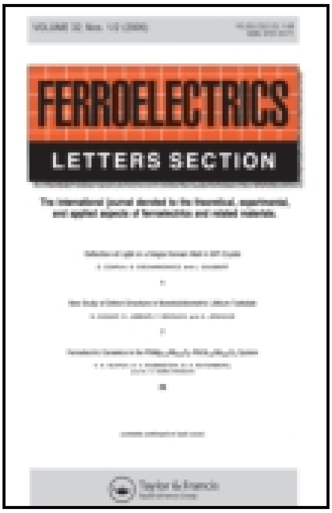

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The thickness vibration of piezoceramic ring is investigated by theoretical analysis, numerical simulation, and experimental measurement. Based on the piezoelectric constitutive equations and the hypothesis that a ring has no strain in circumference when vibrating, the thickness vibration mode of ring is treated as a radial longitudinal extended vibration equivalently, the equations was simplified and the resonant frequency of thickness vibration is derived. The resonant frequency is also simulated with finite element software and measured by Impedance Analyzer. Theoretic calculation of resonant frequency is compared with the simulation and measurement. The errors between them are analyzed.

Keywords Piezoelectric ring; radial thickness vibration; investigation

1. Introduction

As we know, piezoelectric devices are widely used in sensors, actuators, resonators, non-destructive testing devices, and sonars, such as piezoelectric microphone, motor, vibration meter, ultrasonic flaw detector, hydrophone, underwater sound projector. Piezoelectric ring, disc, cylinder is common vibrators used in those devices. The vibrators operate in different vibration modes, such as thickness and radial extensional vibration modes, flexural vibration and shear vibration mode. Because the properties of piezoelectric device depend on vibration mode, therefore many researches are implemented on vibration analysis for piezoelectric vibrators. Chi-Hung Huang [1] investigated vibration characteristics of thin piezoceramic annular disks with stress-free boundary conditions by theoretical analysis, numerical simulation, and experimental measurement. The nonaxisymmetric thickness and radial extensional vibration modes are discussed, but no radial thickness vibration mode. Shuyu Lin [2] studied the radial composite vibrations of a piezoelectric ceramic ring and a metal ring, and their electromechanical equivalent circuits are obtained. For the radial and thickness vibration (out-of-plane) of piezoelectric ceramic thin disks and rings, other work is reported and the design theory is well developed [3–6]. A matrix model of the radial mode of a thin piezoceramic ring capable of predicting the dynamic behavior is described

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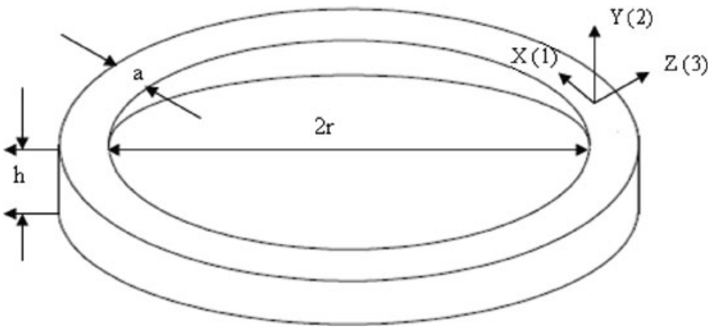

Figure 1. Piezoelectric ring.

[7]. Although there are some reports about vibration mode of ring, but a few analysis for thickness vibration (in-plane) mode for ring.

In this paper, the thickness vibration (in-plane) of piezoceramic ring is investigated, its resonance frequency equation is deduced, and simulation and experiments about it are carried out to validate the equation.

2. Frequency Equation at Thickness Mode (in-plane) for Piezoelectric Ring

A piezoelectric ring, polarized along thickness direction and covered electrode on inner and outer surface, is shown in Fig. 1. The thickness, inner radii, highness of the piezoelectric ring is, respectively, a, r, h. The value of a and h is much less than r. Voltage is applied on inner and outer surface.

Based on piezoelectricity, the linear constitutive equations for a piezoelectric ring are:

$$\begin{cases} S_1 = s_{11}^D T_1 + s_{12}^D T_2 + s_{13}^D T_3 + g_{31} D_3 \\ S_2 = s_{12}^D T_1 + s_{11}^D T_2 + s_{13}^D T_3 + g_{31} D_3 \\ S_3 = s_{13}^D T_1 + s_{13}^D T_2 + s_{33}^D T_3 + g_{33} D_3 \\ S_4 = s_{44}^D T_4 + g_{15} D_2 \\ S_5 = s_{44}^D T_5 + g_{15} D_1 \\ S_6 = 2\left(s_{11}^E - s_{12}^E\right) T_6 \end{cases} \tag{1}$$

$$\begin{cases} E_1 = -g_{15} T_5 + \beta_{11}^T D_1 \\ E_2 = -g_{15} T_4 + \beta_{11}^T D_1 \\ E_3 = -g_{31} T_1 - g_{31} T_2 - g_{33} T_3 + \beta_{33}^T D_3 \end{cases} \tag{2}$$

A ring may be divided into many arc unite cells, the vibration of ring can be figured by vibration of one unit cell, with centre angle $\delta\theta$, as illustrated Fig. 2, because the ring is symmetric and uniform. Symmetric thickness vibration of the ring, along radial direction (in-plane), can be treated as longitudal extensional vibration of unite cell since the r and h is much less than a. When ring vibrate, several modes, radial extensional, non-symmetric thickness mode (out-of-plane) and thickness vibration (in-plane) will appear at the same time, in order to extrude a thickness vibration (in-plane) to simplify analysis, we assume there is only axis-symmetric thickness vibration mode and no other modes exist, i.e.

$$S_1 = 0$$

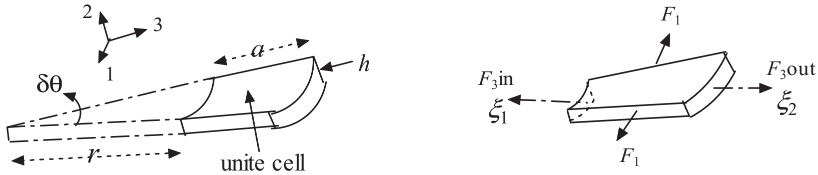

Figure 2. Arc unite cell of ring.

The electric field is applied on inner and outer surface, boundary effect of electric field is ignored, and we can set $D_1 = D_2 = 0$. The up and down surfaces of ring are free, so all stresses on the surfaces are zero. Moreover, the ring vibrates with axial symmetry, i.e., $T_5 = 0$, therefore we have

$$T_2 = T_4 = T_5 = T_6 = 0$$

Above constitutive equations (1) and (2) can be simplified as:

$$\begin{cases} 0 = s_{11}^D T_1 + s_{13}^D T_3 + g_{31} D_3 \\ S_2 = s_{12}^D T_1 + s_{13}^D T_3 + g_{31} D_3 \\ S_3 = s_{13}^D T_1 + s_{33}^D T_3 + g_{33} D_3 \end{cases} \tag{3}$$

$$E_3 = -g_{31} T_1 - g_{33} T_3 + \beta_{33}^T D_3. \tag{4}$$

From fist equation among (3), we have $T_1 = -\frac{s_{13}^D}{s_{11}^D} T_3 - \frac{g_{31}}{s_{11}^D} D_3$, Substituting T_1 into third equation among (3), we can obtain:

$$T_3 = \frac{s_{11}^D}{s_{33}^D s_{11}^D - s_{13}^{D2}} S_3 + \frac{s_{13}^D g_{31} - s_{11}^D g_{33}}{s_{33}^D s_{11}^D - s_{13}^{D2}} D_3$$

From $S_3 = \frac{\partial \xi}{\partial z}$, T_3 can be rewritten as:

$$T_3 = \frac{s_{11}^D}{s_{33}^D s_{11}^D - s_{13}^{D2}} \frac{\partial \xi}{\partial z} + \frac{s_{13}^D g_{31} - s_{11}^D g_{33}}{s_{33}^D s_{11}^D - s_{13}^{D2}} D_3 \tag{5}$$

From Kinematical equation of elastomer:

$$\left. \begin{aligned} \frac{\partial T_1}{\partial x} + \frac{\partial T_6}{\partial y} + \frac{\partial T_5}{\partial z} = \rho \frac{\partial^2 \zeta}{\partial t^2} \\ \frac{\partial T_2}{\partial x} + \frac{\partial T_2}{\partial y} + \frac{\partial T_4}{\partial z} = \rho \frac{\partial^2 \eta}{\partial t^2} \\ \frac{\partial T_5}{\partial x} + \frac{\partial T_4}{\partial y} + \frac{\partial T_3}{\partial z} = \rho \frac{\partial^2 \xi}{\partial t^2} \end{aligned} \right\} \tag{6}$$

Simplifying equation (6) as:

$$\rho \frac{\partial^2 \xi}{\partial t^2} = \frac{\partial T_3}{\partial z} \tag{7}$$

Substituting (5) into equation (7), because no free charge exists in piezoelectric ceramic, we have:

$$\frac{\partial D_3}{\partial z} = 0$$

Therefore equation (7) is translated as:

$$\rho \frac{\partial^2 \xi}{\partial t^2} = \frac{s_{11}^D}{s_{33}^D s_{11}^D - s_{13}^{D2}} \frac{\partial^2 \xi}{\partial z^2}, \text{ writingas } \frac{\partial^2 \xi}{\partial t^2} = \nu^2 \frac{\partial^2 \xi}{\partial z^2} \tag{8}$$

where

$$\nu = \sqrt{\frac{s_{11}^D}{\rho(s_{33}^D s_{11}^D - s_{13}^{D2})}} \tag{9}$$

Based on resonant frequency equation of piezoelectric crystal with longitudinal extensional vibration:

$$f = \frac{\nu}{2a} \tag{10}$$

Substituting (9) into equation (10), we can obtain frequency equation of piezoelectric ring with thickness vibration (in-plane):

$$f = \frac{1}{2a}\sqrt{\frac{s_{11}^D}{\rho(s_{33}^D s_{11}^D - s_{13}^{D2})}} \tag{11}$$

3. Numerical Calculating the Frequency Equation of Ring with Thickness Vibration

Selecting ceramic PZT-5 as material used to make ring, its parameters are followed:

$$\text{PZT}-5 \begin{cases} s_{11}^E = 16.4 \times 10^{-12}\ \text{m}^2/\text{N} \\ s_{12}^E = -5.74 \times 10^{-12}\ \text{m}^2/\text{N} \\ s_{13}^E = -7.22 \times 10^{-12}\ \text{m}^2/\text{N} \\ s_{33}^E = 18.8 \times 10^{-12}\ \text{m}^2/\text{N} \\ \rho = 7.75 \times 10^3\ \text{kg/m}^3 \end{cases}$$

Substituting above parameters into equation (11), the relation of frequency-thickness is calculated and the results are shown in Fig. 3. From it, we can found when thickness increase, the resonant frequency descends fleetly at thickness of less than 5 mm, but decrease smoothly more than 5 mm.

4. Finite Element Analysis for Piezoelectric Ring

The finite element modeling was used to simulate vibration of the ring in order to verify its frequency equation deduced by piezoelectric theory. Because the ring is axisymmetric, a twentieth of the ring (shown in Fig. 4) was modeled with software ANSYS to simplify the computation. It is assumed that the piezoelectric ring behaves as a pure thickness mode (in-plane, on radial direction) vibrator, in the vicinity of the thickness resonance, no radial

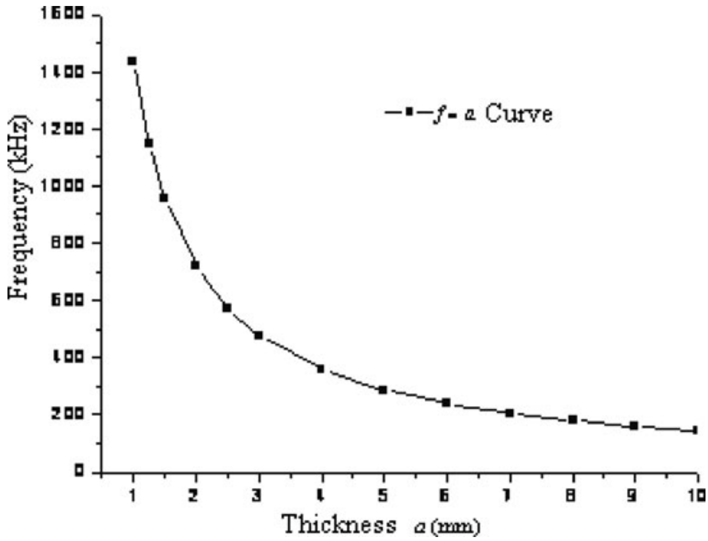

Figure 3. Calculation curve of frequency Vs thickness for PZT-5 ring.

extension/contraction vibration occurs. Therefore, the imposition of rigid boundaries on the lateral surfaces of the unit cell under analysis is justified. The up and down, inner and outer surfaces have free boundaries. Resonant frequency is determined with the potentials on the inner and outer surfaces set equal to zero. PZT-5 material was selected for model,

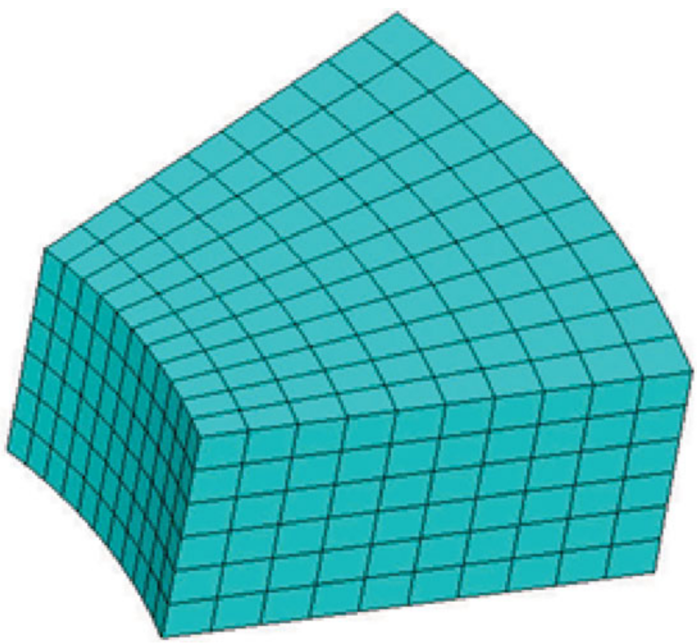

Figure 4. A twentieth model of ring. (Figure available in color online)

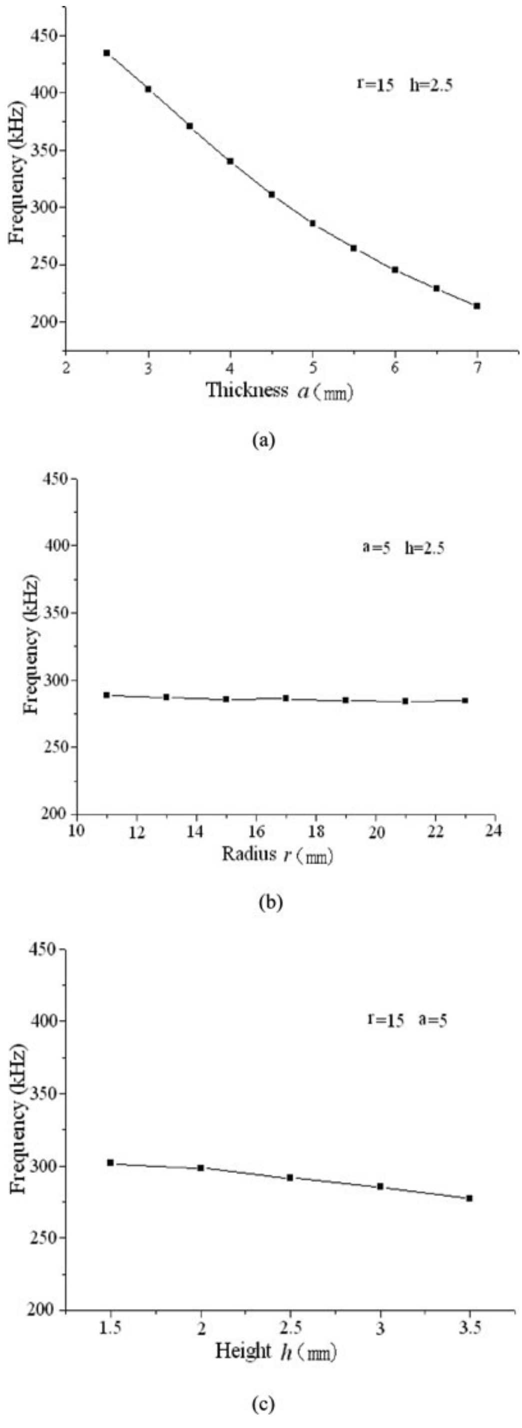

Figure 5. (a) Simulate curve of frequency Vs thickness, (b) simulate curve of frequency Vs radius, (c) simulate curve of frequency Vs height.

its parameters is as follows:

$$\text{Density}\ \rho = 7750\,\text{kg/m}^3 \tag{12}$$

$$\text{Elastic constants matrix}\ C^E = \begin{bmatrix} 11.5 & 7.43 & 7.78 & 0 & 0 & 0 \\ 7.43 & 13.9 & 7.43 & 0 & 0 & 0 \\ 7.78 & 7.43 & 13.9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.56 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3.06 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.56 \end{bmatrix} \times 10^{10}\ \text{N}/\text{m}^2 \tag{13}$$

$$\text{Piezoelectric constants matrix}\ e = \begin{bmatrix} 15.1 & -5.2 & -5.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 12.7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 12.7 \end{bmatrix} \text{N}/(\text{V} * \text{m}) \tag{14}$$

$$\text{Permittivity matrix}\ \varepsilon^S = \begin{bmatrix} 0.561 & & \\ & 0.645 & \\ & & 0.645 \end{bmatrix} e^{-8}\ \text{c}/\text{m} \tag{15}$$

Because geometrical configuration of the ring is key factor that influence resonant frequency, so the function of frequency depended on configuration parameters (radius, thickness, height) of the ring are analyzed. Setting the finite element model of the ring with a series of configuration parameters, determining resonant frequency at each size of the ring, the relation of frequency- parameters (radius, thickness, height) can be gotten (shown in Fig. 5). When calculating, we set one of the parameters (radius, thickness, height) to changing, but the other two unchanged.

From Fig. 5, we can find the resonant frequency descends obviously as thickness increase, the speed of decrease is faster at thickness of less than 5 mm, but become slow more than 5 mm; it also goes smoothly down as radius and height increase, the decrease is small and can be ignored. Thus, the resonant frequency with thickness vibration (on radial direction) depends mainly on thickness of the ring.

Table 1
Configuration parameters and test frequency of the samples

No.	Inner diameter (mm)	Outer diameter (mm)	Thickness (mm)	Height (mm)	Test frequency (kHz)
1	15	19	2	3	510
2	15	21	3	3	421
3	15	23	4	3	350
4	15	25	5	3	290
5	15	29	7	3	220
6	15	33	9	3	170

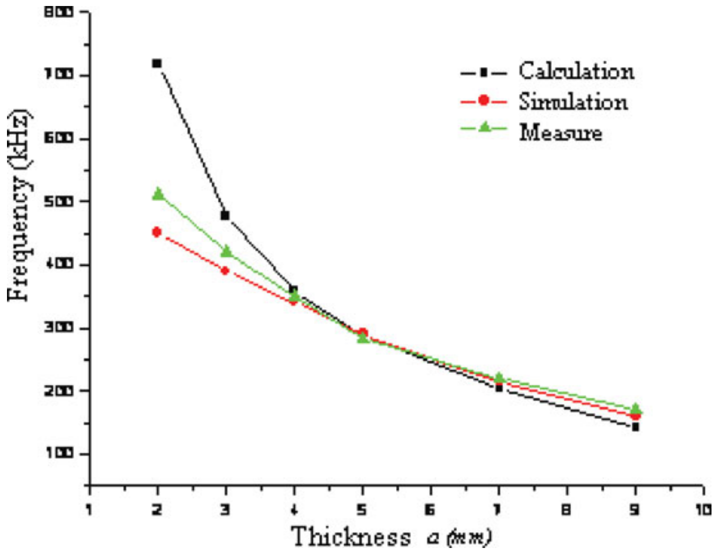

Figure 6. Comparing frequency Vs thickness curves. (Figure available in color online)

5. Experiment

Five PZT5 rings, made of Ninhai electronic ceramic company (Zibo, China), were used as experiment samples. The rings were covered with silver on its inner and outer surfaces and polarized along radial direction, its configuration parameters are shown in Table 1. The resonant frequency of the sample was measured with an HP 4294A impedance analyzer. The results are listed in Table 1.

6. Comparing Calculation and Simulation with Experiment

The resonant frequencies of calculation and simulation were compared with experiment on Fig 6. The result show that three curves descend quickly first and go down slow after the thickness increases, they tend towards the same tracking trend, they are in agreement with each other between the thickness of 4 mm to 9 mm but have evident deflection at the thickness of less than 4 mm.

The error between the calculation and experiment come from the assumption, the unite cell of ring vibrate with longitudinal extensional mode, when deducing frequency equation. The assumption is reasonable if the thickness of ring (length of unite cell) is bigger, but irrational when the thickness is small because the primary vibration of the unite cell transform to thickness extensional mode at that time. In addition, we ignore radial extensional and non-symmetric thickness mode (out-of-plane) in deducing frequency equation.

7. Conclusion

By using the piezoelectric constitutive equations and Kinematical equation, the wave equation of piezoceramic ring is derived and the frequency expressions with thickness vibration of the ring is obtained. To validate the theoretical expressions, the resonant frequency of the ring is simulated numerically and measured experimentally. Theoretical calculation was compared with experiment and simulation, the result show that the calculation is good agree with experiment and simulation between the thickness of 4 mm to 9 mm but have evident deflection at the thickness of less than 4 mm. The error primary comes from the assumption used for deducing frequency equation. In order to reduce the error, the assumption that unite cell vibrate with thickness extensional mode for thin ring is more reasonable. Simulation results also show that the thickness vibration frequency (on radial direction) of piezoceramic ring depends mainly on its thickness, but is almost independent of radius and height.

Acknowledgments

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