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Analysis of Dynamic Spectrum Management for Secondary Network

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Abstract

Cognitive radios (CR) have a great potential to improve spectrum utilization by enabling users to access the spectrum dynamically without disturbing licensed primary users. Spectrum management is an important issue for spectrum sharing. The focus of this paper is spectrum management in secondary network, considering a non cooperative framework. Price utility function based power allocation will be analyzed and robust optimization will be involved in this paper.

Keywords: Cognitive radio, spectrum management, game theory, robust optimization

1. introduction

Cognitive radios (CR) is established in a hierarchical structure, in which differentiate primary users(PUs), who are licensed spectrum holders, from secondary users (SUs), who are permitted to share the licensed spectrum with PUs under the constraints of not introducing intolerable interference to PUs[1]. The cognitive radio comprises two basic operations: radio analysis at the receiver and spectrum management or power allocation at the transmitter. Iterative water-filling (IWF) was first proposed in 2002, where a non-cooperative game was used to model the spectrum management problem with each user maximizing its own rate[2]. It was illustrated that a Nash-equilibrium game such as IWF problem can be reformulated as a variational inequality (VI) problem[3]. The problem about pricing for uplink power control and price based spectrum management in cognitive radio networks was analyzed in [4] and [5]; however, the research did not involve the dynamic problem. In this paper, we study the problem of power allocation in CR networks involving pricing utility and robust optimization due to activities.

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2. System Model

Suppose that the bandwidth shared by the PUs and SUs can be divided into K subcarriers with assuming a frequency-flat fading channel. For the secondary network, the cross-channel transfer function over the k th subcarrier between transmitter of SU i and receiver of SU j is denoted by h_k^{ij}, for $k \in \{1,\cdots,K\}$ and $i, j \in \{1,\cdots,N\}$. The transmit strategy of each SU i is given by the following constraints

$$\mathbf{P^i} = \{p_k^i : \sum\nolimits_{k=1}^{K} p_k^i \le p_{\max}^i ,\ 0 \le p_k^i \le p_k^{peak}\} \tag{1}$$

Where $p_{\max}^i$ is the sum power budget, p_k^{peak} is the peak power limit. Let δ_k^i denote the total noise plus interference from PUs to SU i over channel k. For simplification, the noise plus interference experienced by user i at subcarrier k because of the transmission of other users is

$$I_k^i = \sigma_k^i + \sum\nolimits_{j \ne i} \alpha_k^{ij} p_k^j \ .\ \ \textit{Where}\ \ \sigma_k^i = \delta_k^i \Big/ \left|h_k^{ii}\right|^2 ,\ \ \alpha_k^{ij} = \left|h_k^{ij}\right|^2 \Big/ L \left|h_k^{ii}\right|^2 . \tag{2}$$

The constraint from the primary network is given by $p_k^i + I_k^i \le MIC_k$. While primary service providers charge SUs on the resource consumption such as transmission power can cause interference to PUs. Thus we consider pricing in power allocation problem during the interaction of primary users and secondary users. Thus, the utility of SU is given by

$$u^i = \log(1 + p_k^i / I_k^i) - c_k^i p_k^i \tag{3}$$

3. Game-Theoretic Analysis for Spectrum Sharing Problem

3.1. Pricing and Power Control

On the basis of the pricing utility function in (3), each SU iteratively chooses its power vector to maximize subject to the constraints listed in section 2. Non-cooperative game let SUs to solve the following problem

$$\begin{aligned} &\max_{\mathrm{P}^i} \quad \mathbf{u}^i\left(\mathrm{P}^1,\ldots,\mathrm{P}^N\right) = \sum_{k=1}^{K}\left\{\log(1+p_k^i/I_k^i) - c_k^i p_k^i\right\} \\ &\textit{subject to}:\ \sum\nolimits_{k=1}^{K} p_k^i \le p_{\max}^i ;\ p_k^i + I_k^i \le MIC_k ;\ p_k^i \ge 0 \end{aligned} \tag{4}$$

The Lagrangian function of the optimization problem in (4) for the user i is written as

$$\mathbf{L}\left(\mathrm{P}^1,\ldots,\mathrm{P}^N\right) = -\mathbf{u}^i + \mu^i\left(\sum\nolimits_{k=1}^{K} p_k^i - p_{\max}^i\right) + \sum \gamma_k^i\left(\sigma_k^i + \sum\nolimits_{j=1}^{N}\alpha_k^{ij} p_k^j - MIC_k\right) + \sum \lambda_k^i p_k^i \tag{5}$$

The Karush–Kuhn–Tucker (KKT) conditions[6] for the user i are as follows (where $x \perp y$ means that the two scalars (or vectors) a and b are orthogonal)

$$\begin{aligned} &0 \le p_k^i \perp -1\Big/(p_k^i + \sigma_k^i + \sum\nolimits_{j \ne i}\alpha_k^{ij} p_k^j) + c_k^i + \mu^i + \gamma_k^i \ge 0 \\ &0 \le \mu^i \perp (p_{\max}^i - \sum\nolimits_{k=1}^{K} p_k^i) \ge 0 \\ &0 \le \gamma_k^i \perp MIC_k - \sigma_k^i - \sum\nolimits_{j=1}^{N}\alpha_k^{ij} p_k^j \ge 0 \end{aligned} \tag{6}$$

Proposition : If $c_k^i \le 1\Big/ p_k^i + \sigma_k^i + \sum\nolimits_{j \ne i}\alpha_k^{ij} p_k^j$, A Nash equilibrium game can be reformulated as a VI problem: $\mathbf{P}^* = (\mathrm{P}^{*1^{\mathrm{T}}},\ldots,\mathrm{P}^{*N^{\mathrm{T}}})^{\mathrm{T}}$ is a Nash equilibrium of the game if it is a solution of the following VI problem: $(\mathrm{P} - \mathrm{P}^*)^{\mathrm{T}}\mathbf{F}(\mathrm{P}^*) \ge 0$

Proof: To avoid triviality, we assume that

$$\sum_{j=1}^{N} p_{\max}^{i} < \sum (MIC_k - \sigma_k^i - \sum_{j\neq i} \alpha_k^{ij} p_k^j), \quad \forall i = 1,\ldots,N \tag{7}$$

Since power is nonnegative and it is known that:

$$\sigma_k^i + \sum_{j=1}^{N} \alpha_k^{ij} p_k^j > 0 \ \ \forall k = 1,\ldots,K. \tag{8}$$

If $\mu^i = 0$, then $c_k^i + \gamma_k^i \geq 1\big/(p_k^i + \sigma_k^i + \sum_{j\neq i} \alpha_k^{ij} p_k^j)$. As $c_k^i \leq 1\big/(p_k^i + \sigma_k^i + \sum_{j\neq i} \alpha_k^{ij} p_k^j)$, so $\gamma_k^i > 0$ leads to $\sum_{j=1}^{N} \alpha_k^{ij} p_k^j = MIC_k - \sigma_k^i$. For $0 < \alpha_k^{ij} < 1$, we have $p_{\max}^i \geq \sum_{k=1}^{K} p_k^i = \sum (MIC_k - \sigma_k^i - \sum_{j\neq i} \alpha_k^{ij} p_k^j)$ which contradicts (7). So we must have $\mu^i > 0$, changing the first constraint of (6), we can get

$$-\frac{1 - c_k^i (p_k^i + \sigma_k^i + \sum_{j\neq i} \alpha_k^{ij} p_k^j)}{p_k^i + \sigma_k^i + \sum_{j\neq i} \alpha_k^{ij} p_k^j} + \mu^i + \gamma_k^i \geq 0\,, \quad \begin{cases} \eta^i = \mu^i (p_k^i + \sigma_k^i + \sum_{j\neq i} \alpha_k^{ij} p_k^j) \\ \varphi_k^i = \gamma_k^i (p_k^i + \sigma_k^i + \sum_{j\neq i} \alpha_k^{ij} p_k^j) \end{cases} \tag{9}$$

So we can rewrite first constraint of (6) as

$$\begin{aligned} & 0 \leq p_k^i \perp 1 - c_k^i (p_k^i + \sigma_k^i + \sum_{j\neq i} \alpha_k^{ij} p_k^j) + \eta^i + \varphi_k^i \geq 0 \\ & \eta^i \ \mathit{free}, \quad p_{\max}^i = \sum_{k=1}^{K} p_k^i \\ & 0 \leq \varphi_k^i \perp MIC_k - \sigma_k^i - \sum_{j=1}^{N} \alpha_k^{ij} p_k^j \geq 0 \end{aligned} \tag{10}$$

Holding $\mu^i = \eta^i \big/ (\sigma_k^i + \sum_{j=1}^{N} \alpha_k^{ij} p_k^j)$, $\gamma_k^i = \varphi_k^i \big/ (\sigma_k^i + \sum_{j=1}^{N} \alpha_k^{ij} p_k^j)$. Thus, the mixed linear complementarity problem (MLCP) (6) is the KKT condition for an affine variational inequality (AVI) problem[7], defined by the affine mapping

$$\mathbf{F}(\mathrm{P}) = \mathbf{I} - (\boldsymbol{\sigma} + \boldsymbol{\Psi}\mathbf{P}) \tag{11}$$

The variables in (12) can be concatenated as follows

$$\mathbf{p} = \left[\begin{pmatrix} p_1^1 \\ \vdots \\ p_K^1 \end{pmatrix} \begin{pmatrix} p_1^2 \\ \vdots \\ p_K^2 \end{pmatrix} \cdots \begin{pmatrix} p_1^N \\ \vdots \\ p_K^N \end{pmatrix} \right]^T, \ \boldsymbol{\sigma} = \left[\begin{pmatrix} c_1^1 \sigma_1^1 \\ \vdots \\ c_K^1 \sigma_K^1 \end{pmatrix} \begin{pmatrix} c_1^2 \sigma_1^2 \\ \vdots \\ c_K^2 \sigma_K^2 \end{pmatrix} \cdots \begin{pmatrix} c_1^N \sigma_1^N \\ \vdots \\ c_K^N \sigma_K^N \end{pmatrix} \right]^T \tag{12}$$

$$\boldsymbol{\Psi} = \begin{bmatrix} \boldsymbol{\psi}^{11} & \cdots & \boldsymbol{\psi}^{1N} \\ \vdots & \ddots & \vdots \\ \boldsymbol{\psi}^{N1} & \cdots & \boldsymbol{\psi}^{NN} \end{bmatrix}, \ \boldsymbol{\psi}^{ij} = \begin{bmatrix} c_1^i \alpha_1^{ij} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & c_N^i \alpha_N^{ij} \end{bmatrix} \tag{13}$$

3.2. Robust Optimization Approach

In traditional research, many users co-exist over a long period of time. But in fact, the cognitive radio environment is dynamic. Due to the activities, the interference-plus-noise term in the utility function is time-varying. Similar to paper [8], we let the noise-plus-interference $\tilde{I}_k^i$ term be the summation of two components: a nominal term I_k^i and a perturbation term Δe_k^i which is caused by dynamic of user set and imperfect CSI and Δe_k^i belongs to an elliptical uncertainty region.

In the sense described in [8] is basically a max-min problem[9], [10], in which each user tries to maximize its own utility while the environment and the other users are trying to minimize that user's utility. The problem (7) can be formulated as the following optimization problem:

$$\max_{\mathrm{P}^i} \left\{ \min_{\|\Delta \mathrm{I}^i\| \le \varepsilon} \sum_{k=1}^{K} \left[\log(1+p_k^i / \tilde{I}_k^i) - c_k^i p_k^i \right] \right\}, \quad \tilde{I}_k^i = I_k^i + \Delta e_k^i$$
$$\text{subject to}: \sum_{k=1}^{K} p_k^i \le p_{\max}^i; \; \max_{\|\Delta \mathrm{I}^i\| \le \varepsilon} (p_k^i + \tilde{I}_k^i) \le MIC_k; \; p_k^i \ge 0 \tag{14}$$

From the propositions above, we can use the following distributed algorithms in which the SUs sequentially or simultaneously update their power allocation by solving the optimization problem iteratively to reach the NE. Each SU determines its best behavior by measuring the total noise-plus-interference over all channels. The best response of user is to maximize its utility function (15) subject to the constraints. The same procedure is repeated for all users in the secondary network. The optimization problem can be solved using lots of mathematical method, which is no longer described due to the limitation of length.

4. Numerical Results

In this section, numerical results are now illustrated to support the theoretical analysis of the previous sections. In our experiments, similar to [3], the background noise levels σ_k^i and the normalized interference gains α_k^{ij} are chosen randomly from the intervals $\left(0, 0.1/(N-1)\right)$, $\left(0, 1/(N-1)\right)$, respectively. The total power budgets $p_{\max}^i$ are chosen uniformly from the interval $(K, K/2)$. The chosen values for noise levels and power constraints guarantee that the worst case p_k^i / σ_k^i per subcarrier will be close to 7 dB. We select pricing coefficient c_k^i using in [5], and the formulation can be denoted as

$$c_k^i = \sum\nolimits_{j \ne i} \frac{p_k^j \alpha_k^{ji}}{(\sigma_k^j + \sum\nolimits_{n \ne j} \alpha_k^{jn} p_k^n)(\sigma_k^j + \sum \alpha_k^{jn} p_k^n)} \tag{15}$$

We set up a scenario that addresses a network with $N = 4$ nodes and $K = 2$ available subcarriers, and all of the SUs simultaneously update their transmit powers by our algorithm. At the fifth time-step, a new user joins the network, which increases the interference to the network. The interference gains are also changed randomly at different time instants to consider mobility of the users. The transmit power control simulation performance in a cognitive radio network using the non robust price based algorithm in [5] and our robust approach were presented in this section.

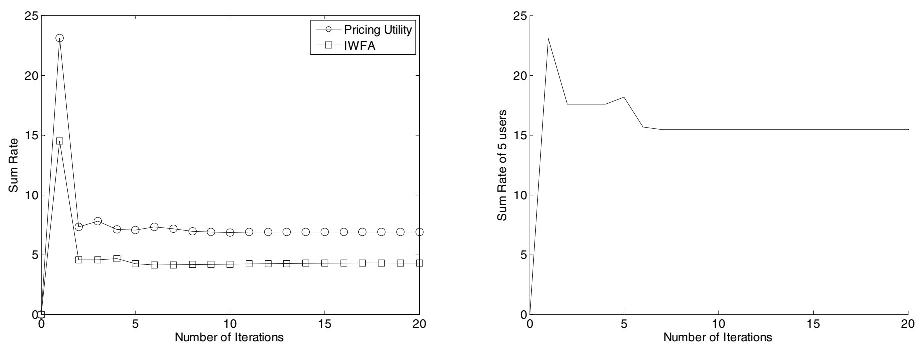

Fig. 1. (a) Sum rate comparison (b) Total data rate with robust algorithm

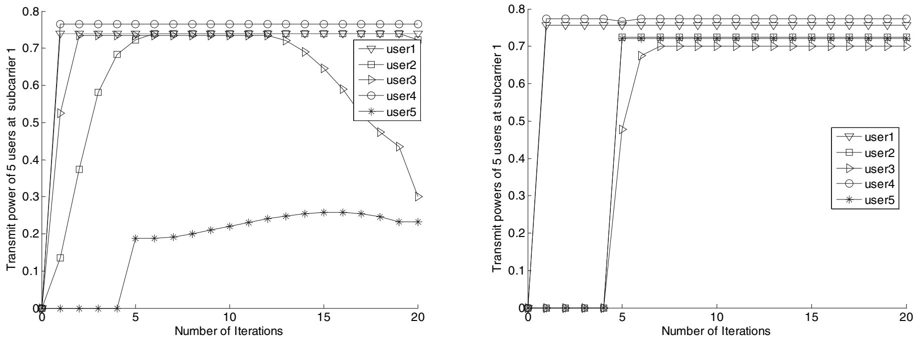

Fig. 2. (a) Transmit powers with non-robust approach (b) Transmit powers with robust approach

Fig. 1(a) illustrated sum rate of pricing utility based algorithm versus classical IWFA. Total data rate of network was demonstrated in fig. 1(b). Fig. 2(a) showed the transmit powers of five users at the first subcarrier using the non robust algorithm in [5]. Obviously, the transmit power of some users in the network couldn’t achieve convergence due to the dynamic nature. The transmit powers allocation using our robust approach was illustrated in Fig. 2(b).

5. Conclusion

In this paper we have formulated a game problem for the pricing issue in a competitive secondary network. Moreover, we have designed a robust CR optimization problem, by taking into account of dynamic characteristic of CR with primary network and multiple non-cooperative SUs. We also proposed an iterative algorithm for the secondary network to achieve the competitive optimality. The best response of each SU has been obtained by solving the optimization problem. As a future work, we are interested in further analyzing the mobility of multiple SUs in the cognitive network.

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