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A two-stage programming approach for water resources management under randomness and fuzziness

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ABSTRACT

In this study, a fuzzy stochastic two-stage programming (FSTP) approach is developed for water resources management under uncertainty. The concept of fuzzy random variable expressed as parameters' uncertainties with both stochastic and fuzzy characteristics was used in the method. FSTP has advantages in uncertainty reflection and policy analysis. FSTP integrates the fuzzy robust programming, chance-constrained programming and two-stage stochastic programming (TSP) within a general optimization framework. FSTP can incorporate pre-regulated water resources management policies directly into its optimization process. Thus, various policy scenarios with different economic penalties (when the promised amounts are not delivered) can be analyzed. FSTP is applied to a water resources management system with three users. The results indicate that reasonable solutions were generated, thus a number of decision alternatives can be generated under different levels of stream flows, α-cut levels and different levels of constraint-violation probability. The developed FSTP was also compared with TSP to exhibit its advantages in dealing with multiple forms of uncertainties.

1. Introduction

Water resources management is related to many technical, social, environmental, institutional, political and financial factors (Zarghaami, 2006). However, in the 21st century, the available water resources are becoming over utilized and there is an urgent need to develop sound management plans (Wheida and Verhoeven, 2007). The competitions for water among municipal, industrial and agricultural users have been intensifying. The disparate groups of water users need to know how much water they can expect in order to make appropriate decisions regarding their various activities and investments. If the promised water cannot be delivered due to insufficient supply, users will have to either obtain water from higher-priced alternatives or curb their development plans (Maqsood et al., 2005).

In water resources management, uncertainties that exist in many system parameters could intensify the conflict-laden issue of water allocation among competing municipal, industrial and agricultural interests (Huang and Chang, 2003). The above complexities could become further compounded through not only interactions among the uncertain parameters but also combinations of the uncertainties as presented in multiple formats. In analyzing water resources systems, inexact optimization methods were considered useful for planning water resources systems under uncertainty (Huang and Loucks, 2000; Maqsood et al., 2005; Jung et al., 2006; Li et al., 2006; Dessai and Hulme, 2007; Wu et al., 2007; Guo and Huang, 2009). Two-stage stochastic programming (TSP) is effective for problems where an analysis of policy scenarios is desired and when the right-hand side coefficients are random with known probability distributions. The advantage of TSP is its capability of guiding corrective actions after a random event has taken place. In TSP, a decision is firstly undertaken before values of random variables are known; then, after the random events have happened and their values are known, a second decision is made in order to minimize "penalties" that may appear due to any infeasibility. The TSP methods were widely explored over the past decades (Kall, 1979, 1982; Wang and Adams, 1986; Beraldi et al., 2000; Dai et al., 2000; Huang and Loucks, 2000; Luo et al., 2003; Ahmed

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et al., 2004; Maqsood et al., 2005; Guo et al., 2008a). Fuzzy optimization is a flexible approach that permits an adequate solution of real-world problems in the presence of vague information. The fuzzy programming (FP) method considers uncertainties as fuzzy sets and is effective in reflecting ambiguity and vagueness in resource availabilities (Li et al., 2007). The chance-constrained programming (CCP) method was used to deal with random uncertainty information. CCP required that all of the constraints be satisfied in a proportion of cases under given probability levels (Loucks et al., 1981). Previously, a number of research works for FP and CCP methods were undertaken (Huang, 1998; Huang et al., 2001; Liu and Iwamura, 1998; Debjani, 2002; Cooper et al., 2004; Yang and Wen, 2005; Guo et al., 2008b, c, 2009).

However, a remarkable limitation of the aforementioned methods is their incapability in reflecting parameter's multiple uncertainties presented as combinations of fuzzy and probability distributions. In many real-world problems, a number of parameters can hardly be expressed in conventional fuzzy or stochastic formats; instead, they may have characteristics of both probability distributions and fuzzy sets (Luhandjula, 2004). The concept of fuzzy random variable (FRV) was first defined by Kwakernaak (1978, 1979). When the parameters have characteristics of both randomness and fuzziness, the fuzzy random variables would be included in the model. Through the fuzzy random variables, the parameters' multiple uncertainties could be expressed. Therefore, to address fuzzy and probabilistic uncertainties, both fuzzy programming and stochastic programming can be integrated within a general optimization framework (Luhandjula and Gupta, 1996; Luhandjula, 2004, 2006; Liu, 1997, 2001; Katagiri et al., 2005; Liu and Liu, 2005; Huang, 2006).

One potential approach for accounting for various uncertainties in the constraints' left- and right-hand sides is to integrate the fuzzy robust programming (FRP), chance-constrained programming (CCP), and two-stage stochastic programming (TSP) within a general optimization framework. This research aims to develop a fuzzy stochastic two-stage programming (FSTP) method for water resources management under uncertainty. FSTP can handle uncertainties in left-hand sides presented as fuzzy sets and those in right-hand sides as probability distributions and fuzzy random variables. The developed FSTP will then be applied to a case of water resources management to demonstrate its applicability. FSTP can incorporate pre-regulated water resources management policies directly into its optimization process. Thus, various policy scenarios with different economic penalties (when the promised amounts are not delivered) can be analyzed. The developed FSTP will also be compared with TSP to exhibit its advantages in dealing with multiple forms of uncertainties.

2. Modeling formulation

2.1. Problem formulation

The water manager is responsible for allocating the scarce water supply to the competing users within multiple periods. The future availability of this water supply is uncertain. The manager needs to create a plan to effectively allocate the uncertain supply of water to the three users in order to maximize the overall system benefit while simultaneously considering the system disruption risk attributable to the uncertainties. Based on the regional water management policies, an allowable flow level to each user must be regulated. If this level is satisfied, it will result in a net benefit to the system. However, if it is not satisfied, the shortage will lead to a decreased net system benefit. Under such a situation, the shortage amount will be the targeted allocation minus the actual allocation amount. Uncertainties of seasonal water flows presented as probability distributions should also be reflected. The distribution of each seasonal water flow can be converted into an equivalent set of discrete values (Huang and Loucks, 2000). This leads to a two-stage stochastic programming (TSP) problem as follows:

$$\max f = \sum_{i=1}^{m} B_i W_i - \sum_{i=1}^{m}\sum_{j=1}^{n} p_j C_i (W_i - A_{ij}) \tag{1a}$$

subject to

$$\sum_{i=1}^{m} A_{ij}(1+\xi) \le q_j, \quad \forall j \tag{1b}$$

[water availability constraints],

$$A_{ij} \le W_i \le W_{i\,\max}, \quad \forall i \tag{1c}$$

[allowable water-allocation constraints],

$$A_{ij} \ge 0, \quad \forall i, j \tag{1d}$$

[non-negativity and technical constraints]

In model (1), the water-allocation targets (W_i) must be set at the first stage before the stream flows (q_j) are known. The water-allocation plan (A_{ij}) will thus be determined during the second stage when the stochastic stream flows are known. ξ is the loss rate of water process during transportation. The $\sum_{i=1}^{m} B_i W_i$ is the first-stage decision. The $\sum_{i=1}^{m}\sum_{j=1}^{n} p_j C_i (W_i - A_{ij})$ is the second-stage recourse when the random event has happened (Kall, 1979). Model (1) can reflect uncertainties in stream flows presented as probability density functions. However, in many real-world problems, the quality of the uncertain information may be more complex, especially, in large-scale models.

2.2. Methodology

The fuzzy robust linear programming (FRLP) involves the optimization of a precise objective function in a fuzzy decision space delimited by constraints with fuzzy coefficients and capacities (Inuiguchi and Sakawa, 1998). Consider a FRLP problem as follows (Leung, 1988):

$$\min f \cong CX \tag{2a}$$

subject to

$$AX \lesssim B \tag{2b}$$

$$X \ge 0 \tag{2c}$$

where $C \in \{R\}^{1\times n}$ and $X \in \{R\}^{n\times 1}$; $\{R\}$ denote a set of numbers; $A \in \{R\}^{m\times n}$ and $B \in \{R\}^{m\times 1}$ are fuzzy sets; symbols $\simeq$ and $\lesssim$ present fuzzy equality and inequality; Let c_j be the jth element of C, $\tilde{a}_{ij}$ be the ith row and jth line element of A, $\tilde{b}_i$ be the ith element of B, x_j be the jth element of X, and f be the objective's aspiration level.

$$A_1 x_1 \oplus A_2 x_2 \oplus \cdots \oplus A_n x_n \lesssim B \tag{3}$$

where A_j ($j = 1, 2, \ldots, n$) and B are fuzzy subsets, and symbol $\oplus$ denotes the addition of fuzzy subsets. Fuzziness of the decision space is due to uncertainties in the coefficients A_j and B. Letting $\underset{\sim}{U}j$ and $\underset{\sim}{V}$ be base variables imposed by fuzzy subsets A_j and B, we have:

$$\mu_{A_j}: \ \underset{\sim}{U}j \to [0, 1] \tag{4a}$$

$$\mu_B: \ \underset{\sim}{V} \to [0, 1] \tag{4b}$$

where μ_{A_j} indicates the possibility of consuming a specific amount of resource by activity j, and μ_B indicates the possible availability of resource B. Fuzzy subset N can be expressed as follows (Dubois and Prade, 1978):

$$\mu_N(x) = \begin{cases} F_L\left(\frac{m-x}{a}\right), & \text{if } -\infty < x < m,\ a > 0 \\ 1, & \text{if } x = m \\ F_r\left(\frac{x-m}{\delta}\right), & \text{if } m < x < +\infty,\ \delta > 0 \end{cases} \tag{5a}$$

where F_L and F_r are membership functions under different x levels (Fig. 1). Based on the nature of water resources management systems, fuzzy set N in (5a) can be simplified as a linear membership function that has the following general format (5b) (Liu et al., 2003).

$$\mu_N(x) = \begin{cases} 0, & \text{if } x < \underline{a} \text{ or } x > \overline{a} \\ 1, & \text{if } x = m \\ 1 - \frac{2|m-x|}{\overline{a}-\underline{a}}, & \text{if } \underline{a} < x < \overline{a} \end{cases}$$

where $[\underline{a}, \overline{a}]$ is a interval imposed by fuzzy subset N. To state precisely the fuzzy restrictions on the corresponding base variables $\underset{\sim}{U}j$ and $\underset{\sim}{V}$ imposed by fuzzy subsets A_j and B, fuzzy constraints in (3) can be regarded as fuzzy inclusive constraints as follows (Leung, 1988):

$$A_1x_1 \oplus A_2x_2 \oplus \cdots \oplus A_nx_n \subseteq B \tag{6}$$

Based on the concept of a level set (fuzzy α-cut) and the representation theorem, constraints in (3) can be represented as follows (Negoita et al., 1976):

$$\left(\underset{\sim}{A}_1\right)_\alpha x_1 \oplus \left(\underset{\sim}{A}_2\right)_\alpha x_2 \oplus \cdots \oplus \left(\underset{\sim}{A}_n\right)_\alpha x_n \subseteq \underset{\sim}{B}_\alpha, \quad \alpha \in [0,1] \tag{7a}$$

where

$$\left(\underset{\sim}{A}_j\right)_\alpha = \left\{a_j \in \underset{\sim}{U}_j \middle| \mu_{A_j}(a_j) \geq \alpha\right\} \tag{7b}$$

$$(\underset{\sim}{B})_\alpha = \{b \in \underset{\sim}{V} | \mu_B(b) \geq \alpha\} \tag{7c}$$

Assume that the fuzzy subsets in (6) are finite and have the following characteristic:

$$\left\{\mu_{A_j}(a_j) | a_j \in \underset{\sim}{U}_j\right\} = \{\alpha_1, \alpha_2, \ldots, \alpha_s\} \tag{8}$$

where $0 \leq \alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_s \leq 1$. Then, for each α_s ($k = 1, 2, \ldots, s$), constraints in (7a) become:

$$\left(\underset{\sim}{A}_1\right)_{\alpha_s} x_1 \oplus \left(\underset{\sim}{A}_2\right)_{\alpha_s} x_2 \oplus \cdots \oplus \left(\underset{\sim}{A}_n\right)_{\alpha_s} x_n \subseteq \underset{\sim}{B}_{\alpha_s}, \quad \alpha_s \in [0,1] \tag{9}$$

where $(\underset{\sim}{A}j)_{\alpha_s}$ ($j = 1, 2, \ldots, n$; $k = 1, 2, \ldots, s$) and $\underset{\sim}{B}\alpha_s$ constitute convex and non-empty fuzzy sets. Then, fuzzy constraints in (9) can be replaced by the following $2s$ precise inequalities, in which s denotes s levels of α-cut (Soyster, 1973; Leung, 1988; Luhandjula and Gupta, 1996).

$$\overline{a}_1^k x_1 + \overline{a}_2^k x_2 + \cdots + \overline{a}_n^k x_n \leq \overline{b}^k, \quad k = 1, 2, \ldots, s \tag{10a}$$

$$\underline{a}_1^k x_1 + \underline{a}_2^k x_2 + \cdots + \underline{a}_n^k x_n \geq \underline{b}^k, \quad k = 1, 2, \ldots, s \tag{10b}$$

where

$$\overline{a}_j^k = \sup\left(a_j^k\right), \ a_j^k \in \left(\underset{\sim}{A}j\right)_{\alpha_s} \tag{10c}$$

$$\underline{a}_j^k = \inf\left(a_j^k\right), \ a_j^k \in \left(\underset{\sim}{A}j\right)_{\alpha_s} \tag{10d}$$

$$\overline{b}^k = \sup\left(b^k\right), \ b^k \in \underset{\sim}{B}\alpha_s \tag{10e}$$

$$\underline{b}^k = \inf\left(b^k\right), \ b^k \in \underset{\sim}{B}\alpha_s \tag{10f}$$

where $\sup(t)$ denotes the superior limit value among t, and $\inf(t)$ represents the inferior limit value among t. Therefore, for a water resources management system with m fuzzy constraints, the decision space, together with the decision variables, can be delimited by the following fuzzy inclusive constraints (Liu et al., 2003):

$$A_{i1}x_1 \oplus A_{i2}x_2 \oplus \cdots \oplus A_{in}x_n \subseteq B_i, \quad i = 1, \ldots, m \tag{11}$$

Represented in terms of level sets, fuzzy constraints in (11) become:

$$\left(\underset{\sim}{A}i_1\right)_{\alpha_s} x_1 \oplus \left(\underset{\sim}{A}i_2\right)_{\alpha_s} x_2 \oplus \cdots \oplus \left(\underset{\sim}{A}_{\text{in}}\right)_{\alpha_s} x_n \subseteq \underset{\sim}{B}_{i\alpha_s},$$
$$i = 1, 2, \ldots, m; \quad \alpha_s \in [0,1] \tag{12a}$$

where

$$\left\{\mu_{A_{ij}}(a_{ij}) \middle| a_{ij} \in \underset{\sim}{U}ij\right\} = \left\{\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{is}\right\} \tag{12b}$$

$$0 \leq \alpha_{i1} \leq \alpha_{i2} \leq \cdots \leq \alpha_{is} \leq 1, \quad i = 1, \ldots, m \tag{12c}$$

Then, following the arguments in (7a)–(10f), the decision space for problem (12a) can be delimited by the following deterministic constraints (Liu et al., 2003):

$$\sum_{j=1}^{n}\left(\overline{a}_{ij}^k x_j\right) \leq \overline{b}_i^k, \quad i = 1, 2, \ldots, m, \ k = 1, 2, \ldots, s \tag{13a}$$

$$\sum_{j=1}^{n}\left(\underline{a}_{ij}^k x_j\right) \geq \underline{b}_i^k, \quad i = 1, 2, \ldots, m, \ k = 1, 2, \ldots, s \tag{13b}$$

$$x_j \geq 0, \quad j = 1, 2, \ldots, n \tag{13c}$$

For fuzzy robust programming with fuzzy random variables in B, constraints (13a) and (13b) become (Luhandjula and Gupta, 1996):

$$\sum_{j=1}^{n}\left(\overline{a}_{ij}^k x_j\right) \leq \overline{b}_i^k(\omega), \quad i = 1, 2, \ldots, m, \ k = 1, 2, \ldots, s \tag{14a}$$

$$\sum_{j=1}^{n}\left(\underline{a}_{ij}^k x_j\right) \geq \underline{b}_i^k(\omega), \quad i = 1, 2, \ldots, m, \ k = 1, 2, \ldots, s \tag{14b}$$

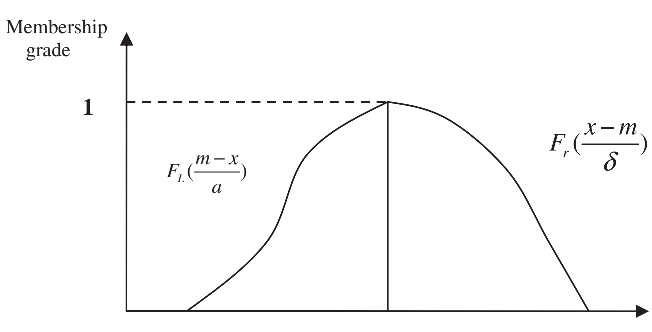

Fig. 1. L/R fuzzy membership function.

When uncertainties of some elements in B can be expressed as probability distributions, methods of chance-constrained programming (CCP) can be used to deal with them. In terms of uncertainties in B, consider a general stochastic linear programming (SLP) problem as follows (Charnes et al., 1972; Charnes and Cooper, 1983):

$$\min \; CX \tag{15a}$$

subject to

$$AX \le B(\omega) \tag{15b}$$

$$X \ge 0 \tag{15c}$$

where $B(\omega)$ are sets with random elements defined on a probability space Ω, $\omega \in \Omega$. The problem can be converted into a deterministic version through: (1) fixing a certain level of probability $(r_i \in [0,1])$ for each constraint (i), and (2) imposing the condition that the constraint is satisfied with at least a probability of $1 - r_i$. The solution set is thus subject to the following constraints (Charnes et al., 1972; Charnes and Cooper, 1983):

$$\Pr[\{\omega | A_i X \le b_i(\omega)\}] \ge 1 - r_i$$

$$A_i \in A, \;\; b_i(\omega) \in B(\omega), \quad i = 1, 2, \ldots m \tag{16}$$

Constraint (16) is generally nonlinear, and the set of feasible constraints is convex only for some particular cases, one of which exists when the left-hand side coefficients (a_{ij}) are deterministic, and the right-hand-side variables (b_j) are random. This leads to an equivalent linear constraint that has the same size and structure as a deterministic term, and the only required information about the uncertainty is the r_i for the unconditional distribution of b_i. Thus, constraint (16) becomes linear (Charnes et al., 1972; Charnes and Cooper, 1983; Huang, 1998):

$$A_i X \le b_i(\omega)^{(\underset{\sim}{p}\, i)}, \quad \forall i \tag{17}$$

where $b_i(\omega)^{(\underset{\sim}{r}\, i)} = F_i^{-1}(\underset{\sim}{r}\, i)$, given the cumulative distribution function of b_i (i.e., $F_i(b_i)$), and the probability of violating constraint i $(\underset{\sim}{r}\, i)$. However, linear constraints can only reflect the case when A is deterministic. If both A and B are uncertain, the set of feasible constraints may become more complicated.

When uncertainties of some elements in the constraints can be expressed as fuzzy random variables and denoted as fuzzy stochastic constraints, methods of fuzzy stochastic programming can be used for dealing with them (Leung, 1988; Inuiguchi and Sakawa, 1998). When the multiple uncertainties in the parameters having fuzziness and randomness features at the same time, fuzzy random variables can be used (Luhandjula and Gupta, 1996). To address a linear program with fuzzy random variables, a fuzzy robust chance-constrained programming (FRCP) model associated with fuzzy random variables can be formulated (Loucks et al., 1981; Huang, 1998; Liu et al., 2003; Luhandjula, 2006):

$$\min \; f = \sum_{j=1}^{n} c_j x_j \tag{18a}$$

subject to

$$\sum_{j=1}^{n} \left(\overline{a}_{ij}^{k} x_j \right) \le \left(\overline{b}_i^{k} \right)^{(\underline{r}_i)}, \quad i = 1, 2, \ldots, m, \;\; k = 1, 2, \ldots, s \tag{18b}$$

$$\sum_{j=1}^{n} \left(\underline{a}_{ij}^{k} x_j \right) \ge \left(\underline{b}_i^{k} \right)^{(\overline{r}_i)}, \quad i = 1, 2, \ldots, m, \;\; k = 1, 2, \ldots, s \tag{18c}$$

$$x_j \ge 0, \quad j = 1, 2, \ldots, n \tag{18d}$$

The FRCP model can deal with left-hand coefficients presented as fuzzy sets and right-hand ones as fuzzy random variables.

A two-stage stochastic linear programming (TSP) model can be formulated as follows:

$$\max f = CX + E(DY) \tag{19a}$$

subject to

$$A_1 X + A_2 Y \le B(t) \tag{19b}$$

$$X \ge 0 \tag{19c}$$

$$Y \ge 0 \tag{19d}$$

where $A_1 \in \{R\}^{m \times n}$, $A_2 \in \{R\}^{m \times n}$, $C \in \{R\}^{1 \times n}$, $D \in \{R\}^{1 \times n}$, $X \in \{R\}^{n \times 1}$ and $Y \in \{R\}^{n \times 1}$; $\{R\}$ denote a set of real numbers; $B(t)$ are random variable with known distribution functions. When uncertainties of some elements in A_1 and A_2 can be expressed as fuzzy sets, and those in B can be expressed as probability distributions and fuzzy random variables, then a fuzzy stochastic two-stage programming (FSTP) model can be formulated through incorporate model (18) with two-stage stochastic programming:

$$\max \; f = CX + E(DY) \tag{20a}$$

subject to

$$\underset{\sim}{A}_1 X + \underset{\sim}{A}_2 Y \le \underset{\sim}{B}(t) \tag{20b}$$

$$X \ge 0 \tag{20c}$$

$$Y \ge 0 \tag{20d}$$

where A_1 and A_2 are fuzzy sets in left-hand side of constraints; X and Y are non-negative decision variables; $B(t)$ are fuzzy random variables with known distribution functions. Let c_j be the jth element of C, d_j be the jth element of D, $\underset{\sim}{a}'_{ij}$ be the ith row and jth line element of A_1, $\underset{\sim}{a}_{ij}$ be the ith row and jth line element of A_2, $\underset{\sim}{b}\, i$ ($i = 1$, 2, …, m, $i \in M$), be the ith element of B which is a fuzzy random variable with a known distribution function, x_j be the jth element of X, and y_j be the jth element of Y.

2.3. Modeling for water resources management

In water resources management systems, fuzzy random variables are introduced into the TSP framework to facilitate communication of the uncertainties into the optimization process, resulting in a fuzzy stochastic two-stage programming (FSTP) model. For example, uncertainties in stream flows may be presented as fuzzy random variables; FSTP can then be used to solve this type of problem. FSTP is an integration of fuzzy robust programming, chance-constrained programming and two-stage programming methods. It can be presented as follows:

$$\max \; f^{\pm} = \text{Objective function} = A - B \tag{21a}$$

where A is the benefit from the pre-regulated water allocated,

$$\sum_{i=1}^{m} B_i W_i$$

B is the economic loss due to the amount of water not delivered,

$$\sum_{i=1}^{m}\sum_{j=1}^{n} p_j C_i (W_i - A_{ij})$$

subject to

Water availability constraints:

$$\sum_{i=1}^{m} A_{ij}\left(1+\overline{\xi}_j^k\right) \le \left(\overline{q}_j^k\right)^{(\underline{r}_h)}, \quad \forall j,k \tag{21b}$$

$$\sum_{i=1}^{m} A_{ij}\left(1+\underline{\xi}_j^k\right) \ge \left(\underline{q}_j^k\right)^{(\overline{r}_h)}, \quad \forall j,k \tag{21c}$$

Allowable water-allocation constraints:

$$A_{ij} \le W_i \le W_{i\,\max}, \quad \forall i \tag{21d}$$

Non-negativity constraints:

$$A_{ij} \ge 0, \quad \forall i,j \tag{21e}$$

where f is the net system benefit (\$); B_i is the net benefit to user i per m^3 of water allocated (\$/m^3) (first-stage revenue parameter); W_i is the target water allocation that is promised to user i (m^3) (first-stage decision variable); C_i is the loss to user i per m^3 of water not delivered, $C_i > B_i$ (\$/m^3) (second-stage cost parameter); A_{ij} is the allocation of water to user i under flow level j (m^3) (second-stage decision variable; when $A_{ij} < W_i$, the amount by which W_i is not met while the seasonal flow is q_j); ξ is the loss rate of water during transportation; $(\underset{\sim}{q}_j)^{(r_h)}$ is the amount of seasonal flow under flow level j (m^3) (fuzzy random variables); $W_{i\,\max}$ is the maximum allowable allocation amount for user i (m^3); p_j is the probability of occurrence of flow level j; m is the total number of water users; n is the total number of flow levels; i is the water user (in the study case, $m = 3$ and $i = 1, 2, 3$, with $i = 1$ for the municipality, $i = 2$ for the industrial user, and $i = 3$ for the agricultural sector); j is the flow level (in the study case, $n = 3$ and $j = 1, 2, 3$, with $j = 1$ representing low flows, $j = 2$ representing medium flows, and $j = 3$ representing high flows); r_h is the probability of violating constraint i.

Both two-stage programming and chance-constrained programming are dealt within the proposed method for the probability distribution. The distribution for the two-stage programming is addressed for the seasonal flow (q_j) under flow level j and the probability of occurrence of flow level j (p_j). Let each seasonal flow takes value q_j with probability p_j, where j is denoted as the level of seasonal water flow rate (for $j = 1, 2, 3$ corresponding to low level, medium level and high level); In this problem, a decision of water-allocation target needs to be made at the beginning facing future uncertainties of river flow; at a future time, when the uncertainties of water flow are disclosed, a recourse action can then be taken. Thus, water allocation at the beginning is called the first-stage decision, and the plan for recourse actions is named the second-stage decision. The distribution for the chance-constrained programming are addressed for violating of water availability constraints corresponding to the lower bound and upper bound under each α - cut level and each level of seasonal flow $((\underset{\sim}{q}_j)^{(r_h)})$ (Fig. 2).

2.4. Algorithm to solve the problem

The fuzzy random variable is a more general definition for uncertainties than fuzzy set and random variable. The significance of this concept is its capability in reflecting more complex uncertainties. Fig. 2 shows the fuzzy random information of the seasonal flows $[(\underset{\sim}{q}_j)^{(r_h)}]$ that have both fuzzy and random characteristics. Fig. 3 gives the framework of the FSTP model.

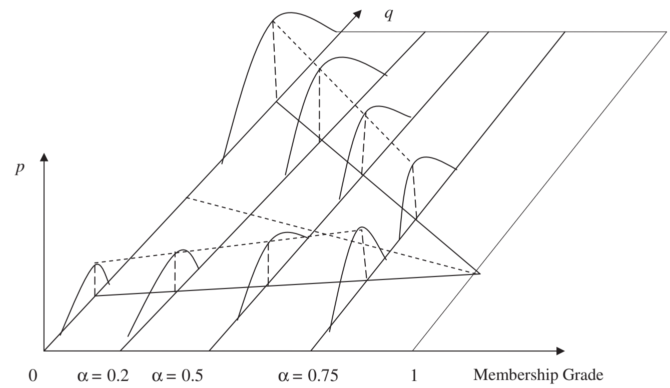

Fig. 2. Fuzzy random information for seasonal flow.

The solutions present as intervals for the objective function value and decision variables under different levels of seasonal flows and r_h levels. They can be interpreted for generating multiple decision alternatives. Model (21) can be solved based on Huang and Loucks (2000), Liu et al. (2003), Maqsood et al. (2005), and Guo et al. (2008c). The detailed solution process can be summarized as follows:

Step 1: Acquire the economic information of water allocated and not delivered.
Step 2: Define the target of water allocation that is promised to user.
Step 3: Acquire distribution information for the seasonal flows in system constraints.
Step 4: Acquire fuzzy information for water loss rate and seasonal flow rates under different α-cut levels.
Step 5: Acquire distribution information of stream flows under different combinations of r_h and α-cut levels.
Step 6: Formulate the FSTP model.
Step 7: Solve the FSTP model to obtain solutions of $A_{ij\,\text{opt}}$ under each r_h level and different levels of seasonal flows.

3. Case study

Consider a region where a manager is responsible for allocating water in a dry season from an unregulated reservoir to three users. In the study system, there are three users, including a municipality, an industry and an agricultural sector. The stream flows vary temporally with random features. The net benefit to user i per m^3 of water allocated and the loss to user i per m^3 of water not delivered also vary temporally and spatially. Table 1 shows the maximum allowable water allocation ($W_{i\,\max}$) (in 10^6 m^3), the net benefit to user per m^3 of water allocated (B_i) (in \$/m^3), the loss to user per m^3 of water not delivered (C_i) (in \$/m^3), and water-allocation target (W_i) (in 10^6 m^3). Table 2 presents values of fuzzy subsets for water loss rate (ξ) and seasonal flow rates (q_j) under different α-cut levels. Assume that the standard deviations are 4% of the expected values; the relevant normal distributions can then be obtained. Table 3 shows expected values and variances of $\underline{q}_j(t)$ and $\overline{q}_j(t)$. Table 4 gives stream flow distributions (in 10^6 m^3) under different combinations of r_h (probability of constraint violation) and α-cut levels.

The decision variables represent probabilistic water-allocation flows from the reservoir to user i under flow level j (denoted as A_{ij}). The objective is to maximize net system benefits through

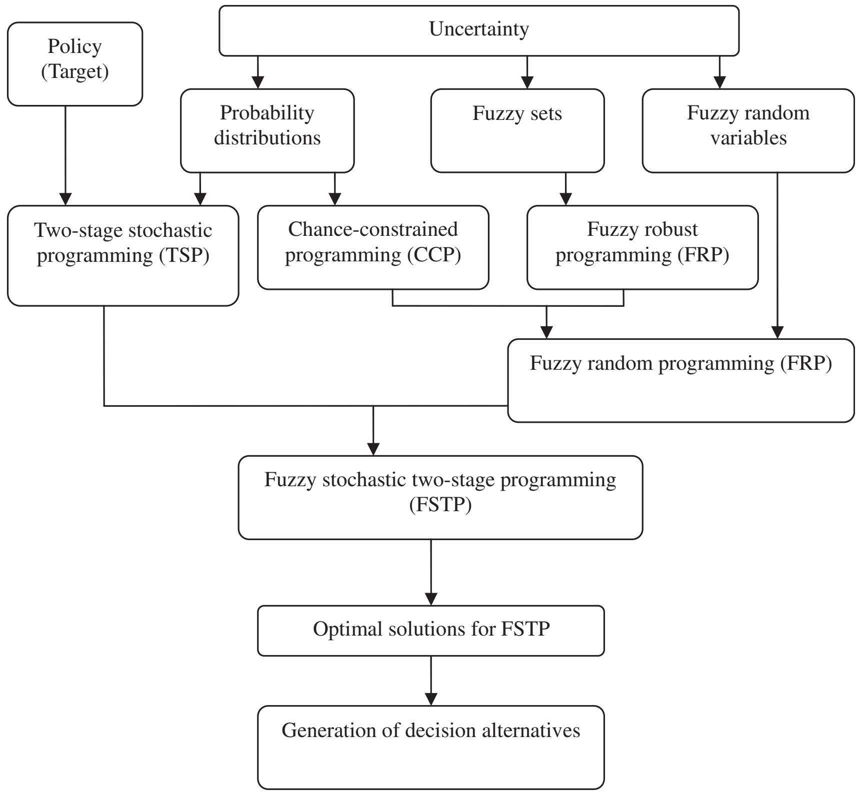

Fig. 3. Framework of the FSTP method.

effectively allocating the water flows from the reservoir to the three users; the constraints involve all relationships among the decision variables and the water-allocation conditions. Because uncertainties exist in a variety of system components and a linkage to the pre-regulated policies as formulated by local authorities is desired, the FSTP method is considered to be a suitable approach for tackling this management problem.

Fig. 4 shows the structure of the water management system. The system consists of a reservoir and three users (Municipality, Industry and Agricultural sector). The expected net system benefit include the net benefit to user per m^3 of water allocated, the loss to user per m^3 of water not delivered and water-allocation target. The problem under consideration is to maximize the net system benefit through effectively allocating the water flows from the reservoir to the three users; the constraints involve relationships among the decision variables and the seasonal flow rates under different conditions.

Table 5 presents the solutions of the FSTP model under different r_h levels and those of the TSP model under optimized water-allocation targets (in 10^6 m^3). Solutions for the objective function value and decision variables are deterministic values. In case of insufficient water, allocation should firstly be guaranteed to the municipality, secondly to the industry, and lastly to the agriculture. This is because municipal use brings the highest benefit when water demand is satisfied and is subject to the highest penalty if the promised water is not delivered; in comparison, the industrial and agricultural uses correspond to lower benefits and penalties. The flow allocation patterns vary among different users with uncertain characteristics. This is due to the uncertainties of the inputting W_i (policies), B_i (benefits), C_i (costs), $q_j(\omega)$ (stream flows), ξ (loss rate of water) and ω (random variable) as well as the complexities of their interactions.

In Table 5, the solutions of $A_{11} = 2.5 \times 10^6$ m^3, $A_{21} = 2.11 \times 10^6$ m^3, and $A_{31} = 0$ (under $r_h = 0.1$) indicate that, under low stream flow levels, there would be no shortage of water (in reference to the optimized water-allocation target of 2.5×10^6 m^3) for municipal water use. However, some shortages of 3.19×10^6 and 6.8×10^6 m^3 may exist (in reference to the optimized water-allocation targets of 5.3×10^6 and 6.8×10^6 m^3) for industrial and agricultural uses,

Table 1
Water-allocation targets (in 10^6 m^3) and the related economic data (in \$/m^3).

Activity	User		
	Municipal ($i = 1$)	Industrial ($i = 2$)	Agricultural ($i = 3$)
Maximum allowable allocation ($W_{i\ max}$)	7	7	7
Water-allocation target (W_i)	2.5	5.3	6.8
Net benefit when water demand is satisfied (B_i)	90	45	28
Reduction of net benefit when demand is not delivered (C_i)	220	60	50

Table 2
Values of fuzzy subsets for water losses and seasonal flows (q_j).

α cut levels	Water loss		Seasonal flow (q_j)					
	$\underline{\xi}$	$\overline{\xi}$	Low ($j = 1$)		Medium ($j = 2$)		High ($j = 3$)	
			$p = 0.2$		$p = 0.6$		$p = 0.2$	
			$(\underline{q}_j)_\alpha$	$(\overline{q}_j)_\alpha$	$(\underline{q}_j)_\alpha$	$(\overline{q}_j)_\alpha$	$(\underline{q}_j)_\alpha$	$(\overline{q}_j)_\alpha$
$\alpha = 0$	0.1	0.2	3	7	7	12	12	22
$\alpha = 0.2$	0.11	0.19	3.4	6.6	7.5	11.5	13	21
$\alpha = 0.5$	0.125	0.175	4	6	8.25	10.75	14.5	19.5
$\alpha = 0.75$	0.1375	0.1625	4.5	5.5	8.875	10.125	15.75	18.25
$\alpha = 1$	0.15	0.15	5	5	9.5	9.5	17	17

Table 3
Expected values and variances of $\underset{\sim}{q}_j(t)$ and $\overline{q}_j(t)$.

Flow level		α-cut level					
		$\alpha = 0.2$		$\alpha = 0.5$		$\alpha = 0.75$	
		$(\underset{\sim}{q}_j)_\alpha(t)$	$(\overline{q}_j)_\alpha(t)$	$(\underset{\sim}{q}_j)_\alpha(t)$	$(\overline{q}_j)_\alpha(t)$	$(\underset{\sim}{q}_j)_\alpha(t)$	$(\overline{q}_j)_\alpha(t)$
Low ($j = 1$)	Expected value	3.4	6.6	4	6	4.5	5.5
	Standard deviation	0.136	0.264	0.16	0.24	0.18	0.22
	Variance	0.019	0.07	0.026	0.058	0.032	0.048
Medium	Expected value	7.5	11.5	8.25	10.75	8.875	10.125
($j = 2$)	Standard deviation	0.3	0.46	0.33	0.43	0.355	0.405
	Variance	0.09	0.212	0.109	0.185	0.126	0.164
High	Expected value	13	21	14.5	19.5	15.75	18.25
($j = 3$)	Standard deviation	0.52	0.84	0.58	0.78	0.63	0.73
	Variance	0.27	0.706	0.336	0.608	0.397	0.533

respectively, with the probability of occurrence being 20%. The results of $A_{12} = 2.5 \times 10^6\ \text{m}^3$, $A_{22} = 5.3 \times 10^6\ \text{m}^3$ and $A_{32} = 0.69 \times 10^6\ \text{m}^3$ (under $r_h = 0.1$) indicate that, under medium flows, there would be no shortages of water for municipal and industrial uses. However, there would be shortages of $6.11 \times 10^6\ \text{m}^3$ for agricultural water use with the probability of 60%. Likewise, the solutions of $A_{11} = 2.5 \times 10^6\ \text{m}^3$, $A_{21} = 5.3 \times 10^6\ \text{m}^3$ and $A_{31} = 6.8 \times 10^6\ \text{m}^3$ (under $r_h = 0.1$) indicate that, under high stream flow levels, there would be no shortage of water with a probability of 20%. The results under $r_h = 0.25$ would be similar to those under $r_h = 0.1$.

A number of decision alternatives under different r_h levels were also generated through the FSTP model. Willingness to accept a low level of constraint-violation probability would guarantee a high system benefit but a high risk of violating the allowable criterion; a strong desire to acquire a low risk level (of violating the criterion) would run into a low system benefit. A lower r_h level should be used under advantageous conditions, and a higher r_h level would correspond to relatively demanding conditions. Therefore, FSTP allows decision makers to incorporate implicit knowledge within the optimization process and thus obtain satisfactory decision alternatives.

Solutions of the FSTP model provide desired water-allocation patterns. The complexities associated with water allocation arise mainly from limited supply and increasing demand. Therefore, the observed variations in the values of W_i could reflect different policies for water resources management. A too optimistic policy corresponding to a high system benefit may be subject to a high risk of system disruption; a too conservative policy may lead to a waste of resources. The solutions can provide ranges of decision variables and objective function value under different levels of stream flows, α-cut levels, and r_h levels. They can be easily interpreted for generating multiple decision alternatives.

If the fuzzy random variables in seasonal flow constraints are substituted by conventional random numbers, the study problem can then be converted into a two-stage stochastic programming (TSP) model. From Table 5, the solution of $A_{21} = 1.85 \times 10^6\ \text{m}^3$ (allocation to industry under low stream flow levels) from TSP is less than those from FSTP ($A_{21} = 2.11 \times 10^6\ \text{m}^3$ and $A_{21} = 2.0 \times 10^6\ \text{m}^3$ under $r_h = 0.1$ and 0.25). This is because a number of multiple uncertainties were neglected in the TSP model.

The stream flows have multiple uncertainties. Firstly, the stream flow (q) is expressed as q_j [i.e., 5, 9.5 and 17 ($10^6\ \text{m}^3$)], under low (probability = 20%), medium (probability = 60%) and high (probability = 20%) stream flow levels, respectively); secondly, q_j are presented as $(\underset{\sim}{q}_j)_\alpha$ [fuzzy sets, i.e., [3.4, 6.6], [4, 6] and [4.5, 5.5] ($10^6\ \text{m}^3$) for low stream flow levels (with a probability of 20%) under $\alpha = 0.2$, 0.5 and 0.75, respectively]; thirdly, $(\underset{\sim}{q}_j)_\alpha$ are expressed as $[(\underset{\sim}{q}_j)_\alpha(t)]^{r_h}$ [fuzzy random variable, i.e., [3.57, 6.26] and [3.49, 6.42] ($10^6\ \text{m}^3$) for low stream flow levels (with a probability of 20%) under $\alpha = 0.2$ and $r_h = 0.1$ and 0.25, respectively].

Fuzzy random variables (FRV), such as $[(\underset{\sim}{q}_j)_\alpha(t)]^{r_h}$, are useful for presenting fuzzy sets and random variables. Both fuzziness and randomness can thus be reflected. Fuzzy stochastic optimization methods were proposed to solve the associated decision problems. Fuzzy robust programming is suitable for handling fuzzy coefficients in both left- and right-hand sides of the constraints. Chance-constrained programming is helpful for dealing with right-hand-side parameters with known probability distributions. To address fuzzy random variables, fuzzy robust programming was firstly used to convert fuzzy random constraints into random ones. Then, chance-constrained programming methods were used to treat the random constraints. Two-stage stochastic programming was useful for dealing with stochastic complexities in water resources planning. Thus, incorporation of the two-stage stochastic programming, fuzzy robust programming and chance-constrained programming within a general setting is advantageous for deal with multiple uncertainties in water resources management.

In general, FSTP has the following advantages: (a) since fuzzy sets are used for reflecting uncertainties in left-hand-side coefficients of the constraints and fuzzy random variables are used for tackling uncertainties in $[(\underset{\sim}{q}_j)_\alpha(t)]^{r_h}$ of the right-hand-side constraints, implications of the constraints are extended; (b) FSTP is applicable to practical problems since the solution methods of fuzzy robust programming and chance-constrained programming do not lead to more complicated intermediate submodels; (c) it provides bases for identifying desired water-allocation plans with reasonable benefit and risk levels.

FSTP can effectively reflect multiple uncertainties described as fuzzy sets and probability distributions in the modeling parameters and the associated impact factors. FSTP can directly incorporate uncertainties within its optimization framework. Its solutions are presented as deterministic values and probability distributions, and thus offer flexibilities in result interpretation and decision-alternative generation. Moreover, solutions of the FSTP model also contain information of system disruption penalties under varying water resources management conditions.

Table 4
Stream flows (in $10^6\ \text{m}^3$) under different combinations of r_h and α-cut levels.

r_h level	Flow level	α					
		$\alpha = 0.2$		$\alpha = 0.5$		$\alpha = 0.75$	
		$[(\underset{\sim}{q}_j)_\alpha(t)]^{r_h}$	$[(\overline{q}_j)_\alpha(t)]^{r_h}$	$[(\underset{\sim}{q}_j)_\alpha(t)]^{r_h}$	$[(\overline{q}_j)_\alpha(t)]^{r_h}$	$[(\underset{\sim}{q}_j)_\alpha(t)]^{r_h}$	$[(\overline{q}_j)_\alpha(t)]^{r_h}$
$r_h = 0.1$	Low ($j = 1$)	3.57	6.26	4.2	5.69	4.73	5.22
	Medium ($j = 2$)	7.88	10.91	8.67	10.2	9.33	9.61
	High ($j = 3$)	13.67	19.92	15.25	18.5	16.56	17.31
$r_h = 0.25$	Low ($j = 1$)	3.49	6.42	4.11	5.84	4.62	5.35
	Medium ($j = 2$)	7.7	11.19	8.47	10.46	9.11	9.85
	High ($j = 3$)	13.35	20.43	14.89	18.97	16.17	17.76

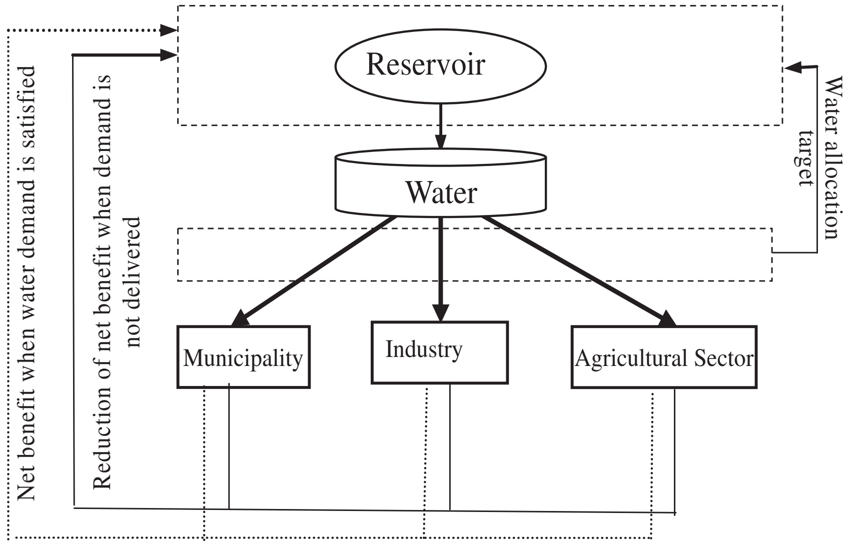

Fig. 4. Structure of the water management system.

Table 5
Solutions of the FSTP and TSP models (in 10^6 m^3).

User (i)	Level (j)	$r_h = 0.1$		$r_h = 0.25$		TSP	
		W_i	A_{ij}	W_i	A_{ij}	W_i	A_{ij}
Municipal	Low	2.5	2.5	2.5	2.5	2.5	2.5
Industrial	Low	5.3	2.11	5.3	2	5.3	1.85
Agricultural	Low	6.8	0	6.8	0	6.8	0
Municipal	Medium	2.5	2.5	2.5	2.5	2.5	2.5
Industrial	Medium	5.3	5.3	5.3	5.3	5.3	5.3
Agricultural	Medium	6.8	0.69	6.8	0.48	6.8	0.46
Municipal	High	2.5	2.5	2.5	2.5	2.5	2.5
Industrial	High	5.3	5.3	5.3	5.3	5.3	5.3
Agricultural	High	6.8	6.8	6.8	6.8	6.8	6.8
f (\$$10^6$)		364.39		356.83		354.3	

4. Conclusions

Fuzzy random variables are presented as a type of more extensive uncertainties than fuzzy and random variables. Such parameters can be described through incorporating fuzzy and probabilistic uncertainties. Fuzzy stochastic programming is introduced to deal with such uncertainties. Two-stage programming (TSP) is useful for reflecting recourse in order to minimize "penalties" that may appear due to any infeasibility, which choose corrective actions after a random event has taken place. A fuzzy stochastic two-stage programming (FSTP) method under multiple uncertainties is thus developed based on fuzzy robust programming (FRP), chance-constrained programming (CCP), and two-stage stochastic programming (TSP) approaches.

FSTP is applied to a case of water resources management where a reservoir and three water users exist. FSTP has advantages in uncertainty reflection and policy analysis. It can provide decision makers with decision support for water resources management under uncertainty and recourse. This involves (1) providing a linkage between pre-defined water policies and the associated economic implications, (2) extending the range of uncertainty which expressed as multiple uncertainties with both fuzziness and randomness characteristics, (3) communicating uncertainties into the optimization process under different stream flows, α-cut levels and levels of constraint-violation probability, and (4) providing a number of alternatives for decision makers to ascertain the schemes for allocating water resources to the users.

According to relative low, medium and high stream flows and different policies for water resource management, the decision maker can evaluate maximize system benefit for allocating water; an optimistic policy may be subject to a high risk of system failure penalties while a policy which is too conservative may lead to a waste of resources. Uncertainty presented as fuzzy random variables, fuzzy membership functions and probability distributions can be tackled by the FSTP method. In addition to its application to water resources management, FSTP can also be extended to other problems of resources and environmental management.

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