research. The authors would like to acknowledge the referee for improving the manuscript a lot.

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