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Multi-objective robust control based on fuzzy singularly perturbed models for hypersonic vehicles

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Abstract In this paper, we propose a multi-objective robust controller based on fuzzy singularly perturbed models (FSPM) for the longitudinal motion of an air-breathing hypersonic vehicle. The control objective is to provide velocity and altitude tracking in the presence of the uncertainties and unknown nonlinearities in the model caused by variations in flight conditions. By approximating the multi-time-scale model with FSPM, ill-posed matrix calculations are circumvented. Under this framework, a fuzzy multi-objective robust controller is developed, and the linear matrix inequalities (LMI) condition is derived. Numerical simulation using the longitudinal model of a hypersonic aircraft is provided to demonstrate the effectiveness of the proposed strategy.

Keywords FSPM, multi-objective robust control, intelligent control, hypersonic vehicle

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1 Introduction

Hypersonic air-breathing vehicles provide a promising and cost-effective technology to meet the objectives of commercial as well as military applications for space access and rapid global reach capabilities. However, the design of flight control systems for hypersonic vehicles poses many challenges stemming from the highly coupled and nonlinear nature of their dynamic behavior. High Mach numbers and altitude make the flight control extremely sensitive to changes in atmospheric conditions as well as physical and aerodynamic parameters. However, a controller designed for hypersonic aircraft must guarantee stability for the system and provide satisfactory control performance. Recently, several representative control approaches based on feedback linearization techniques have been proposed for the longitudinal dynamic model of hypersonic aircraft, such as stochastic robust control [1], dynamic inversion control [2], neural network adaptive control [3], and adaptive sliding mode control [4], etc. Although the simulation results show the effectiveness of these approaches, the complexity of the model inevitably leads to very complicated expressions for the high-order Lie derivatives and thus a robustness analysis cannot be performed. In [5], a robust linear output feedback controller is presented. Although this approach provides robust performance under certain flight conditions, it still lacks adaptive ability due to its linear control nature.

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A nonlinear controller using the Sum-Of-Squares technique combined with the dual of Lyapunov stability theorem for longitudinal dynamics of a hypersonic aircraft model is designed in [6]. However, the problem of how to reduce dimension caused by the intrinsic characteristics of the Sum-Of-Squares technique and accelerate the response characteristics, remains to be solved.

Hypersonic aircraft are typical multi-time-scale systems and the state variables can be divided into fast and slow modes according to the order of difference in the magnitude of the center-of-mass motion and attitude motion. It is well known that fuzzy singularly perturbed models (FSPM) with uncertaintyterms can approximate, to some extent, the multi-time-scale system and reduce the number of fuzzy rules [7]. Therefore, it is natural to describe highly-nonlinear aircraft models using FSPM. On the other hand, multi-objective robust controllers can resist disturbance and guarantee dynamic performance at the same time, which is well suited for flight control. Previous work such as [8–14] dealt with the robust control problem of FSPM with parameter perturbation. However, they only focused on a single perturbation parameter. In fact, FSPM with multiple parameter perturbations is more practical and precise. To the best of our knowledge, there has been no research on this problem. Motivated by this factor, this paper will study multi-objective robust controllers using FSPM with uncertainty terms as well as multiple perturbation parameters. Based on this, a controller for air-breathing hypersonic aircrafts is developed.

The paper is organized as follows. Section 2 introduces the hypersonic vehicle model and presents the formulation of the problem. In Section 3, an LMI condition is derived for a fuzzy multi-objective robust controller which can guarantee the control performance and stability at the same time. Numerical simulation results are given in Section 4. Section 5 draws our conclusions.

2 Problem formulation

Nomenclature

a = speed of sound, ft/s
C_D= drag coefficient
C_L= lift coefficient
$C_M(q)$= pitching moment coefficient due to pitch rate
$C_M(a)$ = pitching moment coefficient due to angle of attack
$C_M(\delta e)$ = pitching moment coefficient due to elevator deflection
C_T = chord
c = reference length, 80ft
D = drag, lbf
h = altitude
I_{yy} = moment of inertia, 7×10^6 slug·ft^2
M = Mach number
M_{yy} = pitching moment, lbf·ft
m = mass, 9375slugs
q = pitch
R_E = radius of the Earth, 20,903,500ft
r = radius distance from Earth's center, ft
S = reference area, 3603ft^2
T = thrust, lbf
V = velocity, ft/s
α = angle of attack, rad
β = throttle setting
γ = flight-path angle, rad
θ_p = pitch angle, rad
δe = elevator deflection
μ = gravitational constant, 1.39×10^{16} ft^3/sec^2
ρ = density of air, slug/ft^3

2.1 Hypersonic air-breathing vehicle model

We consider the longitudinal model of a generic hypersonic vehicle presented in [1, 4], which was developed at the NASA Langley Research Center. The longitudinal dynamics of the air-breathing hypersonic vehicle model can be described by a set of differential equations for velocity, altitude, flight-path angle, angle of attack, and pitch rate as follows:

$$\dot{V} = (T\cos\alpha - D)/m - \mu\sin\gamma/r^2, \tag{1}$$

$$\dot{h} = V\sin\gamma, \tag{2}$$

$$\dot{\gamma} = (L + T\sin\alpha)/(mV) - [(\mu - V^2 r)\cos\gamma]/Vr^2, \tag{3}$$

$$\dot{\alpha} = q - \dot{\gamma}, \tag{4}$$

$$\dot{q} = M_{yy}/I_{yy}, \tag{5}$$

where L, D, T and M_{yy} denote the lift, drag, thrust and pitching moment and are defined as

$$L = \frac{1}{2}\rho V^2 S C_L, \tag{6}$$

$$D = \frac{1}{2}\rho V^2 S C_D, \tag{7}$$

$$T = \frac{1}{2}\rho V^2 S C_T, \tag{8}$$

$$M_{yy} = \frac{1}{2}\rho V^2 S\bar{c}[C_M(\alpha) + C_M(\delta e) + C_M(q)], \tag{9}$$

$$r = h + R_E, \tag{10}$$

where ρ denotes the air density which is calculated as:

$$\rho = 0.00238\exp\left(\frac{-h}{24000}\right)$$

and $C_x, x = L, D, T, M$ in the dynamic equations (6)–(9) are the force and moment coefficients which are given by (see [15]):

$$C_L = \alpha(0.493 + 1.91/M), \tag{11}$$

$$C_D = 0.0082(171\alpha^2 + 1.15\alpha + 1) \times (0.0012M^2 - 0.054M + 1), \tag{12}$$

$$C_T = \begin{cases} C_T^*(1 + 0.15)\beta, & \text{if } \beta < 1, \\ C_T^*(1 + 0.15\beta), & \text{if } \beta \geqslant 1, \end{cases} \tag{13}$$

$$C_M(\alpha) = 10^{-4}(0.06 - \mathrm{e}^{-M/3}) \times (-6565\alpha^2 + 6875\alpha + 1), \tag{14}$$

$$C_M(q) = (q\bar{c}/2V) \times (-0.025M + 1.37) \times (-6.83\alpha^2 + 0.303\alpha - 0.23), \tag{15}$$

$$C_M(\delta\mathrm{e}) = 0.0292(\delta\mathrm{e} - \alpha), \tag{16}$$

where

$$C_T^* = 0.0105[1 - 164(\alpha - \alpha_0)^2](1 + 17/M). \tag{17}$$

M in the above equations is the Mach number defined as $M = V/a$, with the speed of sound a, given by

$$a = 8.99 \times 10^{-9}h^2 - 9.16 \times 10^{-4}h + 996. \tag{18}$$

The engine dynamics are modeled by a second-order system

$$\ddot{\beta} = -2\zeta\omega_n\dot{\beta} - \omega_n^2\beta + \omega_n^2\beta_c. \tag{19}$$

For the purpose of this study, the aerodynamic coefficients are simplified around the typical cruising flight conditions [15] with h_0 being 108500, 110000, 111500ft and v_0 being 14760, 15060, 15360ft/s, respectively. The control inputs are the throttle setting β_c and the elevator deflection δe. The outputs are the velocity V and the altitude h. The desired values of velocity and altitude are denoted by $V_d(t)$ and $h_d(t)$.

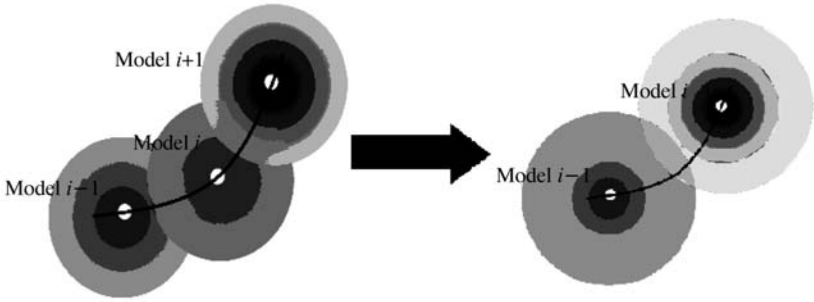

Figure 1 Uncertainty terms reduce the fuzzy rules.

2.2 FSPM for a hypersonic vehicle

A normal TS model with uncertainty and a disturbance term can be written as follows:

$$\begin{aligned}&\text{Rule } i: \text{if } \xi_1(t) \text{ is } F_1^i \text{and } \xi_2(t) \text{ is } F_2^i \cdots \text{ and } \xi_g(t) \text{ is } F_g^i,\\ &\text{then } \dot{\boldsymbol{x}}(t) = (\tilde{\boldsymbol{A}}^i + \Delta\tilde{\boldsymbol{A}}^i)\boldsymbol{x}(t) + (\tilde{\boldsymbol{B}}^i + \Delta\tilde{\boldsymbol{B}}^i)\boldsymbol{u}(t) + \tilde{\boldsymbol{D}}^i\boldsymbol{w}(t), \qquad i = 1, 2, \ldots, r,\end{aligned} \tag{20}$$

where $\tilde{\boldsymbol{A}}^i$, $\tilde{\boldsymbol{B}}^i$ and $\tilde{\boldsymbol{D}}^i$ are all matrices with proper dimensions. $\Delta\tilde{\boldsymbol{A}}^i$ and $\Delta\tilde{\boldsymbol{B}}^i$ are uncertainty terms, which can reduce the rules needed for approximating the nonlinear system as shown in Figure 1. Note that the introduction of uncertainty terms is important for hypersonic aircraft since the reduction in the number of fuzzy rules can significantly improve the real-time performance.

In general, there are two ways to establish the FSPM, namely, the eigenvalue decomposition method and the mechanism analysis method. Since the attitude motion including the angle of attack and the elevation angle are slow variables, whereas the center-of-mass motion such as height and velocity are fast variables [13], the mechanism analysis method is adopted here to obtain the FSPM. Let $\boldsymbol{x}_s$ denote the slow variable and $\boldsymbol{x}_z$ denote the fast variable, then the FSPM can be obtained by row operation as follows:

$$\begin{aligned}&\text{Rule } i: \text{ if } \xi_1(t) \text{is } F_1^i \text{and } \xi_2(t) \text{ is } F_2^i \cdots \text{ and } \xi_g(t) \text{ is } F_g^i,\\ &\text{then} \begin{bmatrix} \dot{\boldsymbol{x}}_s(t) \\ \dot{\boldsymbol{x}}'_z(t) \end{bmatrix} = I_t \cdots I_2 I_1 (\tilde{\boldsymbol{A}}^i + \Delta\tilde{\boldsymbol{A}}^i) I_1^{-1} I_2^{-1} \cdots I_t^{-1} \begin{bmatrix} \boldsymbol{x}_s(t) \\ \boldsymbol{x}'_z(t) \end{bmatrix}\\ &\qquad + I_t \cdots I_2 I_1 (\tilde{\boldsymbol{B}}^i + \Delta\tilde{\boldsymbol{B}}^i)\boldsymbol{u}(t) + I_t \cdots I_2 I_1 \tilde{D}^i w(t), \qquad i = 1, 2, \ldots, r,\end{aligned} \tag{21}$$

where $I_1, I_2, \ldots, I_t$ are row operation matrices. By extracting the perturbation parameters from the fast variables and denoting $\boldsymbol{x}'_z(t) = \boldsymbol{\Lambda}_{\boldsymbol{\varepsilon}}\boldsymbol{x}_z(t)$, where

$$\boldsymbol{\Lambda}_{\boldsymbol{\varepsilon}} = \begin{bmatrix} \varepsilon_1 \boldsymbol{I}_{z_1\times z_1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \varepsilon_2 \boldsymbol{I}_{z_2\times z_2} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \ddots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \varepsilon_H \boldsymbol{I}_{z_H\times z_H} \end{bmatrix}, \ \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_H \end{bmatrix}, 0 < \varepsilon_h \ll 1, h = 1, 2, \ldots, H,$$

eq. (22) is reduced to

$$\begin{aligned}&\text{Rule } i: \text{ if } \xi_1(t) \text{ is } F_1^i \text{ and } \xi_2(t) \text{ is } F_2^i \cdots \text{ and } \xi_g(t) \text{ is } F_g^i,\\ &\text{then } \boldsymbol{E}_{\boldsymbol{\varepsilon}}\dot{\boldsymbol{x}}(t) = (\boldsymbol{A}^i + \Delta\boldsymbol{A}^i)\boldsymbol{x}(t) + (\boldsymbol{B}^i + \Delta\boldsymbol{B}^i)\boldsymbol{u}(t) + \boldsymbol{D}^i\boldsymbol{w}(t); \quad i = 1, 2, \cdots, r,\end{aligned} \tag{22}$$

where

$$\boldsymbol{E}_{\boldsymbol{\varepsilon}} = \begin{bmatrix} \boldsymbol{I}_{n_s\times n_s} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Lambda}_{\boldsymbol{\varepsilon}} n_z \times n_z \end{bmatrix}, \ \boldsymbol{x}(t) = \begin{bmatrix} \boldsymbol{x}_s(t) \\ \boldsymbol{x}_z(t) \end{bmatrix}, \ \boldsymbol{A}^i = \begin{bmatrix} \boldsymbol{A}_{ss}^i & \boldsymbol{A}_{sz}^i \\ \boldsymbol{A}_{zs}^i & \boldsymbol{A}_{zz}^i \end{bmatrix},$$

$$\boldsymbol{B}^i = \begin{bmatrix} \boldsymbol{B}_s^i \\ \boldsymbol{B}_z^i \end{bmatrix}, \ \boldsymbol{D}^i = \begin{bmatrix} \boldsymbol{D}_s^i \\ \boldsymbol{D}_z^i \end{bmatrix}$$

and $\boldsymbol{\Lambda_\varepsilon} \in \mathbb{R}^{n_z \times n_z}$.

By using the fuzzy inference method with a center-average defuzzifier, product inference and singleton-fuzzifier, we get

$$\boldsymbol{E_\varepsilon}\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{r} \mu_i(\boldsymbol{\xi}(t))[(\boldsymbol{A}^i + \Delta\boldsymbol{A}^i)\boldsymbol{x}(t) + (\boldsymbol{B}^i + \Delta\boldsymbol{B}^i)\boldsymbol{u}(t) + \boldsymbol{D}^i\boldsymbol{w}(t)], \tag{23}$$

where $\mu_i(\boldsymbol{\xi}(t)) = \frac{v_i(\boldsymbol{\xi}(t))}{\sum_{i=1}^{r} v_i(\boldsymbol{\xi}(t))}$, $v_i(\boldsymbol{\xi}(t)) = \prod_{j=1}^{g} F_j^i(\xi_j(t))$, $\sum_{i=1}^{r}\mu_i(\boldsymbol{\xi}(t)) = 1$. For simplicity, $\mu_i(\boldsymbol{\xi}(t))$ is denoted as μ_i henceforth. Before proceeding, we first present an assumption and an instrumental Lemma which will be used in the following.

Assumption 1. The uncertainty term is norm-bounded. Define $\Delta\boldsymbol{A}^i = \boldsymbol{H}^i\boldsymbol{L}^i(t)\boldsymbol{E}_1^i$, $\Delta\boldsymbol{B}^i = \boldsymbol{H}^i\boldsymbol{L}^i(t)\boldsymbol{E}_2^i$. $\boldsymbol{L}^i(t)$ is the unknown time-varying matrix and satisfies $\|\boldsymbol{L}_i(t)\| < 1$. $\boldsymbol{H}^i$, $\boldsymbol{E}_1^i$ and $\boldsymbol{E}_2^i$ are all steady matrices which reflect the structure of the model's uncertainty.

Lemma 1 [9]. For any matrix $\boldsymbol{H}$, $\boldsymbol{E}$, $\boldsymbol{L}(t)$ and constant $\beta > 0$, if $\boldsymbol{L}^{\mathrm{T}}(t)\boldsymbol{L}(t) \leqslant \boldsymbol{I}$, then

$$\boldsymbol{H}\boldsymbol{L}(t)\boldsymbol{E} + (\boldsymbol{H}\boldsymbol{L}(t)\boldsymbol{E})^T \leqslant \beta\boldsymbol{H}\boldsymbol{H}^T + \beta^{-1}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{E}. \tag{24}$$

3 Fuzzy multi-objectives robust controller design

In this section, we focus on the multi-objective controller design for the FSPM with multiple perturbation parameters. The multi-objective robust controller provides a tradeoff between the system performance and stability. It is well known that H_∞ control can enhance the robustness and H_2 control can optimize the dynamic performance of the control system. Thus, the combination of the H_∞ performance index:

$$\int_0^{t_f} [\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_1\boldsymbol{x}(t) + \boldsymbol{u}^{\mathrm{T}}(t)\boldsymbol{R}_1\boldsymbol{u}(t)]dt < \gamma^2 \int_0^{t_f} \boldsymbol{w}^{\mathrm{T}}(t)\boldsymbol{w}(t)dt + \boldsymbol{x}_s^{\mathrm{T}}(0)\boldsymbol{P}_{1,ss}\boldsymbol{x}_s(0) + \boldsymbol{O}(\boldsymbol{\varepsilon}) \tag{25}$$

and the H_2 performance index:

$$\int_0^{t_f} [\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_2\boldsymbol{x}(t) + \boldsymbol{u}^{\mathrm{T}}(t)\boldsymbol{R}_2\boldsymbol{u}(t)]dt < \boldsymbol{x}_s^{\mathrm{T}}(0)\boldsymbol{P}_{2,ss}\boldsymbol{x}_s(0) + \boldsymbol{O}(\boldsymbol{\varepsilon}) \tag{26}$$

can guarantee the anti-interference character and dynamic performance at the same time. According to parallel distributed compensation (PDC), the feedback controller can be expressed as follows:

$$\boldsymbol{u}(t) = \sum_{j=1}^{r} \mu_j \cdot [\boldsymbol{K}_s^j\boldsymbol{x}_s(t) + \boldsymbol{K}_z^j\boldsymbol{x}_z(t)] = \sum_{i=1}^{r} \mu_j \cdot \boldsymbol{K}^j\boldsymbol{x}(t). \tag{27}$$

The following Theorem provides the condition of stability while satisfying the H_∞ performance index in (25) at the same time.

Theorem 1. Consider the fuzzy system (23) and the feedback matrix $\boldsymbol{K}^j$ in (27), if there exist common matrices $\boldsymbol{P}_{1,ss}^{\mathrm{T}} = \boldsymbol{P}_{1,ss} > \boldsymbol{0}$, $\boldsymbol{P}_{1,zz}^{\mathrm{T}} = \boldsymbol{P}_{1,zz} > \boldsymbol{0}$ and $\boldsymbol{P}_{1,zs}$ which satisfy

$$\boldsymbol{\Xi}_1^{ii} < \boldsymbol{0}, \qquad i = 1, 2, \ldots, r; \tag{28}$$

$$\boldsymbol{\Xi}_1^{ij} < \boldsymbol{0}, \quad i, j = 1, 2, \ldots, r \quad \text{and} \quad i < j, \tag{29}$$

where

$$\boldsymbol{\Xi}_1^{ii} = \begin{bmatrix} \begin{pmatrix} (\boldsymbol{A}^i + \boldsymbol{B}^i\boldsymbol{K}^i)^{\mathrm{T}}\boldsymbol{P}_1 + \boldsymbol{P}_1^{\mathrm{T}}(\boldsymbol{A}^i + \boldsymbol{B}^i\boldsymbol{K}^i) + \boldsymbol{P}_1^{\mathrm{T}}\boldsymbol{H}^i(\boldsymbol{H}^i)^{\mathrm{T}}\boldsymbol{P}_1 \\ +(\boldsymbol{E}_1^i + \boldsymbol{E}_2^i\boldsymbol{K}^i)^{\mathrm{T}}(\boldsymbol{E}_1^i + \boldsymbol{E}_2^i\boldsymbol{K}^i) + \boldsymbol{Q}_1 + (\boldsymbol{K}^i)^{\mathrm{T}}\boldsymbol{R}_1\boldsymbol{K}^i \end{pmatrix} & * \\ (\boldsymbol{D}^i)^{\mathrm{T}}\boldsymbol{P}_1 & -\gamma^2\boldsymbol{I} \end{bmatrix},$$

$$\boldsymbol{\Xi}_1^{ij} = \left[\begin{array}{cc} \left(\begin{array}{c} (\boldsymbol{A}^i + \boldsymbol{B}^i\boldsymbol{K}^j + \boldsymbol{A}^j + \boldsymbol{B}^j\boldsymbol{K}^i)^{\mathrm{T}}\boldsymbol{P}_1 + \boldsymbol{P}_1^{\mathrm{T}}(\boldsymbol{A}^i + \boldsymbol{B}^i\boldsymbol{K}^j + \boldsymbol{A}^j + \boldsymbol{B}^j\boldsymbol{K}^i) \\ +\boldsymbol{P}_1^{\mathrm{T}}\boldsymbol{H}^i(\boldsymbol{H}^i)^{\mathrm{T}}\boldsymbol{P}_1 + (\boldsymbol{E}_1^i + \boldsymbol{E}_2^i\boldsymbol{K}^j)^{\mathrm{T}}(\boldsymbol{E}_1^i + \boldsymbol{E}_2^i\boldsymbol{K}^j) + \boldsymbol{P}_1^{\mathrm{T}}\boldsymbol{H}^j(\boldsymbol{H}^j)^{\mathrm{T}}\boldsymbol{P}_1 \\ +(\boldsymbol{E}_1^j + \boldsymbol{E}_2^j\boldsymbol{K}^i)^{\mathrm{T}}(\boldsymbol{E}_1^j + \boldsymbol{E}_2^j\boldsymbol{K}^i) + 2\boldsymbol{Q}_1 + (\boldsymbol{K}^i)^{\mathrm{T}}\boldsymbol{R}_1\boldsymbol{K}^i + (\boldsymbol{K}^j)^{\mathrm{T}}\boldsymbol{R}_1\boldsymbol{K}^j \end{array}\right) & * \\ (\boldsymbol{D}^i + \boldsymbol{D}^j)^{\mathrm{T}}\boldsymbol{P}_1 & -2\gamma^2\boldsymbol{I} \end{array}\right],$$

$$\boldsymbol{P}_1 = \left[\begin{array}{cc} \boldsymbol{P}_{1,ss} & \boldsymbol{0} \\ \boldsymbol{P}_{1,zs} & \boldsymbol{P}_{1,zz} \end{array}\right], \boldsymbol{P}_{1,zz} = \mathrm{diag}\{\boldsymbol{P}_{1,z_1\times z_1}, \boldsymbol{P}_{1,z_2\times z_2}, \ldots, \boldsymbol{P}_{1,z_H\times z_H}\},$$

then, $\exists\, \varepsilon_h^* > 0$, $\forall \varepsilon_h \in (0, \varepsilon_h^*]$, $h = 1, 2, \ldots, H$, which satisfy the stability request and the H_∞ performance index in (25).

Proof. Let

$$\boldsymbol{P}_{\boldsymbol{\varepsilon}1} = \left[\begin{array}{cc} \boldsymbol{P}_{1,ss} & \boldsymbol{P}_{1,zs}^{\mathrm{T}}\boldsymbol{\Lambda}_{\boldsymbol{\varepsilon}} \\ \boldsymbol{P}_{1,zs} & \boldsymbol{P}_{1,zz} \end{array}\right],$$

where $\boldsymbol{P}_{1,ss}$ is a positive definite symmetric matrix and $\boldsymbol{P}_{1,zz}$ is a positive definite block to angle matrix, then

$$\boldsymbol{E}_{\boldsymbol{\varepsilon}}\boldsymbol{P}_{\boldsymbol{\varepsilon}1} = \left[\begin{array}{cc} \boldsymbol{P}_{1,ss} & \boldsymbol{P}_{1,zs}^{\mathrm{T}}\boldsymbol{\Lambda}_{\boldsymbol{\varepsilon}} \\ \boldsymbol{\Lambda}_{\boldsymbol{\varepsilon}}\boldsymbol{P}_{1,zs} & \boldsymbol{P}_{1,zz}\boldsymbol{\Lambda}_{\boldsymbol{\varepsilon}} \end{array}\right] = \boldsymbol{P}_{\boldsymbol{\varepsilon}1}^{\mathrm{T}}\boldsymbol{E}_{\boldsymbol{\varepsilon}} > \boldsymbol{0}. \tag{30}$$

A Lyapunov function is chosen as

$$V(\boldsymbol{x}(t)) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{E}_{\boldsymbol{\varepsilon}}\boldsymbol{P}_{\boldsymbol{\varepsilon}1}\boldsymbol{x}(t). \tag{31}$$

From the Lemma 1, we have

$$\begin{aligned} \dot{V}(\boldsymbol{x}(t)) &= \dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{E}_{\boldsymbol{\varepsilon}}\boldsymbol{P}_{\boldsymbol{\varepsilon}1}\boldsymbol{x}(t) + \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_{\boldsymbol{\varepsilon}1}^{\mathrm{T}}\boldsymbol{E}_{\boldsymbol{\varepsilon}}\dot{\boldsymbol{x}}(t) \\ &\leqslant \sum_{i=1}^{r}\mu_i^2\{\boldsymbol{x}^{\mathrm{T}}(t)[(\boldsymbol{A}^i)^{\mathrm{T}}\boldsymbol{P}_{\boldsymbol{\varepsilon}1} + \boldsymbol{P}_{\boldsymbol{\varepsilon}1}^{\mathrm{T}}\boldsymbol{A}^i + (\boldsymbol{B}^i\boldsymbol{K}^i)^{\mathrm{T}}\boldsymbol{P}_{\boldsymbol{\varepsilon}1} + \boldsymbol{P}_{\boldsymbol{\varepsilon}1}^{\mathrm{T}}(\boldsymbol{B}^i\boldsymbol{K}^i) \\ &\quad + \boldsymbol{P}_{\boldsymbol{\varepsilon}1}^{\mathrm{T}}\boldsymbol{H}^i(\boldsymbol{H}^i)^{\mathrm{T}}\boldsymbol{P}_{\boldsymbol{\varepsilon}1} + (\boldsymbol{E}_1^i + \boldsymbol{E}_2^i\boldsymbol{K}^i)^{\mathrm{T}}(\boldsymbol{E}_1^i + \boldsymbol{E}_2^i\boldsymbol{K}^i)]\boldsymbol{x}(t) \\ &\quad + \boldsymbol{w}^{\mathrm{T}}(t)(\boldsymbol{D}^i)^{\mathrm{T}}\boldsymbol{P}_{\boldsymbol{\varepsilon}1}\boldsymbol{x}(t) + \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_{\boldsymbol{\varepsilon}1}^{\mathrm{T}}\boldsymbol{D}^i\boldsymbol{w}(t)\} \\ &\quad + \sum_{\substack{i,j=1 \\ i<j}}^{r}\mu_i\mu_j\{\boldsymbol{x}^{\mathrm{T}}(t)[(\boldsymbol{A}^i)^{\mathrm{T}}\boldsymbol{P}_{\boldsymbol{\varepsilon}1} + \boldsymbol{P}_{\boldsymbol{\varepsilon}1}^{\mathrm{T}}\boldsymbol{A}^i + (\boldsymbol{B}^i\boldsymbol{K}^j)^{\mathrm{T}}\boldsymbol{P}_{\boldsymbol{\varepsilon}1} + \boldsymbol{P}_{\boldsymbol{\varepsilon}1}^{\mathrm{T}}(\boldsymbol{B}^i\boldsymbol{K}^j) \\ &\quad + (\boldsymbol{A}^j)^{\mathrm{T}}\boldsymbol{P}_{\boldsymbol{\varepsilon}1} + \boldsymbol{P}_{\boldsymbol{\varepsilon}1}^{\mathrm{T}}\boldsymbol{A}^j + (\boldsymbol{B}^j\boldsymbol{K}^i)^{\mathrm{T}}\boldsymbol{P}_{\boldsymbol{\varepsilon}1} + \boldsymbol{P}_{\boldsymbol{\varepsilon}1}^{\mathrm{T}}(\boldsymbol{B}^j\boldsymbol{K}^i) \\ &\quad + \boldsymbol{P}_{\boldsymbol{\varepsilon}1}^{\mathrm{T}}\boldsymbol{H}^i(\boldsymbol{H}^i)^{\mathrm{T}}\boldsymbol{P}_{\boldsymbol{\varepsilon}1} + (\boldsymbol{E}_1^i + \boldsymbol{E}_2^i\boldsymbol{K}^j)^{\mathrm{T}}(\boldsymbol{E}_1^i + \boldsymbol{E}_2^i\boldsymbol{K}^j) \\ &\quad + \boldsymbol{P}_{\boldsymbol{\varepsilon}1}^{\mathrm{T}}\boldsymbol{H}^j(\boldsymbol{H}^j)^{\mathrm{T}}\boldsymbol{P}_{\boldsymbol{\varepsilon}1} + (\boldsymbol{E}_1^j + \boldsymbol{E}_2^j\boldsymbol{K}^i)^{\mathrm{T}}(\boldsymbol{E}_1^j + \boldsymbol{E}_2^j\boldsymbol{K}^i)]\boldsymbol{x}(t) \\ &\quad + \boldsymbol{w}^{\mathrm{T}}(t)(\boldsymbol{D}^i)^{\mathrm{T}}\boldsymbol{P}_{\boldsymbol{\varepsilon}1}\boldsymbol{x}(t) + \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_{\boldsymbol{\varepsilon}1}^{\mathrm{T}}\boldsymbol{D}^i\boldsymbol{w}(t) \\ &\quad + \boldsymbol{w}^{\mathrm{T}}(t)(\boldsymbol{D}^j)^{\mathrm{T}}\boldsymbol{P}_{\boldsymbol{\varepsilon}1}\boldsymbol{x}(t) + \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_{\boldsymbol{\varepsilon}1}^{\mathrm{T}}\boldsymbol{D}^j\boldsymbol{w}(t)\}, \end{aligned} \tag{32}$$

$$\begin{aligned} J_\infty &= \int_0^{t_f}[\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_1\boldsymbol{x}(t) + \boldsymbol{u}^{\mathrm{T}}(t)\boldsymbol{R}_1\boldsymbol{u}(t) - \gamma^2\boldsymbol{w}^{\mathrm{T}}(t)\boldsymbol{w}(t)]dt \\ &= \int_0^{t_f}[\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_1\boldsymbol{x}(t) + \boldsymbol{u}^{\mathrm{T}}(t)\boldsymbol{R}_1\boldsymbol{u}(t) - \gamma^2\boldsymbol{w}^{\mathrm{T}}(t)\boldsymbol{w}(t) + \dot{V}(\boldsymbol{x}(t))]dt \\ &\quad + V(\boldsymbol{x}(0)) - V(\boldsymbol{x}(t_f)). \end{aligned} \tag{33}$$

From [10], it follows that

$$J_\infty \leqslant \int_0^{t_f}\left\{\sum_{i=1}^{r}\mu_i^2\left[\begin{array}{c} \boldsymbol{x}(t) \\ \boldsymbol{w}(t) \end{array}\right]^{\mathrm{T}}\boldsymbol{\Xi}_{\boldsymbol{\varepsilon}1}^{ii}\left[\begin{array}{c} \boldsymbol{x}(t) \\ \boldsymbol{w}(t) \end{array}\right] + \sum_{\substack{i,j=1 \\ i<j}}^{r}\mu_i\mu_j\left[\begin{array}{c} \boldsymbol{x}(t) \\ \boldsymbol{w}(t) \end{array}\right]^{\mathrm{T}}\boldsymbol{\Xi}_{\boldsymbol{\varepsilon}1}^{ij}\left[\begin{array}{c} \boldsymbol{x}(t) \\ \boldsymbol{w}(t) \end{array}\right]\right\}dt + V(\boldsymbol{x}(0)), \tag{34}$$

where

$$\boldsymbol{\Xi}_{\varepsilon 1}^{ii}=\begin{bmatrix}\begin{pmatrix}(\boldsymbol{A}^i+\boldsymbol{B}^i\boldsymbol{K}^i)^{\mathrm{T}}\boldsymbol{P}_{\varepsilon 1}+\boldsymbol{P}_{\varepsilon 1}^{\mathrm{T}}(\boldsymbol{A}^i+\boldsymbol{B}^i\boldsymbol{K}^i)+\boldsymbol{P}_{\varepsilon 1}^{\mathrm{T}}\boldsymbol{H}^i(\boldsymbol{H}^i)^{\mathrm{T}}\boldsymbol{P}_{\varepsilon 1}\\ +(\boldsymbol{E}_1^i+\boldsymbol{E}_2^i\boldsymbol{K}^i)^{\mathrm{T}}(\boldsymbol{E}_1^i+\boldsymbol{E}_2^i\boldsymbol{K}^i)+\boldsymbol{Q}_1+(\boldsymbol{K}^i)^{\mathrm{T}}\boldsymbol{R}_1\boldsymbol{K}^i\end{pmatrix} & * \\ (\boldsymbol{D}^i)^{\mathrm{T}}\boldsymbol{P}_{\varepsilon 1} & -\gamma^2\boldsymbol{I}\end{bmatrix},$$

$$\boldsymbol{\Xi}_{\varepsilon 1}^{ij}=\begin{bmatrix}\begin{pmatrix}(\boldsymbol{A}^i+\boldsymbol{B}^i\boldsymbol{K}^j+\boldsymbol{A}^j+\boldsymbol{B}^j\boldsymbol{K}^i)^{\mathrm{T}}\boldsymbol{P}_{\varepsilon 1}+\boldsymbol{P}_{\varepsilon 1}^{\mathrm{T}}(\boldsymbol{A}^i+\boldsymbol{B}^i\boldsymbol{K}^j+\boldsymbol{A}^j+\boldsymbol{B}^j\boldsymbol{K}^i)\\ +\boldsymbol{P}_{\varepsilon 1}^{\mathrm{T}}\boldsymbol{H}^i(\boldsymbol{H}^i)^{\mathrm{T}}\boldsymbol{P}_{\varepsilon 1}+(\boldsymbol{E}_1^i+\boldsymbol{E}_2^i\boldsymbol{K}^j)^{\mathrm{T}}(\boldsymbol{E}_1^i+\boldsymbol{E}_2^i\boldsymbol{K}^j)+\boldsymbol{P}_{\varepsilon 1}^{\mathrm{T}}\boldsymbol{H}^j(\boldsymbol{H}^j)^{\mathrm{T}}\boldsymbol{P}_{\varepsilon 1}\\ +(\boldsymbol{E}_1^j+\boldsymbol{E}_2^j\boldsymbol{K}^i)^{\mathrm{T}}(\boldsymbol{E}_1^j+\boldsymbol{E}_2^j\boldsymbol{K}^i)+2\boldsymbol{Q}_1+(\boldsymbol{K}^i)^{\mathrm{T}}\boldsymbol{R}_1\boldsymbol{K}^i+(\boldsymbol{K}^j)^{\mathrm{T}}\boldsymbol{R}_1\boldsymbol{K}^j\end{pmatrix} & * \\ (\boldsymbol{D}^i+\boldsymbol{D}^j)^{\mathrm{T}}\boldsymbol{P}_{\varepsilon 1} & -2\gamma^2\boldsymbol{I}\end{bmatrix}.$$

Obviously $\boldsymbol{\Xi}_{\varepsilon 1}^{ii}=\boldsymbol{\Xi}_1^{ii}+\mathbf{O}(\boldsymbol{\varepsilon})$, $\boldsymbol{\Xi}_{\varepsilon 1}^{ij}=\boldsymbol{\Xi}_1^{ij}+\mathbf{O}(\boldsymbol{\varepsilon})$, therefore, $\exists\ \varepsilon_h^*>0$, $\forall\varepsilon_h\in(0,\varepsilon_h^*]$, $h=1,2,\ldots,H$, which satisfy

$$\boldsymbol{\Xi}_{\varepsilon 1}^{ii}<\mathbf{0}\quad i=1,2,\ldots,r; \tag{35}$$

$$\boldsymbol{\Xi}_{\varepsilon 1}^{ij}<\mathbf{0}\quad i,j=1,2,\ldots,r \text{ and } i<j. \tag{36}$$

Thus

$$J_\infty<V(\boldsymbol{x}(0)). \tag{37}$$

It is evident that the H_∞ performance in (25) can be satisfied from $V(\boldsymbol{x}(0))=\boldsymbol{x}_s^{\mathrm{T}}(0)\boldsymbol{P}_{1,ss}\boldsymbol{x}_s(0)+\mathbf{O}(\boldsymbol{\varepsilon})$. Finally, when $\boldsymbol{w}(t)\equiv 0$, it is easy to see that

$$\dot{V}(\boldsymbol{x}(t))=\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{E}_{\boldsymbol{\varepsilon}}\boldsymbol{P}_{\varepsilon 1}\boldsymbol{x}(t)+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_{\varepsilon 1}^{\mathrm{T}}\boldsymbol{E}_{\boldsymbol{\varepsilon}}\dot{\boldsymbol{x}}(t)<\mathbf{0}. \tag{38}$$

Therefore, the closed-loop system can also be stabilized. This completes the proof.

Note that (28) and (29) are both bilinear matrix inequalities and should be transformed to the LMI for solution in Matlab. Let $\boldsymbol{X}_1=(\boldsymbol{P}_1)^{-1}$, $\boldsymbol{M}^i=\boldsymbol{K}^i\boldsymbol{X}_1$, $\boldsymbol{M}^j=\boldsymbol{K}^j\boldsymbol{X}_1$. After left multiplying (28) and (29) with $\mathrm{diag}\{\boldsymbol{X}_1^{\mathrm{T}},\boldsymbol{I}\}$ and right multiplying them with $\mathrm{diag}\{\boldsymbol{X}_1,\boldsymbol{I}\}$, the schur complement theorem is applied. Then, the LMI condition can be derived as

$$\begin{bmatrix}\begin{pmatrix}(\boldsymbol{A}^i\boldsymbol{X}_1+\boldsymbol{B}^i\boldsymbol{M}^i)^{\mathrm{T}}+\boldsymbol{A}^i\boldsymbol{X}_1\\ +\boldsymbol{B}^i\boldsymbol{M}^i+\boldsymbol{H}^i(\boldsymbol{H}^i)^{\mathrm{T}}\end{pmatrix} & * & * & * & * \\ \boldsymbol{E}_1^i\boldsymbol{X}_1+\boldsymbol{E}_2^i\boldsymbol{M}^i & -\boldsymbol{I} & * & * & * \\ \boldsymbol{X}_1 & \mathbf{0} & -(\boldsymbol{Q}_1)^{-1} & * & * \\ \boldsymbol{M}^i & \mathbf{0} & \mathbf{0} & -(\boldsymbol{R}_1)^{-1} & * \\ (\boldsymbol{D}^i)^{\mathrm{T}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\gamma^2\boldsymbol{I}\end{bmatrix}<\mathbf{0}, \tag{39}$$

$$\begin{bmatrix}\boldsymbol{\Phi}_{111}^{ij} & * & * & * & * & * & * \\ \boldsymbol{E}_1^i\boldsymbol{X}_1+\boldsymbol{E}_2^i\boldsymbol{M}^j & -\boldsymbol{I} & * & * & * & * & * \\ \boldsymbol{E}_1^j\boldsymbol{X}_1+\boldsymbol{E}_2^j\boldsymbol{M}^i & \mathbf{0} & -\boldsymbol{I} & * & * & * & * \\ \boldsymbol{X}_1 & \mathbf{0} & \mathbf{0} & -(2\boldsymbol{Q}_3)^{-1} & * & * & * \\ \boldsymbol{M}^i & \mathbf{0} & \mathbf{0} & \mathbf{0} & -(\boldsymbol{R}_3)^{-1} & * & * \\ \boldsymbol{M}^j & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -(\boldsymbol{R}_3)^{-1} & * \\ (\boldsymbol{D}^i+\boldsymbol{D}^j)^{\mathrm{T}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -2\gamma^2\boldsymbol{I}\end{bmatrix}<\mathbf{0}, \tag{40}$$

where

$$\begin{aligned}\boldsymbol{\Phi}_{111}^{ij}=&(\boldsymbol{A}^i\boldsymbol{X}_1+\boldsymbol{B}^i\boldsymbol{M}^j+\boldsymbol{A}^j\boldsymbol{X}_1+\boldsymbol{B}^j\boldsymbol{M}^i)^{\mathrm{T}}+\boldsymbol{A}^i\boldsymbol{X}_1+\boldsymbol{B}^i\boldsymbol{M}^j\\&+\boldsymbol{A}^j\boldsymbol{X}_1+\boldsymbol{B}^j\boldsymbol{M}^i+\boldsymbol{H}^i(\boldsymbol{H}^i)^{\mathrm{T}}+\boldsymbol{H}^j(\boldsymbol{H}^j)^{\mathrm{T}}.\end{aligned}$$

We recall that Theorem 1 addresses the H_∞ performance index with an LMI condition. Next, we show that a similar conclusion can also be derived for the H_2 performance index case.

Theorem 2. when $\boldsymbol{w}(t) \equiv 0$, if there exist common matrices $\boldsymbol{P}_{2,ss}^{\mathrm{T}} = \boldsymbol{P}_{2,ss} > \boldsymbol{0}$, $\boldsymbol{P}_{2,zz}^{\mathrm{T}} = \boldsymbol{P}_{2,zz} > \boldsymbol{0}$ and $\boldsymbol{P}_{2,zs}$ which satisfy:

$$\boldsymbol{\Xi}_2^{ii} < \boldsymbol{0} \quad i = 1, 2, \ldots, r, \tag{41}$$

$$\boldsymbol{\Xi}_2^{ij} < \boldsymbol{0} \quad i, j = 1, 2, \ldots, r \ \text{ and } \ i < j, \tag{42}$$

where

$$\begin{aligned}
\boldsymbol{\Xi}_2^{ii} &= (\boldsymbol{A}^i + \boldsymbol{B}^i\boldsymbol{K}^i)^{\mathrm{T}}\boldsymbol{P}_2 + \boldsymbol{P}_2^{\mathrm{T}}(\boldsymbol{A}^i + \boldsymbol{B}^i\boldsymbol{K}^i) + \boldsymbol{P}_2^{\mathrm{T}}\boldsymbol{H}^i(\boldsymbol{H}^i)^{\mathrm{T}}\boldsymbol{P}_2 + (\boldsymbol{E}_1^i + \boldsymbol{E}_2^i\boldsymbol{K}^i)^{\mathrm{T}}(\boldsymbol{E}_1^i + \boldsymbol{E}_2^i\boldsymbol{K}^i) \\
&\quad + \boldsymbol{Q}_2 + (\boldsymbol{K}^i)^{\mathrm{T}}\boldsymbol{R}_2\boldsymbol{K}^i, \\
\boldsymbol{\Xi}_2^{ij} &= (\boldsymbol{A}^i + \boldsymbol{B}^i\boldsymbol{K}^j + \boldsymbol{A}^j + \boldsymbol{B}^j\boldsymbol{K}^i)^{\mathrm{T}}\boldsymbol{P}_2 + \boldsymbol{P}_2^{\mathrm{T}}(\boldsymbol{A}^i + \boldsymbol{B}^i\boldsymbol{K}^j + \boldsymbol{A}^j + \boldsymbol{B}^j\boldsymbol{K}^i) + \boldsymbol{P}_2^{\mathrm{T}}\boldsymbol{H}^i(\boldsymbol{H}^i)^{\mathrm{T}}\boldsymbol{P}_2 \\
&\quad + (\boldsymbol{E}_1^i + \boldsymbol{E}_2^i\boldsymbol{K}^j)^{\mathrm{T}}(\boldsymbol{E}_1^i + \boldsymbol{E}_2^i\boldsymbol{K}^j) + \boldsymbol{P}_2^{\mathrm{T}}\boldsymbol{H}^j(\boldsymbol{H}^j)^{\mathrm{T}}\boldsymbol{P}_2 + (\boldsymbol{E}_1^j + \boldsymbol{E}_2^j\boldsymbol{K}^i)^{\mathrm{T}}(\boldsymbol{E}_1^j + \boldsymbol{E}_2^j\boldsymbol{K}^i) \\
&\quad + 2\boldsymbol{Q}_2 + (\boldsymbol{K}^i)^{\mathrm{T}}\boldsymbol{R}_2\boldsymbol{K}^i + (\boldsymbol{K}^j)^{\mathrm{T}}\boldsymbol{R}_2\boldsymbol{K}^j,
\end{aligned}$$

$$\boldsymbol{P}_2 = \begin{bmatrix} \boldsymbol{P}_{2,ss} & \boldsymbol{0} \\ \boldsymbol{P}_{2,zs} & \boldsymbol{P}_{2,zz} \end{bmatrix}, \boldsymbol{P}_{2,zz} = \mathrm{diag}\{\boldsymbol{P}_{2,z_1\times z_1}, \boldsymbol{P}_{2,z_2\times z_2}, \ldots, \boldsymbol{P}_{2,z_H\times z_H}\}.$$

Then $\exists\varepsilon_h^* > 0$, $\forall\varepsilon_h \in (0, \varepsilon_h^*]$, $h = 1, 2, \ldots, H$ which satisfy the H_2 performance index in (26).

Proof. Let

$$\boldsymbol{P}_{\boldsymbol{\varepsilon}2} = \begin{bmatrix} \boldsymbol{P}_{2,ss} & \boldsymbol{P}_{2,zs}^{\mathrm{T}}\boldsymbol{\Lambda}_{\boldsymbol{\varepsilon}} \\ \boldsymbol{P}_{2,zs} & \boldsymbol{P}_{2,zz} \end{bmatrix},$$

where $\boldsymbol{P}_{2,ss}$ is a positive definite symmetric matrix and $\boldsymbol{P}_{2,zz}$ is a positive definite block to angle matrix, then

$$\boldsymbol{E}_{\boldsymbol{\varepsilon}}\boldsymbol{P}_{\boldsymbol{\varepsilon}2} = \begin{bmatrix} \boldsymbol{P}_{2,ss} & \boldsymbol{P}_{2,zs}^{\mathrm{T}}\boldsymbol{\Lambda}_{\boldsymbol{\varepsilon}} \\ \boldsymbol{\Lambda}_{\boldsymbol{\varepsilon}}\boldsymbol{P}_{2,zs} & \boldsymbol{P}_{2,zz}\boldsymbol{\Lambda}_{\boldsymbol{\varepsilon}} \end{bmatrix} = \boldsymbol{P}_{\boldsymbol{\varepsilon}2}^{\mathrm{T}}\boldsymbol{E}_{\boldsymbol{\varepsilon}} > \boldsymbol{0}. \tag{43}$$

Similar to the proof of Theorem 1, we choose the Lyapunov function as

$$V(\boldsymbol{x}(t)) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{E}_{\boldsymbol{\varepsilon}}\boldsymbol{P}_{\boldsymbol{\varepsilon}2}\boldsymbol{x}(t). \tag{44}$$

From Lemma 1, we have

$$\begin{aligned}
\dot{V}(\boldsymbol{x}(t)) &= \dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{E}_{\boldsymbol{\varepsilon}}\boldsymbol{P}_{\boldsymbol{\varepsilon}2}\boldsymbol{x}(t) + \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_{\boldsymbol{\varepsilon}2}^{\mathrm{T}}\boldsymbol{E}_{\boldsymbol{\varepsilon}}\dot{\boldsymbol{x}}(t) \\
&\leqslant \sum_{i=1}^{r}\mu_i^2\boldsymbol{x}^{\mathrm{T}}(t)[(\boldsymbol{A}^i)^{\mathrm{T}}\boldsymbol{P}_{\boldsymbol{\varepsilon}2} + \boldsymbol{P}_{\boldsymbol{\varepsilon}2}^{\mathrm{T}}\boldsymbol{A}^i + (\boldsymbol{B}^i\boldsymbol{K}^i)^{\mathrm{T}}\boldsymbol{P}_{\boldsymbol{\varepsilon}2} + \boldsymbol{P}_{\boldsymbol{\varepsilon}2}^{\mathrm{T}}(\boldsymbol{B}^i\boldsymbol{K}^i) \\
&\quad + \boldsymbol{P}_{\boldsymbol{\varepsilon}2}^{\mathrm{T}}\boldsymbol{H}^i(\boldsymbol{H}^i)^{T}\boldsymbol{P}_{\boldsymbol{\varepsilon}2} + (\boldsymbol{E}_1^i + \boldsymbol{E}_2^i\boldsymbol{K}^i)^{\mathrm{T}}(\boldsymbol{E}_1^i + \boldsymbol{E}_2^i\boldsymbol{K}^i)]\boldsymbol{x}(t) \\
&\quad + \sum_{\substack{i,j=1 \\ i<j}}^{r}\mu_i\mu_j\boldsymbol{x}^{\mathrm{T}}(t)[(\boldsymbol{A}^i)^{\mathrm{T}}\boldsymbol{P}_{\boldsymbol{\varepsilon}2} + \boldsymbol{P}_{\boldsymbol{\varepsilon}2}^{\mathrm{T}}\boldsymbol{A}^i + (\boldsymbol{B}^i\boldsymbol{K}^j)^{\mathrm{T}}\boldsymbol{P}_{\boldsymbol{\varepsilon}2} + \boldsymbol{P}_{\boldsymbol{\varepsilon}2}^{\mathrm{T}}(\boldsymbol{B}^i\boldsymbol{K}^j) \qquad (45) \\
&\quad + (\boldsymbol{A}^j)^{\mathrm{T}}\boldsymbol{P}_{\boldsymbol{\varepsilon}2} + \boldsymbol{P}_{\boldsymbol{\varepsilon}2}^{\mathrm{T}}\boldsymbol{A}^j + (\boldsymbol{B}^j\boldsymbol{K}^i)^{\mathrm{T}}\boldsymbol{P}_{\boldsymbol{\varepsilon}2} + \boldsymbol{P}_{\boldsymbol{\varepsilon}2}^{\mathrm{T}}(\boldsymbol{B}^j\boldsymbol{K}^i) \\
&\quad + \boldsymbol{P}_{\boldsymbol{\varepsilon}2}^{\mathrm{T}}\boldsymbol{H}^i(\boldsymbol{H}^i)^{\mathrm{T}}\boldsymbol{P}_{\boldsymbol{\varepsilon}2} + (\boldsymbol{E}_1^i + \boldsymbol{E}_2^i\boldsymbol{K}^j)^{\mathrm{T}}(\boldsymbol{E}_1^i + \boldsymbol{E}_2^i\boldsymbol{K}^j) \\
&\quad + \boldsymbol{P}_{\boldsymbol{\varepsilon}2}^{\mathrm{T}}\boldsymbol{H}^j(\boldsymbol{H}^j)^{\mathrm{T}}\boldsymbol{P}_{\boldsymbol{\varepsilon}2} + (\boldsymbol{E}_1^j + \boldsymbol{E}_2^j\boldsymbol{K}^i)^{\mathrm{T}}(\boldsymbol{E}_1^j + \boldsymbol{E}_2^j\boldsymbol{K}^i)]\boldsymbol{x}(t),
\end{aligned}$$

$$\begin{aligned}
J_Q &= \int_0^{t_f}[\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_2\boldsymbol{x}(t) + \boldsymbol{u}^{\mathrm{T}}(t)\boldsymbol{R}_2\boldsymbol{u}(t)]dt \\
&= \int_0^{t_f}[\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_2\boldsymbol{x}(t) + \boldsymbol{u}^{\mathrm{T}}(t)\boldsymbol{R}_2\boldsymbol{u}(t) + \dot{V}(\boldsymbol{x}(t))]dt + V(\boldsymbol{x}(0)) - V(\boldsymbol{x}(t_f)). \qquad (46)
\end{aligned}$$

From [10], it follows

$$J_Q \leqslant \int_0^{t_f} \left\{ \sum_{i=1}^{r} \mu_i^2 \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{\Xi}_{\varepsilon 2}^{ii}\boldsymbol{x}(t) + \sum_{\substack{i,j=1\\ i<j}}^{r} \mu_i\mu_j \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{\Xi}_{\varepsilon 2}^{ij}\boldsymbol{x}(t) \right\} dt + V(\boldsymbol{x}(0)), \tag{47}$$

where

$$\begin{aligned}\boldsymbol{\Xi}_{\varepsilon 2}^{ii} = {} & (\boldsymbol{A}^i + \boldsymbol{B}^i\boldsymbol{K}^i)^{\mathrm{T}}\boldsymbol{P}_{\varepsilon 2} + \boldsymbol{P}_{\varepsilon 2}^{\mathrm{T}}(\boldsymbol{A}^i + \boldsymbol{B}^i\boldsymbol{K}^i) + \boldsymbol{P}_{\varepsilon 2}^{\mathrm{T}}\boldsymbol{H}^i(\boldsymbol{H}^i)^{\mathrm{T}}\boldsymbol{P}_{\varepsilon 2} \\ & + (\boldsymbol{E}_1^i + \boldsymbol{E}_2^i\boldsymbol{K}^i)^{\mathrm{T}}(\boldsymbol{E}_1^i + \boldsymbol{E}_2^i\boldsymbol{K}^i) + \boldsymbol{Q}_2 + (\boldsymbol{K}^i)^{\mathrm{T}}\boldsymbol{R}_2\boldsymbol{K}^i,\end{aligned} \tag{48}$$

$$\begin{aligned}\boldsymbol{\Xi}_{\varepsilon 2}^{ij} = {} & (\boldsymbol{A}^i + \boldsymbol{B}^i\boldsymbol{K}^j + \boldsymbol{A}^j + \boldsymbol{B}^j\boldsymbol{K}^i)^{\mathrm{T}}\boldsymbol{P}_{\varepsilon 2} + \boldsymbol{P}_{\varepsilon 2}^{\mathrm{T}}(\boldsymbol{A}^i + \boldsymbol{B}^i\boldsymbol{K}^j + \boldsymbol{A}^j + \boldsymbol{B}^j\boldsymbol{K}^i) + \boldsymbol{P}_{\varepsilon 2}^{\mathrm{T}}\boldsymbol{H}^i(\boldsymbol{H}^i)^{\mathrm{T}}\boldsymbol{P}_{\varepsilon 2} \\ & + (\boldsymbol{E}_1^i + \boldsymbol{E}_2^i\boldsymbol{K}^j)^{\mathrm{T}}(\boldsymbol{E}_1^i + \boldsymbol{E}_2^i\boldsymbol{K}^j) + \boldsymbol{P}_{\varepsilon 2}^{\mathrm{T}}\boldsymbol{H}^j(\boldsymbol{H}^j)^{\mathrm{T}}\boldsymbol{P}_{\varepsilon 2} + (\boldsymbol{E}_1^j + \boldsymbol{E}_2^j\boldsymbol{K}^i)^{\mathrm{T}}(\boldsymbol{E}_1^j + \boldsymbol{E}_2^j\boldsymbol{K}^i) \\ & + 2\boldsymbol{Q}_2 + (\boldsymbol{K}^i)^{\mathrm{T}}\boldsymbol{R}_2\boldsymbol{K}^i + (\boldsymbol{K}^j)^{\mathrm{T}}\boldsymbol{R}_2\boldsymbol{K}^j,\end{aligned} \tag{49}$$

obviously $\boldsymbol{\Xi}_{\varepsilon 2}^{ii} = \boldsymbol{\Xi}_2^{ii} + \mathbf{O}(\varepsilon)$, $\boldsymbol{\Xi}_{\varepsilon 2}^{ij} = \boldsymbol{\Xi}_2^{ij} + \mathbf{O}(\varepsilon)$, so $\exists \varepsilon_h^* > 0$, $\forall \varepsilon_h \in (0, \varepsilon_h^*]$, $h = 1, 2, \ldots, H$ which satisfy

$$\boldsymbol{\Xi}_{\varepsilon 2}^{ii} < \mathbf{0} \ \ i = 1, 2, \ldots, r; \tag{50}$$

$$\boldsymbol{\Xi}_{\varepsilon 2}^{ij} < \mathbf{0} \ \ i, j = 1, 2, \ldots, r \text{ and } \ i < j. \tag{51}$$

Thus

$$J_Q < V(\boldsymbol{x}(0)). \tag{52}$$

It can be seen that the H_2 performance index in (26) can be satisfied from $V(\boldsymbol{x}(0)) = \boldsymbol{x}_s^{\mathrm{T}}(0)\boldsymbol{P}_{2,ss}\boldsymbol{x}_s(0) + \mathbf{O}(\varepsilon)$. Eq. (41) and (42) are both bilinear matrix inequalities. Let $\boldsymbol{X}_2 = (\boldsymbol{P}_2)^{-1}$, $\boldsymbol{M}^i = \boldsymbol{K}^i\boldsymbol{X}_2$ and $\boldsymbol{M}^j = \boldsymbol{K}^j\boldsymbol{X}_2$, along the same lines with Theorem 1, the LMI condition for Theorem 2 can be derived as

$$\begin{bmatrix} \begin{pmatrix} (\boldsymbol{A}^i\boldsymbol{X}_2 + \boldsymbol{B}^i\boldsymbol{M}^i)^{\mathrm{T}} + \boldsymbol{A}^i\boldsymbol{X}_2 \\ +\boldsymbol{B}^i\boldsymbol{M}^i + \boldsymbol{H}^i(\boldsymbol{H}^i)^{\mathrm{T}} \end{pmatrix} & * & * & * \\ \boldsymbol{E}_1^i\boldsymbol{X}_2 + \boldsymbol{E}_2^i\boldsymbol{M}^i & -\boldsymbol{I} & * & * \\ \boldsymbol{X}_2 & \mathbf{0} & -(\boldsymbol{Q}_2)^{-1} & * \\ \boldsymbol{M}^i & \mathbf{0} & \mathbf{0} & -(\boldsymbol{R}_2)^{-1} \end{bmatrix} < \mathbf{0}, \tag{53}$$

$$\begin{bmatrix} \boldsymbol{\Phi}_{211}^{ij} & * & * & * & * & * \\ \boldsymbol{E}_1^i\boldsymbol{X}_2 + \boldsymbol{E}_2^i\boldsymbol{M}^j & -\boldsymbol{I} & * & * & * & * \\ \boldsymbol{E}_1^j\boldsymbol{X}_2 + \boldsymbol{E}_2^j\boldsymbol{M}^i & \mathbf{0} & -\boldsymbol{I} & * & * & * \\ \boldsymbol{X}_2 & \mathbf{0} & \mathbf{0} & -(2\boldsymbol{Q}_2)^{-1} & * & * \\ \boldsymbol{M}^i & \mathbf{0} & \mathbf{0} & \mathbf{0} & -(\boldsymbol{R}_2)^{-1} & * \\ \boldsymbol{M}^j & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -(\boldsymbol{R}_2)^{-1} \end{bmatrix} < \mathbf{0}, \tag{54}$$

where

$$\begin{aligned}\boldsymbol{\Phi}_{211}^{ij} = {} & (\boldsymbol{A}^i\boldsymbol{X}_2 + \boldsymbol{B}^i\boldsymbol{M}^j + \boldsymbol{A}^j\boldsymbol{X}_2 + \boldsymbol{B}^j\boldsymbol{M}^i)^{\mathrm{T}} + \boldsymbol{A}^i\boldsymbol{X}_2 + \boldsymbol{B}^i\boldsymbol{M}^j + \boldsymbol{A}^j\boldsymbol{X}_2 + \boldsymbol{B}^j\boldsymbol{M}^i + \boldsymbol{H}^i(\boldsymbol{H}^i)^{\mathrm{T}} \\ & + \boldsymbol{H}^j(\boldsymbol{H}^j)^{\mathrm{T}}.\end{aligned}$$

We now have the LMI conditions for H_∞ and H_2 performance indexes. However, the two performance indexes are usually contradictory since it is difficult to guarantee robustness and dynamic performance at the same time. Thus for overall consideration, a trade-off strategy should be employed. This is in essence a multi-objective optimization problem. Next, we show that this problem can be tackled by the estimation of distribution algorithms (EDA). To this end, let us denote $J_1 = \gamma^2$, $J_2 = \boldsymbol{x}_s^T(0)\boldsymbol{P}_{2,ss}\boldsymbol{x}_s(0)$, and the optimization model can be established as (54), where $[J_1^* \quad J_2^*]$ corresponds to the optimization value of $[J_1 \quad J_2]$.

$$\min \sqrt{\left(\frac{J_1(\gamma, \mathbf{Q}_2, \mathbf{R}_2) - J_1^*}{J_1^*}\right)^2 + \left(\frac{J_2(\gamma, \mathbf{Q}_2, \mathbf{R}_2) - J_2^*}{J_2^*}\right)^2}.$$

$$\text{s.t.}\mathbf{X} = \mathbf{X}_1 = \mathbf{X}_2 > \mathbf{0}, \quad \begin{cases} \text{inequality} \ \ (40), \\ \text{inequality} \ \ (41), \\ \text{inequality} \ \ (52), \\ \text{inequality} \ \ (53), \end{cases} \tag{55}$$

where γ, $\boldsymbol{Q}_2$ and $\boldsymbol{R}_2$ are decision variables. When the algorithm is convergent, γ_{opt}, $\boldsymbol{Q}_{2\text{opt}}$ and $\boldsymbol{R}_{2\text{opt}}$ can be obtained.

4 Numerical simulation

In this section, a numerical simulation is conducted to demonstrate the effectiveness of our proposed algorithm. By small disturbance linearization at a certain trim point, we can get the linear state-space equation shown as follows:

$$\begin{bmatrix} \dot{V} \\ \dot{\alpha} \\ \dot{\theta} \\ \dot{q} \\ \dot{\beta} \\ \ddot{\beta} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} \frac{T_V - D_V}{m} & \frac{\mu}{(h_0+R_E)^2} - \frac{D_\alpha}{m} & -\frac{\mu}{(h_0+R_E)^2} & 0 & \frac{T_\beta}{m} & 0 & 0 \\ -\frac{L_V}{mV_0} + \frac{2}{h_0+R_E} & -\frac{L_\alpha}{mV_0} & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{M_V}{I_{yy}} & \frac{M_\alpha}{I_{yy}} & 0 & \frac{M_q}{I_{yy}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -V_0 & V_0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V \\ \alpha \\ \theta \\ q \\ \beta \\ \dot{\beta} \\ h \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{M_{\delta_e}}{I_{yy}} \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \beta_c \\ \delta_e \end{bmatrix}, \tag{56}$$

where $T_V = (\frac{\partial T}{\partial V})_0$, $T_\beta = (\frac{\partial T}{\partial \beta})_0$, $L_\alpha = (\frac{\partial L}{\partial \alpha})_0$, $L_V = (\frac{\partial L}{\partial V})_0$, $D_V = (\frac{\partial D}{\partial V})_0$, $D_\alpha = (\frac{\partial D}{\partial \alpha})_0$, $M_V = (\frac{\partial M_{yy}}{\partial V})_0$, $M_\alpha = (\frac{\partial M_{yy}}{\partial \alpha})_0$, $M_{\delta_e} = (\frac{\partial M_{yy}}{\partial \delta_e})_0$, $M_q = (\frac{\partial M_{yy}}{\partial q})_0$.

We denote (56) as $\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t)$, where $\boldsymbol{x}(t) = [V, \alpha, \theta, q, \beta, \dot{\beta}, h]^T$ and $\boldsymbol{u}(t) = [\beta_c, \delta_e]^T$. More practically, if the uncertainty of the aerodynamic parameter is considered, the state space equation becomes

$$\dot{\boldsymbol{x}}(t) = [\boldsymbol{A}(t_0) + \Delta\boldsymbol{A}(\alpha(t), V(t), q(t), \beta(t), \delta_e)]\boldsymbol{x}(t) + (\boldsymbol{B}(t_0) + \Delta\boldsymbol{B}(\alpha(t), V(t), q(t), \beta(t), \delta_e))\boldsymbol{u}(t). \tag{57}$$

We choose velocity and height to be antecedent variables for the TS model and a set of state space equations can be obtained by changing the initial trim point. For example, the following are obtained at

V_0 =15060 ft/s, $h_0 = 110000$ ft, $\alpha_0 = -\ 0.0312$ rad, $q_0 = 0$ rad/s, δ_{e0} =0.0089 rad, β_0 =0.1657:

$$\boldsymbol{A}(t_0) = \begin{bmatrix} 2.9321\text{e}-007 & 69.6046 & -31.4788 & 0 & 27.3094 & 0 & 0 \\ 2.7634\text{e}-007 & -0.0437 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1.2493 & 1.0909 & 0 & -0.0739 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -15060 & 15060 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{B}(t_0) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 3.3167 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}.$$

The aerodynamic parameter may change tremendously due to the high velocity of the aircraft. Thus, it is more practical to introduce some uncertainty terms to the state variable to describe it. Let $|\Delta V| \leqslant 50$ ft/s, $|\Delta\alpha| \leqslant 0.5$ rad, $|\Delta q| \leqslant 2$ rad/s, $|\Delta\delta_e| \leqslant 1$ rad, $\Delta\beta \in [0, 2]$. The uncertainty matrices $|\Delta\boldsymbol{A}|_{\max}$ and $|\Delta\boldsymbol{B}|_{\max}$ are

$$|\Delta\boldsymbol{A}|_{\max} = \begin{bmatrix} 0.0253 & 688.5955 & 0 & 0 & 23.7709 & 0 & 0 \\ 2.9097\times10^{-6} & 2.9043\times10^{-4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.2493 & 8.5627 & 0 & 0.6243 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$|\Delta\boldsymbol{B}|_{\max} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0.0221 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Clearly, the magnitudes of some elements in the uncertainty matrix are much larger than others. This phenomenon makes it difficult to stabilize the system via the robust control method, since it will also induce the same problem to matrix $\boldsymbol{E}$ in Lemma 1. Therefore, we propose a procedure to tackle this problem as follows:

Step 1. Choose the matrix $\boldsymbol{H}$ in Lemma 1 properly to balance matrix $\boldsymbol{E}$. This is accomplished by multiplying the elements in $\boldsymbol{E}$ which correspond to V' with an amplification factor. Then, the uncertainty

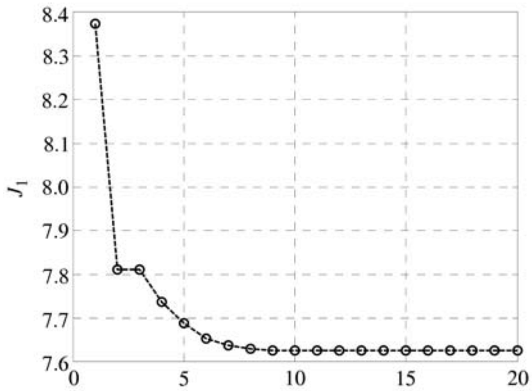

Figure 2 Optimization process of J_1.

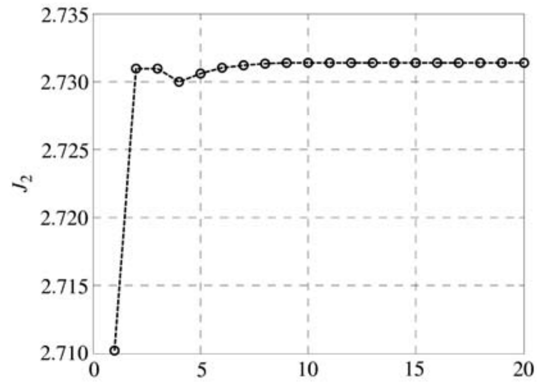

Figure 3 Optimization process of J_2.

matrices are transformed into

$$|\Delta \boldsymbol{A}|_{\max} = \begin{bmatrix} 0.0253 & 68 & 0 & 0 & 23.7709 & 0 & 0 \\ 2.9097\times 10^{-6} & 2.9043\times 10^{-4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.2493 & 8.5627 & 0 & 0.6243 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$|\Delta \boldsymbol{B}|_{\max} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0.0221 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Step 2. Through careful observation of $|\Delta \boldsymbol{A}|_{\max}$and $|\Delta \boldsymbol{B}|_{\max}$ above, we conclude that the impact of $\Delta\alpha$ on $\Delta\dot{V}$ is still much larger than other elements. Therefore, it can be viewed as the disturbance term $\boldsymbol{w}(t)$.

Now, the final state space equation is the same as (20), and the EDA algorithm can be employed to optimize the performance index. Figures 2 and 3 show the optimization process of J_1 and J_2 with $\gamma_{\rm opt} = 2.77$, $\boldsymbol{Q}_{2\rm opt} = 0.0012\boldsymbol{I}_{7\times 7}$, $\boldsymbol{R}_{2\rm opt} = 0.0012\boldsymbol{I}_{7\times 7}$. Figure 4 shows the flight path angle command and its actual trajectory, which are satisfied for the altitude tracking performance requirements, see Figure 5. From Figure 6, we can see that the designed controller can maintain the velocity at 15060 ft/s successfully. Note that velocity changing in a small range may not influence the altitude tracking performance. The control inputs of the throttle setting and elevator deflection, shown in Figures 7 and 8 remain within their feasible ranges.

5 Conclusions

A multi-objective robust controller for an air-breathing hypersonic vehicle is developed. FSPM is used to approximate the nonlinear longitudinal model of an aircraft with different flight conditions. This technique tackles the problem of “multi-time-scale” caused by different types of motion of the hypersonic vehicle. The proof shows that the control law can guarantee the stability of the closed-loop system while

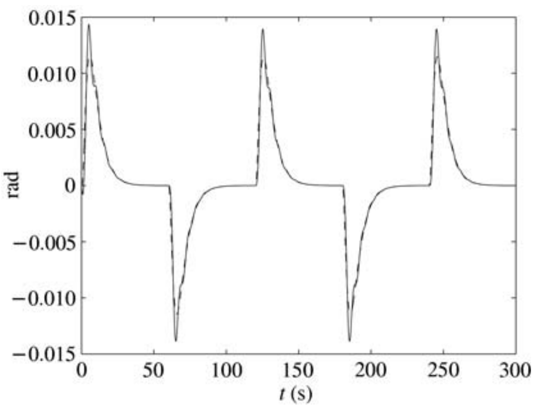

Figure 4 Flight path angle command (dashed line) and its actual trajectory (solid line).

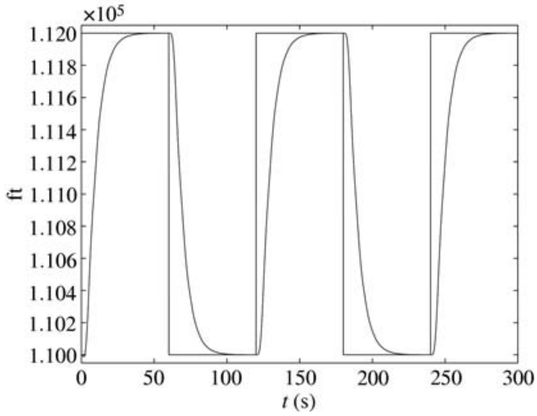

Figure 5 Altitude tracking command (square wave) and its actual trajectory (curve).

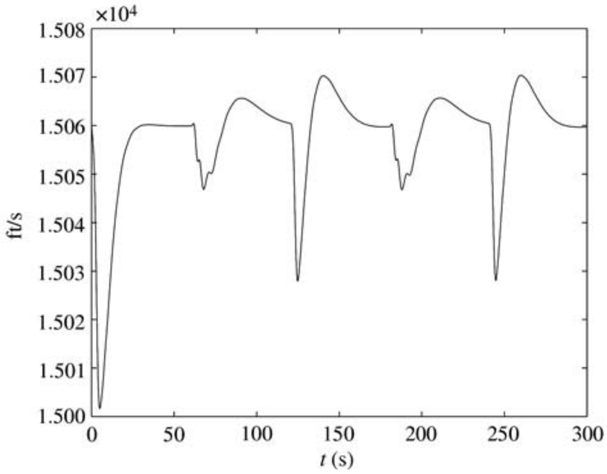

Figure 6 Velocity maintained at 15060 ft/s.

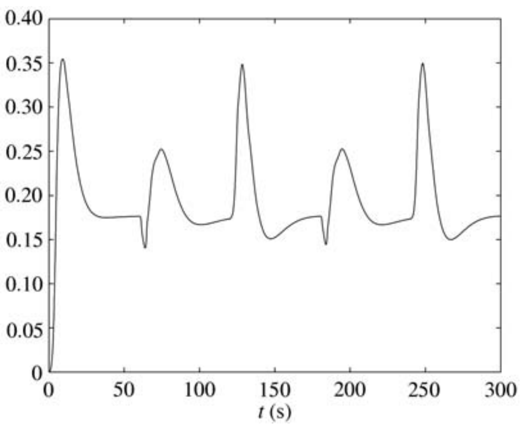

Figure 7 Throttle setting.

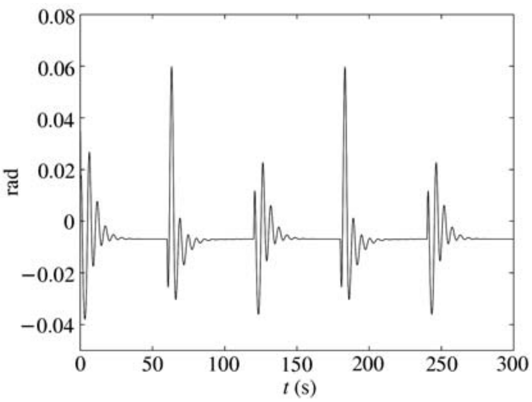

Figure 8 Elevator deflection.

satisfying the H_2 and H_∞ performance indexes at the same time. The simulation results illustrate that good tracking performance can be obtained using the proposed approach.

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