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3 AUTHORS, INCLUDING:

Shiguo Xiao

Southwest Jiaotong University

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A Method Combining Numerical Analysis and Limit Equilibrium Theory to Determine Potential Slip Surfaces in Soil Slopes

XIAO Shiguo[1]*, YAN Liping[2], CHENG Zhiqiang[3]

1 Department of Geological Engineering, Southwest Jiaotong University, Chengdu 610031, China

2 Los Angeles Department of Water and Power, Los Angeles, USA

3 Department of Applied Mechanics and Engineering, Southwest Jiaotong University, Chengdu 610031, China

**Corresponding author, e-mail: xiaoshiguo126@126.com*

Abstract: This paper describes a precise method combining numerical analysis and limit equilibrium theory to determine potential slip surfaces in soil slopes. In this method, the direction of the critical slip surface at any point in a slope is determined using the Coulomb's strength principle and the extremum principle based on the ratio of the shear strength to the shear stress at that point. The ratio, which is considered as an analysis index, can be computed once the stress field of the soil slope is obtained. The critical slip direction at any point in the slope must be the tangential direction of a potential slip surface passing through the point. Therefore, starting from a point on the top of the slope surface or on the horizontal segment outside the slope toe, the increment with a small distance into the slope is used to choose another point and the corresponding slip direction at the point is computed. Connecting all the points used in the computation forms a potential slip surface exiting at the starting point. Then the factor of safety for any potential slip surface can be computed using limit equilibrium method like Spencer method. After factors of safety for all the potential slip surfaces are obtained, the minimum one is the factor of safety for the slope and the corresponding potential slip surface is the critical slip surface of the slope. The proposed method does not need to pre-assume the shape of potential slip surfaces. Thus it is suitable for any shape of slip surfaces. Moreover the method is very simple to be applied. Examples are presented in this paper to illustrate the feasibility of the proposed method programmed in ANSYS software by macro commands.

Keywords: Soil slope; Stress field; Potential slip surface; Slope stability; Factor of safety; Numerical analysis; Limit equilibrium method; ANSYS software

Introduction

In slope stability analyses, numerical methods have the advantage that cannot be replaced by the limit equilibrium method as they can simulate accurately the stress field and the displacement field in a slope. Before application of the shear strength reduction technique, the numerical

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simulation of the slope stability was focused on the analysis of characteristics of stress field and displacement field in the slope (Ding et al. 1995; Duncan 1996; Zhang et al. 1998) and there were no good methods to determine reasonably defined potential slip surfaces and the factor of safety for the slope. The shear strength reduction technique provides a practical method to compute slope factor of safety using numerical methods. However, there exist some unsolved issues and different opinions relating to the condition to end computation (the instability criterion) for the technique (Liu et al. 2005; Luan et al. 2003; Zhao et al. 2005). Some authors compute the stress on a slip surface by interpolating the results obtained from a finite element method (Giam and Donald 1988; Zou et al. 1995), and then compute the factor of safety using the force equilibrium equation. This method avoids some of the assumptions adopted in the limit equilibrium method, but it is quite complicated and not easy for realization. On the basis of the principle of optimality, along with the method of slices, a critical slip field (CSF) in a slope is postulated recently (Zhu 2001). The method can determine the minimum factor of safety of a slope, but it is not easy to analyze complicated slope and it still belongs to a kind of limit equilibrium method. Some authors also propose to compute the slope factor of safety using the finite arc searching method (Yin and Lv 2005). This method needs to assume the slip surface to be a circular arc shape and is not suitable for slip surfaces with other shapes. In reality, the potential slip surfaces of soil slopes are not just the same as circular arc surfaces. Assuming circular slip surfaces is the obvious defect of the classical limit equilibrium method. For this reason, a numerical method is still chosen in this paper. After the stress field in the slope is obtained using the ANSYS software, the critical slip direction of any point in the slope can be defined by the strength principle of the slope material; then positions of the potential slip surfaces can be determined and factors of safety can be further computed using limit equilibrium method. By comparing the factors of safety, the slope stability can be evaluated and no assumption of the shape of slip surfaces is needed. The proposed method can reasonably well predict potential slip surfaces and it can be conveniently realized in computer programming.

1 Analysis of Potential Slip Direction

1.1 Basic assumptions

(1) The geometric shape and the nature of loading for the slope satisfy the conditions of the plane strain state; thus the slope model can be simplified as a plane strain problem.

(2) Coulomb's strength principle is appropriate for the slope material.

1.2 Definition of factor of safety at a point

According to the shear strength criterion described in Rock and Soil Mechanics, whether the instability occurs at a point in the slope depends on the shear stress and the shear strength at the point. The factor of safety at the point i (F_{si}) is defined as the ratio of the shear strength (S_i) to the shear stress (τ_i):

$$F_{si}=S_i/\tau_i \tag{1}$$

For any point in the slope, if $F_{si}>1$, it means the point is stable; if $F_{si}=1$, the point is in the critical state of instability.

1.3 Determination of potential slip direction

The magnitude of shear stress in a point is relevant to its direction. According to the Coulomb's strength principle, the magnitude of shear strength in a point is also relevant to its direction. Therefore, the factor of safety at a point computed from Equation (1) is relevant to its direction. In other word, the factor of safety F_{si} of a point is a function of its direction.

The stress state of a unit body for any point in a slope induced by self weight of soil or other applied loads on the slope is shown in Figure 1. At this point, σ_x, σ_y, τ_{xy} are the normal stress in the x-axis direction, the normal stress in the y-axis direction, the shear stress, respectively; and they can be computed by interpolation of the stress field for the neighboring elements obtained from a numerical simulation. The normal stress and shear stress on an oblique section of the unit body in Figure 1 are expressed by Equation (2) and Equation (3), respectively. The normal direction of the oblique section has an angle of α with the horizontal direction.

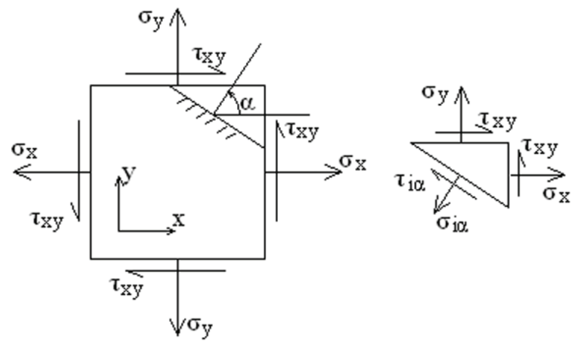


Figure 1 Stress state of unit body in slope

$$\sigma_{i\alpha} = (\sigma_x + \sigma_y)/2 + (\sigma_x - \sigma_y)/2 \times \cos 2\alpha + \tau_{xy} \sin 2\alpha \quad (2)$$

$$\tau_{i\alpha} = (\sigma_x - \sigma_y)/2 \times \sin 2\alpha - \tau_{xy} \cos 2\alpha \quad (3)$$

According to the Coulomb's strength principle, the shear strength along the oblique section is expressed as:

$$S_{i\alpha} = c_i + \sigma_{i\alpha} \tan \phi_i \quad (4)$$

where c_i and φ_i are cohesion and internal friction angle of the slope material at the point i.

Substituting Equations (2)-(4) into Equation (1) leads to the following expression for the factor of safety at the point i in the slope:

$$\begin{aligned} F_{si} &= S_{i\alpha}/\tau_{i\alpha} \\ &= (c_i + \sigma_{i\alpha} \tan \phi_i)/\left|(\sigma_x - \sigma_y)/2 \times \sin 2\alpha - \tau_{xy} \cos 2\alpha\right| \end{aligned} \quad (5)$$

By further arrangement, one can get:

$$F_{si} = [(A_1 - A_2)t^2 + 2A_3 t + A_1 + A_2]/(a_3 t^2 + 2a_2 t - a_3) \quad (6)$$

where coefficient t is equal to tanα, and A_1, A_2, A_3, a_1, a_2, a_3 are the coefficients expressed in Equation (7).

$$\left.\begin{array}{ll} A_1 = c_i + a_1 \tan \phi_i & a_1 = (\sigma_x + \sigma_y)/2 \\ A_2 = a_2 \tan \phi_i & a_2 = (\sigma_x - \sigma_y)/2 \\ A_3 = a_3 \tan \phi_i & a_3 = \tau_{xy} \end{array}\right\} \quad (7)$$

It is obvious that the factor of safety F_{si} of a point changes with angle α in Equation (5). The value of α ranges between 0° and 180°. To be meaningful, F_{si} is taken to be positive. The minimum value of F_{si} is the factor of safety at the point and the corresponding angle α is the critical slip direction.

Performing derivative operation with the coefficient t on both sides of Equation (6) and setting to zero, an extremum equation is derived as:

$$X_1 t^2 - X_2 t - X_3 = 0 \quad (8)$$

where X_1, X_2, X_3 are coefficients as expressed in Equation (9):

$$\left.\begin{array}{l} X_1 = (A_1 - A_2)a_2 - A_3 a_3 \\ X_2 = 2A_1 a_3 \\ X_3 = A_3 a_3 + (A_1 + A_2)a_2 \end{array}\right\} \quad (9)$$

Thus, t can be obtained by solving Equation (8):

$$t = \left(X_2 \pm \sqrt{X_2^2 + 4X_1X_3}\right)/(2X_1) \quad (10)$$

With t computed by Equation (10), the minimum value of F_{si} is obtained by Equation (6). As there exist two solutions of t, there are "two" critical slip directions. By the symmetry of the unit body, the two directions are symmetric about the vertical axis and the absolute values of the two factor of safety along the two directions must be equal. The two solutions of t are represented by t_1 and t_2 and are computed by Equations (11) and (12):

$$t_1 = \left(X_2 + \sqrt{X_2^2 + 4X_1X_3}\right)/(2X_1) \quad (11)$$

$$t_2 = \left(X_2 - \sqrt{X_2^2 + 4X_1X_3}\right)/(2X_1) \quad (12)$$

The above-discussed stress analysis is based on a unit body and the slope direction is not considered. The different slope direction will induce different slip direction. Practically a slope always slips or has a slip tendency forward and down in the free surface. For a two-dimension problem, there are two kinds of configurations, called "left-up and right-down" slope with sign "\" and "right-up and left-down" slope with sign "/". When angle α varies between 0°and 90° in the unit body shown in Figure 1, it corresponds to "left-up and right-down" slope and the critical slip directions of the points on the slope are between 0° and 90°. When angle α varies between 90° and 180° in the unit body, it corresponds to "right-up and left-down" slope and the critical slip directions of the points on the slope are between 90° and 180°. It should be noticed that the critical slip direction on the horizontal segment before the slope toe is in the range of 90°<α<180° for the "left-up and right-down" slope, whereas it is in the range of 0°<α< 90° for the "right-up and left-down" slope. In

Equation (10), taking the positive sign on the right hand side corresponds to the "left-up and right-down" slope and taking the negative sign on the right hand side corresponds to the "right-up and left-down" slope. In practical situations, it is self-evident that t has unique value when the slope direction is defined. Therefore, the critical slip direction of a point in the slope is unique.

Based on the coordinate system of the unit body shown in Figure 1, substituting $t = t_1$ into Equation (6) results in the minimum factor of safety for the "left-up and right-down" slope, which is a positive value; substituting $t=t_2$ into Equation (6) results in the minimum factor of safety for the "right-up and left-down" slope, which is a negative value and the absolute value can be used for practical purposes. For simplification, only the "left-up and right-down" slope model is applied by using coordinate transformation. Therefore, Equation (11) is used directly to determine the critical slip direction in a point and then the minimum factor of safety at the point can be computed by Equation (6). For many available finite element software tools, the basic information obtained from a finite element stress analysis are σ_x, σ_y, and τ_{xy} within each element. The known σ_x, σ_y, and τ_{xy} at the Gauss numerical integration points in each element are projected to the nodes and then averaged at each node. With the σ_x, σ_y, and τ_{xy} known at the nodes, the stresses can be computed at any other points within the element. Since the stress at a selected point can be uniquely worked out by some numerical simulation methods, the critical slip direction at the point in a soil slope can be also uniquely determined.

After the critical slip directions at the selected points in the slope have been defined, these points are connected with a smooth curve along the critical slip directions and this curve is the potential slip surface for the two-dimension problem. Because the potential slip surface is determined by computation, there is no need to make any assumptions. There are countless potential slip surfaces for a slope. According to the geometry and loading conditions of a practical slope, a slip surface always passes through the free surface or the horizontal segment outside the slope toe. Therefore, computation can started from a point on the free surface or the horizontal segment outside the slope toe. First, the critical slip direction of the starting point is determined, then the position of another point is obtained by advancing a small distance step along this critical slip direction into the slope; and then the critical slip direction of the obtained point is determined. Repeating this process until reaching the top of slope can form a potential slip surface with the exit point at the starting point for computation. Choosing another position of the starting point for computation (exit points of sliding surface), another potential slip surface can be obtained by the same process. After further computation of the factors of safety for all slip surfaces, the stability of the slope can be evaluated quantitatively.

2 Computation of Factor of Safety

The above paragraphs describe a method to determine a potential slip surface in a soil slope and to compute the factor of safety of a point in the slope. It should be mentioned that whether instability occurs in the slope is not determined by the factor of safety of a point, but the factor of safety of a potential slip surface. In other words, when instability occurs at some points in the slope, the potential slip surface may not have the stability failure; vice versa, when instability occurs on the potential slip surface, instability occurs surely at each point on the potential slip surface. A slip surface consists of a group of points, whose stability is evaluated by the ratio of the shear strength to the shear stress at the point. To fully represent the stability of the slip surface that is formed by connecting the points, the factor of safety of a potential slip surface F_s can be computed using the limit equilibrium method after the potential slip surface is determined by the above-described method. Considering the potential slip surfaces being not circular, Spencer method (Spencer 1967) which assumes a potential sliding mass to be divided into a number of vertical slices and parallel inter-slice forces can be properly used to calculate the factor of safety of the potential slip surface.

In fact, the approach is combination of the numerical simulation method and the limit equilibrium method. So it can be called the composite method. After the factor of safety of each potential slip surface is obtained, the minimum value is the factor of safety of the slope and the corresponding potential slip surface is the critical slip surface.

Using the above-described method, the factor of safety of each point on the potential slip surface can be computed at the same time as the factor of safety of the potential slip surface is obtained. It reveals quantitatively the fact that the factors of safety are different at different points on the same potential slip surface, which represents the status of stability at each point on the potential slip surface and is helpful to determine the gradual

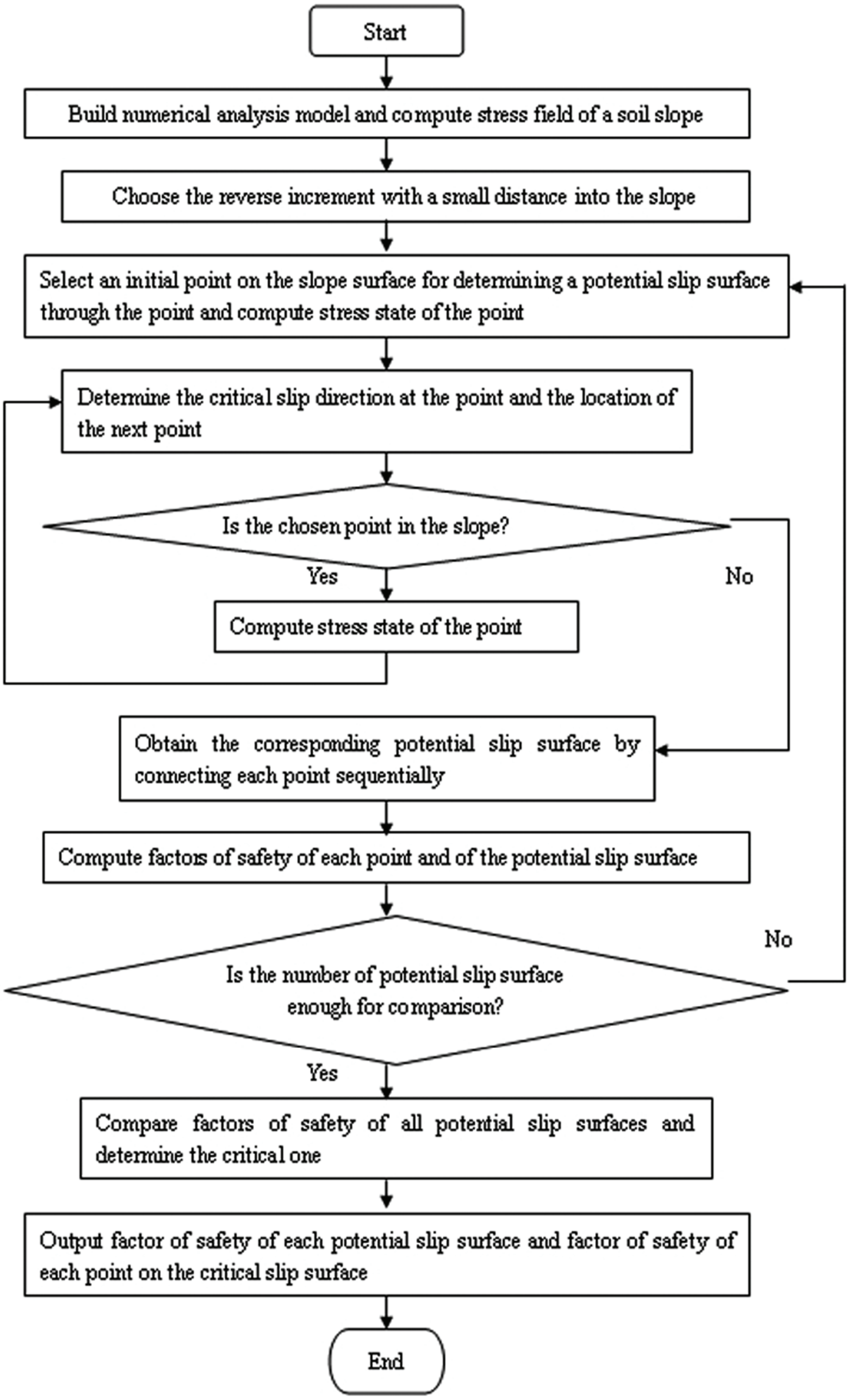


Figure 2 Programming flow chart of the proposed method in this paper

failure process in the slope. This can not be represented by the shear strength reduction method. The proposed method in this paper is beneficial for the analysis of relative stability at different segments along the potential slip surface in the slope and thus it enables the comprehension of the mechanism of the gradual failure process in the slope and stabilization measures can be taken in segments for the complete slope treatment.

The programming flow chart of the proposed method in this paper is shown in Figure 2.

3 Analyses of Practical Examples

To illustrate the use of the proposed method, two soil slopes are chosen as examples for analysis.

1) Example 1

A classical soil slope (Hassiotis et al. 1997; Hull and Puolos 1999) with its dimensions in m is shown in Figure 3. It has a height of 13.7 m, a slope angle of 30°. The slope soil has a unit weight of 19.63 kN/m^3, an internal friction angle of 10°, a cohesion of 23.94 kPa, an elastics modulus of 50 MPa, and a Poisson ratio of 0.35. The load considered is only the self weight of soil slope.

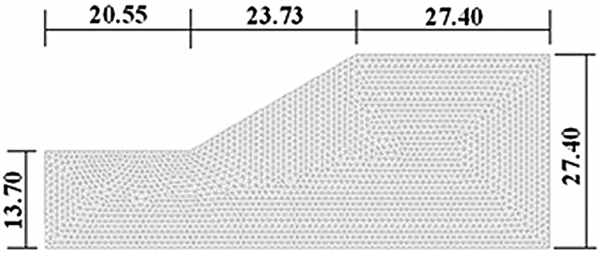


Figure 3 Finite element model of slope in Example 1

A finite-element model was developed using the ANSYS software to analyze the stress field of the above soil slope. ANSYS software is a numerical simulation software with finite element method developed by ANSYS Incorporation, USA. It can carry out many mechanical computations including geomechanic problems. In the paper, the slope material was simulated with the elastoplastic constitutive relationship based on a non-associative plastic flow rule and the Mohr-Coulomb's failure criterion. The finite-element discretization using 6-noded triangular elements is also shown in Figure 3. For the boundary conditions, the left and right sides of model were imposed with horizontal displacement constraint only and the bottom of model was imposed with both horizontal and vertical displacement constraints. After the stress field of the slope was obtained from the numerical analysis, the potential slip surfaces were determined and the factors of safety were computed using the proposed method that was realized by the macro commands programmed in ANSYS software. Some of the potential slip surfaces and their factors of safety computed using Spencer method are shown in Figure 4. Surface C is the critical slip surface and its factor of safety is 1.056. For the same slope, slope stability was analyzed using shear strength reduction method (SSRM), simplified Bishop method (SBM), and simplified Janbu method (SJM); and the resulted factors of safety are 1.100, 1.110, 1.031, respectively. The critical slip surfaces obtained from the three methods are shown in Figure 5.

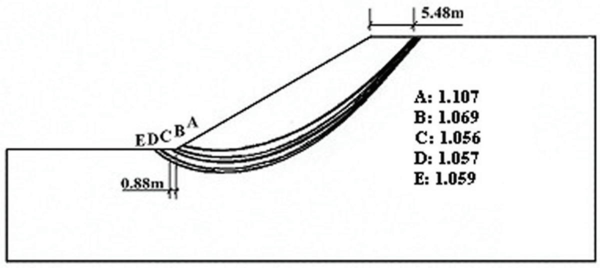
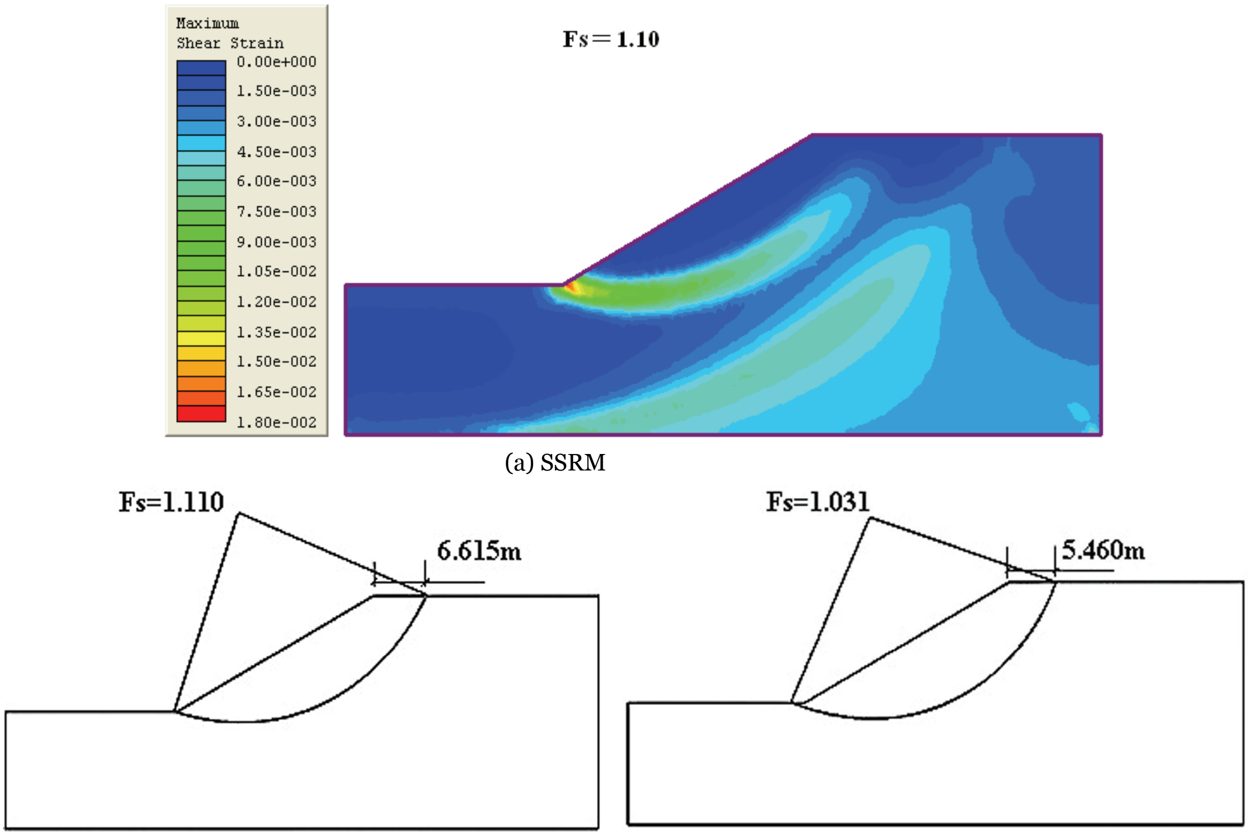


Figure 4 Results of the proposed method in Example 1

By the comparison of Figure 4 and Figure 5, it can be noticed that there are some differences in the predicted critical slip surfaces between the proposed method and the other three methods. The shape of the critical slip surface predicted by the proposed method is not circular arc, which is coincident with the actual situation in projects. The distribution of factor of safety of a point on the critical slip surface along the distance from the slope toe is shown in Figure 6. It can be seen that the stability at these points on the critical slip surface is not the same, which means they do not reach the limit equilibrium state at the same time and the occurrence of failure is not simultaneous at all the points on the critical slip surface but with a sequential gradual process. For the discussed example, the safety factors of the upper part of slope are larger than that of the middle and lower



(a) SSRM

(b) Simplified Bishop method

(c) Simplified Janbu method

Figure 5 Results of the classic methods in Example 2

part of slope, which indicates that slope failure mode that begins at the middle and upper part of slope is a type of towing shear-sliding rupture.

The slope factors of safety computed by different methods are shown in Table 1. Except that the slope safety factor computed by SJM is relatively small, the other results are similar. The

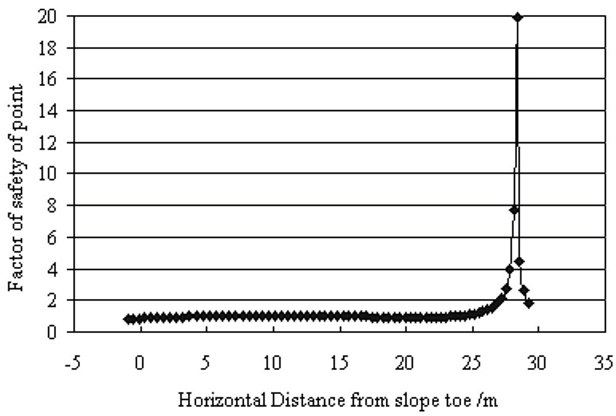


Figure 6 Distribution of factor of safety of point on the critical slip surface in Example 1

slope factor of safety computed by the proposed method is very close to that computed by the friction circle method, which is a good verification for the proposed method.

For the above example, the influence of the step size Ls on the slope factor of safety Fs is shown in Figure 7. As the step size increases, the slope factor of safety gradually decreases in an approximately exponential fashion. The slope factor of safety is of high precision at smaller step size. The slope factor of safety is 1.062 when the step size is 0.1 m, whereas it is 1.043 when the step size is 2.4 m. The slope factor of safety varies among various step sizes. In order to maintain enough precision for a practical slope stability problem, we suggest the step size should not be more than 0.5 m when using the proposed method.

2) Example 2

A soil slope with its sizes in m is shown in Figure 8. The slope material parameters are shown in Table 2. Only loading due to self weight of the soil is considered in the analysis.

Table 1 Comparison of slope factors of safety computed by different methods in Example 1

Calculation method	Friction circle method (Hassiotis et al. 1997)	Taylor Method (Hassiotis et al. 1999)	Limit analysis upper bound method (Nian et al. 2005)	Simplified Bishop method	Simplified Janbu method	Shear strength reduction method	Proposed method in this paper
Safety factor	1.080	1.110	1.107	1.110	1.031	1.100	1.056

Table 2 Material parameters of the slope in Example 3

Stratum	Unit weight (kN/m3)	Cohesion (kPa)	Angle of internal friction (degrees)	Elastic modulus (MPa)	Poisson's ratio
Soil 1	18	7.3	17.9	50	0.33
Soil 2	18.5	14.1	16.7	40	0.35
Soil 3	19	16.1	14.0	100	0.34

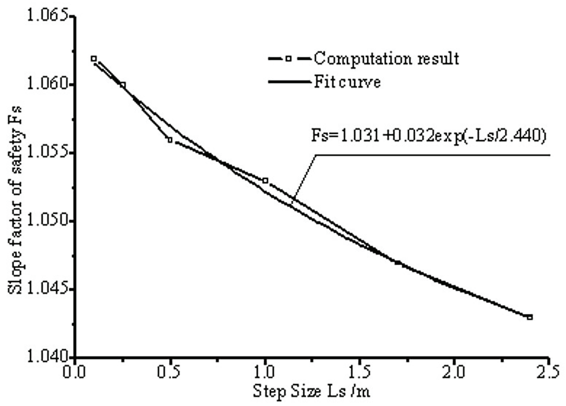


Figure 7 Influence of the step size Ls on the slope factor of safety Fs in Example 1

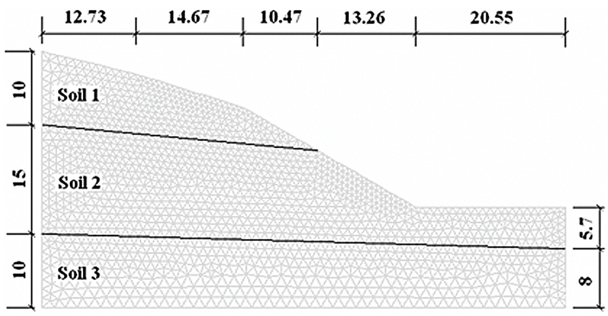


Figure 8 Finite element model of slope in Example 3

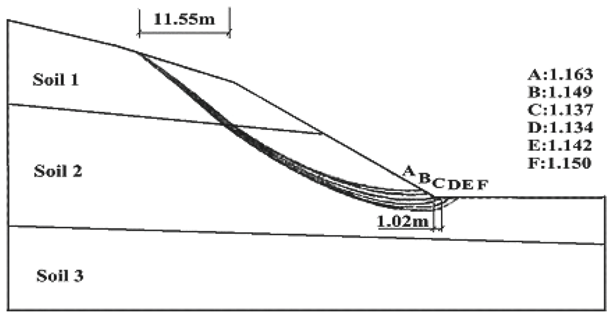
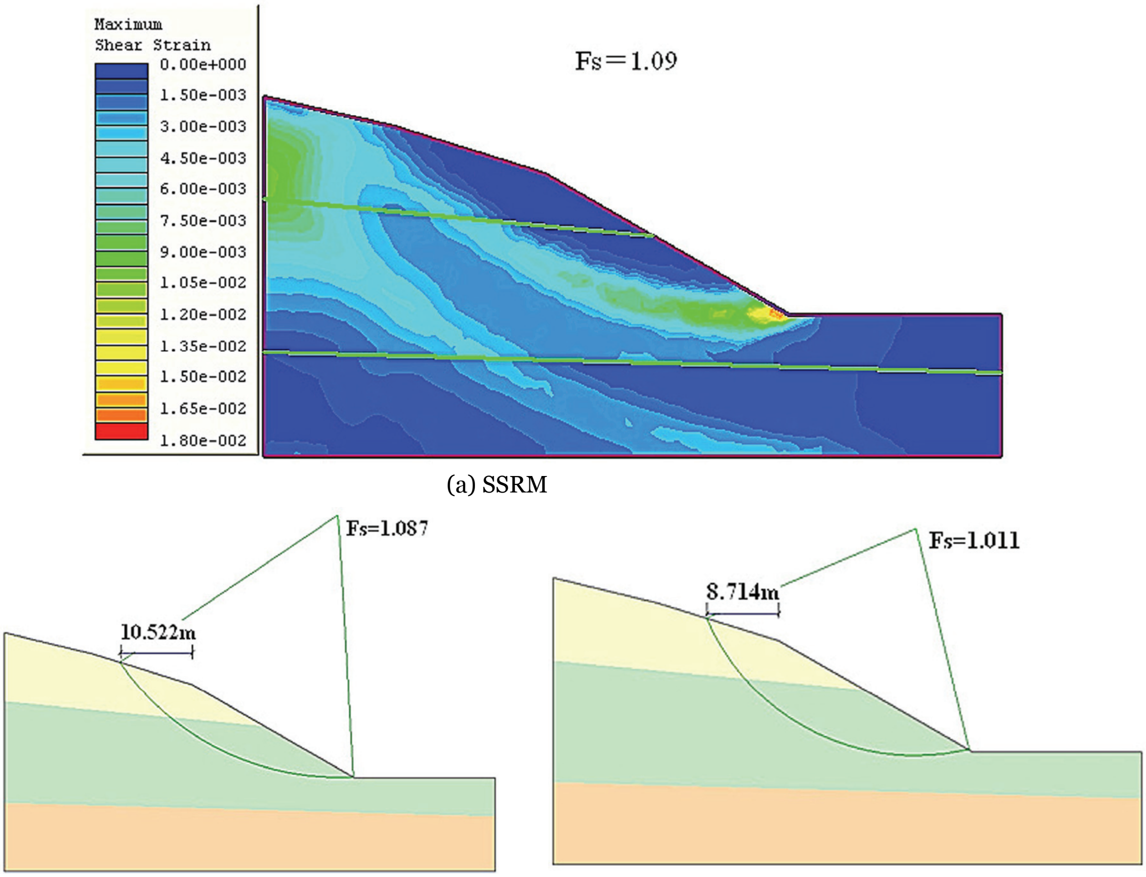


Figure 9 Results of the proposed method in Example 3

Using the proposed method, the predicted potential slip surfaces and their slope factors of safety are shown in Figure 9. Surface D is the critical slip surface and its factor of safety is 1.134. For the same slope, slope stability was analyzed using SSRM, SBM, and SJM; and the resulting factor of safety is 1.090, 1.087, 1.011, respectively. The critical slip surfaces obtained from the three methods are shown in Figure 10. The slope factor of safety computed by the proposed method is only 4.32% bigger than that computed by the simplified Bishop method, which is a good verification for the proposed method. Comparing Figure 9 with Figure 10, it is obvious that there are some differences on the predicted critical slip surfaces between the proposed method and the other three methods. The shape of the critical slip surface predicted by the proposed method is not circular arc too. The distribution of factor of safety of a point on the critical slide surface is shown in Figure 11. It also indicates that the slope failure is a gradual process. For the discussed example, the factors of safety of a point in the behind part of slope are larger than those in the forehead, which means the slope failure initiates at the forehead of slope and the failure mode belongs to the type of towing shear-sliding rupture.

According to the analysis results of the two examples described above, it is obvious that the shape of the slip surface determined using the proposed method is not circular arc, which is different from those determined using some limit equilibrium methods. Moreover the position of the



(a) SSRM

(b) Simplified Bishop method

(c) Simplified Janbu method

Figure 10 Results of the classic methods in Example 3

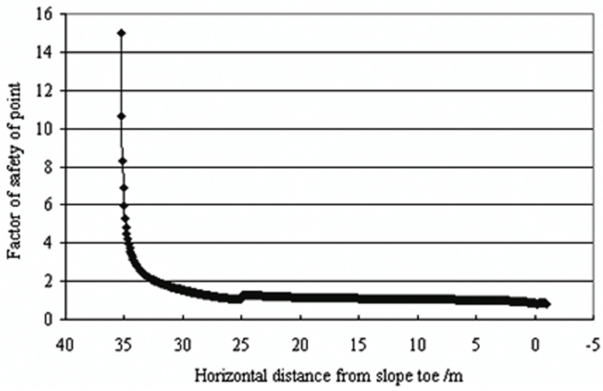


Figure 11 Distribution of factor of safety of point on the critical slide surface in Example 3

critical slip surface obtained using the proposed method is different from those obtained using the classic methods. The slope factor of safety computed from the proposed method is slightly larger than those computed from the classical methods. Based on the analysis of loading condition in the slope, the critical slip surface should be the connection curve of points along their critical slip directions. Normally, this curve is not circular arc. Therefore, the assumption about the circular arc in the classical methods is not accurate.

4 Conclusions

After the stress field of a soil slope is obtained from numerical simulations, the ratio of shear strength to shear stress at any point in the slope can be used as an analysis index and the critical sliding failure direction at that point can be determined by utilizing the Coulomb's strength principle incorporated with the extremum principle. The critical sliding failure direction is in the tangential direction of the potential slip surface passing through that point. Starting from a point on the slope surface or on the horizontal segment outside the slope toe, the reverse computation with a small distance increment step is performed to advance into another point in the slope. This process is repeated step by step until a potential slip surface is formed. Choosing another starting point can result in a different potential slip surface with the same process. Comparing the factors of safety computed using all potential slip surfaces, the slope stability can be evaluated quantitatively. Since there is no need to make any assumptions on the shape of potential slip surfaces, the proposed method is suitable for any shapes of potential slip surfaces. Furthermore, the variation of the factor of safety of point along the critical slip surface can be obtained. Therefore, the characteristics of the gradual failure of the slope can be assessed. The feasibility of the proposed method has been validated by the two practical examples discussed in the paper.

It is necessary to state that although the proposed method for determining a potential slip surface is elucidated for soil slopes in this paper, it is also applicable to rock slopes as long as the stress field of rock slopes is obtained accurately.

The analysis in this paper is only based on two-dimensional slopes. For three-dimensional slopes, further study is needed for the application of the proposed method.

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