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Particle model to optimize resource allocation and task assignment ☆

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Abstract

This paper presents a novel generalized particle model for the parallel optimization of the resource allocation and task assignment in complex environment of enterprise computing. The generalized particle model (GPM) transforms the optimization process into the kinematics and dynamics of massive particles in a force-field. The GPM approach has many advantages in terms of the high-scale parallelism, multi-objective optimization, multi-type coordination, multi-degree personality, and the ability to handle complex factors. Simulations show the effectiveness and suitability of the proposed GPM approach to optimize the enterprise computing.

Keywords: Optimization; Resource allocation; Task assignment; Enterprise computing; Generalized particle; Non-linear dynamics; Parallel algorithm

1. Introduction

The distributed enterprise computing is featured by the geographically distributed resources and jobs, heterogeneous collection of autonomous systems, and collaboration based large-scale problem-solving. Since enterprise computing always involves the resource allocation, task assignment, and behavior coordination, their optimization in complex environment is of great significance for the quality-assurance and performance-improvement of enterprise computing.

Most of optimization methods [1–11] currently used for the resource allocation and task assignment in enterprise computing have the below limitations and disadvantages:

- Do not consider the complex environment related to multi-type coordinate, multi-degree autonomy, multi-objective optimization and multi-granularity coalition [1–4].
- Do not consider complex coordinations such as unilateral, unaware and unconscious coordinations, besides bilateral and conscious cooperation or competition [10].
- Only consider completely unselfish or completely selfish entity which tries to increase either the aggregate utility or personal utility [2,5].

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- Need the global control, global information access and global objective, hence lead to series or small-scale parallel computation [8,9].
- Do not consider stochastic, emergent and concurrent phenomena such as congestion, failure and priority change [11].

To overcome the above limitations, this paper proposes a novel generalized particle model (GPM) which transforms the optimization of enterprise computing into the kinematics and dynamics of massive particles in a force-field. The features of the GPM-based optimization include the large-scale parallelism, multi-objective optimization, multi-type coordination, multi-degree autonomy, multi-granularity coalition, and the ability to deal with complex factors, e.g. congestion, failure, and priority. Simulations show the effectiveness and suitability of the proposed GPM approach to the enterprise computing optimization.

The structure of this paper is as follows. Section 2 elucidates the generalized particle model for the resource allocation and task assignment. Section 3 discusses the generalized particle algorithm and its properties. Section 4 highlights simulations and comparisons. Finally, we conclude in Section 5.

2. Generalized particle model

Consider the parallel distributed resource allocation among users of enterprise computing. Let $G(\tau) = \{G_1, \ldots, G_m\}$ be a finite set of resource users, and $A(t) = \{A_1, \ldots, A_n\}$ be a finite set of resource suppliers in the time session τ. The supplier A_i provides the user G_j with the resource $a_{ij}(t)$ at time t, and meanwhile the user G_j offers the payment $p_{ij}(t)$ for the unit resource of A_i. The supplier A_i uses the intention strength $\zeta_{ij}(t)$ with respect to the user G_j to describe the influence of complex environment, such as interaction, congestion, failure, and priority. We thus obtain an assignment matrix $S(t) = [s_{ik}(t)]_{n\times m}$, as shown in Fig. 1, where $s_{ij}(t) = \langle a_{ij}(t), p_{ij}(t), \zeta_{ij}(t)\rangle$. For convenience, they are normalized such that $0 \leqslant a_{ij}(t) \leqslant 1$, $0 \leqslant p_{ij}(t) \leqslant 1$, and $-1 \leqslant \zeta_{ij} \leqslant +1$.

	G_1	$\cdot$	G_m
A_1	$a_{11}(t), p_{11}(t), \zeta_{11}(t)$	$\cdot$	$a_{1m}(t), p_{1m}(t), \zeta_{1m}(t)$
$\cdot$	$\cdots$	$\cdot$	$\cdots$
A_i	$a_{i1}(t), p_{i1}(t), \zeta_{i1}(t)$	$\cdot$	$a_{im}(t), p_{im}(t), \zeta_{im}(t)$
$\cdot$	$\cdots$	$\cdot$	$\cdots$
A_n	$a_{n1}(t), p_{n1}(t), \zeta_{n1}(t)$	$\cdot$	$a_{nm}(t), p_{nm}(t), \zeta_{nm}(t)$

Fig. 1. The assignment matrix for the enterprise computing optimization, $\mathbf{S(t)} = [s_{ik}(t)]_{n\times m}$, where $s_{ij}(t) = \langle a_{ij}(t), p_{ij}(t), \zeta_{ij}(t)\rangle$.

The conceptual diagram of a GPM for the enterprise computing optimization is shown in Fig. 2, where the particle s_{ik} in a force-field corresponds to the entry s_{ik} in the assignment matrix S. A particle may be driven by several kinds of forces that are produced by the force-field, other particles and itself. The gravitational force produced by the force-field tries to drive a particle to move towards boundaries of the force-field, which embodies the tendency that a particle pursues maximizing the aggregate benefit of systems. The pushing or pulling forces produced by other particles are used to embody social coordinations among resource suppliers and users. The self-driving force produced by a particle itself represents autonomy and personality of individual supplier and users. The resultant force on a particle drives the particle to move in the force-field. In this way, the GPM transforms the optimization problem of resource allocation for enterprise computing into the kinematics and dynamics of particles in a force-field.

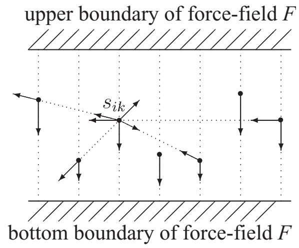

Fig. 2. Generalized particle model for the enterprise computing optimization.

Definition 1. Let $u_{ik}(t)$ be the utility of particle s_{ik} at time t, and let $J(t)$ be the aggregate utility of all particles. They are defined by

$$u_{ik}(t) = \alpha_{ik}[1 - \exp(-p_{ik}(t)a_{ik}(t))], \tag{1}$$

$$J(t) = \alpha \sum_{i=1}^{n} \sum_{k=1}^{m} u_{ik}(t), \tag{2}$$

where α_{ik}, $\alpha \geqslant 0$, and α_{ik} is related to the activities of supplies A_i and user G_j, such as congestion degree, failure rate, and priority level.

Definition 2. At time t, the potential energy function $P(t)$ that is related to the gravitational force of force-field F is defined by

$$P(t) = \varepsilon^2 \ln \sum_{i=1}^{n} \sum_{k=1}^{m} \exp[-u_{ik}^2(t)/2\varepsilon^2] - \varepsilon^2 \ln mn, \quad (3)$$

where $0 < \varepsilon < 1$.

Definition 3. At time t, the potential energy function $Q(t)$ that is related to interactive forces among particles is defined by

$$Q(t) = \xi \sum_{i=1}^{n} \left| \sum_{k=1}^{m} a_{ik}(t) - r_i(t) \right|^2 - \sum_{i,k} \int_0^{u_{ik}} \{[1 + \exp(-\zeta_{ik} x)]^{-1} - 0.5\} \, \mathrm{d}x, \quad (4)$$

where $0 < \xi < 1$; r_i is the capacity of resource supplier A_i. The second term of $Q(t)$ represents social coordinations among them, where $-1 \leqslant \zeta_{ij} \leqslant +1$.

Definition 4. The hybrid energy function of the particle s_{ik} at time t is defined by

$$\Gamma_{ik}(t) = -\lambda_{ik}^{(1)} u_{ik}(t) - \lambda_{ik}^{(2)} J(t) + \lambda_{ik}^{(3)} P(t) + \lambda_{ik}^{(4)} Q(t), \quad (5)$$

where $0 < \lambda_{ik}^{(1)}, \lambda_{ik}^{(2)}, \lambda_{ik}^{(3)}, \lambda_{ik}^{(4)} \leqslant 1$.

Definition 5. Let the coordinate origin be located at the central line between the upper and bottom boundaries of force-field F, and $q_{ik}(t)$ be the vertical coordinate of particle s_{ik} at time t. The dynamic equation for particle s_{ik} is defined by

$$\begin{cases} \mathrm{d}q_{ik}(t)/\mathrm{d}t = \Psi_{ik}^{(1)}(t) + \Psi_{ik}^{(2)}(t), & (6) \\ \Psi_{ik}^{(1)}(t) = -q_{ik}(t) + \gamma v_{ik}(t), & (6a) \\ \Psi_{ik}^{(2)}(t) = I_{ik} + \sum_{j=1}^{m} w_{jk} u_{jk}(t) + \sum_{j=1}^{n} w_{ij} u_{ij}(t), & (6b) \end{cases}$$

where $\gamma > 1$, I_{ik} is a constant bias. The weight w_{jk} represents the polymerization strength of particles, s_{ik} and s_{jk}, and w_{ij} represents the polymerization strength of particles, s_{ik} and s_{ij}. The dynamic state $v_{ik}(t)$ is a piecewise linear function of $q_{ik}(t)$, which is defined by

$$v_{ik}(t) = \begin{cases} 0 & \text{if } q_{ik}(t) < 0, \\ q_{ik}(t) & \text{if } 0 \leqslant q_{ik}(t) \leqslant 1, \\ 1 & \text{if } q_{ik}(t) > 1. \end{cases} \quad (7)$$

Parallel Algorithm (GPMA).

Costep 1. Initiate in parallel $a_{ik}(t_0)$, $p_{ik}(t_0)$ and $q_{ik}(t_0)$ for $i \in \{1, \ldots, n\}$, $k \in \{1, \ldots, m\}$.

Costep 2. By Eq. (1), calculate in parallel the utility $u_{ik}(t)$ of every particle s_{ik} in force-field F at time t.

Costep 3. Calculate in parallel $\Psi_{ik}^{(1)}(t)$ by Eq. (6a), and $\Psi_{ik}^{(2)}(t)$ by Eq. (6b) of every particle s_{ik}.

Costep 4. If all particles reach their equilibrium states at time t, then finish with success; otherwise, modify a_{ik} and p_{ik} by the following Eqs. (8) and (9), respectively, then go to *Costep* 2:

$$\mathrm{d}p_{ik}(t)/\mathrm{d}t = \lambda_{ik}^{(1)} \frac{\partial u_{ik}(t)}{\partial p_{ik}(t)} + \lambda_{ik}^{(2)} \frac{\mathrm{d}J(t)}{\mathrm{d}p_{ik}(t)} - \lambda_{ik}^{(3)} \frac{\mathrm{d}P(t)}{\mathrm{d}p_{ik}(t)} - \lambda_{ik}^{(4)} \frac{\mathrm{d}Q(t)}{\mathrm{d}p_{ik}(t)} + \lambda_{ik}^{(5)} q_{ik}(t), \quad (8)$$

$$\mathrm{d}a_{ik}(t)/\mathrm{d}t = \lambda_{ik}^{(1)} \frac{\partial u_{ik}(t)}{\partial a_{ik}(t)} + \lambda_{ik}^{(2)} \frac{\mathrm{d}J(t)}{\mathrm{d}a_{ik}(t)} - \lambda_{ik}^{(3)} \frac{\mathrm{d}P(t)}{\mathrm{d}a_{ik}(t)} - \lambda_{ik}^{(4)} \frac{\mathrm{d}Q(t)}{\mathrm{d}a_{ik}(t)} + \lambda_{ik}^{(5)} q_{ik}(t), \quad (9)$$

where $0 < \lambda_{ik}^{(5)} \leqslant 1$.

3. Properties of generalized particle model

We summarize properties of GPM to optimize enterprise computing in the below lemmas and theorems, which involve the correctness, convergency and stability of GPM. Their proofs are given in Appendix.

Lemma 1. *The first and second terms of Eqs.* (8), (9) *enable the particle s_{ik} to increase the personal utility of the resource supplier A_i from the user G_k, in direct proportion to* $(\lambda_{ik}^{(1)} + \alpha\lambda_{ik}^{(2)})$.

Lemma 2. *Updating p_{ik} and a_{ik} by Eqs.* (8), (9), *respectively, gives rise to monotonic increase of the aggregate utility of all the particles, in direct proportion to $\alpha\lambda_{ik}^{(2)}$.*

Lemma 3. *If ε is very small, then decreasing the potential energy $P(t)$ of Eq.* (3) *amounts to increasing the minimal utility of all the particles.*

Lemma 4. *The third terms of Eqs.* (8), (9) *enable the particle s_{ik} to increase the minimal utility of all the particles, in direct proportion to $\lambda_{ik}^{(3)}\omega_{ik}^2(t)$, where*

$$\omega_{ik}^2(t) = \exp[-(u_{ik}(t))^2/2\varepsilon^2] \Big/ \sum_{i=1}^{m} \exp[-u_{ik}(t)^2/2\varepsilon^2].$$

Lemma 5. *The fourth terms of Eqs.* (8), (9) *enable the particle s_{ik} to monotonic decrease of the potential energy $Q(t)$, in direct proportion to the value of $\lambda_{ik}^{(4)}$.*

Theorem 1. *Updating p_{ik} and a_{ik} by Eqs. (8), (9), respectively, gives rise to decreasing the hybrid energy function $\Gamma_{ik}(t)$, where every particle may autonomously determine its optimization objective according to its own personality and intention.*

Theorem 2. *The algorithm GPMA can dynamically optimize in parallel the resource allocation for enterprise computing in the context of multi-type social coordination, multi-degree autonomy and multi-objective optimization.*

Lemma 6. *If $\gamma - 1 > -\Psi_{ik}^{(2)}(t) > 0$, $\frac{\partial \Psi_{ik}^{(2)}(t)}{\partial q_{ik}(t)} < 1$ for $q_{ik}(t) < 0$ and $q_{ik}(t) > 1$; and $\frac{\partial \Psi_{ik}^{(2)}(t)}{\partial q_{ik}(t)} \geqslant 1 - \gamma$ for $0 < q_{ik}(t) < 1$ remain valid, then a stable equilibrium point of the particle s_{ik} will be either ($q_{ik}(t) < 0$, $v_{ik}(t) = 0$) or ($q_{ik}(t) > 1, v_{ik}(t) = 1$) (see Fig. 3).*

Lemma 7. *If $\gamma > 1, -\Psi_{ik}^{(2)}(t) < 0$ and $\frac{\partial \Psi_{ik}^{(2)}(t)}{\partial q_{ik}(t)} < 1$ for $q_{ik}(t) > 1$ remain valid, then a stable equilibrium point of the particle s_{ik} will be ($q_{ik}(t) > 1, v_{ik}(t) = 1$).*

Lemma 8. *$\frac{\partial \Psi_{ik}^{(2)}(t)}{\partial q_{ik}(t)} < 1$ for $q_{ik}(t) < 0$ remain valid, then a stable equilibrium point of the particle s_{ik} will be ($q_{ik}(t) < 0, v_{ik}(t) = 0$).*

Lemma 9. *If $\gamma > 1$, $\frac{\partial \Psi_{ik}^{(2)}(t)}{\partial q_{ik}(t)} < 1$ for $q_{ik}(t) = 1^{+0}$ and $\frac{\partial \Psi_{ik}^{(2)}(t)}{\partial q_{ik}(t)} \geqslant 1 - \gamma$ for $q_{ik}(t) = 1^{-0}$ remain valid, then the equilibrium point ($q_{ik}(t) = 1, v_{ik}(t) = 1$) is saddle point. Moreover, if $\gamma > 1$, $\frac{\partial \Psi_{ik}^{(2)}(t)}{\partial q_{ik}(t)} < 1$ for $q_{ik}(t) = 1^{-0}$ and $\frac{\partial \Psi_{ik}^{(2)}(t)}{\partial q_{ik}(t)} \geqslant 1 - \gamma$ for $q_{ik}(t) = 1^{+0}$ remain valid, then the equilibrium point ($q_{ik}(t) = 0, v_{ik}(t) = 0$) is saddle point.*

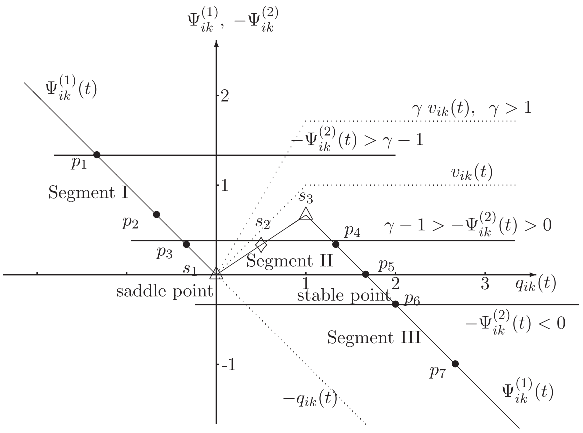

Fig. 3. When $\gamma > 1$, the possible equilibrium points of the dynamic status $v_{ik}(t)$ of a particle s_{ik}. The point where $-\Psi_{ik}^{(2)}(t)$ equals $\Psi_{ik}^{(1)}(t)$ is an equilibrium point. The symbols, •, △ and ◇, denote a stable equilibrium point, saddle point and unstable equilibrium point, respectively.

Theorem 3. *If $\gamma > 1$ and $0 \leqslant q_{ik}(t_0) \leqslant 1$ remain valid, then the dynamical Eq. (6) has a stable equilibrium point iff $0 < -\Psi_{ik}^{(2)}(t) < \gamma - 1$.*

Theorem 4. *If the condition of Theorem 3 remains valid, then GPM will converge to a stable equilibrium state.*

4. Simulations

Some simulation results on the algorithm GPMA for the enterprise computing optimization in the context of resource allocation and task assignment in complex environment are given as follows.

- *Influence of problem size on the utility for enterprise computing optimization*: For different problem sizes, the transients of the minimal personal utility among all the particles and the aggregate utility of all the particles during executing the algorithm GPMA are shown in Figs. 4 and 5, respectively. We can see that for different problem sizes using the GPMA always gives rise to the increase of the minimal personal utility.
- *The influence of problem size on optimization criteria*: In order to evaluate the optimality performance of GPMA, we use the three criteria:

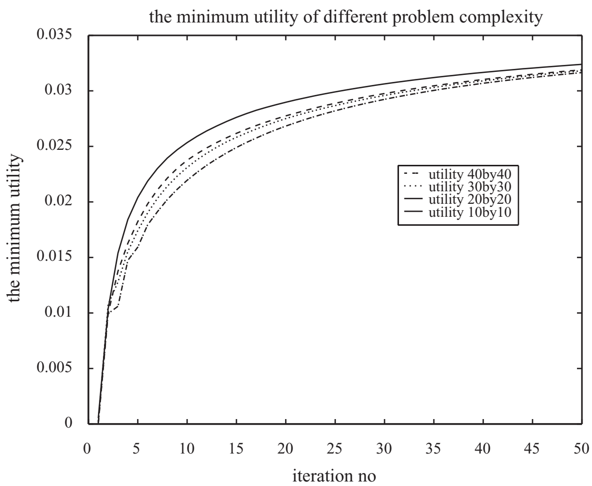

Fig. 4. For different problem sizes, the transient of minimal personal utility among all the particles during executing the algorithm GPMA, where the number of particles 100, 400, 900, 1600 corresponds to problem sizes: 10×10; 20×20; 30×30; 40×40, respectively.

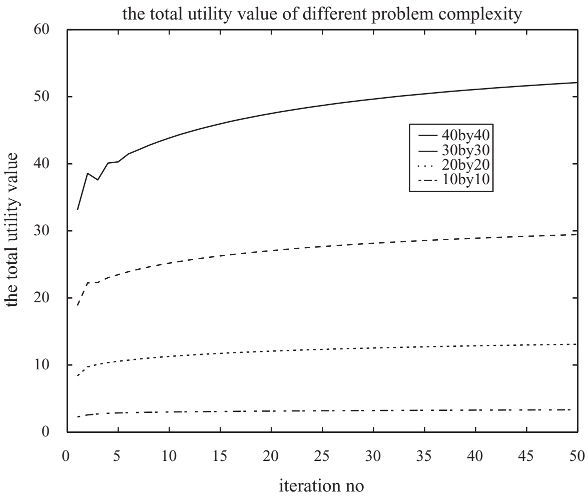

Fig. 5. For different problem sizes, the transient of the aggregate utility of all the particles during executing the algorithm GPM.

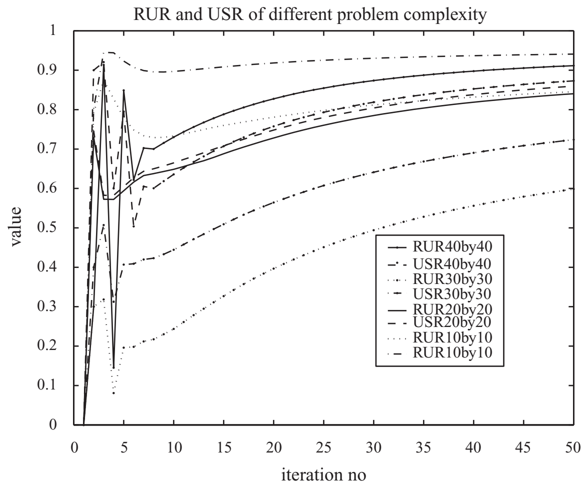

Fig. 6. For different problem sizes, the transients of the aggregate utilization rate of resources and the aggregate satisfactory degree of users during executing the algorithm GPMA.

the fairness FN, resource utilization rate RUR, and user satisfactory degree USD. For different problem sizes, the transients of the aggregate utilization rate of resource suppliers, and aggregate satisfactory degree of resource users are shown in Fig. 6.

- *Comparisons*: The comparisons between the algorithm GPMA and the famous Max–Min algorithm (MMA) are shown in Fig. 7, which demonstrate that, for different problem sizes,

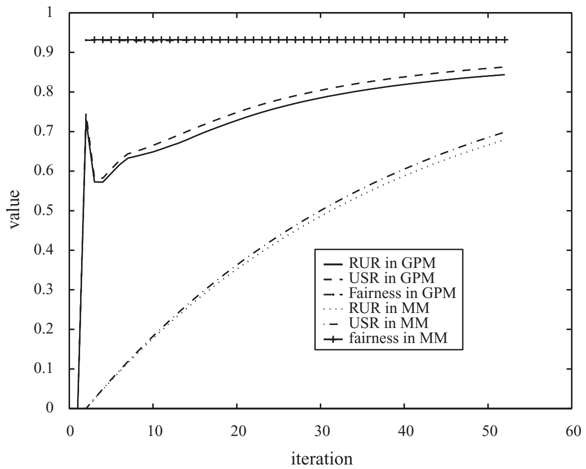

Fig. 7. For 30×30 problem size, the performance comparison between GPAA and the Max–Min Algorithm in terms of transients of the allocation fairness, aggregate utilization rate of resource suppliers, and aggregate satisfactory degree of resource users.

GPMA can converge to a stable equilibrium solution much faster than the MMA. The algorithm GPMA exhibits much better optimality performance than MMA in terms of the aggregate utilization rate of resource suppliers and aggregate satisfactory degree of resource users, whereas they have almost approximately equal allocation fairness.

5. Conclusions

We propose a new generalized particle model (GPM) for parallel optimization of resource allocation and task assignment in complex environment for enterprise computing. We give the GPM-based algorithm and prove its properties in detail. GPM may deal with multi-type coordination, multi-degree autonomy, multi-objective optimization, multi-granularity coalition, and some complex factors such as congestion, failure and priority. The GPM approach also has advantages in terms of parallelism and feasibility for hardware implementation by VLSI technology.

Appendix

Proof of Lemma 1. Denote the jth terms of Eqs. (8) and (9) by $\langle\frac{\mathrm{d}p_{ik}(t)}{\mathrm{d}t}\rangle_j$ and $\langle\frac{\mathrm{d}a_{ik}(t)}{\mathrm{d}t}\rangle_j$, respectively. $\langle\frac{\mathrm{d}a_{ik}(t)}{\mathrm{d}t}\rangle_1$ and $\langle\frac{\mathrm{d}p_{ik}(t)}{\mathrm{d}t}\rangle_1$ give rise to the utility differential of the particle s_{ik} as follows:

$$\langle \mathrm{d}u_{ik}(t)/\mathrm{d}t\rangle_1 = \frac{\partial u_{ik}(t)}{\partial p_{ik}(t)}\left\langle\frac{\mathrm{d}p_{ik}(t)}{\mathrm{d}t}\right\rangle_1 + \frac{\partial u_{ik}(t)}{\partial a_{ik}(t)}\left\langle\frac{\mathrm{d}a_{ik}(t)}{\mathrm{d}t}\right\rangle_1 = \lambda_{ik}^{(1)}\left[\left(\frac{\partial u_{ik}(t)}{\partial p_{ik}(t)}\right)^2 + \left(\frac{\partial u_{ik}(t)}{\partial a_{ik}(t)}\right)^2\right] \geqslant 0.$$

Similarly, $\langle\frac{\mathrm{d}p_{ik}(t)}{\mathrm{d}t}\rangle_2$ and $\langle\frac{\mathrm{d}a_{ik}(t)}{\mathrm{d}t}\rangle_2$ give rise to

$$\begin{aligned}\langle \mathrm{d}u_{ik}(t)/\mathrm{d}t\rangle_2 &= \frac{\partial u_{ik}(t)}{\partial p_{ik}(t)}\left\langle\frac{\mathrm{d}p_{ik}(t)}{\mathrm{d}t}\right\rangle_2 + \frac{\partial u_{ik}(t)}{\partial a_{ik}(t)}\left\langle\frac{\mathrm{d}a_{ik}(t)}{\mathrm{d}t}\right\rangle_2 \\ &= \lambda_{ik}^{(2)}\frac{\partial u_{ik}(t)}{\partial p_{ik}(t)}\frac{\mathrm{d}J(t)}{\mathrm{d}p_{ik}(t)} + \lambda_{ik}^{(2)}\frac{\partial u_{ik}(t)}{\partial a_{ik}(t)}\frac{\mathrm{d}J(t)}{\mathrm{d}a_{ik}(t)} \\ &= \lambda_{ik}^{(2)}\frac{\partial u_{ik}(t)}{\partial p_{ik}(t)}\frac{\partial J(t)}{\partial u_{ik}(t)}\frac{\partial u_{ik}(t)}{\partial p_{ik}(t)} \\ &\quad + \lambda_{ik}^{(2)}\frac{\partial u_{ik}(t)}{\partial a_{ik}(t)}\frac{\partial J(t)}{\partial u_{ik}(t)}\frac{\partial u_{ik}(t)}{\partial a_{ik}(t)} \\ &= \alpha\lambda_{ik}^{(2)}\left[\left(\frac{\partial u_{ik}(t)}{\partial p_{ik}(t)}\right)^2 + \left(\frac{\partial u_{ik}(t)}{\partial a_{ik}(t)}\right)^2\right] \geqslant 0.\end{aligned}$$

Therefore, the joint differential of utility $u_{ik}(t)$ is directly proportional to $(\lambda_{ik}^{(1)} + \alpha\lambda_{ik}^{(2)})$. □

Proof of Lemma 2. It is straightforward from the proof of Lemma 1. □

Proof of Lemma 3. Suppose that $H(t) = \max_{i,k}\{-u_{ik}^2(t)\}$. We have

$$\begin{aligned}[\exp(H(t)/2\varepsilon^2)]^{2\varepsilon^2} &\leqslant \left[\sum_{i=1}^{n}\sum_{k=1}^{m}\exp(-u_{ik}^2(t)/2\varepsilon^2)\right]^{2\varepsilon^2} \\ &\leqslant [mn\exp(H(t)/2\varepsilon^2)]^{2\varepsilon^2}.\end{aligned}$$

Taking the logarithm of both sides of the above inequalities leads to

$$\begin{aligned}H(t) &\leqslant 2\varepsilon^2\ln\sum_{i=1}^{n}\sum_{k=1}^{m}\exp(-u_{ik}^2(t)/2\varepsilon^2) \\ &\leqslant H(t) + 2\varepsilon^2\ln mn.\end{aligned}$$

Since mn is constant and ε is very small, we have

$$\begin{aligned}H(t) &\approx 2\varepsilon^2\ln\sum_{i=1}^{n}\sum_{k=1}^{m}\exp(-u_{ik}^2(t)/2\varepsilon^2) \\ &\quad - 2\varepsilon^2\ln mn = 2P(t).\end{aligned}$$

It results that the potential energy $P(t)$ at the time t represents the minimum among all the $u_{ik}(t)$. Hence decreasing the potential energy $P(t)$ will result in increasing the minimum among all the $u_{ik}(t)$'s. □

Proof of Lemma 4. Similar to the proof of the Lemma 1, the third terms of Eqs. (8), (9) give rise to the utility differential of the particle s_{ik} as follows:

$$\left\langle\frac{\mathrm{d}u_{ik}(t)}{\mathrm{d}t}\right\rangle_3 = -\lambda_{ik}^{(3)}\frac{\partial P(t)}{\partial u_{ik}(t)}\left[\left(\frac{\partial u_{ik}(t)}{\partial a_{ik}(t)}\right)^2 + \left(\frac{\partial u_{ik}(t)}{\partial p_{ik}(t)}\right)^2\right].$$

Thus the third terms of Eqs. (8), (9) cause the component, $\langle\frac{\mathrm{d}P(t)}{\mathrm{d}t}\rangle$, of $\frac{\mathrm{d}P(t)}{\mathrm{d}t}$ as follows:

$$\begin{aligned}\left\langle\frac{\mathrm{d}P(t)}{\mathrm{d}t}\right\rangle &= \frac{\partial P(t)}{\partial u_{ik}(t)}\left\langle\frac{\mathrm{d}u_{ik}(t)}{\mathrm{d}t}\right\rangle_3 \\ &= -\lambda_{ik}^{(3)}\left[\frac{\partial P(t)}{\partial u_{ik}(t)}\right]^2 \\ &\quad\times\left[\left(\frac{\partial u_{ik}(t)}{\partial a_{ik}(t)}\right)^2 + \left(\frac{\partial u_{ik}(t)}{\partial p_{ik}(t)}\right)^2\right] \leqslant 0.\end{aligned}$$

It turns out that the third terms of Eqs. (8), (9) give rise to monotonic decrease of $P(t)$. Then by Lemma 3, they result in the increase of the minimal utility among all the particles, in direct proportion to $\lambda_{ik}^{(3)}\omega_{ik}^2(t)$. □

Proof Lemma 5. Similar to the proof of the Lemma 1, the fourth terms of Eqs. (8), (9) give rise to the utility differential of the particle s_{ik} as follows:

$$\left\langle\frac{\mathrm{d}u_{ik}(t)}{\mathrm{d}t}\right\rangle_4 = -\lambda_{ik}^{(4)}\frac{\partial Q(t)}{\partial u_{ik}(t)}\left[\left(\frac{\partial u_{ik}(t)}{\partial a_{ik}(t)}\right)^2 + \left(\frac{\partial u_{ik}(t)}{\partial p_{ik}(t)}\right)^2\right].$$

Hence we have

$$\begin{aligned}\left\langle\frac{\mathrm{d}Q(t)}{\mathrm{d}t}\right\rangle &= \frac{\partial Q(t)}{\partial u_{ik}(t)}\left\langle\frac{\mathrm{d}u_{ik}(t)}{\mathrm{d}t}\right\rangle_4 \\ &= -\lambda_{ik}^{(4)}\left[\frac{\partial Q(t)}{\partial u_{ik}(t)}\right]^2 \\ &\quad\times\left[\left(\frac{\partial u_{ik}(t)}{\partial a_{ik}(t)}\right)^2 + \left(\frac{\partial u_{ik}(t)}{\partial p_{ik}(t)}\right)^2\right] \leqslant 0. \quad \square\end{aligned}$$

Proof of Theorem 1. It follows that

$$\begin{aligned}&\sum_{j=1}^{4}\langle \mathrm{d}u_{ik}(t)/\mathrm{d}t\rangle_j \\ &= \left[\lambda_{ik}^{(1)} + \lambda_{ik}^{(2)}\frac{\partial J(t)}{\partial u_{ik}(t)} - \lambda_{ik}^{(3)}\frac{\partial P(t)}{\partial u_{ik}(t)}\right. \\ &\quad\left. -\lambda_{ik}^{(4)}\frac{\partial Q(t)}{\partial u_{ik}(t)}\right]\left[\left(\frac{\partial u_{ik}(t)}{\partial a_{ik}(t)}\right)^2 + \left(\frac{\partial u_{ik}(t)}{\partial a_{ik}(t)}\right)^2\right] \\ &= -\frac{\partial \Gamma_{ik}(t)}{\partial u_{ik}(t)}\left[\left(\frac{\partial u_{ik}(t)}{\partial a_{ik}(t)}\right)^2 + \left(\frac{\partial u_{ik}(t)}{\partial p_{ik}(t)}\right)^2\right]\end{aligned}$$

and thus

$$\left\langle \frac{\mathrm{d}\Gamma_{ik}(t)}{\mathrm{d}t} \right\rangle = \frac{\partial \Gamma_{ik}(t)}{\partial u_{ik}(t)} \sum_{j=1}^{4} \langle \mathrm{d}u_{ik}(t)/\mathrm{d}t \rangle_j = -\left[\frac{\partial \Gamma_{ik}(t)}{\partial u_{ik}(t)}\right]^2 \times \left[\left(\frac{\partial u_{ik}(t)}{\partial a_{ik}(t)}\right)^2 + \left(\frac{\partial u_{ik}(t)}{\partial p_{ik}(t)}\right)^2\right] \leqslant 0.$$

Therefore, updating $p_{ik}^{(j)}$ and $a_{ik}^{(j)}$ by Eqs. (8) and (9), respectively, gives rise to monotonically decreasing the hybrid energy function $\Gamma_{ik}(t)$. $\square$

Proof of Theorem 2. It is straightforward from Lemmas 1–5 and Theorem 1. In summary, $(\lambda_{ik}^{(1)} + \alpha\lambda_{ik}^{(2)})$ represents the autonomy degree to maximize the personal utility of individual resource supplier and user; $\lambda_{ik}^{(2)}$ represents the autonomous strength to pursue the aggregate utility of systems; $\lambda_{ik}^{(3)}\omega_{ik}^2(t)$ represents the autonomous degree to maximize the minimal personal utility among all the particles. The $\lambda_{ik}^{(4)}$ represents the personality in terms of interactions. $\square$

Proof of Lemma 6. Let $\gamma > 1$ hold true. $\Psi_{ik}^{(1)}(t)$ of particle s_{ik} is a piecewise linear function of the stimulus $q_{ik}(t)$, as shown by three segments: Segment I, Segment II, and Segment III in Fig. 3. By Eq. (6), a point is an equilibrium point, i.e. $\mathrm{d}q_{ik}(t)/\mathrm{d}t = 0$, iff $-\Psi_{ik}^{(2)}(t) = \Psi_{ik}^{(1)}(t)$ at the point. We see that an equilibrium point may be on Segment I, II or III in the case of $\gamma - 1 > -\Psi_{ik}^{(2)}(t) > 0$.

Suppose that the particle s_{ik} is at an equilibrium point on Segment III or Segment I at time t_0, and an arbitrarily small perturbation Δq_{ik} to the equilibrium point occurs at time t_1, Since $\frac{\partial \Psi_{ik}^{(2)}(t)}{\partial q_{ik}(t)} < 1$ and $\frac{\partial \Psi_{ik}^{(1)}(t)}{\partial q_{ik}(t)} = -1$ for $q_{ik}(t) < 0$ and $q_{ik}(t) > 1$, we have

$$c = \left[\frac{\partial \Psi_{ik}^{(1)}(t)}{\partial q_{ik}(t)} + \frac{\partial \Psi_{ik}^{(2)}(t)}{\partial q_{ik}(t)}\right] < 0$$

and

$$\Delta \frac{\mathrm{d}q_{ik}}{\mathrm{d}t} = \left.\frac{\mathrm{d}q_{ik}}{\mathrm{d}t}\right|_{t_1} - \left.\frac{\mathrm{d}q_{ik}}{\mathrm{d}t}\right|_{t_0} = \left.\frac{\mathrm{d}q_{ik}}{\mathrm{d}t}\right|_{t_1} = \Delta[\Psi_{ik}^{(1)}(t) + \Psi_{ik}^{(2)}(t)] \approx \left[\frac{\partial \Psi_{ik}^{(1)}(t)}{\partial q_{ik}(t)} + \frac{\partial \Psi_{ik}^{(2)}(t)}{\partial q_{ik}(t)}\right] \Delta q_{ik} = -|c| \Delta q_{ik}.$$

Hence $\frac{\mathrm{d}q_{ik}}{\mathrm{d}t}|_{t_1}$ is always against Δq_{ik}, in other words, the perturbation will be suppressed and the particle s_{ik} hence returns to the original equilibrium point.

On the other hand, however, in the case of an equilibrium point on Segment II, because there are $\frac{\partial \Psi_{ik}^{(2)}(t)}{\partial q_{ik}(t)} \geqslant 1 - \gamma$ and $\frac{\partial \Psi_{ik}^{(1)}(t)}{\partial q_{ik}(t)} = \gamma - 1 > 0$, we have

$$c = \left[\frac{\partial \Psi_{ik}^{(2)}(t)}{\partial q_{ik}(t)} + \frac{\partial \Psi_{ik}^{(1)}(t)}{\partial q_{ik}(t)}\right] \geqslant 0$$

and

$$\left.\frac{\mathrm{d}q_{ik}}{\mathrm{d}t}\right|_{t_1} \approx \left[\frac{\partial \Psi_{ik}^{(1)}(t)}{\partial q_{ik}(t)} + \frac{\partial \Psi_{ik}^{(2)}(t)}{\partial q_{ik}(t)}\right] \Delta q_{ik} = |c| \Delta q_{ik},$$

which leads to the perturbation unchanged or intensified, so that the particle s_{ik} departs from the original equilibrium point on Segment II. Therefore, an equilibrium point $(q_{ik}(t) < 0, v_{ik}(t) = 0)$ on Segment I or $(q_{ik}(t) > 1, v_{ik}(t) = 1)$ on Segment III, e.g. points p_3, p_4 in Fig. 3, is stable; and an equilibrium point on Segment II is unstable, e.g. the point s_2. $\square$

Proof of Lemma 7. Due to $\gamma > 1$ and $-\Psi_{ik}^{(2)}(t) < 0$, an equilibrium point must be on Segment III, e.g. p_6 in Fig. 3. Moreover, as stated in the proof of Lemma 6, $\frac{\partial \Psi_{ik}^{(2)}(t)}{\partial q_{ik}(t)} < 1$ for $q_{ik}(t) > 1$ guarantees that any equilibrium point $(q_{ik}(t) > 1, v_{ik}(t) = 1)$ on Segment III is stable. $\square$

Proof of Lemma 8. By assumptions, an equilibrium point must be on the Segment I. By Lemma 6, it follows that an equilibrium point $(q_{ik}(t) < 0, v_{ik}(t) = 0)$ is stable. $\square$

Proof of Lemma 9. Consider the equilibrium point s_3 in Fig. 3, i.e. $(q_{ik}(t) = 1, v_{ik}(t) = 1)$. It follows from the proof of Lemma 6 that a positive perturbation of q_{ik} will be suppressed, whereas a negative perturbation intensified, that is, s_3 is saddle point.

Similarly, we can prove that the equilibrium point $(q_{ik}(t) = 0, v_{ik}(t) = 0)$ is also saddle point. $\square$

Proof of Theorem 3. Note that $\Psi_{ik}^{(2)}(t)$ is not an explicit function of $q_{ik}(t)$. Thus, $\frac{\partial \Psi_{ik}^{(2)}(t)}{\partial q_{ik}(t)} = 0$ holds true and the condition $1 - \gamma \leqslant \frac{\mathrm{d}\Psi_{ik}^{(2)}(t)}{\mathrm{d}q_{ik}(t)} \leqslant 1$ of Lemmas 6–8 is satisfied. It follows that an equilibrium point on Segment II is unstable, the points s_1 and s_3 in Fig. 3

are saddle points, and a stable equilibrium point must be on Segment I or III.

Sufficiency Proof . $-\Psi_{ik}^{(2)}(t) < \gamma - 1$ implies that at the point s_3 there is $\frac{dq_{ik}}{dt} = \Psi_{ik}^{(1)}(t) + \Psi_{ik}^{(2)}(t) = \gamma - 1 + \Psi_{ik}^{(2)}(t) > 0$, i.e. $\Psi_1(t) \neq -\Psi_2(t)$. Thus the point s_3 is no equilibrium point of Eq. (6). Moreover, $\frac{dq_{ik}}{dt} > 0$ at the point s_3 leads to the increase of q_{ik} from value 1. It turns out that it is impossible that the dynamic state of the particle s_{ik} evolves into Segment II from Segment III via the point s_3.

On the other hand, $-\Psi_2(t) > 0$ means that at the point s_1 there is $\frac{dq_{ik}}{dt} = \Psi_{ik}^{(1)}(t) + \Psi_{ik}^{(2)}(t) = \Psi_{ik}^{(2)}(t) < 0$. Thus the point s_1 is no equilibrium point of Eq. (6). Moreover, $\frac{dq_{ik}}{dt} < 0$ at the point s_1 leads to the decrease of q_{ik} from zero. It turns out that it is impossible that the dynamic state of the particle s_{ik} evolves into Segment II from Segment I via the point s_1.

Because an equilibrium point on Segment II is unstable and the points, s_1 and s_3, are not equilibrium points, the dynamic state of the particle s_{ik} must evolve irreversibly into Segment I from Segment II via the point s_1, or irreversibly into Segment III from Segment II via the point s_3. Therefore, Eq. (6) has a stable equilibrium point on Segment I or III.

Necessity Proof . Suppose that the Eq. (6) has a stable equilibrium point. By contrary, we assume $-\Psi_{ik}^{(2)}(t) \geqslant \gamma - 1$ or $-\Psi_{ik}^{(2)}(t) \leqslant 0$. In the case of $-\Psi_{ik}^{(2)}(t) \geqslant \gamma - 1$, the condition $0 < q_{ik}(t_0) < 1$ leads to that the dynamic state of particle s_{ik} never evolves into Segment III (where $-\Psi_{ik}^{(2)}(t) \leqslant \gamma - 1$) from Segment II via the saddle point s_3. In the case of $-\Psi_{ik}^{(2)}(t) \leqslant 0$, the condition $0 < q_{ik}(t_0) < 1$ leads to the dynamic state of particle s_{ik} never evolves into Segment I (where $-\Psi_{ik}^{(2)}(t) \geqslant 0$) from Segment II via the saddle point s_1. Hence the state of particle s_{ik} always stagnates on Segment II, in contradiction with the assumption of existing stable equilibrium point. Therefore, $0 < -\Psi_{ik}^{(2)}(t) < \gamma - 1$ holds true. □

Proof of Theorem 4. We define a Lyapunov function $L(t)$ of GPM by

$$L(t) = \sum_{i,k} \left\{ [0.5(1-\gamma) v_{ik}(t)^2] + \int_0^t \frac{dv_{ik}(x)}{dx} [-\Psi_{ik}^{(2)}(x)] \, dx \right\}.$$

By the conditions, $\gamma > 1$, and $-\Psi_{ik}^{(2)}(t) < \gamma - 1$, we have

$$\begin{aligned} |L(t)| &\leqslant \sum_{i,k} (\gamma - 1) |v_{ik}(t)^2| + \sum_{i,k} \int_0^t \left| \frac{dv_{ik}(x)}{dx} \right| |\Psi_{ik}^{(2)}(x)| \, dx \\ &\leqslant \sum_{i,k} (\gamma - 1) |v_{ik}(t)^2| + \sum_{i,k} \int_0^t \left| \frac{dv_{ik}(x)}{dx} \right| (\gamma - 1) \, dx \\ &\leqslant \sum_{i,k} (\gamma - 1) |v_{ik}(t)^2| + (\gamma - 1) \sum_{i,k} |v_{ik}(x)|. \end{aligned}$$

Since $0 \leqslant v_{ik}(t) \leqslant 1$ and $\gamma > 1$, $|L(t)|$ is bounded.

Moreover, since

$$\frac{dv_i(t)}{dq_i(t)} = \begin{cases} 1 & \text{if } 0 < q_i(t) < 1, \\ 0 & \text{otherwise,} \end{cases}$$

we obtain

$$\begin{aligned} \frac{dL(t)}{dt} &= -\sum_{i,k} (\gamma - 1) v_{ik}(t) \frac{dv_{ik}(t)}{dt} - \sum_{i,k} \frac{dv_{ik}(t)}{dt} \Psi_{ik}^{(2)}(t) \\ &= -\sum_{i,k} \frac{dv_{ik}(t)}{dt} [(\gamma - 1) v_{ik}(t) + \Psi_{ik}^{(2)}(t)] \\ &= -\sum_{i,k} \frac{dv_{ik}(t)}{dt} \frac{dq_{ik}(t)}{dt} \\ &= -\sum_{i,k} \frac{dv_{ik}(t)}{dq_{ik}(t)} \left(\frac{dq_{ik}(t)}{dt} \right)^2 \leqslant 0. \end{aligned}$$

The bounded $L(t)$ will monotonically decrease as the time elapses. Thus, the dynamics of GPM must converge to a stable equilibrium state. □

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