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Noncommutative Chern–Simons Description of the Fractional Quantum Hall Edge *

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Noncommutative Chern–Simons (NCCS) theory is a workable description for the fractional quantum Hall fluid. We apply and generalize the NCCS theory to the physically important case with an edge. From relabeling symmetry of electrons and incompressibility of the fluid, we obtain a constraint and reduce the two-dimensional NCCS theory to a one-dimensional chiral Tomonaga–Luttinger liquid theory, which contains additional interaction terms. Further, we calculate one-loop corrections to the boson and electron propagators and obtain a new tunneling exponent, which agrees with experiments.

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The chiral Tomonaga–Luttinger liquid at the fractional quantum Hall (FQH) edge has become an active subject since Wen's hydrodynamic and effective field formulation.[1−3] The effective edge theory is derived from the bulk effective Chern–Simons theory. It predicts a nonlinear current-voltage relationship $I \sim V^\alpha$ with a universal exponent $\alpha = 2n + 1$ at the filling fraction $\nu = \frac{p}{2np+1}$, where n and p are positive integers.

A number of experiments[4−6] have established the existence of Tomonaga–Luttinger-liquid-like behavior. However, the tunneling exponent measured is different from the prediction. At $\nu = 1/3$, Ref. [4] measured a value of 2.7 ± 0.06 and Ref. [5] measured 2.85 ± 0.15. Using the highest quality samples, Ref. [6] saw a plateau region that occurs at $\alpha \approx 2.7$ and extends from $1/\nu = 2.76$ to 3.33.

The discrepancy between experiment and theory has been addressed in Refs. [7,9-14]. Many works have attempted to explain the discrepancy. For $\nu = 1/3$ and other Jain fractions, some papers suggested that the discrepancy is due to edge reconstruction.[8−10] In contrast, some propose that the exponent is not universal, since the discrepancy persists even in the absence of edge reconstruction.[11−14] It is still an open question.

An elementary derivation of the Chern–Simons description of the FQH effect was given by Susskind,[15] wherein he claimed that the noncommutative version of the description is exactly equivalent to the Laughlin theory. Noncommutative Chern–Simons (NCCS) theory was shown to reproduce basic features of the FQH effect. A regularized version of it, the Chern–Simons matrix model, was proposed to describe a finite number of electrons.[16] The states of the matrix model are in one-to-one correspondence with the Laughlin[17,18] and Jain hierarchical[19] states. Recently, the FQH hierarchy[20] and edge[21] were incompletely discussed with NCCS theory. As many successes on the connection between two theories have been achieved, we would stress that NCCS theory is a workable description for the FQH fluid.

In this Letter, we try to pursue two questions: whether the edge states in fractional quantum Hall effects could be described by NCCS theory; whether a more reliable and sounder exponent α could be derived.

Based on Susskind's derivation,[15] we will reformulate Wen's edge theory.[2,3] First we give a constraint by considering microscopic dynamics: relabeling symmetry of electrons and incompressibility of the fluid. It should be more exact and convincing than that given by choosing a gauge-fixing condition.[2] Second, we solve the constraint exactly. It is amazing that the solution as well as the action has a total differential form. Finally, we reduce the (2+1)-dimensional NCCS theory to a (1+1)-dimensional noncommutative chiral Tomonaga–Luttinger liquid theory, which contains interaction terms expanding to all orders in the noncommutative parameter θ. The commutative limit of it is Wen's theory.

Furthermore, as our theory contains interaction terms, it will predict a new exponent and may provide a solution to the discrepancy mentioned above. We calculate the one-loop Feynman diagrams caused by interaction among chiral bosons. Because of the existence of the shortest incompressible distance, an ultraviolet cutoff is imposed to evaluate the integrals. Then, we obtain one-loop corrections to the boson and electron propagators. The electron propagator still exhibits a power-law correlation, but with a newly corrected prediction of the exponent, which is in good agreement with the experimental results. This is a support to our derivation and the NCCS theory.

We begin with a review of Susskind's derivation.[15] Consider a two-dimensional electron system, the dis-

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crete electrons should be labeled with a discrete index α. Under the relabeling (or permutation) of electrons, the real space coordinates $x_i^\alpha(t)$ $(i=1,2)$ and the Lagrangian remain invariant. We can replace α by a continuous y space, and naturally choose the coordinates so that the electrons are evenly distributed in y with a constant density ρ_0. The relabeling symmetry of α is replaced by the area preserving diffeomorphism (APD) of y.

Assuming that the system is adiabatic so that short range forces lead to an equilibrium and the potential is ρ dependent ($\rho=\rho_0|\partial y/\partial x|$ is the real space density), in a background magnetic field B, the Lagrangian is

$$L=\int d^2y\rho_0\Big[\frac{m}{2}\dot{x}^2-V(\rho)+\frac{eB}{2}\epsilon_{ab}\dot{x}_a x_b\Big]. \qquad (1)$$

Consider an infinitesimal APD transformation from y to y' with unit Jacobian, the Lagrangian transforms as

$$\delta L=\frac{d}{dt}\Big(\frac{\partial L}{\partial\dot{x}_a}\delta x_a\Big)+\Big(-\frac{d}{dt}\frac{\partial L}{\partial\dot{x}_a}+\frac{\partial L}{\partial x_a}\Big)\delta x_a=0. \quad (2)$$

Besides the equation of motion, we arrive at a conserved quantity and the constraints

$$g(y)=\begin{cases}\frac{\partial}{\partial y_j}(\epsilon_{ij}\dot{x}_a\frac{\partial x_a}{\partial y_i}) & \text{if } B \text{ is absent,}\\ \frac{1}{2}\epsilon_{ij}\epsilon_{ab}\frac{\partial x_b}{\partial y_j}\frac{\partial x_a}{\partial y_i}=|\frac{\partial x}{\partial y}| & \text{if } B \text{ is strong,}\end{cases} \qquad (3)$$

where $g(y)$ is an arbitrary time independent function. When the magnetic field is strong, the kinetic term is dropped; $\rho(x,t)=\rho_0/g(y)=\rho(x,0)$.

In the strong magnetic field, the system behaves as a quantum Hall fluid. Consider a fractional quantum Hall fluid with a filling factor $\nu=1/(2n+1)$, where n is a positive integer. Due to electron-electron interaction,[22] the electrons are incompressible with a minimal area $2\pi l_B^2/\nu$ ($l_B=1/\sqrt{eB}$ is the magnetic length).[3] In the absence of vortices (no quasiparticle excitation), we can assume that at $t=0$ the electrons are in equilibrium and uniformly occupy the minimal area. Thus $\rho(x,0)$ is constant, $g(y)$ can be set to unity. The constraint becomes

$$1=\big|\partial x/\partial y\big|. \qquad (4)$$

As $\rho_0=\rho(x,0)=(2\pi l_B^2/\nu)^{-1}$, the factor $\nu=2\pi\rho_0/eB$ is truly the ratio of electrons to magnetic flux quanta.

Consider small deviations from the equilibrium solution $x_i=y_i$,

$$x_i(y,t)=y_i+\epsilon_{ij}\frac{A_j(y,t)}{2\pi\rho_0}\equiv y_i+\theta\epsilon_{ij}A_j. \qquad (5)$$

Substituting it and dropping total time derivatives gives the Chern–Simons action

$$S=\frac{eB}{2}\int dtd^2y\rho_0\epsilon_{ij}\dot{x}_i x_j=\frac{1}{4\pi\nu}\int dtd^2y\epsilon_{ij}\dot{A}_iA_j. \qquad (6)$$

Notice that y space has a basic area quantum $\theta=1/(\nu eB)=l_B^2/\nu$. It means that y space is noncommutative. Equations (4) and (6) are the first-order truncations of the NCCS theory.[15] By expanding to higher order in θ, we can involve the noncommutativity of y space and capture the discrete character of the electron system.

From the constraint, the condition for A is

$$A_j+\frac{1}{2}\theta\epsilon_{mn}A_m\partial_jA_n=\partial_j\phi(y,t), \qquad (7)$$

where $\phi(y,t)$ is an arbitrary scalar field. We must stress that the constraint is more exact and convincing than its linear approximation that is given by choosing the gauge-fixing condition.[2,3] Expanding to all orders in θ, the exact solution is $A_i=\sum_{n=0}^{\infty}\theta^nf_i^{(n)}$ with

$$f_i^{(0)}=\partial_i\phi,\quad f_i^{(n)}=\frac{1}{2}\epsilon_{ab}\sum_{m=0}^{n-1}\partial_if_a^{(m)}f_b^{(n-1-m)}. \quad (8)$$

Substituting them into Eq. (6) gives a noncommutative action

$$S=\frac{1}{4\pi\nu}\int dtd^2y\sum_{n=0}^{\infty}\theta^ns^{(n)}, \qquad (9)$$

where $s^{(n)}=\sum_{m=0}^{n}\epsilon_{ab}\partial_tf_a^{(m)}f_b^{(n-m)}$.

The edge states are the only gapless excitations of incompressible fluid, and govern transport properties. We should study whether S could describe this important one-dimensional case. The first and difficult step is to find a total differential form of the action.

Construct $F_\mu^{(n)}$ with total differentials ($\mu=0,1,2$ and $\partial_0\equiv\partial_t$): $F_\mu^{(0)}=\partial_\mu\phi$ and for $n\geq1$

$$\begin{aligned}F_\mu^{(n)}=&\,\partial_a\Big(\frac{1}{2}\epsilon_{ab}F_\mu^{(n-1)}F_b^{(0)}\Big)+\frac{1}{3}\partial_\mu\Big(\frac{1}{2}\epsilon_{ab}F_a^{(n-1)}F_b^{(0)}\Big)\\&-\frac{1}{3}\sum_{m=1}^{n-1}\partial_a\Big(\frac{1}{2}\epsilon_{ab}F_\mu^{(m)}F_b^{(n-1-m)}\Big).\end{aligned} \qquad (10)$$

We find that $f_i^{(0)}=F_i^{(0)}$, $f_i^{(1)}=F_i^{(1)}$, etc. Using Mathematica, the equivalence has been checked up to $n=7$. Logically, we make a conjecture: for all natural numbers n, $f_i^{(n)}=F_i^{(n)}$.

Similarly, every order of the Lagrangian density is a total differential, $s^{(n)}=2F_0^{(n+1)}$. When the total time derivative is dropped,

$$s^{(n)}=\partial_a(\epsilon_{ab}F_0^{(n)}F_b^{(0)})-\frac{1}{3}\sum_{m=1}^{n}\partial_a\big(\epsilon_{ab}F_0^{(m)}F_b^{(n-m)}\big). \qquad (11)$$

Being integration of a total differential, S is nonzero and nontrivial if and only if a boundary exists. Hence, we continue with an edge. Consider a finite system Σ confined by a simple potential well: an electric field $\boldsymbol{E}$. The electrons drift in the direction perpendicular to $\boldsymbol{E}$ and B and form an edge. In the context of special relativity, in the frame x moving with $v_i\equiv\epsilon_{ij}E_j/B$, the electric field vanishes so that the electrons can be treated the same as that in bulk. The real space x^R is

$$x_i^R = x_i + v_i t = y_i + \theta\epsilon_{ij}A_j + v_i t. \quad (12)$$

Substituting it into the edge action and dropping total time derivatives gives

$$\begin{aligned}S_\Sigma &= \int_\Sigma dt d^2y \rho_0 \Big(\frac{eB}{2}\epsilon_{ij}\partial_t x_i^R x_j^R - eE_i x_i^R\Big)\\ &= \int_\Sigma dt d^2y \rho_0 \Big[\frac{eB}{2}\epsilon_{ij}\partial_t\Big(x_i + 2\epsilon_{ia}\frac{E_a}{B}t\Big)x_j - eE_i x_i\Big]\\ &= \int_\Sigma dt d^2y \rho_0 \frac{eB}{2}\epsilon_{ij}\dot{x}_i x_j = S. \quad (13)\end{aligned}$$

It confirms that the electric field vanishes in the frame x and the co-moving coordinates y. Thus we can use the same Chern–Simons theory as in bulk.

Notice the relationship of the co-moving coordinates y and the laboratory frame y^R

$$\begin{aligned}&y_i^R = y_i + v_i t, \quad t^R = t,\\ &\partial_t = \partial_t^R + v_i\partial_i^R, \quad \partial_i = \partial_i^R. \quad (14)\end{aligned}$$

In terms of y^R, the edge action acquires the form

$$S_\Sigma = \frac{1}{4\pi\nu}\int_\Sigma dt^R d^2y^R \epsilon_{ij}(\partial_t^R + v_a\partial_a^R)A_iA_j. \quad (15)$$

In the laboratory frame, ignoring R for ease of notation, choosing $\boldsymbol{E} = E\hat{y}_2$ and restricting the fluid to $y_2 \le 0$ for convenience, we can reduce the edge action to a $1+1$ dimensional chiral boson theory

$$\begin{aligned}S_\chi &= \frac{1}{4\pi\nu}\int dt dy_1 \phi(\partial_t + v\partial_1)\partial_1\phi + O(\theta)\\ &= \frac{1}{4\pi\nu}\int dt dy_1 \sum_{n=0}^{\infty}\theta^n\chi^{(n)}, \quad (16)\end{aligned}$$

where the edge velocity $v = E/B$ appears naturally and

$$\chi^{(n)} = -F_0^{(n)}F_1^{(0)} + \frac{1}{3}\sum_{m=1}^{n}F_0^{(m)}F_1^{(n-m)}, \quad (17)$$

with redefined $F_0^{(0)} = (\partial_t + v\partial_1)\phi$ and for $n \ge 1$

$$\begin{aligned}F_0^{(n)} &= \partial_a\Big(\frac{\epsilon_{ab}}{2}F_0^{(n-1)}F_b^{(0)}\Big)\\ &+ (\partial_t + v\partial_1)\Big(\frac{\epsilon_{ab}}{6}F_a^{(n-1)}F_b^{(0)}\Big)\\ &- \sum_{m=1}^{n-1}\partial_a\Big(\frac{\epsilon_{ab}}{6}F_0^{(m)}F_b^{(n-1-m)}\Big). \quad (18)\end{aligned}$$

If we ignore the discrete character of the fluid, $\theta \propto l_B^2 \to 0$, we obtain the commutative limit of our description, which coincides with the phenomenological effective theory on the edge effect.[2,3]

In fact, we obtain a noncommutative chiral Tomonaga–Luttinger liquid theory which contains interaction terms. Interaction among chiral bosons makes things different: vertices and loop Feynman diagrams emerge and correct the boson propagator.

Further, we calculate the loop corrections to the boson and electron propagators with S_χ.

Following Wen's hydrodynamic formulation,[1−3] with the equation of motion and commutation relation

$$\Big[\frac{1}{2\pi}\partial_1\phi(y_1), \phi(y_1')\Big] = -i\nu\delta(y_1 - y_1'), \quad (19)$$

we can calculate the retarded Green's function

$$V_2(p) \equiv \tilde{D}_R(p) = \frac{-i2\pi\nu}{(\omega_p - vp)p}. \quad (20)$$

To deal with ∂_2 in $\chi^{(n)}$ ($n \ge 1$), we assume an undetermined distribution $\phi \propto \exp[h(y_2)]$. It should decrease along the negative y_2 axis with a characteristic length, the radius $\sqrt{2l_B^2/\nu}$ occupied by every electron at the filling fraction ν. Using Diagrammar[23] with notations $y_\mu = (y_1, it)$, $p_\mu = (p, i\omega_p)$ and $\int d^2p = i\int dp d\omega_p$, we can spell out the Feynman rules from the action times i with the replacement $\phi(y) = \int \frac{d^2p}{(2\pi)^2}\tilde{\phi}(p)e^{i(py_1 - \omega_p t)}e^{h(y_2)}$. For three-boson and four-boson vertices, the Feynman rules are ($w_p = \omega_p - vp$, the δ functions omitted)

$$V_3(p,q) = \frac{-\theta}{4\pi\nu}h'\sum_{l=p,q,r}l^2 w_l, \quad (21)$$

$$\begin{aligned}V_4(p,q,r) = \frac{i\theta^2}{4\pi\nu}\Big[&\frac{1}{4}(h'^2 + 2h'')\sum_{l=p,q,r,k}l^2\sum_{l=p,q,r,k}lw_l\\ &- (h'^2 + h'')\sum_{l=p,q,r,k}l^3 w_l\Big]. \quad (22)\end{aligned}$$

Let $G(p)$ denote the sum of all 1PI diagrams with two external lines. The full boson propagator is

$$V_2 + V_2GV_2 + \cdots = \frac{-i2\pi\nu}{(\omega_p - vp)p + i2\pi\nu G}. \quad (23)$$

The one-loop (second order in θ) contributions are

$$G_1 = \frac{1}{2}\int\frac{d^2q}{(2\pi)^2}V_3(p,q)V_2(q)V_3(-p,-q)V_2(p+q), \quad (24)$$

$$G_2 = \frac{1}{2}\int\frac{d^2q}{(2\pi)^2}V_4(p,q,-p)V_2(q), \quad (25)$$

where G_n corresponds to the nth diagram in Fig. 1.

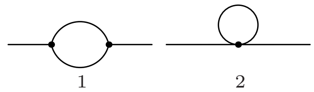

Fig. 1. One-loop Feynman diagrams.

Because y space has the area quantum θ and shortest incompressible distance l_B, we can impose an ultraviolet cutoff $|q| \le \Lambda$ and $|\omega_q| \le v\Lambda$ (due to the dispersion relation $\omega_q = vq$). Via the uncertainty principle, $\Lambda = l_B^{-1}$. To evaluate loop integrals, notice that: integrals over polynomials give zero, $\int dq(q^2)^a = 0$, where a is a nonnegative integer;[23] to the leading order, $\int dqd\omega_q \frac{q}{\omega_q - vq} = -2\Lambda^2$, $\int dqd\omega_q \frac{q^2}{(\omega_q - vq + \omega_p - vp)(\omega_q - vq)} = 2\Lambda^2 \frac{1-\ln\Lambda}{v}$. Then,

$$G_1 = -\frac{i}{(2\pi)^2}\theta^2\Lambda^2 h'^2\Big(pw_p + w_p^2\frac{\ln\Lambda - 1}{4v}\Big), \quad (26)$$

$$G_2 = -\frac{i}{(2\pi)^2}\frac{1}{2}\theta^2\Lambda^2(h'^2 + 2h'')pw_p. \quad (27)$$

To one-loop order, the full boson propagator has the form

$$\frac{-i2\pi\nu}{pw_p(1+c_1) + w_p^2 c_2 \frac{\ln\Lambda - 1}{v}} = \frac{-i2\pi\tilde{\nu}}{p(\omega_p - vp)} - \frac{-i2\pi\tilde{\nu}}{p(\omega_p - v_n p)},$$

where $c_1 = \frac{\nu}{2\pi}\theta^2\Lambda^2(\frac{3}{2}h'^2 + h'')$, $c_2 = \frac{\nu}{8\pi}\theta^2\Lambda^2 h'^2$, $\tilde{\nu} = \frac{\nu}{1+c_1}$ and $v_n = v(1 - \frac{1+c_1}{c_2}\frac{1}{\ln\Lambda - 1})$.

Notice that $\theta = l_B^2/\nu$; $\Lambda = l_B^{-1}$; h'^2 and h'' are proportional to $\nu/(2l_B^2)$, according to the characteristic length $\sqrt{2l_B^2/\nu}$. So c_1 and c_2 are constants independent of ν and l_B. As a perturbation-theory correction should not be too large, we have $|c_1| \ll 1$. Evidently, $c_2 \geq 0$ so that v_n is slightly smaller than v. Because of the damping of the electric field caused by the presence of the electrons, v decreases by a small amount along the negative y_2 axis. Without loss of generality, we can choose v_n to be the next-door neighbor of v, and reconsider the full boson propagator: the second term of the propagator with v cancels the first term with v_n, and so on; the second term with v_1 can be ignored while it is nonchiral and cancels the propagator in bulk with $v_0 = 0$; a sum over all slices gives the overall full boson propagator

$$\frac{-i2\pi\tilde{\nu}}{p(\omega_p - vp)}. \quad (28)$$

In the position representation, it takes the form

$$\langle\phi(y_1,t)\phi(0)\rangle = -\tilde{\nu}\ln(y_1 - vt) + \text{const}. \quad (29)$$

We can determine the distribution by solving $c_1 = \frac{l_B^2}{2\pi\nu}(\frac{3}{2}h'^2 + h'')$. As $\exp[h(y_2)]$ should decrease along the negative y_2 axis, the only solution is $h(y_2) = \sqrt{\frac{4\pi\nu}{3l_B^2}c_1}y_2$ at $c_1 > 0$. Naturally, we choose it and confirm the characteristic length of this exponential distribution to be $\sqrt{2l_B^2/\nu}$. Hence, $c_1 = 3/8\pi$ and $\tilde{\nu} = \frac{\nu}{1+c_1} \approx 0.893\nu$.

With the electron operator (fermionic when $1/\nu$ is odd)

$$\Psi \propto e^{i(1/\nu)\phi},\ \Psi(y_1)\Psi(y_1') = (-1)^{1/\nu}\Psi(y_1')\Psi(y_1), \quad (30)$$

the electron propagator can be calculated via

$$\langle T\{\Psi^\dagger(y_1,t)\Psi(0)\}\rangle = \exp\Big[\frac{\langle\phi(y_1,t)\phi(0)\rangle}{\nu^2}\Big] \propto \frac{1}{(y_1 - vt)^\alpha}, \quad (31)$$

where the tunneling exponent is

$$\alpha = \frac{\tilde{\nu}}{\nu^2} \approx 0.893\frac{1}{\nu}. \quad (32)$$

It exhibits a new nontrivial power-law correlation, which indicates Tomonaga–Luttinger-liquid-like behavior.[7]

At $\nu = 1/3$, the prediction of Eq. (32) is $\alpha \approx 2.68$. It is in good agreement with the value measured in Refs. [4,6], $\alpha \approx 2.7$. It also agrees with Ref. [5] qualitatively.

Strictly, the formalism in this letter only applies to Laughlin fractions. If we abandon Fermi statistics,[13] we can extend the calculation to Jain fractions $\nu = 2/5$ and $3/7$ giving $\alpha \approx 2.23$ and 2.08, close to the measured exponents 2.3 and 2.1.[14] It is interesting but further precise study is needed.

In summary, we have derived the NCCS description of the FQH edge, a new chiral Tomonaga–Luttinger liquid theory containing additional terms that represent interaction among chiral bosons. An important consequence of such terms is a correction to the predicted universal exponent of the electron propagator, which has been measured experimentally and found to be nonuniversal. Without any adjustable parameter, the discrepancy between experiment and theory is resolved.

Furthermore, the edge of hierarchial liquids, higher order corrections and other related topics[24] remain as future subjects.

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