AC Alternating-Current Loss Analyses of a Thin High-Temperature Superconducting Tube Carrying AC Transport Current in AC External Magnetic Field *

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Numerical simulations on the AC loss characteristics in a thin high-temperature superconducting (HTS) tube are presented. Geometry of the HTS conductor is modeled as a tube with negligible thickness, and assumed to carry a transport current with the same phase as an AC externally applied magnetic field perpendicular to the axis. Based on the classical theory of AC loss with the Bean critical current model, the distribution of critical current density j_c *and AC loss* Q *are obtained by means of numerical analysis. The results are in very good agreement with experiments. A double-peak profile is observed in the curve of the critical current density distribution along the azimuth angle. This numerical simulation method is suitable for a thin HTS tube, which may be applicable on a thin tube configuration consisting of coated superconductors.*

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With the development of superconducting materials, BSCCO tapes and YBCO CC have been realized to be valuable in the field of commercialization. In recent years, much progress has been making on new types of superconductors.[1−3] Compared with conventional power equipment, the high temperature superconducting (HTS) one is compact, efficient, energy saving, and therefore has promising applications in the electrical power industry. In recent years, HTS cables,[4,5] transformers,[6−8] fault current limiters (FCL),[9,10] generators and motors,[11] superconducting magnetic energy systems (SMES), etc. have been successively demonstrated.

The AC loss in an HTS conductor is described well by the critical state theory, originally developed by Bean[12] for a strip geometry and later extended by many other researches in recent decades. The AC loss due to an AC transport current flowing in a thin HTS strip was first calculated by Norris.[13] Several explicit analytical equations for AC loss as functions of perpendicular and parallel external magnetic fields are given in the case of carrying AC current.[14,15] Those equations have successfully been fitted to experimental data obtained from measurements on coated-conductor type.[16]

A thin HTS tube is frequently used in power devices with a large current capacity such as power transmission coated superconductors. Although many studies on the AC loss of the cylindrical surface HTS tapes have been presented,[17] the AC loss behavior of an HTS conductor with a thin tube has not been established because of its complexities.[18]

In this Letter, we analyze the AC loss of a thin HTS tube carrying an AC current in an AC magnetic field with the same phase as the current by means of a numerical method based on the critical current state and a thin strip approximation.

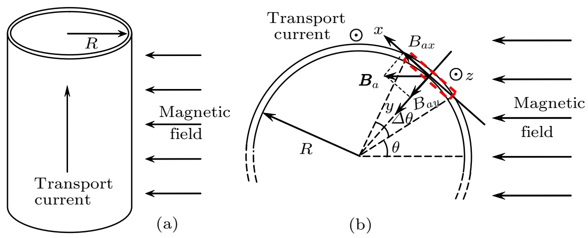

Fig. 1. (a) A thin HTS tube carrying an AC current and exposed to an AC magnetic field with the same phase as the current. (b) Schematic illustration of numerical model for a thin HTS tube.

The side view and cross-section of the thin HTS tube in numerical analysis are shown in Figs. 1(a) and 1(b), respectively. The thin tube is divided into $N = 100$ units along the azimuth direction, each unit can be regarded as an infinitely long HTS strip of finite thickness d and width $2w = R\Delta\theta$. Here the following assumptions are considered: every thin HTS strip fills the space $|x| \le w$, $|z| < \infty$, $|y| < d/2$ with $d \ll w$ and the thickness of the strip d exceeds the London penetration depth (as shown in Fig. 2), the AC external magnetic field with the peak value B_a above the strip is applied at a angle θ to the y axis ($B_{ax} = B_a \sin\theta$,

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$B_{ay} = B_a \cos\theta$, $B_{az} = 0$). Thus the perpendicular component $B_\perp$ of the magnetic field is

$$B_\perp = B_a \cos\theta. \tag{1}$$

The total AC transport current flowing in the thin HTS tube is

$$I_a = \int_0^{2\pi} j(\theta) R d\theta. \tag{2}$$

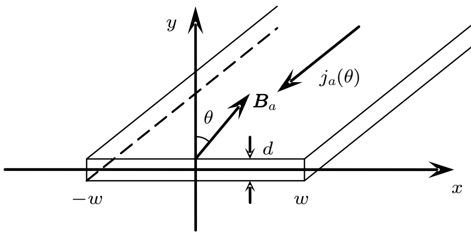

Fig. 2. Schematic illustration of every unit, which can be regarded as an infinitely long thin HTS strip with thickness d and half-width $w (d \ll w)$.

Accordingly, the critical current I_c is determined by

$$I_c = \int_0^{2\pi} j_c(\theta) R d\theta, \tag{3}$$

where $j_c(\theta)$ denotes the critical current density at azimuth angle θ. For the sake of convenience, the normalized transport current $\alpha = I_a/I_c$ is used thereafter. The tube discussed here is very thin and long, therefore the self-field produced by transport current can be calculated by the ampere circuit rule approximately. Based on the rule, the peak value of self-magnetic field B_0 is obtained with transport current I_a,

$$B_0 \approx \frac{\mu_0 I_a}{2\pi R}. \tag{4}$$

For the tape, the parallel components of self-field on both sides have the same magnitudes in opposite directions. However, the self-field generated by the current in the tube is parallel to its surface. Thus the self-field B_0 circumventing the tube should be added to the parallel component of the external field in a unit strip.

$$B_\parallel = B_a \sin\theta + B_0. \tag{5}$$

The characteristic magnetic field for each strip, B_d, is related to j_c as[14]

$$B_d = \frac{\mu_0 j_c}{\pi}. \tag{6}$$

For convenience, a parameter $\beta_1 = B_\perp / B_d$ is defined. In the low-current high-field regime, i.e.,

$$\alpha \le \tanh(\beta_1). \tag{7}$$

The equation of AC loss of each strip due to the perpendicular component of the magnetic field is given by[14]

$$\begin{aligned}\Delta Q = \frac{f\mu_0 I_c^2}{\pi N^2}\bigg\{ & 2\cosh^{-1}\left(\frac{1-p_0^2-a_0^2}{q_1 q_2}\right) \\ & - \frac{1}{4}\big[(1+p_0)q_1 + (1-p_0)q_1\big] \\ & \times \left[\cosh^{-1}\left(\frac{1+p_0}{a_0}\right) + \cosh^{-1}\left(\frac{1-p_0}{a_0}\right)\right] \\ & + \frac{1}{2}(q_1 - q_2) \times \left[(1+p_0)\cosh^{-1}\left(\frac{1+p_0}{a_0}\right)\right. \\ & \left. + (1-p_0)\cosh^{-1}\left(\frac{1-p_0}{a_0}\right)\right] \\ & + \frac{1}{4}(q_1-q_2)^2 - \frac{1}{2}(q_1-q_2)(q_1+q_2)\bigg\},\end{aligned} \tag{8}$$

where a_0, p_0, q_1 and q_2 are defined as

$$a_0 = \frac{\sqrt{1-\alpha^2}}{\cosh\beta_1}, \tag{9a}$$

$$p_0 = \alpha \tanh\beta_1, \tag{9b}$$

$$q_1 = \sqrt{(1+p_0)^2 - a_0^2}, \tag{9c}$$

$$q_2 = \sqrt{(1-p_0)^2 - a_0^2}. \tag{9d}$$

In the high-current low-field regime, i.e., where the relation

$$\alpha \ge \tanh(\beta_1), \tag{10}$$

is valid, we have[14]

$$\begin{aligned}\Delta Q = \frac{f\mu_0 I_c^2}{\pi N^2}\bigg\{ & -2\cosh^{-1}\left(\frac{1-p_0^2-a_0^2}{q_1 q_2}\right) \\ & - \frac{1}{4}\big[(1+p_0)q_1 - (1-p_0)q_2\big] \\ & \times \left[\cosh^{-1}\left(\frac{1+p_0}{a_0}\right) - \cosh^{-1}\left(\frac{1-p_0}{a_0}\right)\right] \\ & + \frac{1}{2}(q_1 + q_2) \times \left[(1+p_0)\cosh^{-1}\left(\frac{1+p_0}{a_0}\right)\right. \\ & \left. + (1-p_0)\cosh^{-1}\left(\frac{1-p_0}{a_0}\right)\right] \\ & - \frac{1}{4}(q_1-q_2)^2 - \frac{1}{2}(q_1-q_2)(q_1+q_2)\bigg\}.\end{aligned} \tag{11}$$

One can notice that the high degree of symmetry between the two solutions can be expressed by Eqs. (8) and (11). The two equations for ΔQ are valid for a thin HTS strip carrying an AC transport current in the same phase as the externally applied AC magnetic field perpendicular to the strip.

A Bi-2223/Ag HTS tape or YBCO coated conductor have high anisotropy in the presence of magnetic field. In order to describe more accurately, there are some models for anisotropic $j(B)$ with varying angle. In this study, we use a model for anisotropic $j(B, \theta)$ dependence, which is based on measurements of I_c, made on Bi-2223/Ag tapes in DC magnetic field up to 300 mT with varying angle. The $j(B, \theta)$ relation is modeled as[19−21]

$$j_c(B, \theta) = \frac{j_{c0}}{48 - 6.8e^{-B_{\parallel}/B_0} - 40.2e^{-B_{\perp}/B_0}}, \quad (12)$$

where j_{c0} is the critical current density of the HTS tape without the external magnetic field, $B_{\parallel}$ and $B_{\perp}$ are the absolute values of the parallel and perpendicular component of the magnetic field (in Tesla) respectively. $B_0 = 1\,\mathrm{T}$ is a normalizing constant.

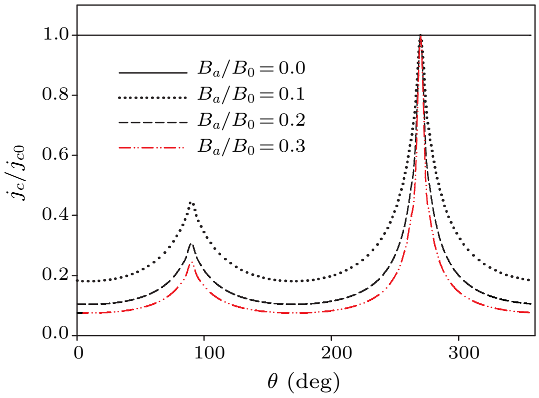

Fig. 3. The distribution of critical current density j_c/j_{c0} vs angle θ.

The distribution of critical current density in different magnetic fields is shown in Fig. 3. It can be seen that the critical current density decreases with magnetic field increasing, and each curve has two peaks, maximizing at $\theta = 90°$ and $\theta = 270°$, minimizing at $\theta = 0°$ and $\theta = 180°$. Thus it is concluded that the perpendicular component of the magnetic field has more influence on the critical current density than the parallel one. Because of the magnetic field superposition $B_{\parallel}$ at $\theta = 90°$ is greater than $\theta = 270°$, the critical current density at $\theta = 90°$ is smaller than $\theta = 270°$.

Under the condition that the shielding current is far from saturation, the AC loss induced by parallel field, much smaller than the loss induced by perpendicular field, is then essentially negligible.[14,15] Thus the total loss Q is simply expressed as a linear sum of the AC loss of each strip due to the perpendicular component of the magnetic field. The magnetic field exists in both sides of the tape symmetrically but only outside of the tube, therefore the total loss Q should be half of the sum,

$$Q = \frac{1}{2}\sum_{i=1}^{N} \Delta Q. \quad (13)$$

To visualize the behavior of AC loss described by Eq. (13), different plots are shown in Fig. 4. For convenience, AC loss is normalized with the factor $Q_{\rm ref} = f\mu I_c^2/\pi$.

As shown in Fig. 4(a), AC loss rises with magnetic field increasing. The current has little influence on AC loss in the low-current regime, and more influence in the high-current regime. Figure 4(b) indicates that the greater the current is, the less the magnetic field has influence on AC loss, which shows a linear field dependence at the high-field regime. When $I_a/I_c \to 1.0$, the HTS tube transfers to normal conductor, so the curve is very flat at $I_a/I_c = 1.0$. Figure 4(c) shows the 3D mesh plot of AC loss versus B_a/B_d and I_a/I_c.

In this study, the half-with of unit strip is

$$w = \frac{\pi R}{N}. \quad (14)$$

For infinite thin strip, $d/w \ll 1$. Let

$$\frac{d}{w} = n. \quad (15)$$

The optimal N in numerical calculation is determined by

$$N = \frac{n\pi R}{d}. \quad (16)$$

AC loss with different N at $I_a/I_c = 0.5$ is shown in Fig. 5. The results are very close at $N = 100$, 120, 150, respectively. It indicates that the accuracy is high enough as $N = 100$ (chosen in this study).

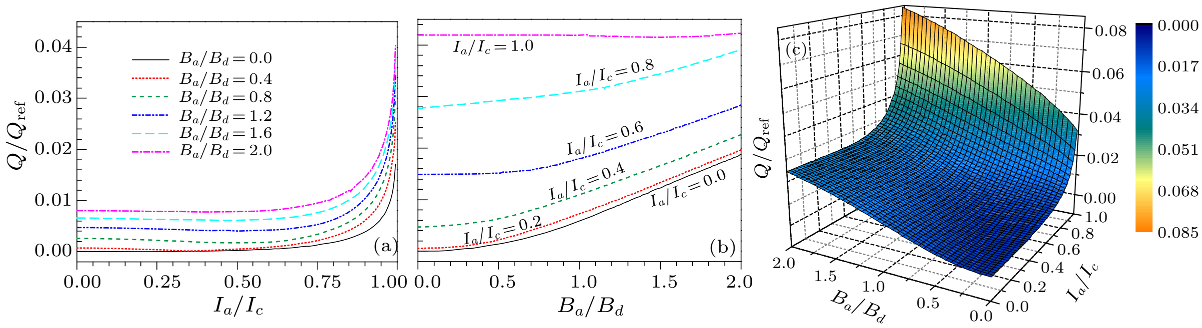

Fig. 4. (a) $Q/Q_{\rm ref}$ as a functions of I_a/I_c for different values of B_a/B_d. (b) $Q/Q_{\rm ref}$ as a function of B_a/B_d for different values of I_a/I_c. (c) $Q/Q_{\rm ref}$ as functions of B_a/B_d and I_a/I_c.

The experimental tests were made using a cylindrical current lead of BiSCCO by Douine *et al.*[22] The dimensions of this sample are $R_i = 3.8$ mm, $R_e = 5$ mm, and $L = 11.7$ cm, as seen in Fig. 6. AC loss $Q/Q_{\rm ref}$ simulated versus I_a/I_c at $B_a/B_d = 0.2$ with linear scale and log-log scale is presented in Fig. 7. The insets are the experimental results.[22] Obviously, our results are in very good agreement with the experiments.[22] The thickness of the tube discussed in this study is far less than the radius of the tube, so this numerical simulation method to calculate AC loss is only suitable for a thin high-temperature superconducting tube carrying an AC transport current in an AC external magnetic field.

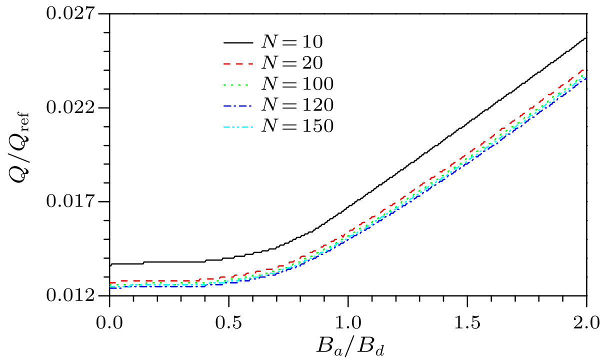

Fig. 5. AC loss with N = 10, 40, 100, 120, 150 at $I_a/I_c = 0.5$.

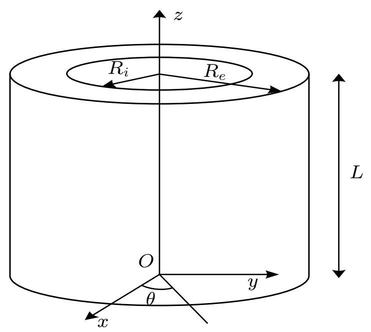

Fig. 6. Diagram of the studied superconducting tube.

In summary, the critical current density and AC loss distributions in a thin HTS tube carrying a transport current in the same phase as the externally applied magnetic field perpendicular to its axis are analyzed based on the infinite thin strip and anisotropy model. We divide the tube into N (here $N = 100$) segments with the same width. According to previous equations of AC loss, numerical equations for AC loss of a thin HTS tube are performed. The results are in very good agreement with experiments. A double-peak profile is observed in the curve of the critical current density distribution, and it is found that the AC loss is mainly determined by the perpendicular component of the magnetic field.

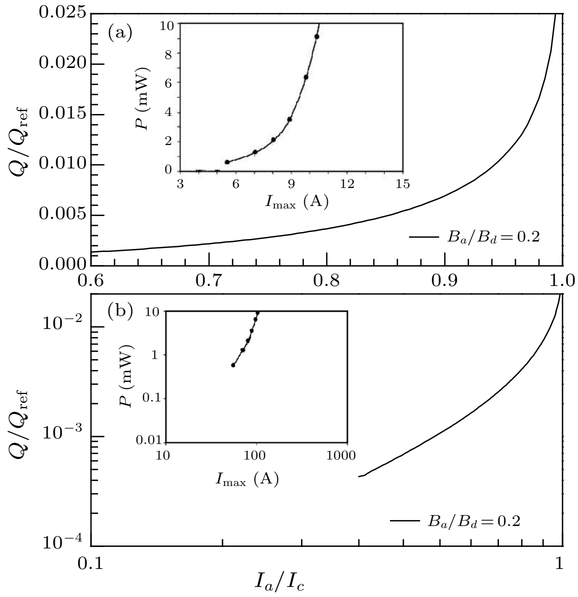

Fig. 7. Compared AC loss between the experimental measurement (see Ref. [22]) and our simulation with (a) linear scale and (b) log-log scale.

The thickness of the tube discussed is very small, so this numerical simulation method for calculating the AC loss is only suitable for a thin tube. This method may be applicable on a tube configuration consisting of coated superconductors.

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