Degree of polarization in Young's double-slit interference experiment formed by stochastic electromagnetic beams

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Received November 6, 2006; revised January 31, 2007; accepted February 22, 2007; posted March 7, 2007 (Doc. ID 76791); published June 13, 2007

We analyze the behavior of the degree of polarization in the interference field of Young's double-slit experiment. We analyze the degree of polarization in Young's double-slit interference experiment illuminated by stochastic electromagnetic beams. The distribution of the degree of polarization in the interference field for different correlation lengths and different slit widths is investigated. Furthermore, it is shown that the degree of polarization for a fixed observation point may take on values different from those it takes in the slits, depending not only on the value of the correlation length but also on the width of the slit. © 2007 Optical Society of America

OCIS codes: 030.1640, 260.5430.

1. INTRODUCTION

The degree of polarization is an important characteristic of the optical field. In the past few years a considerable number of papers have been published on the theory of the polarization of electromagnetic fields. It was found that the degree of polarization of a random electromagnetic beam may change on propagation [1–4].

Young's double-slit interference experiment is one of the most fundamental experiments of all physics; it is widely applied to physical optics and quantum optics. In recent years a great deal of research has been done concerning the degree of polarization and coherence in Young's interference experiment [5–11]. It is shown by Roychowdhury and Wolf that the degree of polarization in the observation plane may be different from that in the pinhole, depending on the value of the degree of coherence of the light incident on the pinholes [5]. However, how this changes the degree of polarization in the whole interference field was unknown until now. Moreover, to the best of our knowledge, in the previous papers the width of the slit of Young's double-slit interference experiment was neglected.

In this paper, we investigate the degree of polarization as well as the slit width in Young's interference experiment. We first derive an expression for the 2×2 electric cross-spectral density matrix of the electric field in the interference field in terms of the cross-spectral density matrix of the electric field in the slits. We then present the spatial distribution of the degree of polarization in the interference field. We specifically study the effect of the correlation length and the degree of polarization of light in the source plane, as well as the width of the slit, on the degree of polarization of the light in a Young's interference experiment. The distribution of the degree of polarization in the whole interference field is illustrated. It is found that the degree of polarization for a fixed point in the interference field may differ, in general, from the degree of polarization of the light in the slit; the difference depends not only on the correlation length in the slit but also on the width of slit, and the distribution of the degree of polarization experiences drastic change with the change in the correlation length in the slit and (or) the width of the slit.

2. THEORY

Young's double-slit interference experiment for theoretical analysis is shown in Fig. 1. Suppose that the two slits are placed across plane A, denoted as the $z=0$ plane. The two slits have identical widths, and the inner distance and outer distance are $2b$ and $2a$, respectively. The observation point (u,z) is located in plane B, which is parallel to plane A. We define the parameter $\varepsilon=b/a$ $(0\leqslant\varepsilon<1)$, representing the width of the two slits.

The cross-spectral density matrix of the electric field in the slit is defined as [12]

$$\overleftrightarrow{W}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=\begin{bmatrix} W_{xx}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) & W_{xy}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) \\ W_{yx}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) & W_{yy}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) \end{bmatrix}, \qquad (1)$$

where

$$W_{ij}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=\langle E_i^{*}(\boldsymbol{\rho}_1,\omega)E_j(\boldsymbol{\rho}_2,\omega)\rangle, \qquad (i=x,y;\ j=x,y). \qquad (2)$$

Here $E_i(\boldsymbol{\rho},\omega)$ and $E_j(\boldsymbol{\rho},\omega)$ are Cartesian components of the frequency component ω of the complex electric vector at a point specified by the transverse position vector $\boldsymbol{\rho}$, the asterisk denotes the complex conjugate, and the angle brackets denote the ensemble average.

We consider an electromagnetic Gaussian Schell-model beam [13,14] propagating close to the z axis. For such a

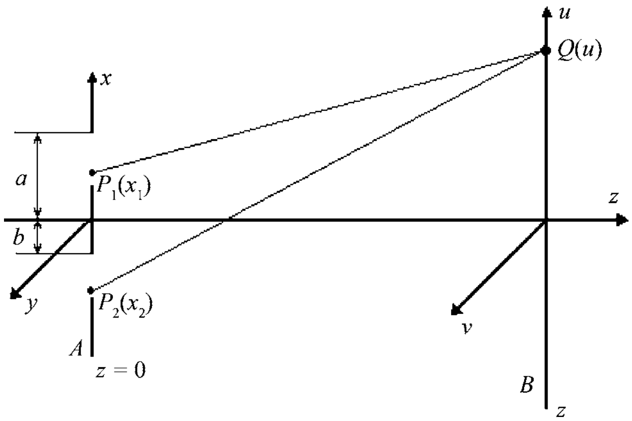


Fig. 1. Notation relating to Young's double-slit interference experiment.

beam, the elements of the cross-spectral density matrix of the field in the slit are given by

$$W_{ij}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=\sqrt{S_i^{(0)}(\boldsymbol{\rho}_1,\omega)}\sqrt{S_j^{(0)}(\boldsymbol{\rho}_2,\omega)}\mu_{ij}^{(0)}(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1,\omega), \tag{3}$$

where spectral densities of the electric field components are given by expression in the form

$$S_x^{(0)}(\boldsymbol{\rho},\omega)=A_x(\omega)\exp(-\boldsymbol{\rho}^2/2\sigma_x^2),$$

$$S_y^{(0)}(\boldsymbol{\rho},\omega)=A_y(\omega)\exp(-\boldsymbol{\rho}^2/2\sigma_y^2), \tag{4}$$

and the degree of coherence between the i and j components of the electric field has the form

$$\mu_{ij}^{(0)}(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1,\omega)=B_{ij}\exp(-|\boldsymbol{\rho}_2-\boldsymbol{\rho}_1|^2/2\delta_{ij}^2),$$

$$(i=x,y,\ j=x,y). \tag{5}$$

The parameter δ_{ij} is related to the correlation length, which represents the correlation between the i component of the electric field in one slit and the j component of the field in another slit. It can be found from Eq. (5) that the correlation length is associated with the degree of coherence. $\delta_{ij}\to 0$ corresponds to mutually incoherent, and $\delta_{ij}\to\infty$ corresponds to completely mutually coherent.

To simplify the analysis we will take

$$A_x(\omega)=A_y(\omega)\equiv A(\omega), \tag{6}$$

$$B_{ij}=1 \qquad (\text{if } i=j),$$

$$B_{ij}=0 \qquad (\text{if } i\neq j), \tag{7}$$

$$\sigma_x=\sigma_y\equiv\sigma. \tag{8}$$

Therefore, the elements of the matrix in Eq. (1) can be written as

$$W_{xx}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=A(\omega)\exp\left[-\frac{\boldsymbol{\rho}_1{}^2+\boldsymbol{\rho}_2{}^2}{4\sigma^2}\right]\exp\left[-\frac{(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)^2}{2\delta_{xx}{}^2}\right], \tag{9a}$$

$$W_{yy}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=A(\omega)B\exp\left[-\frac{\boldsymbol{\rho}_1{}^2+\boldsymbol{\rho}_2{}^2}{4\sigma^2}\right]\times\exp\left[-\frac{(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)^2}{2\delta_{yy}{}^2}\right], \tag{9b}$$

$$W_{xy}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=W_{yx}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=0, \tag{9c}$$

where the parameters B, σ, δ_{xx}, and δ_{yy} are independent of position. The degree of polarization of the field is given by the formula [12]

$$\mathcal{P}(\boldsymbol{\rho},\omega)=\sqrt{1-\frac{4\,\mathrm{Det}\overleftrightarrow{W}(\boldsymbol{\rho},\boldsymbol{\rho},\omega)}{[\mathrm{Tr}\,\overleftrightarrow{W}(\boldsymbol{\rho},\boldsymbol{\rho},\omega)]^2}}, \tag{10}$$

where Det denotes the determinant and Tr the trace.

On substituting Eqs. (1) and (9) into Eq. (10), one can readily obtain the polarization in the slit:

$$\mathcal{P}^{(0)}=\left|\frac{1-B}{1+B}\right|. \tag{11}$$

The elements of the cross-spectral density matrix at two points $(\mathbf{r}_1,z)$ and $(\mathbf{r}_2,z)$ in a transverse plane $z=$const. >0 can be written as

$$W_{ij}(\mathbf{r}_1,\mathbf{r}_2,\omega)=\left(\frac{k}{2\pi z}\right)^2\iint \mathrm{d}^2\boldsymbol{\rho}_1\iint \mathrm{d}^2\boldsymbol{\rho}_2 W_{ij}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)\times\exp\left[-ik\frac{(\boldsymbol{\rho}_1-\mathbf{r}_1)^2-(\boldsymbol{\rho}_2-\mathbf{r}_2)^2}{2z}\right], \tag{12}$$

where $k=2\pi/\lambda=\omega/c$ is the wavenumber associated with the frequency ω, λ being the wavelength and c the speed of light in vacuum.

Regarding Young's double-slit interference experiment shown in Fig. 1, we can rewrite Eq. (12) as

$$W_{ij}(u_1,u_2,z,\omega)=\left(\frac{k}{2\pi z}\right)\iint W_{ij}^{(0)}(x_1,x_2,\omega)\times\exp\left[-ik\frac{(x_1-u_1)^2-(x_2-u_2)^2}{2z}\right]\mathrm{d}x_1\mathrm{d}x_2. \tag{13}$$

On substituting Eq. (9) into Eq. (13), the elements of the cross-spectral density matrix $W(u_1,u_2,z,\omega)$ with $u_1=u_2=u$ are evidently given by the expression

$$W_{xx}(u,z,\omega)=\left(\frac{\omega a^2}{2\pi zc}\right)\iint_A A(\omega)\exp\left[-\frac{x_1^2+x_2{}^2}{4\sigma_0{}^2}\right]\times\exp\left[-\frac{(x_1-x_2)^2}{2\delta_{xx0}{}^2}\right]\times\exp\left[-i\frac{\omega a^2(x_1{}^2-x_2{}^2)-2u_0(x_1-x_2)}{2zc}\right]\times\mathrm{d}x_1\mathrm{d}x_2, \tag{14a}$$

$$W_{yy}(u,z,\omega)=\left(\frac{\omega a^2}{2\pi zc}\right)\int\int_A A(\omega)B \times\exp\left[-\frac{{x_1}^2+{x_2}^2}{4{\sigma_0}^2}\right]\exp\left[-\frac{(x_1-x_2)^2}{2{\delta_{yy0}}^2}\right] \times\exp\left[-i\frac{\omega a^2({x_1}^2-{x_2}^2)-2u_0(x_1-x_2)}{2zc}\right] \times \mathrm{d}x_1\mathrm{d}x_2, \quad (14b)$$

$$W_{xy}(u,z,\omega)=W_{yx}(u,z,\omega)=0, \quad (14c)$$

where $\sigma_0=\sigma/a$, $\delta_{xx0}=\delta_{xx}/a$, $\delta_{yy0}=\delta_{yy}/a$, $\varepsilon=b/a$, and $u_0=u/a$.

Finally, we obtain for the degree of polarization at a point $[u,z]$

$$\mathcal{P}(u,z,\omega)=\frac{W_{xx}(u,z,\omega)-W_{yy}(u,z,\omega)}{W_{xx}(u,z,\omega)+W_{yy}(u,z,\omega)}. \quad (15)$$

It is evident from Eqs. (14) and (15) that the degree of polarization for a point $Q(u,z)$ in the interference field depends on the following four factors:

(1) The position where the observation point was located.
(2) The degree of polarization $\mathcal{P}^{(0)}$ in the slit.
(3) The correlation length δ_{xx0} and δ_{yy0}.
(4) The parameter ε, which denotes the width of the slit.

3. NUMERICAL CALCULATION RESULTS

We will now illustrate the results by some numerical examples. The figures show the behavior of the degree of po-

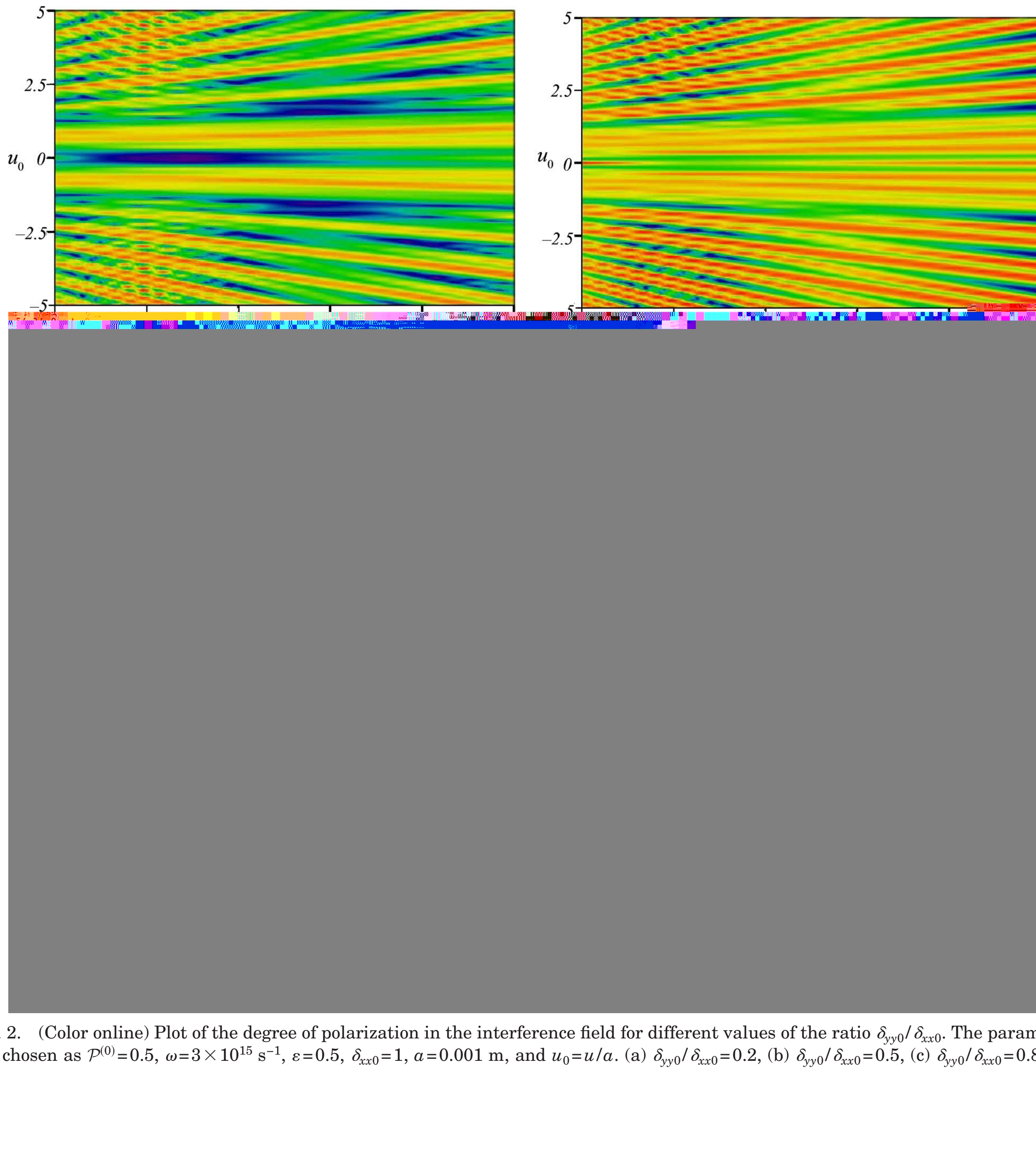


Fig. 2. (Color online) Plot of the degree of polarization in the interference field for different values of the ratio $\delta_{yy0}/\delta_{xx0}$. The parameters are chosen as $\mathcal{P}^{(0)}=0.5$, $\omega=3\times10^{15}\ \mathrm{s}^{-1}$, $\varepsilon=0.5$, $\delta_{xx0}=1$, $a=0.001$ m, and $u_0=u/a$. (a) $\delta_{yy0}/\delta_{xx0}=0.2$, (b) $\delta_{yy0}/\delta_{xx0}=0.5$, (c) $\delta_{yy0}/\delta_{xx0}=0.8$.

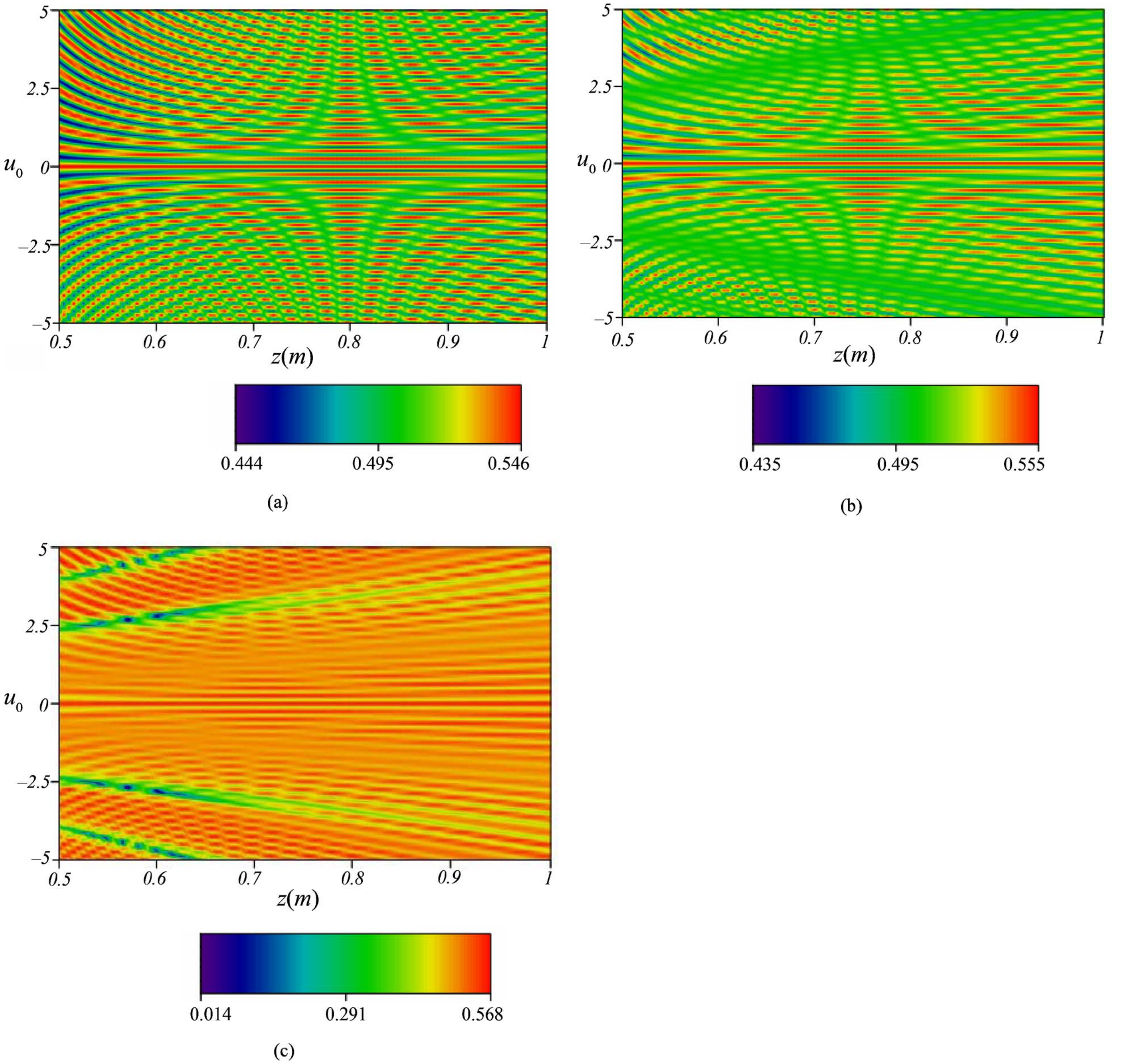


Fig. 3. (Color online) Plot of degree of polarization $\mathcal{P}$ in the interference field for different values of the slit width denoted by ε. (a) $\varepsilon=0.999$, (b) $\varepsilon=0.9$, (c) $\varepsilon=0.8$. The other parameters are the same as in Fig. 2 except that $\delta_{yy0}=0.5\delta_{xx0}$.

larization of the light in the interference field. In Figs. 2 and 3 we present the distribution of the degree of polarization in the interference field. The curves in Figs. 4–6 show the effect of the correlation length and the degree of polarization in the source plane and the effect of the slit width on the degree of polarization of the light in the observation point.

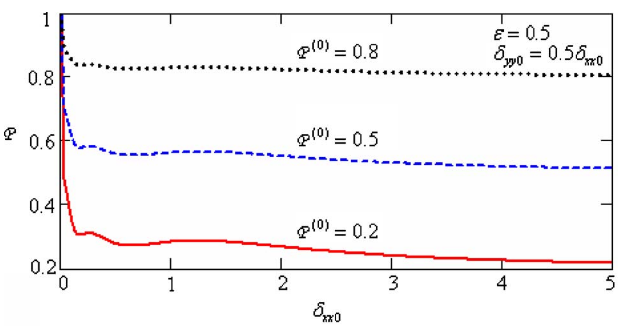
Figure 2 shows the behavior of the degree of polarization in the interference field, calculated from Eq. (15), for the case of $\mathcal{P}^{(0)}=0.5$ (i.e., the case of partially polarized light) and for different values of the ratio $\delta_{yy0}/\delta_{xx0}$. From Fig. 2, we can get a clear picture of the distribution of the degree of polarization in the interference field. It is interesting to find that for the case $\delta_{yy0}/\delta_{xx0}=0.2$ and $\delta_{yy0}/\delta_{xx0}=0.5$, the degree of polarization may drop to zero, which means that light that emerges from a partially polarized source may become completely unpolarized at some position of the interference field.

In Fig. 3, we plot the distribution of the degree of polarization in the interference field for different values of the parameter ε for the case of $\mathcal{P}^{(0)}=0.5$. The parameters of the slit in Fig. 3(a) are chosen as $\varepsilon=0.999$, which can be considered as the width of the slit that is neglected. For a fixed observation plane ($z=$const.) in Fig. 3(a), the degree of polarization reaches the maximum ($\mathcal{P}=0.546$) and minimum ($\mathcal{P}=0.444$) with periodicity as the observation point goes away from the axis. Moreover, the maximum and the minimum keep the same value regardless of the location of the observation plane, which is similar to the discussion in [9]. In Figs. 3(b) and 3(c) the parameters are chosen as $\varepsilon=0.9$ and $\varepsilon=0.8$. By comparing these figures with Fig. 3(a), we find that the behavior of the degree of polarization in the interference field is different; the maximum value will become larger and the minimum value will become smaller as ε becomes smaller (corresponding to the wider slit). We also find that the degree of polarization of the on-axis observation point stays invariant for a certain slit width.

Next we will discuss the variation of the degree of polarization for a fixed observation point ($u_0=1$, $z=1$ m) as the slit width, the correlation length, and the degree of polarization in the slit varied. The other parameters for these figures are chosen as $\omega=3\times10^{15}\ \mathrm{s}^{-1}$ and $a=0.001$ m.

Figure 4 presents the degree of polarization for the fixed observation point ($u_0=1$, $z=1$ m) as a function of the correlation length δ_{xx0}. As shown in Eq. (9), $\delta_{xx0}=0$ corresponds to completely incoherent, while $\delta_{xx0}\rightarrow\infty$ corresponds to fully coherent. It can be found from Fig. 4(a), regardless of the degree of polarization in the slits, that degree of polarization in the observation point is equal to unity when $\delta_{xx0}=0$. However, when $\delta_{xx0}\rightarrow\infty$, the degree of polarization in the observation point is equal to that of the slit. The same results can be also found in Fig. 4(b), where the curves are obtained in the case of different slit widths.

The degree of polarization as a function of the degree of polarization in the slit for the fixed observation point ($u_0=1$, $z=1$ m) is shown in Fig. 5. It can be seen in Fig. 5(a) that the degree of polarization in the observation point is equal to that in the slit when $\delta_{yy0}=\delta_{xx0}$, indicating that the correlation in the x component is the same as in the y component. The different curves in Fig. 5(b) correspond to different values of the slit width.



(a)

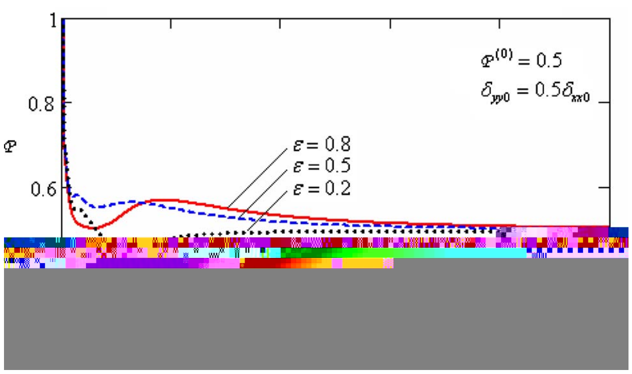
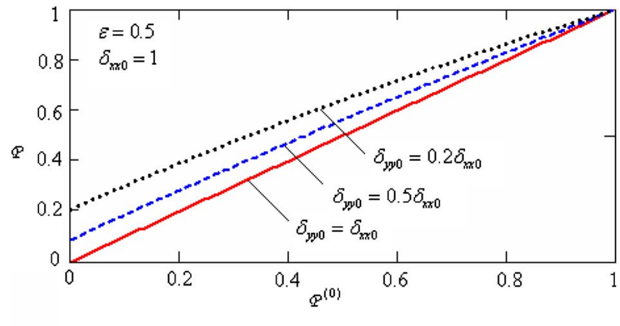
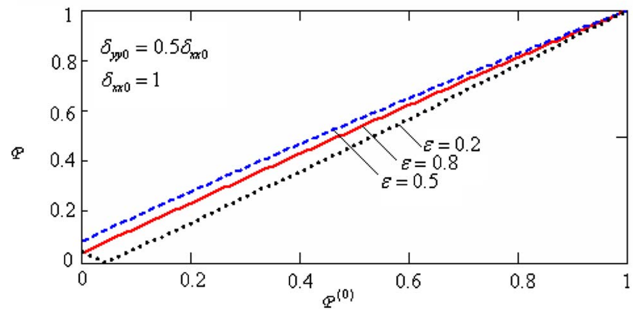


Fig. 4. (Color online) Degree of polarization as a function of correlation length for the fixed observation point ($u_0=1$, $z=1$ m). The curves in (a) are associated with different values of the parameter $\mathcal{P}^{(0)}$, which characterize the degree of polarization in the slit. The curves in (b) are associated with different values of the parameter ε, which characterize the width of the slit.



(a)



(b)

Fig. 5. (Color online) Degree of polarization as a function of degree of polarization in the slit for the fixed observation point ($u_0=1$, $z=1$ m). The curves in (a) are associated with different values of the parameter $\delta_{yy0}/\delta_{xx0}$. The curves in (b) are associated with different values of the parameter ε.

Fig. 6. (Color online) Degree of polarization as a function of the slit width for a fixed observation point ($u_0=1$, $z=1$ m). The curves in (a) are associated with different values of the parameter $\delta_{yy0}/\delta_{xx0}$. The curves in (b) are associated with different values of the parameter $\mathcal{P}^{(0)}$.

Finally, we show in Fig. 6 the variation of the degree of polarization in the fixed observation point ($u_0=1$, $z=1$ m) with ε. We can see from Fig. 6(a) that the degree of polarization in the observation point is invariant when $\delta_{yy0}=\delta_{xx0}$, even when the slit width is changing, which is similar to the results obtained from the solid curve in Fig. 5(a). In addition, the degree of polarization oscillates as the slit width varies, as shown in Figs. 6(a) and 6(b) (dashed and dotted curves). This indicates that for a fixed observation point in the interference field, the degree of polarization may change with change in slit width.

4. CONCLUSIONS

In this paper, we have investigated the behavior of the degree of polarization in the interference field of Young's double-slit pattern. The results show that the degree of polarization in the interference field may be equal to zero in certain cases, differing from that in the slit. It has been found that the degree of polarization for a fixed observation point in the interference field may change with the change of some parameters, such as the slit width, the correlation length, and the degree of polarization in the slit. However, if $\delta_{yy0}=\delta_{xx0}$, the degree of polarization will remain the same, equal to that in the slit, even when the slit width is changed.

ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China (grant 60477041) and the Fujian Natural Science Foundation of China (grant A0510018).

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