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RESEARCH PAPER

Multi-objective design method based on evolution game and its application for suspension

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Abstract Through research and bionics of biology survival mode, game players with competition, cooperation and self-adaptation capacity are introduced in the multi-objective design. The dynamic behavior and bounded rationality in game processes for players are considered according to Chinese saying "In success, commit oneself to the welfare of the society; in distress, maintain one's own integrity". An evolution rule, Poor-Competition-Rich-Cooperation (short for PCRC), is proposed. Then, the corresponding payoff functions of competition and cooperation behavior are established and a multi-objective design method based on evolution game is proposed. The calculation steps are as follows: 1) Taking the design objectives as different game players, and calculating factors of the design variables to objective and fuzzy clustering. The design variables are divided into multiple strategy subsets owned by each game player. 2) According to the evolution rule, each player determines its behavior and payoff function in this game round. 3) In their own strategy subsets, each game player takes their payoff as mono-objective for optimization. It gives the best strategy upon other players. And so the best strategies of all players conform the group strategy in this round. The final equilibrium solution is obtained through multi-round game based on convergence criterion. The validity and reliability of this method are shown by the results of an example of a tri-objective optimization design of passive suspension parameters.

Keywords Multi-objective optimization · Game · Evolution rule · Passive suspension

1 Introduction

Many structural design problems have multi-objective optimization issues. The essential characteristics of multi-objective optimization are: 1) there exist several objective interests; 2) The status of the various objectives are different and have conflicts. Therefore, how to balance the interests of the objectives is the key to solve the multi-objective problem. The predominant methods include the method of reduction of dimension (main objective method), the evaluation function methods (linear weighting method, min–max method, ideal point method, weight-square method and virtual objective method), all of which convert the multi-objective optimization problem into a single-objective optimization problem. In addition to the above methods, other frequently used methods are the sorting method, feasible direction method, the center method and the interactive programming method, which are used to transform multi-objective optimization problem into multiple single-objective optimization problems. All these methods belong to traditional areas of mathematical programming theory. In recent years, considering the similarity between multi-objective design and games, a multi-objective game method has been used to solve multi-objective design problems (Périaux et al. 2001; Xie et al. 2005).

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A typical minimum multi-objective optimal design can be described as follows:

$$\begin{cases} \textit{the design variables}: X = (x_1, x_2, \cdots, x_n) \in \Omega^{\mathrm{n}} \\ \textit{let the objective functions be minimized}: \\ \quad F(X) = (F_1(X), F_2(X), \cdots, F_m(X)) \rightarrow \min \\ \textit{subject to constraint conditions}: \\ \quad g_k(X) \leq 0 \, (k = 1, 2, \cdots, q) \end{cases} \tag{1}$$

Where: n is the number of design variables. m is the number of objective functions. q is the number of constraint conditions. Ω^n is the feasible space of design variables.

Meanwhile, definition of game: G represents one game. If G has m players (Illustration: the implication of number of players is equal to the number of objective functions), the sets of available strategies are denoted by $\mathbf{S}_1, \cdots, \mathbf{S}_m$, then S_i is the strategy subset of player i. The payoff functions are $u_1, \cdots, u_m$ and u_i is the payoff function of player i. Hence, the game with m players can be written as $G = (S_1, \cdots, S_m;\ u_1, \cdots, u_m)$.

When we use the game method to solve multi-objective optimization problems, the key technique is to establish the mapping relationships between factors of the multi-objective optimization model and factors of the game model (where, factors of multi-objective optimization model include optimization objectives and objective functions, design variables and constraints; factors of the game model include game players and payoff functions, strategy subsets and game constraints). We put forward the following three mapping relationships: 1) Bionics mapping. m design objectives are regarded as m game players with the intelligent behaviors such as competition, cooperation and adaptive behavior. 2) Payoff mapping. The mapping relationship between payoff functions u and objective functions F need to be constructed based on the different behaviors. 3) Set mapping. Through the specific technological means, the design variables $\mathbf{X} = [x_1, x_2, \cdots, x_n]$ can be divided into each game players strategy subsets: $S_1, S_2, \cdots, S_m$. Where, $S_1 = \{x_i \ \cdots \ x_j\}, \cdots, S_m = \{x_k \ \cdots \ x_l\}$ are strategy subset of m players and satisfy $S_1 \cup \cdots \cup S_m = X;\ S_a \cap S_b = 0 \, (a, b = 1, \cdots, m;\ a \neq b)$. Besides, the constraints in multi-objective problems can be regarded as constraints in the game method.

The differences between the traditional methods and the game methods are as follows: 1) The traditional methods generally get solutions by merging multiple objective functions but the game methods get solutions by splitting design variables. The design variables are divided into multiple strategy subsets owned by each game player, which can reveal the correlations between each optimization goal and the corresponding design variables. Besides, the original high-dimensional optimization problem is transformed into multiple low-dimensional optimization problems, which can reduce the complexity of problem. 2) The traditional multi-objective optimization methods give consideration to multiple objectives mainly through constructing evaluation functions but the game methods give attention to multiple objectives mainly through bionic mapping. For some complex engineering optimization problems, designers and engineering experts may have not much experience for how to construct the evaluation functions. But they can get solutions and deal with the target status by assigning the appropriate behaviors to game players and constructing game patterns between objectives based on the game methods.

Currently, researches on solving multi-objective optimization problems by game theory are as follows:

1. The most critical step for multi-objective game method is dividing the design variables set into strategy subsets of each player. A reasonable decomposition of strategy subsets is critical for the computational efficiency, accuracy and convergence of the multi-objective game method (Chen and Li 2002). Unfortunately, in most references, because of specialty in engineering projects, there is an obvious physical association and affiliation between the design variables and each objective so that this step has been done empirically by researchers' experience. But they don't realize that this step is the key technology for multi-objective game approach to be universal. At present, computational methods about how to decompose the design variables set into strategy subsets for each player mainly contain an adaptive method proposed by Clarich et al. (2004), which has the characteristic that the strategy subset owned by each player is dynamic during the whole game process. Furthermore, a correlation analysis method (Xie et al. 2010) and a fuzzy clustering method (Lu et al. 2010) are proposed by Neng-gang Xie et al. and a sensitivity analysis method by Hu and Rao (2009).
2. About the behaviors, the main behaviors have competition and cooperation. The typical cooperative behaviors have three types, which are known as the "benefit oneself but do not harm people", "you win to have me, I win to have you", "all for one and one for all" (Chen and Li 2002; Zhili and Jun 2009; Xie et al. 2007a, b; Dhingra and Rao 1995; Maali 2009; Chen et al. 2009). Game patterns between objectives mainly have pure competitive pattern (Clarich et al. 2012; Özyildirim and Alemdar 2000), and pure cooperative pattern (Spallino and Rizzo 2002; Sim et al. 2004; Wang et al. 2003). Pure competitive pattern is defined as that each game player benefits from competitive behavior and pure cooperative pattern is known as that each game player benefits from cooperative behavior.

For the design objectives with unequal status (designers have target preference), we can construct "principal and subordinate" game pattern, such as Stackelberg Oligopoly game model. The Stackelberg Oligopoly game belongs to the dynamic game with the complete and perfect information, which is defined as: u_S is the payoff function of the strong game player and u_W is the payoff function of the weak game player. S_S is the strategy space owned by the strong game player and S_W is the strategy space owned by the weak game player. $s_S \in S_S$, $s_W \in S_W$, where, s_S is an arbitrary strategy, which is adopted by the strong game player and s_W is an arbitrary strategy, which is adopted by the weak game player. If there exists $\sup_{s_W^* \in R(s_S^*)} u_S\left(s_S^*, s_W^*\right) \geq \sup_{s_W^* \in R(s_S)} u_S\left(s_S, s_W^*\right) \quad \forall s_S \in S_S$, then $\left(s_S^*, s_W^*\right)$ is called as the Stackelberg game solution. Where, $R(S_S)$ is the reaction function of the weak game player to the strong game player. Because there exist both the strong game player and the weak game player in the Stackelberg Oligopoly game model, the satisfaction degree of the game players is different. The strong game player can obtain greater and better satisfaction than the weak game player. Hence, we can take the preferred target as the strong game player and take the other target as the weak game player. But it is a kind of ideal situation that all the game players adopt the same behavior. In nature and real life, game players have a variety of behaviors and form the hybrid game pattern. Through the bionics of the survival mechanisms of reproduction of lizard species (Sinervo et al. 2006), a typical mixed game model is presented (Lu et al. 2010), which consists of both competitive behavior, and cooperative behavior of "all for one and one for all" and "benefit oneself but do not harm other people". It should be explained that although the behavior diversity and difference of all game players are considered in the hybrid game pattern, the behavior of each game player keeps unchanged in the whole game process. So, the game adaptive ability and the dynamic characteristics of behavior are not taken into account in the hybrid game pattern. The players with active adaptive capacities can interact with their surroundings and other players continuously. On this basis, they can learn and gather experience. Besides, they can adjust and change their expectations and behavior correspondingly (namely dynamic behavior). Self-adaptation is important ability of the player, which is the basic push factor to development and evolution of the system. The method of this paper is based on competition and cooperation behavior and takes an adaptive ability into account, which is called evolution game model. Hence, the differences between the hybrid game method and the evolution game method are as follows: 1) Game players have not the adaptive ability and the behavior of each game player keeps unconverted in the hybrid game method. But for the evolution game method, game players have the adaptive ability and the behavior of each game player keeps changed. 2) For the hybrid game method, the key technology is that game players have been assigned the appropriate and different behaviors in order to form the mutual coordination relationship. But for the evolution game method, the key technology is that the evolution rules of behavior are constructed felicitously.

Currently, the most prevalent evolution rules are "TFT" (Tit For Tat) (Axelord and Dion 1988) and "WSLS" (win-stay-lose-shift) (Nowak and Sigmund 1993). TFT rule means a player initially adopts cooperative behavior, and then responds according to the opponent's previous response. That is, if the opponent previously was cooperative, the player is cooperative. If not, the player is competitive. WSLS rule means a player repeats the previous behavior if the payoff has met its aspiration level. If not, the player will change the previous behavior.

According to Chinese saying "In success, commit oneself to the welfare of the society; in distress, maintain one's own integrity", an evolution rule, Poor-Competition-Rich-Cooperation (short for PCRC), is proposed in this paper and a multi-objective optimization design model for passive suspension parameters is solved.

2 The multi-objective method based on evolution game

2.1 The basic idea

Bionics mapping, payoff mapping and set mapping need to be built first when we solve the multi-objective optimization based on game methods. The basic idea for multi-objective method based on evolution game is: 1) there are m design objectives which are seen as m players and the design variables $\boldsymbol{X}$ is divided into strategy subsets $\boldsymbol{S}_1, \cdots, \boldsymbol{S}_m$ of the corresponding players by certain technical methods. 2) According to evolution rule, behavior (cooperation or competition) of each player in one round is determined and mapping relationships is established between the payoff functions u and objective functions $\boldsymbol{F}$ corresponding to such behavior. 3) Each player takes its own payoff function as its objective and gets a single-objective optimal solution in its own strategy subset. So this player obtains the best strategy versus other players. The best strategies of all players consist of the strategy permutation in one round game. Conclusively, the final equilibrium solutions can be obtained through multi-round game according to the convergence criterion.

The evolution game method proposed by this paper can be used to solve multi-objective design problems, but there still exist the following key technologies that need to be solved: 1) The decomposition method of strategy subset for each player. 2) Determining the behavior modes and constructing a payoff function u for each player. 3) The evolution rule and the corresponding algorithm structure.

2.2 Game player's strategy subset computation

By computing the design variable impact factor on the players and fuzzy clustering, then get each player's strategy subset $\boldsymbol{S}_1, \cdots, \boldsymbol{S}_m$.

Computation steps (Lu et al. 2010):

1. Optimize m mono-objectives, then obtain optimal solution $F_1\left(X_1^*\right), F_2\left(X_2^*\right), \cdots, F_m\left(X_m^*\right)$, where, $\mathbf{X}_i^* = \left\{x_{1i}^*,\ x_{2i}^*,\ \cdots,\ x_{ni}^*\right\}\quad (i = 1, 2, \cdots, m)$.
2. Every x_j is divided into T fragments with step length Δx_j in its feasible space; Δ_{ji} is impact factor, which presents x_j affecting on the objective f_i and is shown as:

$$\Delta_{ji} = \frac{\sum_{t=1}^{T}\left|F_i\left(x_{1i}^*, \cdots, x_{(j-1)i}^*, x_j(t), x_{(j+1)i}^*, \cdots x_{ni}^*\right) - F_i\left(x_{1i}^*, \cdots, x_{(j-1)i}^*, x_j(t-1), x_{(j+1)i}^*, \cdots x_{ni}^*\right)\right|}{T \cdot \Delta x_j} \tag{2}$$

To avoid the different function's self-affecting, make impact factors dimensionless.

$$\Delta_{ji} = \frac{\Delta_{ji}}{\left|F_i\left(X_i^*\right)\right|} \tag{3}$$

3. Let all samples classification $\Delta = \{\Delta_1, \Delta_2, \cdots, \Delta_n\}$, the classification of j is $\Delta_j = \left\{\Delta_{j1}, \cdots, \Delta_{jm}\right\}$ $(j = 1, \cdots, n)$, Δ_j means the impact factor set of j on all the other players. The purpose is classifying highly similar samples as one classification; this paper uses similar degree approach to reflect the samples' similarity relation. Select any two samples Δ_k and Δ_l, analyze their similarity relation; define a fuzzy relation function by normal distribution.

$$\mu_i\left(\Delta_k, \Delta_l\right) = \exp\left(-\frac{\left|\Delta_{ki} - \Delta_{li}\right|}{\frac{1}{m}\sum_{i=1}^{m}\left|\Delta_{ki} - \Delta_{li}\right|}\right) \quad (k, l = 1, 2, \cdots, n;\quad k \neq l;\quad i = 1, 2, \cdots, m) \tag{4}$$

Where, $\mu_i(\Delta_k, \Delta_l)$ presents the fuzzy relation between $\boldsymbol{\Delta}_k$ and $\boldsymbol{\Delta}_l$ in the i_{th} objective function.

The hamming distance of $\boldsymbol{\Delta}_k$ and $\boldsymbol{\Delta}_l$ is:

$$d\left(\Delta_k, \Delta_l\right) = \frac{1}{m}\sum_{i=1}^{m}\left|\mu_i\left(\Delta_k, \Delta_l\right) - 1\right| \quad (k, l = 1, 2, \cdots, n; k \neq l) \tag{5}$$

The fuzzy closeness of $\boldsymbol{\Delta}_k$ and $\boldsymbol{\Delta}_l$ is:

$$\sigma\left(\Delta_k, \Delta_l\right) = 2\sum_{i=1}^{m}\frac{\mu_i\left(\Delta_k, \Delta_l\right)}{\left[m + \sum_{i=1}^{m}\mu_i\left(\Delta_k, \Delta_l\right)\right]} \quad (k, l = 1, 2, \cdots, n; k \neq l) \tag{6}$$

The correlation degree of $\boldsymbol{\Delta}_k$ and $\boldsymbol{\Delta}_l$ is:

$$r\left(\Delta_k, \Delta_l\right) = \frac{1}{m}\sum_{i=1}^{m}\xi_i\left(\Delta_k, \Delta_l\right) \quad (k, l = 1, 2, \cdots, n; k \neq l) \tag{7}$$

Where, $\xi_i(\Delta_k, \Delta_l)$ is the correlation coefficient of $\boldsymbol{\Delta}_k$ and $\boldsymbol{\Delta}_l$, and can be expressed as:

$$\xi_i\left(\Delta_k, \Delta_l\right) = \frac{\min\limits_{i\in\{1,2,\cdots,m\}}\left|1 - \mu_i\left(\Delta_k, \Delta_l\right)\right| + 0.5\max\limits_{i\in\{1,2,\cdots,m\}}\left|1 - \mu_i\left(\Delta_k, \Delta_l\right)\right|}{\left|1 - \mu_i\left(\Delta_k, \Delta_l\right)\right| + 0.5\max\limits_{i\in\{1,2,\cdots,m\}}\left|1 - \mu_i(\Delta_k, \Delta_l)\right|} \quad (k, l = 1, 2, \cdots, n;\quad k \neq l;\quad i = 1, 2, \cdots, m) \tag{8}$$

Considering the above index, establish the similar approach degree of Δ_k and Δ_l.

$$t_{kl} = \omega_d \left[1 - d\left(\Delta_k, \Delta_l\right)\right] + \omega_\sigma \sigma\left(\Delta_k, \Delta_l\right) + \omega_r r\left(\Delta_k, \Delta_l\right) \quad (k, l = 1, 2, \cdots, n; \quad k \neq l) \tag{9}$$

Where, ω_d is hamming distance weight, ω_σ fuzzy closeness weight, ω_r correlation degree weight, $\omega_d + \omega_\sigma + \omega_r = 1$.

4. Establish the matrix $\boldsymbol{T}$ based on t_{kl} and do fuzzy clustering to matrix $\boldsymbol{T}$.

$$\boldsymbol{T} = \begin{vmatrix} t_{11} & t_{12} & \cdots & t_{1n} \\ t_{21} & t_{22} & \cdots & t_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ t_{n1} & t_{n2} & \cdots & t_{nn} \end{vmatrix}$$

Classification results of $\boldsymbol{\Delta}$ represent the classification results of $\boldsymbol{X}$ because of one to one relationship between $\boldsymbol{\Delta} = \{\boldsymbol{\Delta}_1, \boldsymbol{\Delta}_2, \cdots, \boldsymbol{\Delta}_n\}$ and $\boldsymbol{X} = \{x_1, x_2, \cdots, x_n\}$.

5. According to fuzzy clustering, divide the design variables $\boldsymbol{X}$ into strategy subsets $\boldsymbol{S}_1, \cdots, \boldsymbol{S}_m$, and assign the strategy subset to the corresponding player by the average value of impact factors.

If the number of design variables (n) and the number of objectives (m) are few, divide set $\boldsymbol{X}$ into player's strategy subsets $\boldsymbol{S}_1, \cdots, \boldsymbol{S}_m$ directly according to impact factor value. Otherwise, it needs fuzzy clustering. Meanwhile, according to experience, variables with strong correlation can be first classified as a sample to reduce the complexity of clustering analysis.

2.3 Clustering steps

Input system's classification control value M and maximal sample number P; each sample as one classification, the system is $\boldsymbol{\Delta}_1, \boldsymbol{\Delta}_2, \cdots, \boldsymbol{\Delta}_n$. The steps of clustering are as follows (Lu et al. 2010):

1. Calculate similar approach degree t_{kl}, and build similar approach degree matrix $\boldsymbol{T}^{(0)}$; attention: $t_{kl} = t_{lk}$, $t_{kl} > 0$.
2. Set maximum value of matrix $\boldsymbol{T}^{(0)}$ to be t_{ab}, $t_{ab} = \max\limits_{k,l \in \{1,2,\cdots,n\}} t_{kl}$, classify Δ_a and Δ_b into a new classification Δ_s; if the sample number is larger than $\boldsymbol{P}$, then combine the second maximal value of $\boldsymbol{T}^{(0)}$, and take this as analogizing;
3. Combine Δ_c $(c = 1, 2, \cdots, n; c \neq a, c \neq b)$ and Δ_s into a new classification system, calculate its similar approach degree and build a new similar approach degree matrix $T^{(1)}$, the similar approach degree of any classification Δ_c and Δ_s is $t_{cs} = \min\{t_{ca}, t_{cb}\}$.
4. Repeat procedures 1–3) until system classification number equals control value M.

2.4 Evolution rule

According to Chinese saying "In success, commit oneself to the welfare of the society; in distress, maintain one's own integrity", an evolution rule, Poor-Competition-Rich-Cooperation (PCRC for short), is proposed by the paper as follows: 1) When the value of objective function representing for the player in this round is worse than that of the initial design, the player will use competitive behavior in next round; 2) When the value of objective function representing for the player in this round is better than that of the initial design, cooperation behavior will be adopted in next round; 3) In the first round of the game, all game players adopt cooperative behavior. The advantages of this evolution rule are: (1) "Automatic". Determine the evolution of behaviors according to the satisfaction degree of their own goals. (2) "Prompt". It can quickly do adjustment in the next round according to the satisfaction degree in the current round. (3) "Friendly". All game players first adopt cooperative behavior. (4) "Explicit". Evolution of the rules is clear and easy to understand.

2.5 Behavior modes and construction of game payoff functions

2.5.1 *Competitive behavior mode*

The characteristic of competitive behavior mode is egoism and its corresponding game payoff function is as follows:

$$u_i = \frac{F_i}{\overline{F}_i} \tag{10}$$

Where: $\overline{F}$ is a reference value, which can eliminate the differences in the magnitude for each objective function. In this paper, the initial design value is chosen to be $\overline{F}$.

2.5.2 *Cooperative behavior mode*

Game players with cooperative behavior consider not only their own payoff but also other game player's payoff when they pursue payoff, that is, "you win to have me, I win to have you".

Its corresponding game payoff function is as follows:

$$u_i = w_{ii}\frac{F_i}{\bar{F}_i} + \sum_{j=1(j\neq i)}^{m} w_{ij}\frac{F_j}{\bar{F}_j} \tag{11}$$

Where, $\sum_{j=1}^{m} w_{ij} = 1$, value of w_{ii} reflects the degree of cooperator concerning about the extent of its own interest. The greater the value is, the lower the cooperative degree is.

2.6 The algorithm of finding an equilibrium solution for a game

When bionics mapping, payoff mapping and set mapping have been built, the multi-objective optimization model is changed into game model.

Definition of equilibrium solution of game: In a game $G = (S_1, \cdots, S_m;\ u_1, \cdots, u_m)$, each player's strategy can be assembled into a strategy permutation $\left(s_1^*, s_2^* \cdots, s_m^*\right)$. If an arbitrary game player i's strategy s_i^* is the best strategy to all the other players' strategy permutation $\left(s_1^*, \cdots, s_{i-1}^*, s_{i+1}^* \cdots, s_m^*\right)$, then for any $s_{ij} \in S_i$, there exists

$$\begin{aligned} & u_i\left(s_1^*, \cdots, s_{i-1}^*, s_i^*, s_{i+1}^*, \cdots, s_m^*\right) \\ & \quad \leq u_i\left(s_1^*, \cdots, s_{i-1}^*, s_{ij}, s_{i+1}^*, \cdots, s_m^*\right) \end{aligned} \tag{12}$$

Where, $\left(s_1^*, s_2^* \cdots, s_m^*\right)$ is called an equilibrium solution of game G. (Illustration: The only one difference between the definition of equilibrium solution here and the meaning of the definition of Nash equilibrium solution is that the "$\geq$" in (12) is given in the definition of Nash equilibrium solution. The payoff of the game is minimized as the pursuit for each player in the paper, which is matched with the minimization multi-objective problem in (1)).

At present, the main algorithms have the negotiations algorithm and colonial competition algorithm. The algorithm based on evolution game is as follows:

1) Obtain strategy subset $S_1, \cdots, S_m$ attached to each player through computing impact factor indicators of design variables to payoffs and conduct fuzzy clustering to these indicators.
2) Generate the initial feasible strategies in the strategy set of each player randomly and then form a strategy permutation $s^{(0)} = \left\{s_1^{(0)}, s_2^{(0)}, \cdots, s_m^{(0)}\right\}$.
3) Payoff function u_i to any i_{th} player $(i = 1, 2, \cdots, m)$ is constructed as follows according to evolution rule proposed by this paper:

Each player uses cooperative behavior in the first round, that is, $u_i = w_{ii}\frac{F_i}{\bar{F}_i} + \sum_{j=1(j\neq i)}^{m} w_{ij}\frac{F_j}{\bar{F}_j}$;
In the k_{th} round of the game,

$$\begin{aligned} & when\ \ F_i^{(k-1)} \leq \overline{F}_i \qquad then\ cooperate \\ & \qquad\qquad u_i = w_{ii}\frac{F_i}{\bar{F}_i} + \sum_{j=1(j\neq i)}^{m} w_{ij}\frac{F_j}{\bar{F}_j} \\ & when\ \ F_i^{(k-1)} > \overline{F}_i \qquad then\ compete\ \ u_i = \frac{F_i}{\bar{F}_i} \end{aligned} \tag{13}$$

4) Let $\bar{s}_1^{(0)}, \bar{s}_2^{(0)}, \cdots, \bar{s}_m^{(0)}$ be the corresponding complementary set of $s_1^{(0)}, s_2^{(0)}, \cdots, s_m^{(0)}$ in $s^{(0)}$. For any player $i\,(i = 1, 2, \cdots, m)$, solve the optimal strategy $s_i^* \in \mathbf{S}_i$, and make payoff minimum $u_i\left(s_i^*, \bar{s}_i^{(0)}\right) \to \min$;
5) Define strategy permutation $s^{(1)} = s_1^* \cup s_2^* \cup \cdots \cup s_m^*$. Then judge the feasibility of $s^{(1)}$. If $g_k\left(s^{(1)}\right) \leq 0\,(k = 1, 2, \cdots, q)$ doesn't satisfy, turn to step 2). Otherwise, compute the distance between $s^{(1)}$ and $s^{(0)}$ which is called the Euclidean norm. Then examine whether the distance satisfies the convergence criterion $\left\|s^{(1)} - s^{(0)}\right\| \leq \varepsilon$ or not (ε is a decimal parameter given in advance). If it satisfies, the game is over; if not, let $s^{(1)}$ displace $s^{(0)}$ and turn to step 3) to repeat.

3 Tri-objective optimization of parameters for passive suspension

3.1 Dynamic model of 8 degrees of freedom (DOF) for full vehicle suspension

A full vehicle model with 8 DOF is considered for analysis, as shown in Fig. 1. All the symbols are shown in nomenclature Appendix. The kinetic equation of the suspension system is given as follows (Lu et al. 2010).

$$[\mathbf{M}]\{\ddot{Z}\} + [\mathbf{C}]\{\dot{Z}\} + [\mathbf{K}]\{Z\} = [\mathbf{F}] \tag{14}$$

Where, $\{Z\}$ is a displacement array; $\left\{\dot{Z}\right\}$ is a speed array and $\left\{\ddot{Z}\right\}$ is an acceleration array. $\{Z\} = \{z_1\ \ z_2\ \ \cdots\ \ z_8\}^T$. $[\mathbf{M}]$ is a mass matrix, $[\mathbf{C}]$ is a damping matrix, $[\mathbf{K}]$ is a stiffness matrix, $[\mathbf{F}]$ is a pavement excitation matrix. $[\mathbf{M}] = diag\left\{m_1\ m_2\ I_p\ I_r\ m_3\ m_4\ m_5\ m_6\right\}$; I_p is moment of inertia for pitch and I_r is moment of inertia for roll.

$$[C]=\begin{bmatrix} c_1 & -c_1 & c_1 l_s & c_1 b_s & 0 & 0 & 0 & 0 \\ -c_1 & c_1+c_2+c_3+c_4+c_5 & -c_1 l_s - c_2 l_f - c_3 l_f + c_4 l_r + c_5 l_r & -c_1 b_s + c_2 b_r - c_3 b_l - c_4 b_l + c_5 b_r & -c_2 & -c_3 & -c_4 & -c_5 \\ c_1 l_s & -c_1 l_s - c_2 l_f - c_3 l_f + c_4 l_r + c_5 l_r & c_1 l_s^2 + c_2 l_f^2 + c_3 l_f^2 + c_4 l_r^2 + c_5 l_r^2 & c_1 l_s b_s - c_2 b_r l_f + c_3 b_l l_f - c_4 b_l l_r + c_5 l_r b_r & c_2 l_f & c_3 l_f & -c_4 l_r & -c_5 l_r \\ c_1 b_s & -c_1 b_s + c_2 b_r - c_3 b_l - c_4 b_l + c_5 b_r & c_1 b_s l_s - c_2 b_r l_f + c_3 b_l l_f - c_4 b_l l_r + c_5 b_r l_r & c_1 b_s^2 + c_2 b_r^2 + c_3 b_l^2 + c_4 b_l^2 + c_5 b_r^2 & -c_2 b_r & c_3 b_l & c_4 b_l & -c_5 b_r \\ 0 & -c_2 & c_2 l_f & -c_2 b_r & c_2+c_6 & 0 & 0 & 0 \\ 0 & -c_3 & c_3 l_f & c_3 b_l & 0 & c_3+c_7 & 0 & 0 \\ 0 & -c_4 & -c_4 l_r & c_4 b_l & 0 & 0 & c_4+c_8 & 0 \\ 0 & -c_5 & -c_5 l_r & -c_5 b_r & 0 & 0 & 0 & c_5+c_9 \end{bmatrix}$$

$$[K]=\begin{bmatrix} k_1 & -k_1 & k_1 l_s & k_1 b_s & 0 & 0 & 0 & 0 \\ -k_1 & k_1+k_2+k_3+k_4+k_5 & -k_1 l_s - k_2 l_f - k_3 l_f + k_4 l_r + k_5 l_r & -k_1 b_s + k_2 b_r - k_3 b_l - k_4 b_l + k_5 b_r & -k_2 & -k_3 & -k_4 & -k_5 \\ k_1 l_s & -k_1 l_s - k_2 l_f - k_3 l_f + k_4 l_r + k_5 l_r & k_1 l_s^2 + k_2 l_f^2 + k_3 l_f^2 + k_4 l_r^2 + k_5 l_r^2 & k_1 l_s b_s - k_2 b_r l_f + k_3 b_l l_f - k_4 b_l l_r + k_5 l_r b_r & k_2 l_f & k_3 l_f & -k_4 l_r & -k_5 l_r \\ k_1 b_s & -k_1 b_s + k_2 b_r - k_3 b_l - k_4 b_l + k_5 b_r & k_1 b_s l_s - k_2 b_r l_f + k_3 b_l l_f - k_4 b_l l_r + k_5 b_r l_r & k_1 b_s^2 + k_2 b_r^2 + k_3 b_l^2 + k_4 b_l^2 + k_5 b_r^2 & -k_2 b_r & k_3 b_l & k_4 b_l & -k_5 b_r \\ 0 & -k_2 & k_2 l_f & -k_2 b_r & k_2+k_6 & 0 & 0 & 0 \\ 0 & -k_3 & k_3 l_f & k_3 b_l & 0 & k_3+k_7 & 0 & 0 \\ 0 & -k_4 & -k_4 l_r & k_4 b_l & 0 & 0 & k_4+k_8 & 0 \\ 0 & -k_5 & -k_5 l_r & -k_5 b_r & 0 & 0 & 0 & k_5+k_9 \end{bmatrix}$$

$$[F]=\begin{bmatrix} 0 & 0 & 0 & 0 & c_6 \dot{f}_1(t)+k_6 f_1(t) & c_7 \dot{f}_2(t)+k_7 f_2(t) & c_8 \dot{f}_3(t)+k_8 f_3(t) & c_9 \dot{f}_4(t)+k_9 f_4(t) \end{bmatrix}^T$$

3.2 Multi-objective optimization design model

3.2.1 Design variables

Vehicle seat, suspension damping and stiffness are selected as design variables. The damping and stiffness have the same values on the left and right side due to vehicle symmetry, that is $k_3 = k_2$, $k_5 = k_4$, $c_3 = c_2$, $c_5 = c_4$, and the design variables are $X = \{x_1, x_2, x_3, x_4, x_5, x_6\} = \{k_1, c_1, k_2, c_2, k_4, c_4\}$.

3.2.2 Objective functions

Take ride comfort (RMS of acceleration of the seat), damage of vehicles on the road (RMS of the tire relative to dynamic load) and ride comfort (the maximum dynamic stroke suspension) as objective functions (Lu et al. 2010), denoted as F_1, F_2, F_3.

Let RMS of acceleration of the seat be optimization objective F_1.

$$F_1 = \left[\frac{1}{T}\int_0^T \ddot{z}_1^2(t)\,dt\right]^{\frac{1}{2}} \to \min \tag{15}$$

Where, T is driving time.

Let RMS of the tire relative to dynamic load be optimization objective F_2.

$$F_2 = \left[\frac{1}{T}\int_0^T \left\{\frac{\left(\frac{F_{d1}(t)}{G_1}+\frac{F_{d2}(t)}{G_2}+\frac{F_{d3}(t)}{G_3}+\frac{F_{d4}(t)}{G_4}\right)}{4}\right\}^2 dt\right]^{\frac{1}{2}} \to \min \tag{16}$$

Where, $G_1 = m_2 g \frac{l_r}{l_f+l_r}\frac{b_l}{b_l+b_r}$, $G_2 = m_2 g \frac{l_r}{l_f+l_r}\frac{b_r}{b_l+b_r}$, $G_3 = m_2 g \frac{l_f}{l_f+l_r}\frac{b_r}{b_l+b_r}$ and $G_4 = m_2 g \frac{l_f}{l_f+l_r}\frac{b_l}{b_l+b_r}$ are static loads of four wheels. F_{d1}, F_{d2}, F_{d3}, F_{d4} are dynamic loads of four wheels.

$$\begin{aligned} F_{d1} &= c_2\left(\dot{z}_5-\dot{z}_2+l_f\dot{z}_3-b_r\dot{z}_4\right)+k_2\left(z_5-z_2+l_f z_3-b_r z_4\right) \\ &\quad + m_3\ddot{z}_5 \\ F_{d2} &= c_3\left(\dot{z}_6-\dot{z}_2+l_f\dot{z}_3+b_l\dot{z}_4\right) \\ &\quad + k_3\left(z_6-z_2+l_f z_3+b_l z_4\right)+m_4\ddot{z}_6 \\ F_{d3} &= c_4\left(\dot{z}_7-\dot{z}_2-l_r\dot{z}_3+b_l\dot{z}_4\right)+k_4\left(z_7-z_2-l_r z_3+b_l z_4\right) \\ &\quad + m_5\ddot{z}_7 \\ F_{d4} &= c_5\left(\dot{z}_8-\dot{z}_2-l_r\dot{z}_3-b_r\dot{z}_4\right) \\ &\quad + k_5\left(z_8-z_2-l_r z_3-b_r z_4\right)+m_6\ddot{z}_8 \end{aligned}$$

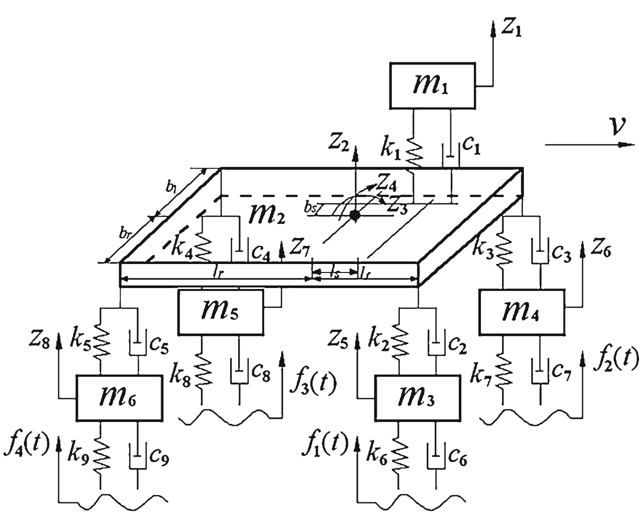

Fig. 1 Dynamic model of 8 DOF for full vehicle suspension

Table 1 The processes of evolution game

Round of the game	Behavior of players			Objective function values		
	F_1	F_2	F_3	F_1(m/s^2)	F_2	F_3(mm)
Initial strategy	–	–	–	0.081457	0.026446	5.24494
1	cooperation	cooperation	cooperation	0.061261	0.025677	4.20556
2	cooperation	cooperation	cooperation	0.053452	0.027349	4.24319
3	cooperation	competition	competition	0.063605	0.025777	4.78693
4	cooperation	cooperation	cooperation	0.066909	0.025060	4.16212
5	cooperation	cooperation	cooperation	0.057370	0.026645	4.02969
6	cooperation	competition	cooperation	0.081928	0.024708	4.43856
7	competition	cooperation	cooperation	0.055801	0.024894	4.14170
8	cooperation	cooperation	cooperation	0.055791	0.024895	4.14174

Let the maximum value of dynamic travel among four suspensions of right front, left front, left rear and right rear be optimization objective F_3.

$$F_3 = \max_{i \in \{1,2,3,4\}} \left\{ \max_{t \in [0,T]} \left[f_{di}\,(t) \right] \right\} \rightarrow \min \tag{17}$$

Where, $f_{d1} = z_2 - l_f z_3 + b_r z_4 - z_5$, $f_{d2} = z_2 - l_f z_3 - b_l z_4 - z_6$, $f_{d3} = z_2 + l_r z_3 - b_l z_4 - z_7$ and $f_{d4} = z_2 + l_r z_3 + b_r z_4 - z_8$ are suspension dynamic travel distance.

3.2.3 Constraint conditions

The suspension stroke f_d is defined as the maximum compression distance allowed by the suspension from the equilibrium position of vehicle. Suspension stroke f_d should be appropriate with $[f_d]$.Otherwise, the suspension will hit against the block frequently. Let the suspension stroke be constraint condition: $0 \leq f_{di} \leq \left[f_d \right] (i = 1, 2, 3, 4)$.

4 Calculation and analysis

4.1 Computational illustrations

According to a particular vehicle, the parameters of the paper are in Guclu (2005) and the values are in Appendix nomenclature. Time-domain data of roughness for the left and right front wheels can be seen in the Figs. 3 and 4 of Lu et al. (2010).

4.2 Computation steps

Refer to Lu et al. (2010), $S_1 = \{x_1, x_2\}$ is the strategy subset of F_1. $S_2 = \{x_3, x_6\}$ is the strategy subset of F_2. $S_3 = \{x_4, x_5\}$ is the strategy subset of F_3.

The computation steps are as follows:

1) Take the corresponding values of the initial design in strategy subsets S_1, S_2 and S_3 as the initial feasible strategies $s_1^{(0)}$, $s_2^{(0)}$ and $s_3^{(0)}$. Then form a strategy permutation $s^{(0)} = \left\{ s_1^{(0)}, s_2^{(0)}, s_3^{(0)} \right\}$

Table 2 Parameters comparison of game design and initial design

Design plan	x_1 (kN/m)	x_2 (N·s/m)	x_3 (kN/m)	x_4 (kN·s/m)	x_5 (kN/m)	x_6 (kN·s/m)	F_1 (m/s^2)	F_2	F_3 (mm)
Initial design	15.000	150.000	15.000	2.500	17.000	2.500	0.0815	0.0264	5.245
Fuzzy optimization (Lu et al. 2008)	10.679	172.645	16.280	2.989	22.489	2.561	0.0679	0.0253	4.989
Nash equilibrium game	10.532	220.217	20.432	3.716	23.277	2.019	0.0699	0.0246	4.402
Cooperation game	8.270	184.169	22.291	3.614	21.705	3.424	0.0624	0.0254	4.172
Mixed game (Lu et al. 2010)	10.112	211.625	20.433	3.614	21.705	2.018	0.0681	0.0248	4.148
Evolution game	10.533	220.217	22.288	3.716	23.274	3.426	0.0558	0.0249	4.142

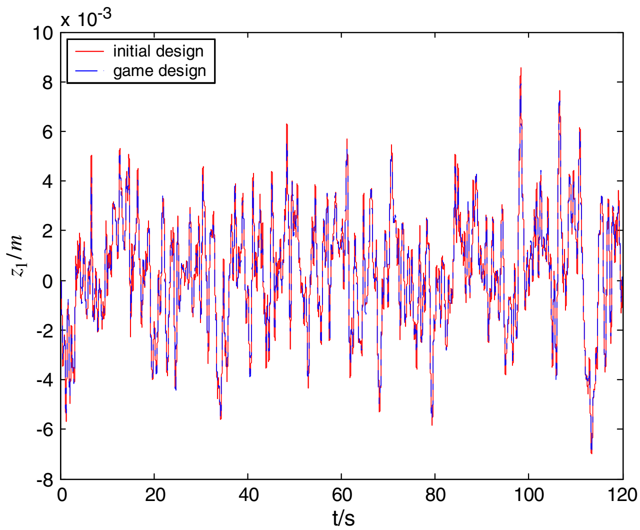

Fig. 2 Comparison of seat displacement between evolution game design and initial design

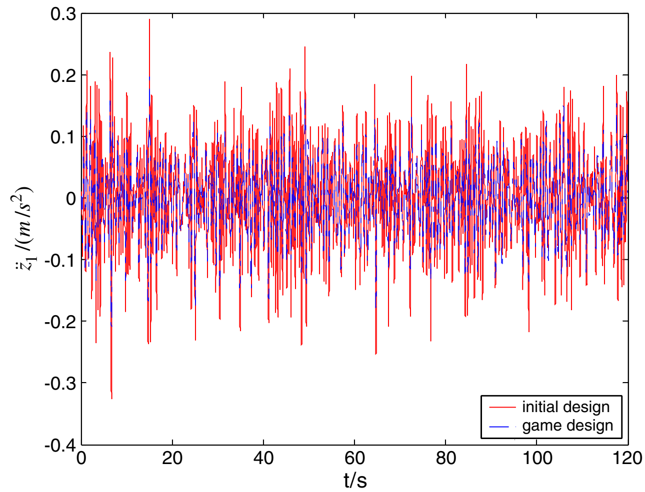

Fig. 4 Comparison of seat acceleration between evolution game design and initial design

2) Perform the following three single-objective optimization

   a) Seek the optimal strategy $s_1^* \in S_1$ and minimize the payoff of game, $u_1\left(s_1^*, s_2^{(0)}, s_3^{(0)}\right) \to \min$
   b) Seek the optimal strategy $s_2^* \in S_2$ and minimize the payoff of game, $u_2\left(s_1^{(0)}, s_2^*, s_3^{(0)}\right) \to \min$
   c) Seek the optimal strategy $s_3^* \in S_3$ and minimize the payoff of game, $u_3\left(s_1^{(0)}, s_2^{(0)}, s_3^*\right) \to \min$

   The construction method of the payoff functions u_1, u_2 and u_3 is as follows:

For the first round of the game

$$u_i = w_{ii}\frac{F_i}{\bar{F}_i} + \sum_{j=1(j\neq i)}^{3} w_{ij}\frac{F_j}{\bar{F}_j}; \quad (i = 1, 2, 3)$$

For the k_{th} round of the game,

$$\begin{aligned}
&when\ \ F_i^{(k-1)} \le \overline{F} \quad then\ cooperate \\
&\qquad\qquad u_i = w_{ii}\frac{F_i}{\bar{F}_i} + \sum_{j=1(j\neq i)}^{m} w_{ij}\frac{F_j}{\bar{F}_j}\ ; \\
&when\ \ F_i^{(k-1)} > \overline{F} \quad then\ compete \\
&\qquad\qquad u_i = \frac{F_i}{\bar{F}_i} \\
&\quad (i = 1, 2, 3)
\end{aligned}$$

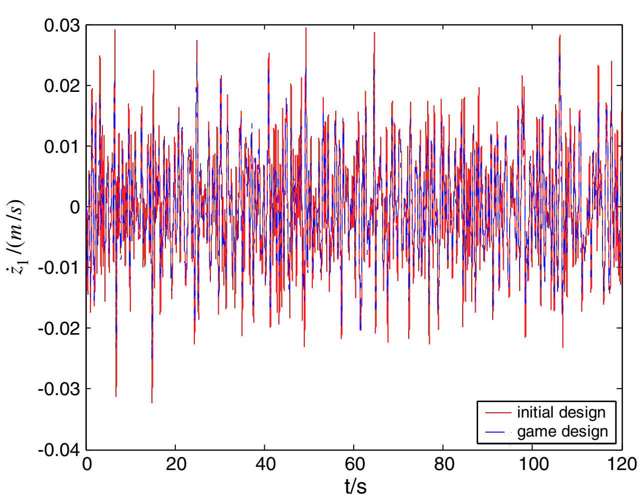

Fig. 3 Comparison of seat velocity between evolution game design and initial design

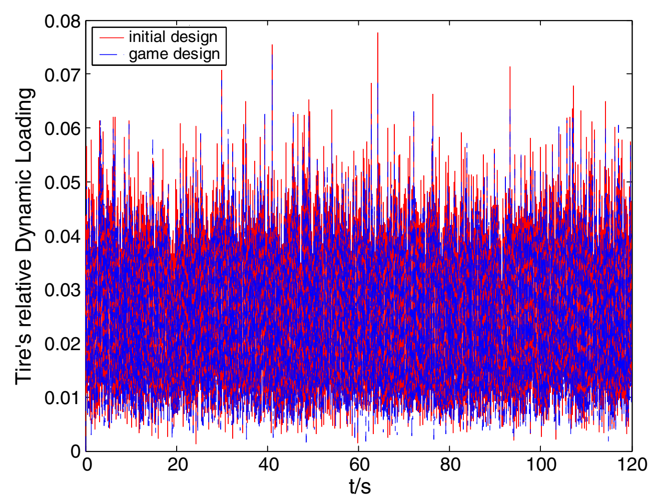

Fig. 5 Comparison of relative dynamic loading for tire between evolution game design and initial design

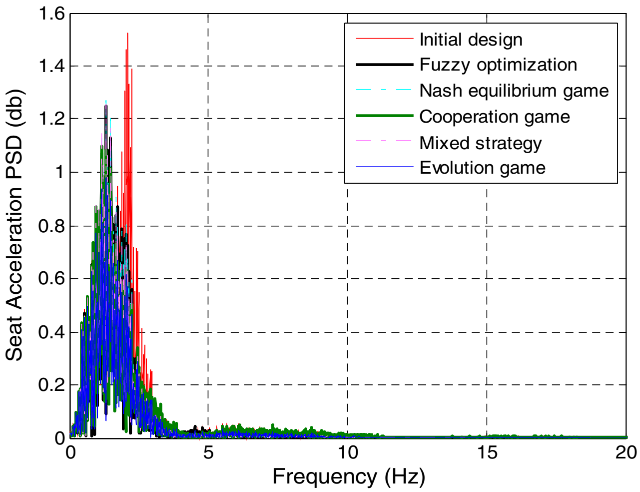

Fig. 6 Comparison of the power spectral density for seat acceleration

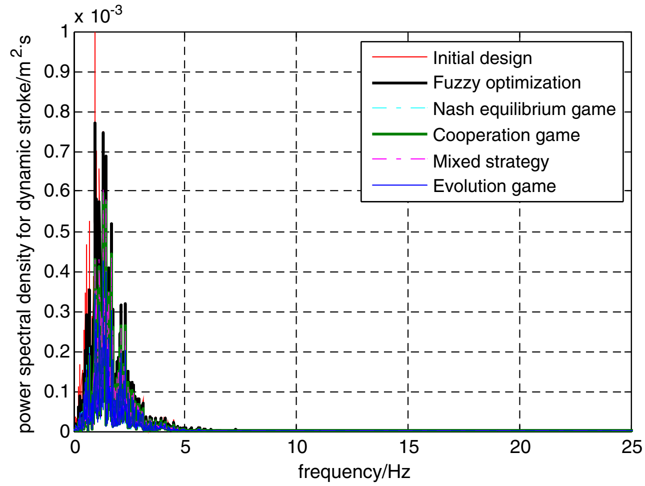

Fig. 8 Comparison of the power spectral density for dynamic stroke (the left front wheel)

Where, $\overline{F}_i\ (i = 1, 2, 3)$ is chosen from the corresponding objective value of the initial design. Weight coefficients are $w_{11} = w_{22} = w_{33} = 0.5$ and $w_{12} = w_{21} = w_{13} = w_{31} = w_{23} = w_{32} = 0.25$. These weight coefficients are determined in accordance with the following principles: One is the principle of equality, including two meanings: 1) treat self-interest and altruism equally, that is, $w_{ii} = 0.5$; 2) treat objective interests of other two players equally except that of its own, that is, $w_{ij} = 0.25 (i \neq j)$; The other is the "reciprocate" principle. How the other players treat objective interest of one player, so does the player, that is, $w_{ij} = w_{ji}$.

3) Define strategy permutation $s^{(1)} = s_1^* \cup s_2^* \cup s_3^*$. Then justify the feasibility of $s^{(1)}$. If $s^{(1)}$ doesn't satisfy constraint conditions, turn to step 1). Otherwise, compute $\sqrt{\sum_{j=1}^{6} \left\{ \left[\left(x_j^{(1)} - x_j^{(0)} \right) / x_j^{(0)} \right]^2 / 6 \right\}}$ and examine whether it satisfies the convergence precision ε (ε is 0.0001 in this paper). If it satisfies, the game is over; if not, let $s^{(0)} = s^{(1)}$ and turn to step 2) to iteration loop.

4.3 Computation results

The game processes of evolution game are shown in Table 1. For comparison, Nash equilibrium game model (pure competitive pattern), cooperation game model (pure cooperative pattern, cooperative behavior of "you win to have me, I win to have you".), mixed game model in Lu et al. (2010) (where: matching behavior are F_1– co-opetition behavior of

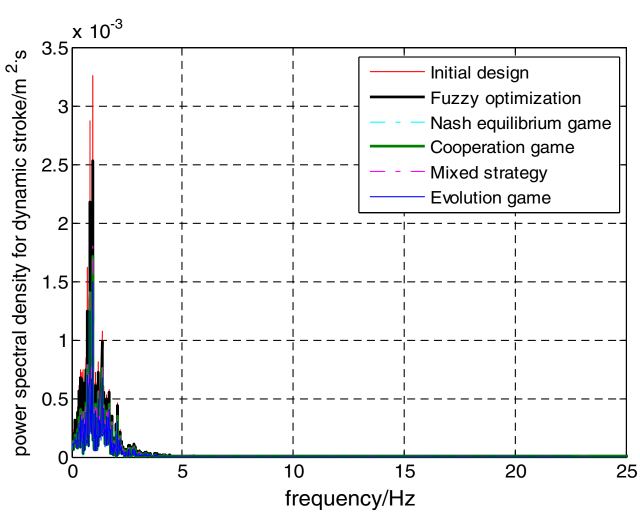

Fig. 7 Comparison of the power spectral density for dynamic stroke (the right front wheel)

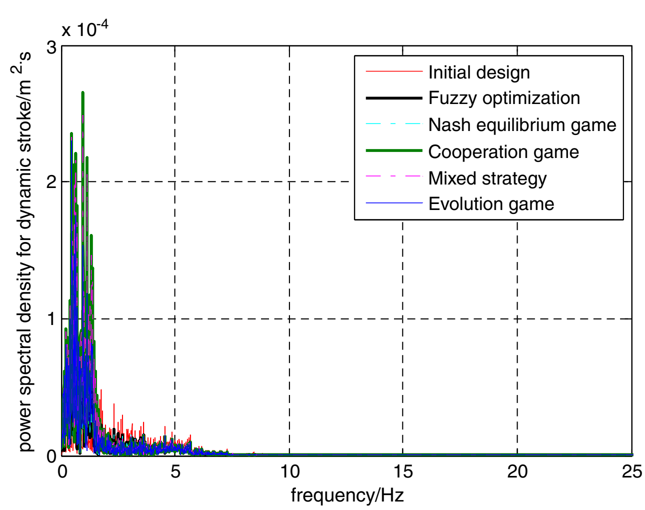

Fig. 9 Comparison of the power spectral density for dynamic stroke (the left rear wheel)

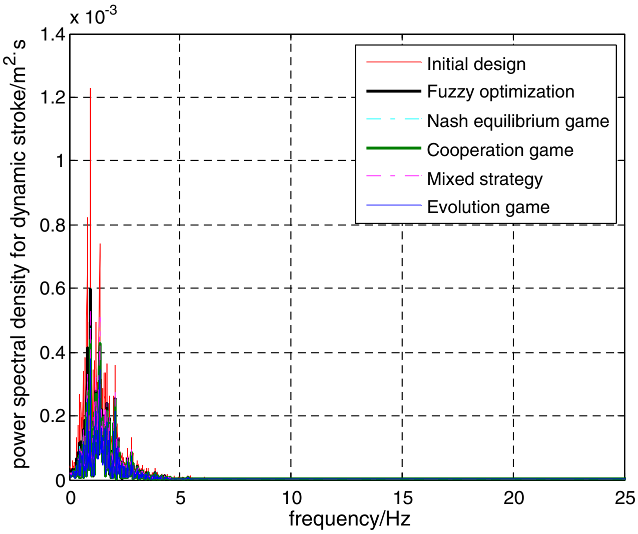

Fig. 10 Comparison of the power spectral density for dynamic stroke (the right rear wheel)

"benefit oneself but do not harm people", F_2– competition behavior and F_3– cooperative behavior of "you win to have me, I win to have you".) and multi-objective fuzzy optimization method (one of traditional multi-objective optimization method) (Lu et al. 2008) are proposed in this paper to solve the problem. All results are shown in Table 2. From Table 2 we can see, the initial design values of the tri-objective functions F_1, F_2 and F_3 are 0.0815, 0.0264 and 5.245, and the design values of the evolution game model are 0.0558, 0.0249 and 4.142, where F_1, F_2 and F_3 are decreased by 31.53%, 5.68% and 21.03% respectively compared with initial values. Compared with mixed game method, traditional multi-objective optimization method and other game methods (shown in Table 2), the evolution game design has comprehensive advantage on tri-objective functions.

As shown from Figs. 2, 3, 4 and 5, the seat displacement, velocity, acceleration, and relative dynamic loading of the tire are significantly decreased.

In the automobile design, power spectral density is used to describe the distributing situation of the power along with the frequency for the accumulation index. Figure 6 demonstrates the distributing situation of the power spectral density along with the frequency for the seat acceleration. The distributing situations of the power spectral density along with the frequency for the dynamic stroke of the wheel are shown from Figs. 7, 8, 9 and 10. From Figs. 6, 7, 8, 9 and 10, we can see that the result obtained by the evolution game method is the best and its response is the least. As human body is very sensitive to vertical vibration and the most sensitive frequency range is $4 \sim 8$ Hz, the peak values of the power spectral density for the seat acceleration are given in Table 3. From Table 3 and Fig. 6 we can see that the peak value of the initial design is the maximum and is close to 4 Hz. The peak value of the evolution game design is the minimum and is far from 4 Hz, so the ride comfort has been improved.

5 Conclusion

1) Optimization objectives are considered to be the different game players with competitive, cooperative and adaptive behaviors in the evolution game method. The design variables are divided into strategy subset owned by the corresponding game players and strategies can be seen as bargaining resource held by game players (In this paper, F_1 (game player 1) can only change the value of x_1 and x_2 to improve its own payoff and F_2 (game player 2) can only change the value of x_3 and x_6 to improve its own payoff and F_3(game player 3) can only change the value of x_4 and x_5 to improve its own payoff). Optimization results are seen as game players' mutual negotiation and compromise, in which game players decide to compete or cooperate with other players based on PCRC evolution rules. The relatively satisfied equilibrium solution can be obtained through the iterative loop.
2) Currently, pure competitive game model and pure cooperative game model are used to solve multi-objective optimization problems, in which the behavior keeps unchanged. On this basis for further consideration, in Lu et al. (2010), a mixed game model is proposed according to the diversity of behaviors caused by differences in the resources and endowment of each player. However, regardless of pure competitive game model, pure cooperative game model, or mixed game model, they only consider competition and cooperation in the behaviors without considering the player's adaptive ability. Self-adaptation is also the very important

Table 3 Comparison of the power spectral density for the seat acceleration

The power spectral density for the seat acceleration	Initial design	Fuzzy optimization	Nash equilibrium game	Cooperation game	Mixed game	Evolution game
Peak value/db	1.522	1.252	1.268	1.098	1.256	0.979

capacity for players, which helps the system development and evolution. In response to this deficiency, this paper presents an evolution game model of PCRC according to Chinese saying that "In success, commit oneself to the welfare of the society; in distress, maintain one's own integrity" by comprehensively considering competition, cooperation and adaptive capacities of the players.

3) Comparing this paper and Lu et al. (2010), we find that it is the same for hybrid game method and evolution game method to split the design variables to game players. But algorithm structure of two methods is obviously different. For the evolution game method, due to the behavior adaptive ability of all design objectives, the game pattern of game players is decided by evolution rules. For the hybrid game method, although behaviors of all game players exist difference and diversity and game players form the mutual coordination hybrid pattern, the behavior of each game player keeps unchanged. Because the simulation approximation degree of natural system decides the rationality and effectiveness of the game solutions, calculation results of the evolution game method are more reasonable than the hybrid game method, which is shown in Tables 2 and 3.

4) Through dynamic evolution of the behavior between mutual competition and cooperation of the players, the method based on an evolution game model is that at the end of each round of the game, all players (design objectives) can adjust their behaviors according to evolution rules, and then adjust payoff functions. Eventually, game results in the next round will be affected and each player will achieve the survival adaptation (corresponding payoffs). From the methodological point of view, as the decision-making process of the game is an objective natural selection, its aim is to achieve a harmonious existence of things. Therefore, equilibrium and coordination of the ultimate combination of strategy for each player determines the stability of the existence of things. From the theoretical point of view, strategy combination (equilibrium solution) through game decision theory has a consistent projection, which means the solution is stable and self-enforced. Because the solution based on the traditional methods may be the fragile optimal solution under specific conditions and it is extremely sensitive to the change of material properties or external load. So, it will be dangerous if this seemingly optimum, but fragile solution is adopted. In consideration of external complex engineering environment, it is necessary to adopt the stable solutions based on the game method.

5) Taking tri-objective optimization of parameters for passive suspension as an example. It is revealed that the correlations of F_1 with x_1 and x_2, F_2 with x_3 and x_6, F_3 with x_4 and x_5 are strong, which is not revealed through the traditional multi-objective optimization method. Evolution game results show that the objectives of the root mean square values of the acceleration of seat and relative dynamic load of tire and the maximum dynamic stroke suspension are better than the traditional multi-objective optimization method (Lu et al. 2008), which shows the effectiveness of this method. For the complex problems of more design variables and reanalysis of the structure in engineering optimization, the advantages of game method, compared with traditional optimization method, are that the design variables are decomposed into the strategy subset owned by each player and that the original high-dimensional optimization problem is transformed into three low-dimensional optimization problems, which can reduce the complexity of problem.

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Appendix

Symbol	Parameters Name	Unit	Value
z_1	vertical driver seat DOF	m	dynamic varying range
z_2	vertical body displacement DOF	m	dynamic varying range
z_3	Pitching DOF	rad	dynamic varying range
z_4	Roll DOF	rad	dynamic varying range
z_5	front right wheel's vertical DOF	m	dynamic varying range
z_6	front left wheel's vertical DOF	m	dynamic varying range
z_7	back right wheel's vertical DOF	m	dynamic varying range
z_8	back left wheel's vertical DOF	m	dynamic varying range

Symbol	Parameters Name	Unit	Value
q_1(t)	front right wheel road roughness incentives	m	input data
q_2(t)	front left wheel road roughness incentives	m	input data
q_3(t)	back left wheel road roughness incentives	m	input data
q_4(t)	back right wheel road roughness incentives	m	input data
k_1	driver Seat's spring stiffness coefficient	kN/m	$7.5 \leq k_1 = x_1 \leq 22.5$ initial 15
k_2	front right suspension spring stiffness coefficient	kN/m	$7.5 \leq k_2 = x_3 \leq 22.5$ initial 15
k_3	front left suspension spring stiffness coefficient	kN/m	$7.5 \leq k_3 = x_3 \leq 22.5$ initial 15
k_4	back left suspension spring stiffness coefficient	kN/m	$8.5 \leq k_4 = x_5 \leq 25.5$ initial 17
k_5	back right suspension spring stiffness coefficient	kN/m	$8.5 \leq k_5 = x_5 \leq 25.5$ initial 17
k_6	front right tire spring stiffness coefficient	kN/m	250
k_7	front left tire spring stiffness coefficient	kN/m	250
k_8	back left tire spring stiffness coefficient	kN/m	250
k_9	back right tire spring stiffness coefficient	kN/m	250
c_1	driver Seat's damping coefficient	N·s/m	$75 \leq c_1 = x_2 \leq 225$ initial 150
c_2	front right Suspension damping coefficient	kN·s/m	$1.25 \leq c_2 = x_4 \leq 3.75$ initial 2.5
c_3	front left Suspension damping coefficient	kN·s/m	$1.25 \leq c_3 = x_4 \leq 3.75$ initial 2.5
c_4	back left Suspension damping coefficient	kN·s/m	$1.25 \leq c_5 = x_6 \leq 3.75$ initial 2.5
c_5	back right Suspension damping coefficient	kN·s/m	$1.25 \leq c_6 = x_6 \leq 3.75$ initial 2.5
c_6	front right tire spring damping coefficient	kN·s/m	0.15
c_7	front left tire spring damping coefficient	kN·s/m	0.15
c_8	back left tire spring damping coefficient	kN·s/m	0.15
c_9	back right tire spring damping coefficient	kN·s/m	0.15
m_1	driver seat mass	kg	90
m_2	suspension mass	kg	1100
m_3	front right tire's mass	kg	25
m_4	front left tire's mass	kg	25
m_5	back left tire's mass	kg	45
m_6	back right tire's mass	kg	45
I_r	roll moment of inertia	kg·m^2	550
I_p	pitch moment of inertia	kg·m^2	1848
l_f	Body mass center to front axle distance	m	1.2
l_r	Body mass center to back axle distance	m	1.4
l_s	vertical body mass distance between seats	m	0.3
b_l	body mass to left wheel distance	m	1.0
b_r	body mass to right wheel distance	m	0.5
b_s	body mass seat to the horizontal distance	m	0.25
v	vehicle's driving velocity	m/s	20
$[f_d]$	the suspension's restricted maximum compression stroke	m	0.1
T	running time	s	120

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