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# A two-step method for spatial circle orientation with a structured light vision sensor and error analysis 

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#### Abstract

A novel two-step method for spatial circle orientation with a structured light vision sensor is proposed for a 3D flexible visual inspection system guided by an industrial robot. Firstly the $z$ coordinate of a spatial circle center is estimated, secondly the $x$ and $y$ coordinates are estimated with the center orientation relative to the camera optic center, and then its radius is computed. Simultaneously, the $x, y$ and $z$ coordinate orientation errors are analyzed in detail. It shows that the method is feasible and valid, and the orientation accuracy for the spatial circle exceeds 0.15 mm by experiment. It eliminates the bottleneck of the traditional orientation method with a stereovision sensor, and greatly expands the application of the structured light visual inspection system.


Keywords: structured light vision sensor, spatial circle, orientation, two-step method, error analysis
(Some figures in this article are in colour only in the electronic version)

## 1. Introduction

A spatial circle is an elemental geometrical shape on measured objects, such as the location hole of workpieces or parts. So spatial circle orientation is very important for automatic inspection and field assembly [1].

Computer vision has developed rapidly in recent years with advantages of non-contact, higher precision and efficiency, on-line measurement and so on, and corresponding inspection technologies are applied widely for three-dimensional measurements on product lines. In a typical inspection system, a binocular stereo measurement model is generally adopted to inspect the spatial edge, round hole or similar round hole with a stereo image pair [2, 3]. But a stereovision sensor is composed of two cameras, and it is relatively larger, heavier and has a high cost, so there are many restrictions for industrial field applications, especially for a 3D flexible visual inspection system guided by an industrial
robot due to the limitation of the snatched load and movement flexibility of the robot arm.

A structured light vision sensor contains only one camera and a line-structured laser. Compared to the stereovision sensor, the structured light vision sensor is much lighter and smaller, has a lower cost and more flexibility, and avoids the socalled correspondence problem [4-9]. Since the early 1970s [10] there has been active research on shape reconstruction and object recognition by projecting structured light stripes onto objects, especially for flexible visual inspection systems guided by industrial robots [11, 12]. A structured light vision sensor can inspect the spatial edge easily but not the spatial circle, because feature points on the spatial edge can be conveniently located on the structured light plane modulated by the measured surfaces, so their 3D coordinates can be inspected easily with the mathematical model of a structured light vision sensor. However, the measured spatial circle center cannot be laid on the structured light plane accurately, so the


Figure 1. Measurement model of a structured light vision sensor.
orientation technology of a spatial circle with a structured light vision sensor is much more complex, and severely restricts the application of the structured light visual inspection system.

In this paper, a two-step method for spatial circle orientation with a structured light vision sensor is proposed, and the $x, y$ and $z$ coordinate orientation errors are analyzed in detail. It eliminates the bottleneck of the traditional spatial circle inspection method with a stereovision sensor, and greatly expands the actual application of the structured light visual inspection system.

## 2. Mathematical model of a structured light vision sensor

Figure 1 shows the mathematical model of the structured light vision sensor. Note that $O_{\mathrm{n}}$ is the center of the camera image plane $\pi_{\mathrm{n}}, O_{\mathrm{c}}$ is the projection center of the camera and the $z_{\mathrm{c}}$ axis is the optical axis of the camera lens. The relative coordinate frames are defined as follows: $O_{\mathrm{w}} x_{\mathrm{w}} y_{\mathrm{w}} z_{\mathrm{w}}$ is the 3D world coordinate frame, $O_{\mathrm{c}} x_{\mathrm{c}} y_{\mathrm{c}} z_{\mathrm{c}}$ is the 3D camera coordinate frame and $O_{\mathrm{n}} x_{\mathrm{n}} y_{\mathrm{n}}$ is the 2D image plane coordinate frame. Let $O_{\mathrm{c}} x_{\mathrm{c}} y_{\mathrm{c}} z_{\mathrm{c}}$ be the sensor measurement coordinate frame, and the mathematical model of a structured light vision sensor can be represented by the equation of the light plane $\pi_{\mathrm{s}}$ in the 3D camera coordinate frame $O_{\mathrm{c}} x_{\mathrm{c}} y_{\mathrm{c}} z_{\mathrm{c}}$.

Given an arbitrary point $P_{\mathrm{s}}$ on the light plane $\pi_{\mathrm{s}}$, its homogeneous coordinates in $O_{\mathrm{c}} x_{\mathrm{c}} y_{\mathrm{c}} z_{\mathrm{c}}$ are denoted by $\tilde{P}_{\mathrm{c}}=$ $\left(\begin{array}{llll}x_{\mathrm{c}} & y_{\mathrm{c}} & z_{\mathrm{c}} & 1\end{array}\right)^{T}$. The light plane $\pi_{\mathrm{s}}$ in $O_{\mathrm{c}} x_{\mathrm{c}} y_{\mathrm{c}} z_{\mathrm{c}}$ can be expressed by

$$
\begin{equation*}
a x_{\mathrm{c}}+b y_{\mathrm{c}}+c z_{\mathrm{c}}+d=0 \tag{1}
\end{equation*}
$$

In (1), $\left[\begin{array}{llll}a & b & c & d\end{array}\right]$ are the equation coefficients of the light plane and should be pre-calibrated [13].
$P_{\mathrm{n}}$ is the ideal projection of $P_{\mathrm{s}}$ in the camera image plane, while $P_{\mathrm{d}}$ is the real projection because of lens distortion. We use $\tilde{p}_{\mathrm{n}}=\left(\begin{array}{lll}x_{\mathrm{n}} & y_{\mathrm{n}} & 1\end{array}\right)^{T}$ to denote $P_{\mathrm{n}}$ 's homogeneous coordinates in $O_{\mathrm{n}} x_{\mathrm{n}} y_{\mathrm{n}}$. The equation of the straight line $O_{\mathrm{c}} P_{\mathrm{n}}$ is given by

$$
\begin{equation*}
\frac{x_{\mathrm{c}}}{x_{\mathrm{c}}-x_{\mathrm{n}}}=\frac{y_{\mathrm{c}}}{y_{\mathrm{c}}-y_{\mathrm{n}}}=\frac{z_{\mathrm{c}}}{z_{\mathrm{c}}-1} \tag{2}
\end{equation*}
$$

From (1) and (2), we can acquire the 3D coordinates of an arbitrary point on the light plane $\pi_{\mathrm{s}}$ in $O_{\mathrm{c}} x_{\mathrm{c}} y_{\mathrm{c}} z_{\mathrm{c}}$, if its homogeneous image coordinates and the light plane equation in $O_{\mathrm{c}} x_{\mathrm{c}} y_{\mathrm{c}} z_{\mathrm{c}}$ are known exactly.


Figure 2. Measurement model of spatial circle orientation.

## 3. Principle of the two-step method

Figure 2 shows the measurement model of the spatial circle with a structured light vision sensor. The camera views the spatial circle straightly and the laser projector lights slantwise in figure 2.

### 3.1. Notation

(1) $O_{\mathrm{c}}$ is the projection center of the camera, $O_{1}$ is the center of the structured light source, and the measurement coordinate frame is overlapped with the camera coordinate frame $O_{\mathrm{c}} x_{\mathrm{c}} y_{\mathrm{c}} z_{\mathrm{c}}$.
(2) $O_{\mathrm{h}}$ is the center of the measured spatial circle and its 3D coordinates in $O_{\mathrm{c}} x_{\mathrm{c}} y_{\mathrm{c}} z_{\mathrm{c}}$ are denoted by $(x, y, z) . P$ is the intersection point between line $O_{\mathrm{c}} O_{\mathrm{h}}$ and light plane $\pi_{\mathrm{s}}$. $P$ 's 3D coordinates are denoted by ( $x_{\mathrm{P}}, y_{\mathrm{P}}, z_{\mathrm{P}}$ ) in $O_{\mathrm{c}} x_{\mathrm{c}} y_{\mathrm{c}} z_{\mathrm{c}}$. $O_{\mathrm{hi}}$ is the point of intersection of the line $O_{\mathrm{c}} O_{\mathrm{h}}$ and the image plane $\pi_{n}$.
(3) The chord $A B$ denotes the intersection line between the light plane $\pi_{\mathrm{s}}$ and the measured spatial circle, $Q$ is the midpoint of $A B$ and its 3D coordinates in $O_{\mathrm{c}} x_{\mathrm{c}} y_{\mathrm{c}} z_{\mathrm{c}}$ are denoted by $\left(x_{\mathrm{Q}}, y_{\mathrm{Q}}, z_{\mathrm{Q}}\right) . A_{\mathrm{i}}$ and $B_{\mathrm{i}}$ are the corresponding images of $A$ and $B$.
(4) The 3D measurement coordinates of the spatial circle center $O_{\mathrm{h}}$ are denoted by $\left(x_{\mathrm{m}}, y_{\mathrm{m}}, z_{\mathrm{m}}\right)$.
(5) To simplify the analysis, assume that the light plane $\pi_{\mathrm{s}}$ is parallel to the $y_{\mathrm{c}}$ axis in $O_{\mathrm{c}} x_{\mathrm{c}} y_{\mathrm{c}} z_{\mathrm{c}}$.
(6) The anticlockwise angle is defined as positive, and the clockwise angle as negative.
(7) The coordinate system obeys the right-hand rule of Cartesian coordinates.

### 3.2. Measurement condition and principle

For accurate spatial circle orientation, the proper work distance and field depth of the structured light vision sensor are generally required. In addition, the vision sensor and the measured 3D hole should meet the ideal measurement pose as follows.
(1) The camera image plane $\pi_{\mathrm{n}}$ should be as parallel to the measured spatial circle plane $\Gamma_{\mathrm{c}}$ as possible.
(2) The light plane $\pi_{\mathrm{s}}$ should intersect the measured spatial circle plane $\Gamma_{\mathrm{c}}$ (the intersecting line $A B$ may cross the spatial circle center $O_{\mathrm{h}}$ or not).
According to the perspective projection model of spatial ellipse [14], if the camera image plane parallels the spatial circle plane $\Gamma_{\mathrm{c}}$, then the image center fitted by the ellipse image is equal to the actual projection center of the spatial circle or ellipse. Otherwise, there must be perspective projection distortion when they are not parallel, and the bigger the angle between the two planes, the larger the projection distortion becomes. Thus, if condition (1) is met approximately, we can get an almost ideal projection image and reduce the 3D coordinate errors of the spatial circle center due to the perspective projection distortion. Moreover, the $z$ orientation error based on the novel two-step method can be decreased further. Condition (2) is the necessary condition for spatial circle orientation with a structured light vision sensor and can be achieved easily.

With the above two presuppositions, the principle of the two-step method for spatial circle orientation with a structured light vision sensor can be described as follows.

- Step 1: estimate the 3D coordinates ( $x_{\mathrm{Q}}, y_{\mathrm{Q}}, z_{\mathrm{Q}}$ ) of the midpoint $Q$ on an intersection line $A B$ according to the mathematical model of a structured light vision sensor as described in section 2 , and assign $z_{\mathrm{Q}}$ to the $z$ coordinate of the measured spatial circle center $O_{\mathrm{h}}$.
- Step 2: estimate the $x$ and $y$ coordinates of the measured spatial circle center $O_{\mathrm{h}}$ according to the known $z$ coordinate and the orientation of the measured spatial circle center $O_{\mathrm{h}}$ relative to the camera projection center $O_{\mathrm{c}}$.

Consequently, the radius of the spatial circle can be computed easily with the above estimated parameters.

## 4. Implementation of the two-step method

### 4.1. Estimation of the z coordinate

The ellipse projection of the measured spatial circle on the image plane $\pi_{\mathrm{n}}$ is defined as

$$
\begin{equation*}
\frac{\left(x-x_{\mathrm{n}}\right)^{2}}{a^{2}}+\frac{\left(y-y_{\mathrm{n}}\right)^{2}}{b^{2}}=1 \tag{3}
\end{equation*}
$$

where $\left(x_{\mathrm{n}}, y_{\mathrm{n}}\right)$ is the ellipse center, and $a$ and $b$ are the semimajor axis and semiminor axis, respectively.

The light stripe $A B$ is defined by the following equation:

$$
\begin{equation*}
y=k x+l, \tag{4}
\end{equation*}
$$

where $k$ is the slope of the light stripe $A B$, and $l$ is the intercept on the $y$ axis.

From (3) and (4), the image coordinates of points $A$ and $B$ can be acquired with the location algorithm of image features; then based on the mathematical model of a structured light vision sensor, the 3D coordinates $\left(x_{\mathrm{A}}, y_{\mathrm{A}}, z_{\mathrm{A}}\right),\left(x_{\mathrm{B}}, y_{\mathrm{B}}, z_{\mathrm{B}}\right)$ of points $A$ and $B$ in the camera coordinate frame $O_{\mathrm{c}} x_{\mathrm{c}} y_{\mathrm{c}} z_{\mathrm{c}}$ can
be easily estimated. Since $Q$ is the midpoint of the chord $A B$, we have

$$
\left\{\begin{array}{l}
x_{\mathrm{Q}}=\frac{x_{\mathrm{A}}+x_{\mathrm{B}}}{2}  \tag{5}\\
y_{\mathrm{Q}}=\frac{y_{\mathrm{A}}+y_{\mathrm{B}}}{2} \\
z_{\mathrm{Q}}=\frac{z_{\mathrm{A}}+z_{\mathrm{B}}}{2}
\end{array}\right.
$$

According to the presupposition and principle of the two-step method, $z_{\mathrm{Q}}$ is the approximate $z$ coordinate of the measured spatial circle center $O_{\mathrm{h}}$, and the $z$ coordinate measurement error exceeds $\pm 0.2 \mathrm{~mm}$ with error analysis under field inspection conditions, so we have

$$
\begin{equation*}
z \approx z_{\mathrm{m}}=z_{\mathrm{Q}} \tag{6}
\end{equation*}
$$

### 4.2. Estimation of $x$ and $y$ coordinates

The image coordinate of the measured spatial circle center $O_{\mathrm{h}}$ can be acquired by image processing. If the calibration parameters of the light plane in the camera coordinate frame $O_{\mathrm{c}} x_{\mathrm{c}} y_{\mathrm{c}} z_{\mathrm{c}}$ are known exactly, the 3D coordinates ( $x_{\mathrm{P}}, y_{\mathrm{P}}, z_{\mathrm{P}}$ ) in $O_{\mathrm{c}} x_{\mathrm{c}} y_{\mathrm{c}} z_{\mathrm{c}}$ of point $P$, which is on the light plane $\pi_{\mathrm{s}}$, can be estimated uniquely according to the mathematical model of a structured light vision sensor.

As shown in figure 2, we draw a perpendicular $P P^{\prime}$ to the plane $O_{\mathrm{c}} x_{\mathrm{c}} y_{\mathrm{c}}$ through $P$ and the intersection is denoted by $P^{\prime}$; then we draw $P^{\prime} P_{x}^{\prime}$ and $P^{\prime} P_{y}^{\prime}$ in the plane $O_{\mathrm{c}} x_{\mathrm{c}} y_{\mathrm{c}}$ through $P^{\prime}$ which are perpendicular to the $x_{\mathrm{c}}$ and $y_{\mathrm{c}}$ axis, respectively, and the intersections are denoted by $P^{\prime}{ }_{x}$ and $P^{\prime}{ }_{y}$. Let $\angle P_{y}^{\prime} P P^{\prime}=\theta$, $\angle P_{x}^{\prime} P P^{\prime}=\varphi$, so the orientation between $P$ and the camera optic center $O_{\mathrm{c}}$ can be described by $\theta$ and $\varphi$. From the notation described in section 3.1 and the geometric relationship as shown in figure 2, we have

$$
\begin{align*}
& \tan \theta=\frac{\left|P^{\prime} P_{y}^{\prime}\right|}{\left|P P^{\prime}\right|}=\frac{\left|x_{\mathrm{P}}\right|}{z_{\mathrm{P}}}  \tag{7}\\
& \tan \varphi=\frac{\left|P^{\prime} P_{x}^{\prime}\right|}{\left|P P^{\prime}\right|}=\frac{\left|y_{\mathrm{P}}\right|}{z_{\mathrm{P}}} . \tag{8}
\end{align*}
$$

Simultaneously, we draw a perpendicular $O_{\mathrm{h}} O_{\mathrm{h}}^{\prime}$ to the plane $O_{\mathrm{c}} x_{\mathrm{c}} y_{\mathrm{c}}$ intersecting at point $O_{\mathrm{h}}^{\prime}$; then we draw $O_{\mathrm{h}}^{\prime} O^{\prime}{ }_{\mathrm{h} x}$ and $O_{\mathrm{h}}{ }^{\prime} O_{\mathrm{h} y}^{\prime}$ in the plane $O_{\mathrm{c}} x_{\mathrm{c}} y_{\mathrm{c}}$ which are perpendicular to the $x_{\mathrm{c}}$ and $y_{\mathrm{c}}$ axis, respectively, and the intersections are denoted by points $O_{\mathrm{h} x}^{\prime}$ and $O_{\mathrm{h} y}^{\prime}$. As shown in figure 2, $\angle O_{\mathrm{h} y}^{\prime} O_{\mathrm{h}} O_{\mathrm{h}}^{\prime}=\angle P_{y}^{\prime} P P^{\prime}=\theta, \angle O_{\mathrm{h} x}^{\prime} O_{\mathrm{h}} O_{\mathrm{h}}^{\prime}=\angle P_{x}^{\prime} P P^{\prime}=\varphi$, so the orientation between $O_{\mathrm{h}}$ and $O_{\mathrm{c}}$ can be described by $\theta$ and $\varphi$ as follows:

$$
\begin{align*}
& \tan \theta=\frac{\left|O_{\mathrm{h}}^{\prime} O_{\mathrm{h} y}^{\prime}\right|}{\left|O_{\mathrm{h}} O_{\mathrm{h}}^{\prime}\right|}=\frac{|x|}{z}  \tag{9}\\
& \tan \varphi=\frac{\left|O_{\mathrm{h}}^{\prime} O_{\mathrm{h} x}^{\prime}\right|}{\left|O_{\mathrm{h}} O_{\mathrm{h}}^{\prime}\right|}=\frac{|y|}{z} . \tag{10}
\end{align*}
$$

Because the symbols $x$ and $y$ are the same as $x_{\mathrm{P}}$ and $y_{\mathrm{P}}$, so from (6)-(10), we have

$$
\begin{align*}
x_{\mathrm{m}} & =\frac{x_{\mathrm{P}}}{z_{\mathrm{P}}} z_{\mathrm{Q}}  \tag{11}\\
y_{\mathrm{m}} & =\frac{y_{\mathrm{P}}}{z_{\mathrm{P}}} z_{\mathrm{Q}} \tag{12}
\end{align*}
$$

### 4.3. Estimation of radius

With the above two-step method, the center coordinates of the ellipse or incomplete circle feature can be estimated. If the measured part is a spatial round hole, its radius can also be computed. Let $\left(x_{\mathrm{A}}, y_{\mathrm{A}}, z_{\mathrm{A}}\right)$ and $\left(x_{\mathrm{B}}, y_{\mathrm{B}}, z_{\mathrm{B}}\right)$ be the 3 D coordinates of the intersection points $A$ and $B$ in $O_{\mathrm{c}} x_{\mathrm{c}} y_{\mathrm{c}} z_{\mathrm{c}}$ and $\left(x_{\mathrm{m}}, y_{\mathrm{m}}, z_{\mathrm{m}}\right)$ be the 3D measurement coordinates of the circle center $O_{\mathrm{h}}$ in $O_{\mathrm{c}} x_{\mathrm{c}} y_{\mathrm{c}} z_{\mathrm{c}}$, so we have

$$
\begin{align*}
& r_{1}=\sqrt{\left(x_{\mathrm{m}}-x_{\mathrm{A}}\right)^{2}+\left(y_{\mathrm{m}}-y_{\mathrm{A}}\right)^{2}+\left(z_{\mathrm{m}}-z_{\mathrm{A}}\right)^{2}}  \tag{13}\\
& r_{2}=\sqrt{\left(x_{\mathrm{m}}-x_{\mathrm{B}}\right)^{2}+\left(y_{\mathrm{m}}-y_{\mathrm{B}}\right)^{2}+\left(z_{\mathrm{m}}-z_{\mathrm{B}}\right)^{2}} .
\end{align*}
$$

We denote the radius of the spatial circle by $r$ :

$$
\begin{equation*}
r=\left(r_{1}+r_{2}\right) / 2 \tag{14}
\end{equation*}
$$

## 5. Error analysis

According to the principle of the two-step method as described in section 3, if the camera image plane is parallel to the measured spatial circle plane $\Gamma_{\mathrm{c}}$, we can get the 3D coordinates of the spatial circle center $O_{\mathrm{h}}$ without theoretical error. While the above condition cannot be met completely in the actual operation, it is necessary to analyze the $x, y$ and $z$ orientation errors of the spatial circle in detail. The notation is as follows.
(1) $O_{\mathrm{p}}$ denotes the ideal working point of the sensor, which is the intersection of the camera optic axis and the light plane $\pi_{\mathrm{s}},\left|O_{\mathrm{p}} O_{\mathrm{c}}\right|$ is the ideal working distance of the sensor, and is denoted by $L$, and $\alpha$ is the intersecting angle between the camera optic axis and the light plane $\pi_{s}$.
(2) $x_{\mathrm{d}}$ is the $x$ offset of the measured spatial circle center $O_{\mathrm{h}}$, and letting the optic center $O_{\mathrm{c}}$ be the origin, $x_{\mathrm{d}}$ is defined as positive if its direction is the same as the $x_{\mathrm{c}}$ axis; otherwise it is negative.
(3) $y_{\mathrm{d}}$ is the $y$ offset of the measured spatial circle center $O_{\mathrm{h}}$, and letting the optic center $O_{\mathrm{c}}$ be the origin, $y_{\mathrm{d}}$ is defined as positive if its direction is the same as the $y_{\mathrm{c}}$ axis; otherwise it is negative.
(4) $z_{\mathrm{d}}$ is the $z$ offset of the measured spatial circle center $O_{\mathrm{h}}$, and letting the ideal working point $O_{\mathrm{p}}$ be the origin, $z_{\mathrm{d}}$ is defined as positive if its direction is the same as the $z_{c}$ axis; otherwise it is negative.
(5) The actual $x$ coordinate of $O_{\mathrm{h}}$ is denoted by $x$, that is, $x=x_{\mathrm{d}}$, and the $x$ orientation error is denoted by $\Delta x$, that is, $\Delta x=x_{\mathrm{m}}-x=x_{\mathrm{m}}-x_{\mathrm{d}}$.
(6) The actual $y$ coordinate of $O_{\mathrm{h}}$ is denoted by $y$, that is, $y=y_{\mathrm{d}}$, and the $y$ orientation error is denoted by $\Delta y$, that is, $\Delta y=y_{\mathrm{m}}-y=y_{\mathrm{m}}-y_{\mathrm{d}}$.
(7) The actual $z$ coordinate of $O_{\mathrm{h}}$ is denoted by $z$, that is, $z=$ $L+z_{\mathrm{d}}$, and the $z$ orientation error is denoted by $\Delta z$, that is, $\Delta z=z_{\mathrm{m}}-z=z_{\mathrm{m}}-L-z_{\mathrm{d}}$.

### 5.1. Error analysis of the $z$ coordinate

Let figure 2 project on the plane $x_{\mathrm{c}} O_{\mathrm{c}} z_{\mathrm{c}}$, as shown in figure 3, the paper plane is defined as the plane $x_{\mathrm{c}} O_{\mathrm{c}} z_{\mathrm{c}}$ and the $y_{\mathrm{c}}$ axis comes into the paper plane vertically. The line $O_{p} O_{l}$ is the projection of the light plane $\pi_{\mathrm{s}}$ on the plane $x_{\mathrm{c}} O_{\mathrm{c}} z_{\mathrm{c}}$. If the


Figure 3. Error analysis scheme of $z$ coordinate orientation.


Figure 4. The ideal curve of $x_{\mathrm{d}}$ and $\mathrm{z}_{\mathrm{d}}$.
camera image plane is not parallel to the spatial circle plane, there will be a projection angle $\beta$ between the line $O_{\mathrm{h}} Q$ and the plane $x_{\mathrm{c}} O_{\mathrm{c}} y_{\mathrm{c}}$. As shown in figure $3, M, N, I$ and $J$ are the perpendicular points of the points $O_{\mathrm{h}}$ and $Q$ to the $x_{\mathrm{c}}$ axis and $z_{\mathrm{c}}$ axis, respectively. The intersection point of the lines $Q N$ and $O_{\mathrm{h}} I$ is denoted by $S$.

From the above notation and the geometric relationship shown in figure 3, we have $x_{\mathrm{d}}=-\left|M O_{\mathrm{c}}\right|, z_{\mathrm{d}}=\left|O_{\mathrm{p}} I\right|$, $\alpha=\angle O_{1} O_{\mathrm{p}} O_{\mathrm{c}}$, where $\alpha$ can be calculated with the line equation of the camera optic axis and the light plane equation accurately based on the mathematical model of a structured light vision sensor and its calculated parameters. The angle $\alpha$ of the vision sensor in experimental application is $20.7712^{\circ}$.

According to the principle of the two-step method and the geometric relationship in figure 3 , the $z$ orientation error is represented by

$$
\begin{equation*}
\Delta z=z_{\mathrm{m}}-z=-\frac{x_{\mathrm{d}}+z_{\mathrm{d}} \cdot \tan \alpha}{1+\tan \alpha \cdot \tan \beta} \cdot \tan \beta \tag{15}
\end{equation*}
$$

From (15), we have the following cases. (1) When $\beta=0^{\circ}$, that is, the camera image plane is parallel to the spatial circle plane, $\Delta z=0$. (2) When $x_{\mathrm{d}}+z_{\mathrm{d}} \cdot \tan \alpha=0$, that is, the light plane $\pi_{\mathrm{s}}$ just gets through the spatial circle center $O_{\mathrm{h}}, \Delta z=$ 0 . In such a case, $x_{\mathrm{d}}$ and $z_{\mathrm{d}}$ should meet the curve shown in figure 4.

In an actual measuring application, the position and orientation of the measured workpiece change only in a relatively small range because there are clamps and a corresponding orientation technique to ensure it in production lines, therefore, we can reduce $\beta$, and $x_{\mathrm{d}}, y_{\mathrm{d}}$ and $z_{\mathrm{d}}$ offsets as possible through the pre-adjusting procedure. Generally, the variational ranges of $\beta, x_{\mathrm{d}}$ and $z_{\mathrm{d}}$ are $-2^{\circ} \leqslant \beta \leqslant 2^{\circ}$,


Figure 5. The curve of $\Delta z$ in the practical measurement.


Figure 6. Error analysis scheme of $x$ coordinate orientation.
$-5 \mathrm{~mm} \leqslant x_{\mathrm{d}} \leqslant 5 \mathrm{~mm}$ and $-5 \mathrm{~mm} \leqslant z_{\mathrm{d}} \leqslant 5 \mathrm{~mm}$ respectively. Let Temp $1=x_{\mathrm{d}}+z_{\mathrm{d}} \cdot \tan \alpha$; then equation (15) can be transformed as follows:

$$
\begin{equation*}
\Delta z=-\frac{\text { Temp1 }}{1+0.3793 \cdot \tan \beta} \cdot \tan \beta \tag{16}
\end{equation*}
$$

From (16) and the value ranges of $\beta$ and Temp1, we can draw the curve of $\Delta z$. As shown in figure 5, the $z$ orientation error may exceed $\pm 0.25 \mathrm{~mm}$.

### 5.2. Error analysis of the $x$ coordinate

To analyze the $x$ orientation error, figure 2 is simplified to figure 6 . As shown in figure $6, O_{\mathrm{h}}$ is the actual center of the measured spatial circle, and $P$ is the intersection of the line $O_{\mathrm{c}} O_{\mathrm{h}}$ and the light plane $\pi_{\mathrm{s}} . \quad M, N, I$ and $J$ are perpendicular points of points $O_{\mathrm{h}}$ and $P$ to the $x_{\mathrm{c}}$ axis and $z_{\mathrm{c}}$ axis, respectively. From the notation and the geometric relationship shown in figure 6 , we have $x_{\mathrm{d}}=-\left|M O_{\mathrm{c}}\right|$, $z_{\mathrm{d}}=\left|O_{\mathrm{p}} I\right|,\left|O_{\mathrm{p}} O_{\mathrm{c}}\right|=L, \alpha=\angle O_{\mathrm{l}} O_{\mathrm{p}} O_{\mathrm{c}}$, where the ideal working distance $L$ of the sensor can be acquired accurately by calculating the $z$ coordinate of the intersection of the camera optic axis and the light plane based on the mathematical model of a structured light vision sensor and its calculated parameters. The ideal working distance $L$ of the vision sensor in the experimental application is 253.0417 mm .

According to the geometric relationship in figure 6 and the notation, the $x$ orientation error is given by

$$
\begin{equation*}
\Delta x=-\frac{x_{\mathrm{d}} \cdot\left(x_{\mathrm{d}}+z_{\mathrm{d}} \cdot \tan \alpha\right)}{\left(L+z_{\mathrm{d}}\right) \cdot(1+\tan \alpha \cdot \tan \beta)} \cdot \tan \beta . \tag{17}
\end{equation*}
$$



Figure 7. The curve of $\Delta x$ in the practical measurement.


Figure 8. Error analysis scheme of $y$ coordinate orientation.
From (17), we have the following cases. (1) When $\beta=0^{\circ}$, that is, the camera image plane is parallel to the spatial circle plane, $\Delta x=0$. (2) When $x_{\mathrm{d}}=0$, that is, the image of the spatial circle center $O_{\mathrm{h}}$ lies on the $y$ axis in $O_{\mathrm{n}} x_{\mathrm{n}} y_{\mathrm{n}}, \Delta x=0$. (3) When $x_{\mathrm{d}}+z_{\mathrm{d}} \cdot \tan \alpha=0$, that is, the light plane $\pi_{\mathrm{s}}$ just gets through the spatial circle center $O_{\mathrm{h}}, \Delta x=0$. In such a case, $x_{\mathrm{d}}$ and $z_{\mathrm{d}}$ should meet the curve shown in figure 4.

Let Temp $2=x_{\mathrm{d}}\left(x_{\mathrm{d}}+z_{\mathrm{d}} \cdot \tan \alpha\right) /\left(L+z_{\mathrm{d}}\right)$; then equation (17) can be transformed as follows:

$$
\begin{equation*}
\Delta x=-\frac{\text { Temp } 2}{1+\tan \alpha \cdot \tan \beta} \cdot \tan \beta \tag{18}
\end{equation*}
$$

From (18) and the value ranges of $\beta$ and Temp2, the curve of $\Delta x$ can be drawn. As shown in figure 7, if we consider only the orientation accuracy of the two-step method and do not take other measurement factors such as image processing precision, etc into account, the $x$ orientation error may exceed $\pm 0.006 \mathrm{~mm}$.

### 5.3. Error analysis of the y coordinate

Similarly, figure 8 is the projection of figure 2 to the plane $y_{\mathrm{c}} O_{\mathrm{c}} z_{\mathrm{c}}$ and $O_{\mathrm{h}}$ is the actual center of the measured spatial circle. $M, N, I$ and $J$ are perpendicular points of the points $O_{\mathrm{h}}$ and $P$ to the $y_{\mathrm{c}}$ axis and $z_{\mathrm{c}}$ axis, respectively. From the notation and the geometric relationship shown in figure 8, we have $y_{\mathrm{d}}=\left|M O_{\mathrm{c}}\right|, z_{\mathrm{d}}=\left|O_{\mathrm{p}} I\right|,\left|O_{\mathrm{p}} O_{\mathrm{c}}\right|=L$.

So the $y$ orientation error can be described by

$$
\begin{equation*}
\Delta y=\frac{y_{\mathrm{d}} \cdot\left(x_{\mathrm{d}}+z_{\mathrm{d}} \cdot \tan \alpha\right)}{\left(L+z_{\mathrm{d}}\right) \cdot(1+\tan \alpha \cdot \tan \beta)} \cdot \tan \beta \tag{19}
\end{equation*}
$$

From (19), we have the following cases. (1) When $\beta=0^{\circ}$, that is, the camera image plane is parallel to the spatial circle


Figure 9. The curve of $\Delta y$ in the practical measurement.


Figure 10. The field orientation scheme.
plane, $\Delta y=0$. (2) When $y_{\mathrm{d}}=0$, that is, the image of the spatial circle center $O_{\mathrm{h}}$ lies on the $x$-axis in $O_{\mathrm{n}} x_{\mathrm{n}} y_{\mathrm{n}}, \Delta y=0$. (3) When $x_{\mathrm{d}}+z_{\mathrm{d}} \cdot \tan \alpha=0$, that is, the light plane $\pi_{\mathrm{s}}$ just gets through the spatial circle center $O_{\mathrm{h}}, \Delta y=0$. In such a case, $x_{\mathrm{d}}$ and $z_{\mathrm{d}}$ should meet the curve shown in figure 4 .

In the field measurement, the variational range of $y_{\mathrm{d}}$ is generally $-5 \mathrm{~mm} \leqslant y_{\mathrm{d}} \leqslant 5 \mathrm{~mm}$. Let Temp $=\left(x_{\mathrm{d}}+z_{\mathrm{d}}\right.$. $\tan \alpha) /\left(L+z_{\mathrm{d}}\right)$, Temp $3=y_{\mathrm{d}} \cdot$ Temp, equation (19) can be transformed as follows:

$$
\begin{equation*}
\Delta y=-\frac{\text { Temp } 3}{(1+\tan \alpha \cdot \tan \beta)} \cdot \tan \beta \tag{20}
\end{equation*}
$$

From (20) and the value ranges of $\beta$ and Temp3, the curve of $\Delta y$ can be drawn. As shown in figure 9 , if we only consider the orientation accuracy of the two-step method and do not take other measurement factors such as image processing precision, etc into account, the $y$ orientation error may exceed $\pm 0.006$ mm in field application.

In the practical measurement, the pre-assumed variational ranges of $\beta$ and $x_{\mathrm{d}}, y_{\mathrm{d}}$ and $z_{\mathrm{d}}$ offsets may be a little enhanced because of the various positions and orientation of the measured workpiece. But as a whole, it will affect the $z$ orientation error much more than $x$ and $y$ orientation errors.

## 6. Experiments

Field orientation for the spatial circle of a white car-body with a structured light vision sensor based on the industrial robot platform is shown in figure 10. The arrangement of the vision sensor is shown in figure 11 and the sensor consists of a Toshiba Teli CCD camera (CS8620CI) with a 25 mm custombuilt lens and a band-pass filter, a 650 nm semiconductor linestructured laser projector and an annulated LED light source.


Figure 11. Arrangement of the vision sensor.


Figure 12. (a) The measured circle image. (b) The processed light stripe image.

Figure 12(a) shows the image of the measured spatial hole, and the processed light stripe image used in the two-step method is shown in figure $12(b)$.

The structured light vision sensor must be calibrated before it is used to inspect the spatial circle. According to the mathematical model of the sensor, the structured light stripe plane coefficient parameters (including the camera intrinsic parameters based on a camera model [3]) are as follows.

$$
\begin{gathered}
{\left[\begin{array}{cccc}
a & b & c & d
\end{array}\right]=\left[\begin{array}{lll}
1.0 & -0.000395 & 0.555042-134.560160
\end{array}\right]} \\
f_{x}=3484.782780 \text { pixels, } \quad f_{y}=3472.023859 \text { pixels } \\
u_{0}=310.046518 \text { pixels, } \quad v_{0}=312.197608 \text { pixels } \\
k_{1}=0.299731, \quad k_{2}=3.218002, \quad p_{1}=0.013074 \\
p_{2}=-0.023963
\end{gathered}
$$

According to the proposed two-step method with the structured light vision sensor and the complete calibrated parameters, the center coordinate of the spatial circle in figure 12(a) can be estimated. We inspect it five times repeatedly, and the orientation results are listed in table 1.

As shown in table 1, the standard deviation of the repeated orientation to the spatial circle with the two-step method exceeds 0.015 mm .

In addition, we estimate the center coordinates of seven other spatial circles in the white car-body with the proposed two-step method, and compare them with the ground truth inspected by a coordinate measuring machine (CMM), and the orientation results are listed in table 2.

As we can see from table 2, the 3D coordinate measurement accuracy of the spatial circle with the two-step method exceeds 0.15 mm . It is shown that the method is valid and can satisfy the field inspection tasks commendably.

Table 1. Orientation repeatability evaluation for the spatial circle in figure 11(a).

| Number | 1 | 2 | 3 | 4 | 5 | Standard deviation (mm) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x_{\mathrm{i}}(\mathrm{mm})$ | 3390.8318 | 3390.8185 | 3390.8147 | 3390.8258 | 3390.7943 | 0.0143 |
| $y_{\mathrm{i}}(\mathrm{mm})$ | -591.1496 | -591.1471 | -591.1535 | -591.1533 | -591.1505 | 0.0027 |
| $z_{\mathrm{i}}(\mathrm{mm})$ | 768.7088 | 768.6957 | 768.696 | 768.6916 | 768.6867 | 0.0082 |

Table 2. Orientation accuracy evaluation for the spatial circle.

| Point | With the two-step method |  |  | By CMM |  |  | $\begin{aligned} & \Delta x \\ & (\mathrm{~mm}) \end{aligned}$ | $\begin{aligned} & \Delta y \\ & (\mathrm{~mm}) \end{aligned}$ | $\begin{aligned} & \overline{\Delta z} \\ & (\mathrm{~mm}) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $x_{\mathrm{i}}(\mathrm{mm})$ | $y_{\mathrm{i}}(\mathrm{mm})$ | $z_{\mathrm{i}}(\mathrm{mm})$ | $x_{\text {c }}(\mathrm{mm})$ | $y_{\mathrm{c}}(\mathrm{mm})$ | $z_{\mathrm{c}}(\mathrm{mm})$ |  |  |  |
| 1 | 3390.8318 | -591.1496 | 768.7088 | 3390.8610 | -591.1090 | 768.6260 | -0.0292 | -0.0406 | 0.0828 |
| 2 | 2566.4065 | 446.0086 | 0 | 2566.3690 | 446.0560 | 0 | 0.0375 | -0.0474 | 0 |
| 3 | 0 | 649.6343 | 320.0449 | 0 | 649.6750 | 319.9720 | 0 | -0.0407 | 0.0729 |
| 4 | 3307.7576 | 806.7249 | 511.8275 | 3307.7470 | 806.7560 | 511.6960 | 0.0106 | -0.0311 | 0.1315 |
| 5 | 1048.8712 | -589.5614 | 1111.7062 | 1048.9000 | -589.5100 | 1111.6940 | -0.0288 | -0.0514 | 0.0122 |
| 6 | 176.4689 | 0 | 705.9960 | 176.5010 | 0 | 705.9520 | -0.0321 | 0 | 0.0440 |
| 7 | 1116.1797 | 0 | 1198.6764 | 1116.1500 | 0 | 1198.6350 | 0.0297 | 0 | 0.0414 |
| 8 | 0 | 0 | 659.0409 | 0 | 0 | 659.0850 | 0 | 0 | -0.0441 |

## 7. Conclusions

A novel two-step method for spatial circle orientation with a structured light vision sensor has been presented for 3D visual inspection, especially for a system guided by an industrial robot. The principle and implementation of the two-step method have been introduced, and the $x, y$ and $z$ coordinate orientation errors have been analyzed in detail. The field experiment reveals that the 3D coordinate measurement accuracy of spatial circle exceeds 0.15 mm , and the standard deviation of repeated orientation exceeds 0.015 mm . It shows that the method is feasible and valid in the vision inspection applications. The proposed approach eliminates the bottleneck of the traditional spatial circle inspection method with a stereovision sensor, and greatly expands the actual application of the structured light visual inspection system.

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