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DISPOSING THE LEFTOVERS UNDER THE CONSIGNMENT CONTRACT WITH REVENUE SHARING: RETAILER VS SUPPLIER*

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Abstract This paper studies the consignment contract with revenue sharing where the retailer offers two revenue share schemes between himself and his supplier from the viewpoint of inventory ownership: One is that the retailer takes charge of the unsold items, the other one is that the retailer returns the unsold items to the supplier at the end of the selling period, and the supplier disposes those overstockings. In each contract, the retailer deducts a percentage from the selling price for each sold item and transfers the balance to the supplier. The supplier solves a two-stage problem: She first chooses contract, then decides retail price and delivery quantity according to the terms of the contract chosen. With an iso-price-elastic demand model, the authors derive the retailer and suppliers' optimal decisions for both schemes. In addition, the authors characterize how they are affected by disposing cost. The authors compare the decisions between the two schemes for disposing cost turn out to be holding cost or salvage value, respectively. The authors use numerical examples to show the supplier's first-stage optimal decision depends critically on demand price elasticity, the disposing cost and the retailer's share for channel cost.

Key words Consignment sales, revenue sharing, stackelberg game, supply chain management.

1 Introduction

We study a model in which a supplier produces a product at a constant marginal cost and sells it to a market through a common retailer. The retailer provides the supplier with a market medium (e.g., the physical shelf-space in a retail store or the internet marketplace at taobao.com, etc.) for selling her product, for any sold item he deducts a percentage from the selling price. There are two kinds of retailers who take different actions on the unsold items, one will dispose the leftovers for the supplier, the other will return the unsold items to the supplier, and then the supplier will dispose the overstockings (note that the dispose cost either

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positive, which represents holding cost, or negative, which represents salvage value). In two different situations, the retailer deducts different percentage. From now, we transfer the model into studying the case that there is one retailer in the market and he offers the supplier two consignment-sales contracts: One is that the retailer retains the ownership of the overstockings and he allocates a revenue share from the selling price for each item sold; the other is that he returns the unsold items to the supplier, then the supplier takes charge of the leftovers and retailer charges another revenue share. We believe this transformation allows us to capture the essence of the problem but not trivialize the true nature of this problem.

The supplier faces a two-stage problem. In the first stage, she must choose one contract to accept or reject the two contracts in order to earn positive profit. If she chooses one contract, based on the terms in the contract, in the second stage, she then has to decide the retail price and delivery quantity. Consignment-sales contract of this type is used for e-business on the internet as well as under traditional retail store settings[1]. For brief, the deals between them proceed sequentially as follows: a) The retailer offers two different consignment-sales contracts which specify who attains the ownership of the overstocking and the corresponding revenue share allocation. b) The supplier first chooses to accept one contract or rejects both. If she doesn't accept any of them then nothing will happen. If she accepts one of the contracts, she then has to decide the product's retail price and delivery quantity based on the terms in the contract she chooses. c) The retailer sells the product at the price chosen by the supplier, for each sold item, he deducts a percentage from the selling price and remits the balance to the supplier. With respect to the leftovers, they conform to the contract chosen by the supplier. Demand is price sensitive and uncertain, we employ a deterministic, iso-price-elastic demand model multiplied by a random factor with general probability distribution to capture the price sensitivity and uncertainty of demand. This demand function has been adopted in the literatures on studying joint pricing-production decisions for supply chain systems.

We model the decision making of the two firms as a Stackelberg game: The retailer, acting as the leader, designs two contracts to maximize his own profit in anticipating the supplier's behavior for himself disposing the leftovers or supplier disposing the leftovers, respectively. The supplier, acting as a follower, first chooses to take it or leave it. If she accepts one, she goes on to decide how many units of the product to produce and the retail price. Wang, et al.[2] considered a supply chain structure where a downstream retailer offers a consignment-sales contract with revenue sharing to a supplier, who then makes production-pricing decisions. Using a multiplicative and iso-price-elastic demand model, they derived an equilibrium solution for the channel and closed-form performance measures. In [2], it was assumed that any unsold product at the end of the season bears no salvage value or holding cost. For most of the cases, that model has limitations, so we extend it by considering two ways of disposing the leftovers. The supplier may dispose or she may request the retailer to dispose the unsold items. Then the retailer offers two consignment-sales contracts with different revenue share allocation. Our key contributions in this paper are: Under a very mild restriction on the distribution function of the random factor, for both of the two consignment contracts we derive the retailer's optimal revenue share allocation and the supplier's optimal decisions. We also derive some beautiful properties of those optimal solutions. Furthermore, we use numerical examples to show that the supplier's first stage decision depends on the price elasticity index, retailer's share of the total channel cost and the unit dispose cost. Under some circumstances, the supplier always chooses the first contract, under other circumstances, she always rejects the two contracts, and for most cases, the supplier's decisions depend critically on the parameters.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 characterizes the optimal decisions of the retailer and the supplier when retailer disposes leftovers. Section 4 analyzes the scenario that supplier disposes leftovers. We compare the above results and use uniform distribution to do our numerical experiments in Section 5. Section 6 concludes the paper with future research directions.

2 Model's Description

We consider that one supplier sells the product to market through a retailer with consignment contract. The retailer offers the supplier contracts which specify the percentage of revenue allocation between them. He may charge different revenue allocation depending on who will dispose the overstock. In each contract, the supplier and retailer consist a Stackelberg game, and the supplier acts as a follower, choosing the quantity and the retail price. The retailer faces uncertain and price-sensitive demand, assuming the demand for the product D, then has the following multiplicative functional form: $D(p,\varepsilon) = y(p)\varepsilon$[3], where $y(p)$ is a decreasing function of the product's selling price p, and ε is a random variable defined on $[A, B]$ with $B > A \geq 0$, with CDF $F(\cdot)$, PDF $f(\cdot)$. Let $r(x) = f(x)/[1 - F(x)]$ denote the failure rate function of the demand distribution and $y(p) = ap^{-b}$ $(a > 0,\ b > 1)$, which is common in the economics literatures, and called iso-elastic demand, the parameter b is the price-elasticity index of demand, the larger the value of b is, the more sensitive the demand is to a change in price. If the price-elasticity index is greater than 1, then a product is defined as price elastic; otherwise, a product is defined as inelastic. As in [2], we focus on price-elastic products by assuming $b > 1$.

The product is produced at a constant cost of c_s per unit, and there is a cost of c_r per unit incurred at the retail stage for inventory handling, shelf-space usage, etc. We define $c = c_s + c_r$ as the total cost per unit, out of which $\alpha = c_r/c$ is incurred at the retail stage and the rest at the product stage. Any leftover is disposed at h per unit, note that h may be negative $h \geq -c$, in which case it represents a per-unit salvage. In the case of shortage, unsatisfied demand carries no additional penalty except for the loss of sales revenue. The retailer offers two consignment contracts to dispose the unsold items: One is that the retailer will handle the leftovers and he charges a percentage from the selling price, the other is that the retailer will return the unsold products back to the supplier and the supplier will dispose these items and then retailer also charges another percentage from the selling price. Based on the model, we will investigate the optimal strategies of the retailer and supplier, and we will also analyze which consignment contract the supplier prefers to.

3 Retailer Disposes Leftovers

In this section, we consider retailer disposes leftovers. We model the decision making of the supplier and the retailer as a Stackelberg game. The retailer acts as the leader and the supplier acts as the follower, so we concentrate on this problem by first considering the supplier's problem.

3.1 Supplier's Problem

For a given revenue share φ, $0 \leq \varphi \leq 1$, the supplier's problem is to choose the retail price and the production quantity to maximize her expected profit. As in [4], we define the stocking factor of inventory as $z = q/y(p)$. Then choosing (p, q) is equivalent to choosing (p, z). So the supplier's profit $\Pi_{1,s}$ (subscript 1 is used to denote the scenario that retailer disposes leftovers) can be written as:

$$\begin{aligned} \Pi_{1,s}(p,z) &= -(1-\alpha)cq + (1-\varphi)p\ E[\min\{q, D\}] \\ &= y(p)\{(1-\varphi)p[z - \Lambda(z)] - (1-\alpha)cz\}, \end{aligned} \tag{1}$$

where $\Lambda(z) = \int_A^z (z-x)f(x)dx$. We first optimize p for a given z, and then search over the resulting optimal trajectory to maximize $E[\Pi_{1,s}(p_1^*(z), z)]$.

Theorem 1 *For any fixed z such that $A \le z \le B$, the unique optimal price $p_1^*(z)$ is given by*

$$p_1^*(z) = \frac{(1-\alpha)bc}{(b-1)(1-\varphi)} \cdot \frac{z}{z-\Lambda(z)},$$

and if $d[xr(x)]/dx = r(x)+xdr(x)/dx > 0$, then the optimal value $z = z_1^*$ (maximizes $\Pi_{1,s}(p_1^*(z), z)$) is uniquely determined by

$$F(z_1^*) = \frac{z_1^* + (b-1)\Lambda(z_1^*)}{bz_1^*}.$$

Proof The proof is simpler than that of Theorem 3, so we omit the detail here. ∎

3.2 Retailer's Decision

Knowing the supplier chooses (p_1^*, z_1^*) according to Theorem 1 in response to a given revenue share allocation φ, the retailer decides φ to maximize his expected profit $\Pi_{1,r}$, which is given by

$$\begin{aligned}\Pi_{1,r}(\varphi) &= -\alpha c q_1^* + \varphi p_1^* \ E[\min\{q_1^*, D\}] - h\ E(q_1^* - D)^+ \\ &= y(p_1^*)\{\varphi p_1^*[z_1^* - \Lambda(z_1^*)] - \alpha c z_1^* - h\Lambda(z_1^*)\}.\end{aligned} \tag{2}$$

We can further rewrite $\Pi_{1,r}(\varphi)$ as

$$\Pi_{1,r}(\varphi) = a\frac{(b-1)^{b-1}[z_1^* - \Lambda(z_1^*)]^b}{((1-\alpha)bcz_1^*)^b} g_1(\varphi), \tag{3}$$

where

$$g_1(\varphi) = (1-\varphi)^{b-1}\{[(b-\alpha)cz_1^* + (b-1)h\Lambda(z_1^*)]\varphi - [\alpha(b-1)cz_1^* + (b-1)h\Lambda(z_1^*)]\}. \tag{4}$$

We find that z_1^* chosen by the supplier does not depend on φ, so maximizing $\Pi_{1,r}(\varphi)$ over φ is equivalent to maximizing $g_1(\varphi)$.

Theorem 2 *The optimal revenue share allocation for the retailer is unique, which is given by*

$$\varphi_1^* = \frac{(1+\alpha b - 2\alpha)cz_1^* + (b-1)h\Lambda(z_1^*)}{(b-\alpha)cz_1^* + (b-1)h\Lambda(z_1^*)}.$$

Proof From (4), we can get that

$$g_1'(\varphi) = b(1-\varphi)^{b-2}[(1+\alpha b-2\alpha)cz_1^* + (b-1)h\Lambda(z_1^*) - [(b-\alpha)cz_1^* + (b-1)h\Lambda(z_1^*)]\varphi].$$

Since $b(1-\varphi)^{b-2} > 0$ for $b > 1$ and $0 < \varphi < 1$, the function $g_1(\varphi)$ is increasing for $0 < \varphi < \varphi_1^*$ and is decreasing for $\varphi_1^* < \varphi < 1$. Therefore, $g_1(\varphi)$ reaches its maximum at $\varphi_1^* = ((1+\alpha b - 2\alpha)cz_1^* + (b-1)h\Lambda(z_1^*))/((b-\alpha)cz_1^* + (b-1)h\Lambda(z_1^*))$. ∎

The optimal revenue share φ_1^* in Theorem 2 depends on four system parameters: The demand price-elasticity index b, the whole cost c, the retailer's cost share α and the unit dispose cost h. The following proposition will show how φ_1^* changes with h given α, c and b.

Proposition 1 *Given* $0 < \alpha < 1,\ c > 0$ *and* $b > 1$,
i) *the retailer's optimal revenue share* φ_1^* *is increasing in* h;
ii) *if* $h \geq 0$, *then* $1 > \varphi_1^* \geq \frac{1+\alpha b-2\alpha}{b-\alpha} > \alpha$;
iii) *if* $-c \leq h < 0$, *then* $0 < \varphi_1^* < \frac{1+\alpha b-2\alpha}{b-\alpha}$.
Proof From Theorem 2 and knowing z_1^* is independent of h, we can obtain

$$\frac{d\varphi_1^*}{dh} = \frac{(1-\alpha)(b-1)^2 z_1^* \Lambda(z_1^*)}{[(b-\alpha)cz_1^* + (b-1)h\Lambda(z_1^*)]^2} > 0.$$

By substituting $h = 0$ in Theorem 2, we can prove the proposition. ∎

Proposition 1 i) illustrates that the retailer's revenue share is increasing in the dispose cost. An intuitive explanation for the fact is that: Now, retailer takes charge of the overstocks, he will spend more money if the dispose cost is higher, what he cares about is his net profit, so it is beneficial for him to allocate a higher revenue share. From Proposition 1 ii) and iii), we can observe that the retailer always allocates himself a share of the revenue that is strictly larger than his share of the channel cost when the dispose cost appears in the form of holding cost. We refer $(1 + \alpha b - 2\alpha)/(b - \alpha)$ as critical share throughout our paper, when the retailer can gain from the salvage value of the leftovers, he will charge smaller revenue share which is less than that critical share.

Now, we have the retailer's optimal revenue share allocation, the supplier can accept or reject the contract. If she accepts it, how her optimal decisions and corresponding profit are affected by h? The following proposition answers these questions.

Proposition 2 *For given* $0 < \alpha < 1,\ c > 0$ *and* $b > 1$,
i) *the supplier's optimal stocking factor of inventory is irrelevant with* h;
ii) *the supplier's optimal selling price is increasing in* h;
iii) *the supplier's optimal expected profit is decreasing in* h.
Proof From Theorem 1, we easily find z_1^* is irrelevant with h, then combine (1), Theorems 1 and 2, the optimal selling price and profit can be formulated as

$$p_1^* = \frac{b[(b-\alpha)cz_1^* + (b-1)h\Lambda(z_1^*)]}{(b-1)^2[z_1^* - \Lambda(z_1^*)]}, \tag{5}$$

$$\Pi_{1,s}(p_1^*, z_1^*) = \frac{(1-\alpha)ac}{b-1}(p_1^*)^{-b} z_1^*, \tag{6}$$

so the monotone of p_1^* and $\Pi_{1,s}(p_1^*, z_1^*)$ with respect to h are obvious. ∎

For the unsold items, the dispose cost will incurred at the retailer, then the supplier's optimal inventory is irrelevant with unit dispose cost h, which is very intuitive. From Proposition 1, we know that the retailer's optimal revenue share is increasing in h, so the supplier will charge a higher retailer price respond to her lower revenue allocation. Though the supplier's price is increasing in h, considering demand is a decreasing function of the selling price, together with smaller channel profit allocation, thus the supplier's profit is decreasing in h.

4 Supplier Disposes Leftovers

In this section, we consider supplier disposes the leftovers, the logic of Section 3 is also applied here, so we begin our analysis by focusing on the supplier's decisions.

4.1 Supplier's Problem

For a given revenue share φ, $0 \leq \varphi \leq 1$, the supplier's problem is to choose (p, z) to maximize her expected profit. Then the supplier's profit $\Pi_{2,s}$ (we use subscript 2 denotes the scenario that the supplier dispose leftovers) can be written as:

$$\begin{aligned}\Pi_{2,s}(p,z) &= -(1-\alpha)cq + (1-\varphi)pE[\min\{q,D\}] - hE(q-D)^+ \\ &= y(p)\{(1-\varphi)p[z-\Lambda(z)] - (1-\alpha)cz - h\Lambda(z)\}. \end{aligned} \tag{7}$$

We present the optimal decision (p_2^*, z_2^*) of the supplier in the following theorem.

Theorem 3 *For any fixed z such that $A \leq z \leq B$, the unique optimal price $p_2^*(z)$ is given by*

$$p_2^*(z) = \frac{(1-\alpha)bc}{(b-1)(1-\varphi)} + \frac{b[(1-\alpha)c+h]}{(b-1)(1-\varphi)}\frac{\Lambda(z)}{z-\Lambda(z)},$$

and if $2r(x)^2 + dr(x)/dx > 0$ and $b \geq 2$ the optimal $z = z_2^*$ (maximizes $\Pi_{2,s}(p_2^*(z), z)$) is uniquely determined by

$$F(z_2^*) = \frac{(1-\alpha)cz_2^* + [bh + (1-\alpha)(b-1)c]\Lambda(z_2^*)}{[(1-\alpha)bc + (b-1)h]z_2^* + h\Lambda(z_2^*)}.$$

Proof First, for any fixed z with $A \leq z \leq B$, it follows from (7) that

$$\frac{\partial \Pi_{2,s}(p,z)}{\partial p} = ap^{-b-1}[(1-\alpha)bcz + bh\Lambda(z) - (b-1)(1-\varphi)(z-\Lambda(z))p].$$

Since $ap^{-b-1} > 0$ for $0 < p < \infty$, so we can get the former part of the theorem for $\partial\Pi_{2,s}(p,z)/\partial p = 0$. $p_2^*(z)$ is the unique maximizer of $\Pi_{2,s}(p,z)$, because $\partial\Pi_{2,s}(p,z)/\partial p > 0$ for all $p < p_2^*(z)$, and $\partial\Pi_{2,s}(p,z)/\partial p < 0$ for all $p > p_2^*(z)$.

Next, we try to find z_2^* to maximize $\Pi_{2,s}(p_2^*(z), z)$. By the chain rule, we have

$$\begin{aligned}\frac{d\Pi_{2,s}(p_2^*(z),z)}{dz} &= \frac{\partial \Pi_{2,s}(p_2^*(z),z)}{\partial z} + \frac{\partial \Pi_{2,s}(p_2^*(z),z)}{\partial p}\frac{dp_2^*(z)}{dz} \\ &= a(p_2^*(z))^{-b}[1-F(z)]G(z)\end{aligned}$$

with

$$G(z) = (1-\varphi)p_2^*(z) + h - \frac{(1-\alpha)c+h}{1-F(z)},$$

where we have used the fact that $\partial\Pi_{2,s}(p_2^*(z),z)/\partial p = 0$ due to the optimality of $p_2^*(z)$. Since $a(p_2^*(z))^{-b}[1-F(z)]$ is always positive for $A < z < B$, first-order condition requires that the optimal z_2^* satisfy $G(z) = 0$. Now, we go on to prove $z_2^* \in (A, B)$ is unique.

a) $G(z)$ is continuous, $G(A) = (1-\alpha)c/(b-1) > 0$ and $G(B) = (1-\varphi)p_2^*(B) + h - (h + (1-\alpha)c)/(1-F(B)) \to -\infty < 0$;

b) We have

$$\frac{dG(z)}{dz} = \frac{b[(1-\alpha)c+h]}{b-1}\frac{zF(z)-\Lambda(z)}{[z-\Lambda(z)]^2} - \frac{[(1-\alpha)c+h]r(z)}{1-F(z)},$$

$$\frac{d^2G(z)}{dz^2} = -[(1-\alpha)c+h]\left[\frac{2r(z)^2 + \frac{dr(z)}{dz}}{1-F(z)} + \frac{b-2}{b-1}\frac{r(z)}{z-\Lambda(z)}\right] - \left[\frac{2(1-F(z))}{z-\Lambda(z)} + r(z)\right]\frac{dG(z)}{dz}.$$

so when $2r(z)^2 + dr(z)/dz > 0$ and $b \geq 2$, $G''(z) < 0$ at $G'(z) = 0$, which implies that $G(z)$ is a unimodal function. This in conjunction with $G(A) > 0,\ G(B) < 0$, guarantees the uniqueness of z_2^*. ∎

Unlike z_1^* is independent of h, we can clearly describe how z_2^* changes with h in the following proposition.

Proposition 3 *If $2r(x)^2 + dr(x)/dx > 0$ and $b \geq 2$, then z_2^* is decreasing in h.*

Proof From the proof of Theorem 2, we have that z_2^* satisfies

$$G(z_2^*) = (1-\varphi)p_2^*(z) + h - \frac{(1-\alpha)c + h}{1 - F(z_2^*)} = 0.$$

By the implicit function rule, we have

$$\frac{dz_2^*}{dh} = -\frac{\frac{\partial G(z_2^*)}{\partial h}}{\frac{\partial G(z_2^*)}{\partial z_2^*}},$$

where

$$\begin{aligned}\frac{\partial G(z_2^*)}{\partial h} &= \frac{b}{b-1}\frac{\Lambda(z_2^*)}{z_2^* - \Lambda(z_2^*)} - \frac{F(z_2^*)}{1-F(z_2^*)} \\ &= \frac{b}{b-1}\frac{\Lambda(z_2^*)}{z_2^* - \Lambda(z_2^*)} - \frac{(1-\alpha)cz_2^* + [(1-\alpha)(b-1)c + bh]\Lambda(z_2^*)}{(b-1)[(1-\alpha)c + h][z_2^* - \Lambda(z_2^*)]} \\ &= -\frac{(1-\alpha)c}{(b-1)[(1-\alpha)c + h]} < 0.\end{aligned}$$

Because $G(A) > 0,\ G(B) < 0$ and z_2^* solving $G(z_2^*) = 0$ is unique, $\partial G(z_2^*)/\partial z_2^* < 0$. Thus $dz_2^*/dh < 0$. ∎

If supplier takes charge of the overstocking, she will invest more when the unit dispose cost h is higher, so in this situation, she may choose to produce less product to save cost, then her optimal stocking factor of inventory is decreasing in h, which is obvious.

4.2 Retailer's Decision

Knowing the supplier will choose (p_2^*, z_2^*) according to Theorem 3 in response to a given revenue share allocation φ, the retailer decides φ to maximize his expected profit $\Pi_{2,r}$, which is given by

$$\begin{aligned}\Pi_{2,r}(\varphi) &= -\alpha c q_2^* + \varphi p_2^*\ E[\min\{q_2^*, D\}] \\ &= y(p_2^*)\{\varphi p_2^*[z_2^* - \Lambda(z_2^*)] - \alpha c z_2^*\}.\end{aligned} \tag{8}$$

We can further rewrite $\Pi_{2,r}(\varphi)$ as

$$\Pi_{2,r}(\varphi) = a(b-1)^{b-1}\left[\frac{z_2^* - \Lambda(z_2^*)}{(1-\alpha)bcz_2^* + bh\Lambda(z_2^*)}\right]^b g_2(\varphi), \tag{9}$$

where

$$g_2(\varphi) = (1-\varphi)^{b-1}\{[(b-\alpha)cz_2^* + bh\Lambda(z_2^*)]\varphi - \alpha(b-1)cz_2^*\}. \tag{10}$$

We find that z_2^* chosen by the supplier does not depend on φ, so maximizing $\Pi_{2,r}(\varphi)$ over φ is equivalent to maximizing $g_2(\varphi)$.

Theorem 4 *The optimal revenue share allocation for the retailer is unique, which is given by*

$$\varphi_2^* = \frac{(1+\alpha b - 2\alpha)cz_2^* + h\Lambda(z_2^*)}{(b-\alpha)cz_2^* + bh\Lambda(z_2^*)}. \tag{11}$$

Proof From (10) we can get

$$g_2'(\varphi) = (1-\varphi)^{b-2}\{[(b-\alpha)cz_2^* + \alpha(b-1)^2cz_2^* + bh\Lambda(z_2^*)] - b[(b-\alpha)cz_2^* + bh\Lambda(z_2^*)]\varphi\}.$$

Along the line in Theorem 2, we can finally find $g_2(\varphi)$ reaches its maximum at φ_2^* given by (11). ∎

We can show that z_2^* is decreasing in h, however, the way by which φ_2^* depends on h is more complex and is not monotone any more. What we can do is to find the upper bound or lower bound of φ_2^* for h.

Proposition 4 *Given* $0 < \alpha < 1,\ c > 0$ *and* $b \geq 2$, *we have*

i) *if* $h \geq 0$, *then* $\frac{1+\alpha b-2\alpha}{b-\alpha} \geq \varphi_2^* > \frac{1}{b}$;

ii) *if* $-c \leq h < 0$, *then* $\varphi_2^* > \frac{1+\alpha b-2\alpha}{b-\alpha} > \alpha$.

Proof Because $b - \alpha < (1+\alpha b - 2\alpha)b$, so $\varphi_2^* > \frac{1}{b}$, and

$$\frac{(1+\alpha b - 2\alpha)cz_2^* + h\Lambda(z_2^*)}{(b-\alpha)cz_2^* + bh\Lambda(z_2^*)} \leq (>) \frac{1+\alpha b - 2\alpha}{b-\alpha},$$

which is derived by following some straightforward calculus steps for $h \geq (<) 0$. ∎

Proposition 4 states that in equilibrium, the retailer always extracts more than his share of the channel cost when the supplier takes salvage from the leftovers. When the supplier costs in holding the overstock, the retailer will charge revenue share less than the critical revenue.

Combine (7), Theorems 3 and 4, we can derive the supplier's optimal selling price and profit, which are given as:

$$p_2^* = \frac{b[(b-\alpha)cz_2^* + bh\Lambda(z_2^*)]}{(b-1)^2[z_2^* - \Lambda(z_2^*)]}, \tag{12}$$

$$\Pi_{2,s}(p_2^*, z_2^*) = a(p_2^*)^{-b}\left[\frac{(1-\alpha)c}{b-1}z_2^* + \frac{h}{b-1}\Lambda(z_2^*)\right]. \tag{13}$$

Regrettably, we cannot clearly characterize how p_2^* and $\Pi_{2,s}(p_2^*, z_2^*)$ are affected by h as we do in Proposition 2, because the stock factor z_2^* is a function of h under this circumstance.

5 Retailer vs Supplier for Disposing Leftovers

In the section, we solve the supplier's first-stage problem: Decide which contract to accept or reject both. We begin our analysis by focusing on theoretical results first.

5.1 Theoretical Comparisons

In the following proposition, we compare the optimal stock decision of the supplier and the optimal revenue allocation of the retailer for two situations: Retailer disposes leftovers and supplier disposes leftovers. From Theorems 1 and 3, we have the following result.

Proposition 5 i) *If* $-c \leq h < 0$, *then* $z_1^* < z_2^*$ *and* $\varphi_1^* < \varphi_2^*$;

ii) *If* $h \geq 0$, *then* $z_1^* \geq z_2^*$ *and* $\varphi_1^* \geq \varphi_2^*$.

Proof Compare the second part of Theorem 1 and Theorem 3, we can get $z_1^* = z_2^*$ when $h = 0$. Combine Proposition 2 i) with Proposition 3, we can easily draw if $-c \leq h < 0$, then $z_1^* < z_2^*$, otherwise $z_1^* \geq z_2^*$.

From Proposition 1 ii) and Proposition 4 i), we have if $h \geq 0$, then $\varphi_1^* \geq \frac{1+\alpha b-2\alpha}{b-\alpha} \geq \varphi_2^*$. Compare Proposition 1 iii) and Proposition 4 ii), we have $\varphi_1^* < \frac{1+\alpha b-2\alpha}{b-\alpha} < \varphi_2^*$ under $-c \leq h < 0$. The proof is completed. ▮

For two different circumstances: Retailer disposes leftovers vs. supplier disposes leftovers. The supplier's optimal stocking factor of inventory in the latter case is greater than the corresponding value in the former case when dispose cost is negative. The reason is that if dispose cost appears in the form of salvage value, then one takes charge of the overstockings can compensate from the leftovers, so supplier will produce more if she disposes leftovers and vice versa. For the same reason, the retailer will extract less revenue share allocation if he can gain from the leftovers.

We cannot compare p_1^*, p_2^* and $\Pi_{1,s}(p_1^*, z_1^*), \Pi_{2,s}(p_2^*, z_2^*)$ as we do in Proposition 5, thus we will do simulations to continue our analysis.

5.2 Numerical Comparisons under Uniform Distribution

In this section, we analyze which consignment contract the supplier will choose when the random factor ε follows a uniform distributed on $[0, B]$ and assumes the product is elastic enough (i.e., $b \geq 2$). Obviously, they are both satisfied with the conditions of Theorem 1 and Theorem 3. Then we have $f(x) = 1/B,\ F(x) = x/B,$ and $\Lambda(x) = x^2/2B$.

From Theorem 1, (5) and (6), we can derive the optimal solution when retailer takes charge of the overstocks:

$$\begin{aligned} z_1^* &= \frac{2B}{b+1}, \\ p_1^* &= \frac{(b+1)(b-\alpha)c + (b-1)h}{(b-1)^2}, \\ \Pi_{1,s}(p_1^*, z_1^*) &= \frac{2(1-\alpha)acB}{(b+1)(b-1)}\left[\frac{(b+1)(b-\alpha)c + (b-1)h}{(b-1)^2}\right]^{-b}. \end{aligned} \tag{14}$$

If supplier takes charge of the leftovers, then we have the optimal stock factor which satisfies:

$$\frac{z}{B} = \frac{(1-\alpha)c + [(1-\alpha)(b-1)c + bh]\frac{z}{2B}}{(1-\alpha)bc + (b-1)h + h\frac{z}{2B}},$$

so if $h = 0$, we have $z_1^* = z_2^*,\ p_1^* = p_2^*$ and $\Pi_{1,s}(p_1^*, z_1^*) = \Pi_{2,s}(p_2^*, z_2^*)$; otherwise, we let

$$\delta = \sqrt{(1-\alpha)^2(b+1)^2c^2 + 2(1-\alpha)(b^2-b+2)ch + (b-2)^2h^2} - (1-\alpha)(b+1)c - (b-2)h,$$

and

$$\begin{aligned} z_2^* &= \frac{B\delta}{2h}, \\ p_2^* &= \frac{b[4(b-\alpha)ch + bh\delta]}{(b-1)^2(4h-\delta)}, \\ \Pi_{2,s}(p_2^*, z_2^*) &= \frac{aB\delta}{8(b-1)h}[4(1-\alpha)c + \delta]\left[\frac{b[4(b-\alpha)ch + bh\delta]}{(b-1)^2(4h-\delta)}\right]^{-b}. \end{aligned} \tag{15}$$

We let x denote the ratio of dispose cost to the total channel cost h/c, after some algebra, we find a, B, c do not play important role in comparison between $\Pi_{1,s}(p_1^*, z_1^*)$ and $\Pi_{2,s}(p_2^*, z_2^*)$, so we fix $a = 5,\ B = 30,\ c = 3$. Since the shape of $\Pi_{2,s}(p_2^*, z_2^*)$ depends on x, the price sensitivity b and the retailer's cost share α, we set x as X-axis, from practical point x was varied from -1 to 10 in steps of 0.11. We label profit as Y-axis and depict $\Pi_{1,s}(p_1^*, z_1^*)$, $\Pi_{2,s}(p_2^*, z_2^*)$ in the form of solid line and broken line, respectively.

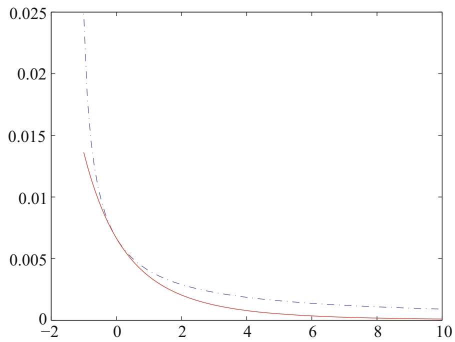

Figure 1 $\alpha = 0,\ b = 5$

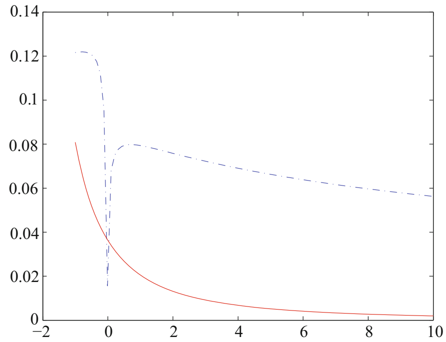

Figure 2 $\alpha = 0.99,\ b = 2$

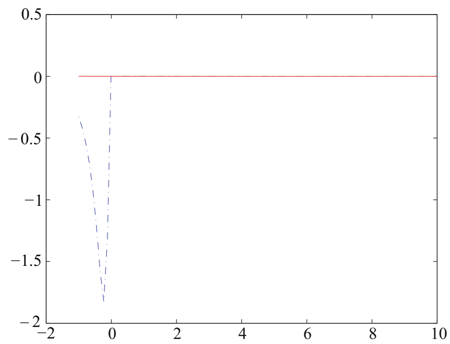

Figure 3 $\alpha = 0.99,\ b = 5$

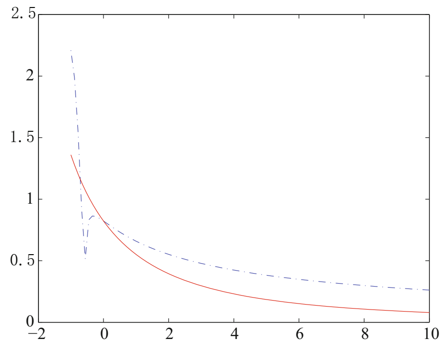

Figure 4 $\alpha = 0.5,\ b = 2$

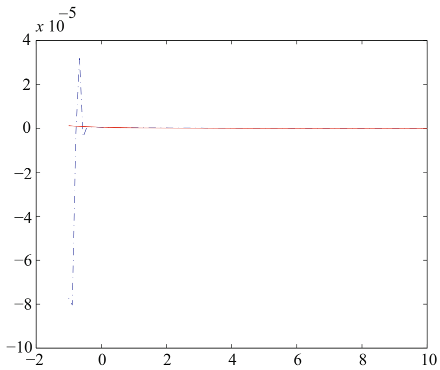

Figure 5 $\alpha = 0.5,\ b = 12$

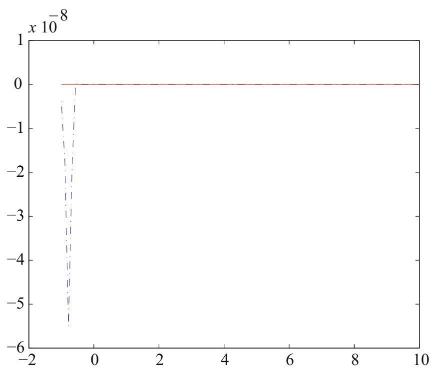

Figure 6 $\alpha = 0.5,\ b = 22$

We run simulations by choosing $\alpha \in \{0, 0.5, 0.99\}$ and changing the value of b. When $\alpha = 0$, $\Pi_{2,s}(p_2^*, z_2^*)$ always lies above $\Pi_{1,s}(p_1^*, z_1^*)$ for all $b \geq 2$. We take $b = 5$ as an example in Figure 1. So if the retailer does not incur any cost, the supplier always chooses to take charge of the overstockings herself. When $\alpha = 0.99$, the curves perform different in price sensitivity b. when $2 \leq b < 5$ the curves perform similarly, for $b \geq 5$ the overall trends are almost the same. We offer two typical figures for $b = 2, 5$, which are Figures 2 and 3. We notice that if retailer incurs almost the total channel cost, when $2 \leq b < 5$ only for x in a very small range of $[-1, 0]$, the supplier will choose the first contract. Otherwise, it is optimal for her to accept the second contract. If $b \geq 5$, it is desirable for the supplier to reject both contracts. When $\alpha = 0.5$, the curves can be divided into three classes: $2 \leq b < 12, 12 \leq b < 22, b \geq 22$. In each class, the trends of figure are extreme similar, we present $b = 2, 12, 22$ as typical examples. From Figure 4, we know for x in a very small range of $[-1, 0]$, the supplier will choose the first contract; Otherwise, she will accept the second contract. In Figure 5, only for x in a very small range of $[-1, 0]$ the supplier will choose the second contract, otherwise, she will reject both contracts. $\Pi_{1,s}(p_1^*, z_1^*)$ is approximate to 0 and $\Pi_{2,\ s}(p_2^*, z_2^*)$ always lies below 0 in Figure 6, so we can conclude if the product is more sensitive to a change in price, the supplier will incline to reject the two contracts.

Remark i) If $\alpha = 0$, then the supplier will always choose herself dispose the leftovers, no matter whether x is positive or negative, and the price elasticity index does not affect her decision.

ii) When α is approximate to 1, if $b \geq 5$, the supplier will reject both of the contracts; if $2 \leq b < 5$ only for x in a small range of $[-1, 0]$, the supplier will choose the first contract, otherwise, the supplier prefers to the second contract.

iii) If $\alpha = 0.5$, then the best strategy depends on b and x, when $b < 22$ for different x, the supplier takes different actions. Because the curves perform very similar for $b \geq 22$, thus if the product is elastic enough, the supplier will not accept any contract.

iv) We also consider ε follows a normal distribution $N(20, 3^2)$ truncated on $[0, 30]$ with $a = 5,\ c = 3$. We observe that with some minor differences exist, the overall trends of the two sets of figures are very similar. So here we omit the detailed figures from truncated normal distribution.

6 Concluding Remarks

In this paper, we study the consignment contract with revenue share where the retailer offers two revenue share schemes between them from the viewpoint of inventory ownership: One is the retailer takes charge of the unsold items and he deducts a percentage from the selling price. The other is that at the end of the selling period, the retailer returns the unsold items to the supplier. Then the supplier disposes those overstockings. In this situation, the retailer deducts a percentage from the selling price which may be different from the counterpart in the first one. We model the supplier's problem in two stages: a) She first chooses to accept one contract or rejects the two contracts, and the supplier accepts the contract as long as she can earn a positive profit; b) If she chooses one contract, she then decides the retail price and delivery quantity for her product according to the terms of the contract chosen in the previous stage.

We model the decision making of the two firms as a Stackelberg game: The retailer acting as the leader, offers the supplier two contracts, which specifies the percentage allocation of sales revenue between himself and his supplier and who will dispose the leftovers. The supplier acting as a follower, chooses which contract to be accepted, how many units of the product to produce and the retail price. We solve the problem backward. In our second-stage problem, we give the retailer's optimal revenue share allocation and the supplier's corresponding optimal retailer price and delivery quantity for both schemes. We derive some elegant properties with the optimal decisions: If the supplier chooses the first contract, then the retailer's optimal revenue share allocation is decreasing in dispose cost h; if h appears in the form of holding cost, the retailer charges a revenue share strictly bigger than his share of channel cost; and if h appears in the form of salvage value, he then charges revenue share which is less than the critical share $(1+\alpha b-2\alpha)/(b-\alpha)$. The supplier's optimal stocking factor is irrelevant with h, but her optimal retail price and profit is decreasing in h. If the supplier chooses the second contract, the retailer's optimal revenue share is not monotone with h any more, but if $h>0$ he charges revenue share smaller than the critical share. Otherwise, his revenue share is bigger than his share of channel cost. The supplier's optimal stocking factor is decreasing in h, but the way by which supplier's optimal price and profit depend on h is complicated and not monotone in general.

We characterize the supplier's first-stage problem by assuming ε follows a uniform distribution. The performance curve of $\Pi_{2,s}(p_2^*, z_2^*)$ depends on price-elasticity b and the retailer's share of total cost α, so we present our numerical experiments for different combinations of b and α. we show that when $\alpha=0$, the supplier will always choose the second contract, no matter h is positive or negative, and the price elasticity index has nothing to do with her decision; if α is approximate to 1, for $b\geq 5$ the supplier will reject both of the contracts. When $2\leq b<5$ only for h in a small range of $[-c,0]$ the supplier will choose the first contract. Otherwise, the supplier prefers to the second contract; for middle α the best strategy depends upon b and x. When $b<22$ for different h, the supplier takes different action, because for $b\geq 22$ the curves perform very similar, so when the product is elastic enough, the supplier chooses not to accept any contract.

There are several avenues for extension. An extension to the model is to consider the case of asymmetric information, for example, the two firms may not have full information about each other's cost. It would be interesting to explore how the insights gained based on the consignment contract with revenue sharing will change when different contracts are considered. Using the models in this paper, one can easily show that, after receiving the delivery quantity from the supplier, a retailer may exert a sales' effort and has incentive to sell them at a price higher than the price chosen by the supplier. So how to solve these problems challenge us.

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