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DYNAMIC EVOLUTION OF FINANCIAL NETWORK AND ITS RELATION TO ECONOMIC CRISES

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The static topology properties of financial networks have been widely investigated since the work done by Mantegna, yet their dynamic evolution with time is little considered. In this paper, we comprehensively study the dynamic evolution of financial network by a sliding window technique. The vertices and edges of financial network are represented by the stocks from S&P500 components and correlations between pairs of daily returns of price fluctuation, respectively. Furthermore, the duration of stock price fluctuation, spanning from January 4, 1985 to September 14, 2009, makes us to carefully observe the relation between the dynamic topological properties and big financial crashes. The empirical results suggest that the financial network has the robust small-world property when the time evolves, and the topological structure drastically changes when the big financial crashes occur. This correspondence between the dynamic evolution of financial network and big financial crashes may provide a novel view to understand the origin of economic crisis.

Keywords: Financial network; correlation matrix; evolutional dynamics; financial crashes.

1. Introduction

The financial market has been considered as a typical complex system constructed with an amount of interacting individuals. The stock price fluctuation is usually regarded as one of the chief representatives of economic activity in financial market, which directly reflect the operation state of a company. From the correlations of different time series of stock price, we can hence figure out the relations between these companies. Exploring the evolution dynamics and intrinsic mechanism of financial market have been attracted much attentions of scientists in various fields. Thanks to the development of complex network science starting from Watts–Strögatz small-world model[1] and Barabási–Albert scale-free model,[2] its theory is applied to empirically analyze and model a series of complex systems.[3–9] Mantegna first studied the connectivity pattern of stocks by minimal spanning tree (MST) obtained from their matrix of correlations coefficients, which showed the hierarchical structure of topological space and provides a meaningful economic taxonomy.[10] After that, a lot of works have investigated and modeled the topology of correlation-based MST, which both suggested that the empirical tree has features of a complex network, such as scale-free structure of vertex distribution, community, assortative mixing patterns, etc.[11–15]

Most of previous works so far mainly focus on the topological structure of the financial networks, which contains global information of the financial market during a specific period. Since the economics is improved along with the social development, it is significant to study the the time evolution of the financial market with dynamic network, from which one can deduce economic events, such as economic crises. However, few work has been involved except that the dynamic asset MST was built by Onnela *et al.* to find that the basic structure of tree topology seems robust, yet the scaling exponents of scale-free degree distribution were different for usual business and financial crash periods.[16] The most recent work done by Person and his cooperators suggested that the robustness of evolving financial networks quantified based on entropy-related measurement became weakened when the economic activity in financial market was unstable.[17]

In our work, we comprehensively study the US stock markets by constructing fully dynamic financial networks in a filtering procedure with static and dynamic threshold, respectively. From the topological structures of the dynamic financial networks, it can be found that a high correspondence between the fluctuations of characteristic parameters (including average degree, shortest path length, and clustering coefficient) and big financial crashes, and the financial network shows a robust small-world property.

The paper is organized as follows: in Sec. 2, the database of US stock markets, constructed method and measurement of complex network are described in detail. The empirical results of dynamic evolution of financial networks and its relation to economic crisis will be comprehensively discussed in Sec. 3. At last, we conclude our work.

2. Materials and Methods

2.1. *Stock market database*

We selected $N = 160$ continuously trading stocks of S&P500 components during the 14-year period from January 4, 1985 to September 14, 2009, and the financial networks are constructed by the use of the daily close prices of stocks that are obtained from *Yahoo*!.[18] The whole length of stock price time series is 6230.

2.2. *Constructed method of network*

The network construction of financial market is similar to the approach based on correlation matrix.[10–13,15–17] Thus, the degree of similarity between synchronous time series of each pair of stock prices should be first quantified. We do not immediately compute the correlations between pairs of time series of stock price because of the trend implying in them, and an alternative method is to use the time series of daily logarithmic return, which is usually defined as $R_i(t) = \ln P_i(t) - \ln P_i(t-1)$. $P_i(t)$ describes the close price of stock i at day t. The 160×160 correlation matrix is then obtained by computing the correlation coefficient $\rho_{i,j}(T)$ for each pair of returns of stock i and j,

$$\rho_{i,j}(T) = \frac{1}{\Delta T} \sum_{t=(T-1)*\delta T+1}^{(T-1)*\delta T+\Delta T} \frac{(R_i(t) - \langle R_i \rangle)(R_j(t) - \langle R_j \rangle)}{\sigma_i(t)\sigma_j(t)}, \tag{1}$$

where $\sigma_i = \sqrt{\langle R_i^2(t)\rangle - \langle R_i(t)\rangle^2}$ and $\sigma_j = \sqrt{\langle R_j^2(t)\rangle - \langle R_j(t)\rangle^2}$ are the standard deviation of $R_i(t)$ and $R_j(t)$, respectively. By the definition, the correlation coefficients vary from -1 (complete anti-correlation) to 1 (complete correlation). When $\rho_{i,j} = 0$, the two stocks are uncorrelated. It is noted that the matrix of correlation coefficients is obviously symmetric, and we use the Kronecker delta function to exclude the autocorrelations of returns (i.e. the diagonal elements of matrix are defined as 0).

On the other hand, the correlation coefficient matrix cannot be straightly used to detect the topology structure of financial network because it does not fulfill the three axioms that define a metric.[10] To accurately analyze the dynamics of topology structure, we restrict the correlation coefficients to be distances with the criterion

$$d_{i,j}(T) = \sqrt{2(1 - \rho_{i,j}(T))}, \tag{2}$$

$d_{i,j}(T)$ is equivalent to the Euclidean distance between two vectors, and its interval is [0, 2].

There are several ways to establish the financial network. MST is most widely used because it has a simple structure which greatly simplifies the calculation, and it is easy for an MST to find the hierarchical structure and successfully cluster companies in financial markets. Another way to add edges between vertices is filtering procedure proposed by Boginski.[19] They filter the correlations with static and dynamic threshold values, so that only the correlations stronger than the threshold value is taken into account. By applying a slide window with length $\Delta T = 600$ days and considering the filtering procedure, a series of dynamic financial networks are achieved by thresholding the time-dependent distance matrices to adjacency matrices. For example, we choose a fraction as the threshold value w, and the network can then be constructed as the following process: We traverse the distance matrix and compare $d_{i,j}$ with w, if the formula $d_{i,j} < w$ is true, the two nodes i and j are connected, whereas, they are disconnected. In each sliding step, the starting displacement of window moves $\delta T = 5$ days away from the prior one, which means, if the prior network associates with the matrix of distance $d_{i,j}(T)$, the subsequent network will be constructed through the distance matrix whose components are

$d_{i,j}(T+\delta T)$ during the period from $T+\delta T$ to $T+\Delta T+\delta T$. For whole time scales, we are therefore able to obtain 1222 adjacency matrices, which suggest the dynamic evolution of financial network.

2.3. *Measurement of complex network*

The constructed financial networks are characterized by the measurement of complex network. The three global parameters characterizing topological structure, average degree centrality (AD), average shortest path length (PL), and average cluster coefficient (CC), evolve with time, which are used to suggest the dynamic evolution of financial network. To keep our description as self-contained as possible, we simply introduce the definition of these parameters as follows:

(i) The degree centrality of vertex v is defined by

$$k_v = \sum_{i=1}^{N} A_{v,i}, \tag{3}$$

which suggests the number of neighbors and hence reflects the local adjacency information in a topological structure. The average degree centrality, $\frac{\sum_{i=1}^{N} A_{v,}i}{N}$, describes the connectivity of whole network.

(ii) The shortest path length between a pair of vertices is given by

$$L = \frac{1}{N(N-1)} \sum_{i,j=1}^{N} d_{i,j}, \tag{4}$$

where $d_{i,j}$ is the minimal number of hops (edges) that takes to move from vertex i to j. Thus, the shortest path length of network is the average value of L between all pairs of vertices.

(iii) The cluster coefficient of vertex i denoted as C_i is a measure of the probability that the neighbors of vertex i are mutually connected, i.e. they tend to form local clusters of whole network. It can be suggested by the ratio of all existing edges e_i and maximum possible number of edges among neighbor vertices[1]

$$C_i = \frac{e_i}{k_i(k_i-1)}. \tag{5}$$

Thus, the CC of whole system is an average value of C_i over all vertices in the network. The PL and CC are important because they determine whether the connectivity of network vertices are in a random way or a small-world way.

3. Empirical Results

Because the filtering procedure is applied to transfer the distance matrices into adjacency ones, we should pay much attention on choosing a proper threshold value to robustly and accurately quantify the topological structures of financial networks.

The network would be separated into a number of isolated small clusters under a lower threshold value. On the other hand, the overlarge one would make the network include the redundancy edges of stocks introduced by noise trades in financial market. We quantify the size of giant component (the largest connected sub-network) under a certain threshold value w, and the percolation phase transition of size occurs when w changes from small value to large one. The critical w corresponding to point of phase transition makes the size of giant component exactly equals to the number of vertices of financial network. In Refs. 15 and 20, it suggests that the proper threshold value should be selected around the critical w.

In the experiments, we use both dynamic and static threshold values to construct the financial networks. Figure 1 shows the critical w evolving with time, which are used to be dynamic threshold values. As shown in Fig. 1, the range of dynamical threshold values finely fluctuates from 1.24 to 1.38, which suggests that the positive correlations between stocks generally exist in the constructed financial networks, and the local maximums (denoted by red lines) associate with the big financial crashes. For instance, the US economic crisis (1985−1987) starting from the strong dollar policy caused by the Plaza Accord was collapsed on October 19, 1987 (i.e. famous "Black Monday"). In this period, the capitals of stocks were quickly and synchronously growing, and they were highly correlated with each other that leads to a number of lower and smooth dynamic threshold values. The similar case can also be found at the end of time evolution of dynamic threshold values because the much bigger financial crash of the US subprime Lending crisis in 2008.

The time evolution of interactions among 160 US stocks of S&P500 components is first studied through a series of financial networks constructed based on dynamic threshold values and the sliding window displaced along with time. First, the three

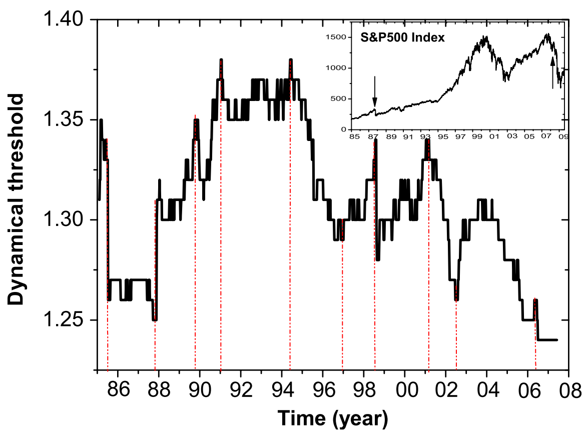


Fig. 1. (Color online) The critical threshold values evolve with time, at which the whole 160 stocks are completely connected in a giant component. The red dashed lines label the large fluctuations (or local maximums), which associate with the big financial crashes. The inner panel shows the the time evolution of S&P500 index, which is used to compare with that of critical threshold values.

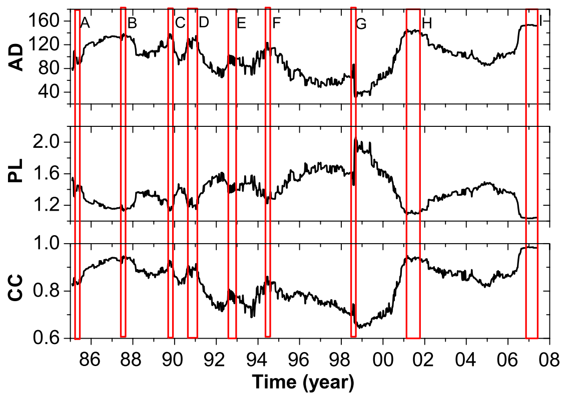


Fig. 2. (Color online) The dynamic evolution of three important global topological parameters of financial networks, average degree centrality (upper panel), average shortest path length (middle panel), average cluster coefficient (lower panel), which immediately suggest the time evolution of 160 US stocks of S&P500 components from January 4, 1985 to September 14, 2009. The red rectangles (denoted by A to I) correspond to financial crashes influencing the financial markets: (A) The starting of US economic crisis caused by the Plaza Accord; (B) Black Monday on October 19, 1987; (C) the US savings and loan crisis and the drastic change in Eastern Europe in 1989; (D) the collapse of Japanese asset price bubble in 1990; (E) the European Exchange Rate Mechanism Crisis 1992; (F) the Mexico's financial crisis in 1994; (G) Asian financial crisis and the collapse of hedge fund Long-Term Capital Market in 1998; (H) 9/11 attacks and the Argentine financial crisis in 2001; (I) the US subprime Lending crisis in 2007−2008.

important global topological parameters AD, PL and CC are shown in Fig. 2. The local optimal regions of three parameters during their dynamic evolution are roughly denoted by red rectangles that associate with a number of financial crashes (see in Fig. 2). Moreover, in the stock market, most of the stock price fluctuations are strongly associated with the index fluctuation (such as S&P500 index) because the index is the straight composition of the stock prices. Therefore, these results suggest that the financial crashes clearly reflected by the drastic index fluctuation (see in the inner panel of Fig. 1) are able to result in the similar trends (i.e. synchronicity) of these stock price fluctuations, which improves the correlations among them. Concretely speaking, for the AD, it drastically fluctuates with time, and keeps very large values during the financial crashes immediately associating with US economics events (e.g. the subprime Lending crisis in 2008). The lowest value is found at the end of Asian financial crisis and the collapse of hedge fund long-term capital market in 1998 because these financial crashes strongly attack the rapid development of US economics from 1995 to 2000 and shortly disturbs the synchronicity of stock price fluctuations. These results also imply in the dynamic evolutions of PL and CC. Furthermore, the large AD makes the financial networks have a lower PL and larger CC, which suggests that the robust small-world property exists in financial networks.

To quantify more accurately and clearly the irregularity of dynamic evolution of topological structures of financial networks, we estimate the discrete curvatures of AD, PL and CC, which both show obviously clustering behaviors (see in Fig. 3). For the AD, the largest magnitude of discrete curvature almost approaches to 2×10^4, which covers the small cluster behaviors. However, from the discrete curvatures of PL and CC, we can clearly find that they have the similar trend and their magnitudes of clusters well corresponding to the financial crashes are much larger than these of usual business day. These results also suggest that the financial crashes may bring the irregularity of dynamics evolution of financial markets, which means that the economic crises highly associate with the collective dynamics of stocks in financial market.

As shown above, the dynamic evolution of financial network based on dynamic threshold value shows a rich phenomenon, and well associates with the economic crises. However, the financial network can also be constructed by a static threshold value, and it is interesting to evaluate how much the dynamic evolution of financial networks is influenced by different static threshold values. According to the critical threshold values in Fig. 1, we mainly choose various static threshold values $w = 1.25$, 1.28, 1.30 and 1.32 to construct the financial networks with different sliding windows displaced along with time, respectively. In Fig. 4, we first compare the ADs, PLs and CCs as a function of time with different static threshold values, which correspond to those analysis of dynamic threshold values. Their dynamic evolutions both show a similar trend, and the locations of anomalous fluctuations in dynamic evolution indicate the collapses of economic crises, regardless of the threshold values. We also take the dynamic evolution of AD as a example. On one hand, the values of AD, especially the bottom part of curves, increase with w, and it interests us that the slight financial crashes reflected by anomalous fluctuations turn to be much more prominent, like the Asian financial crisis in 1998. The reason may be that the weak correlations among stocks with small capitals are introduced with larger static threshold value, so that the values of AD are obviously improved in local regions. On

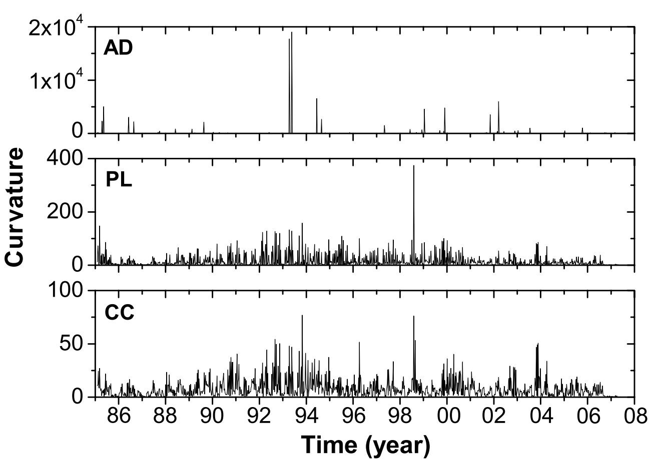
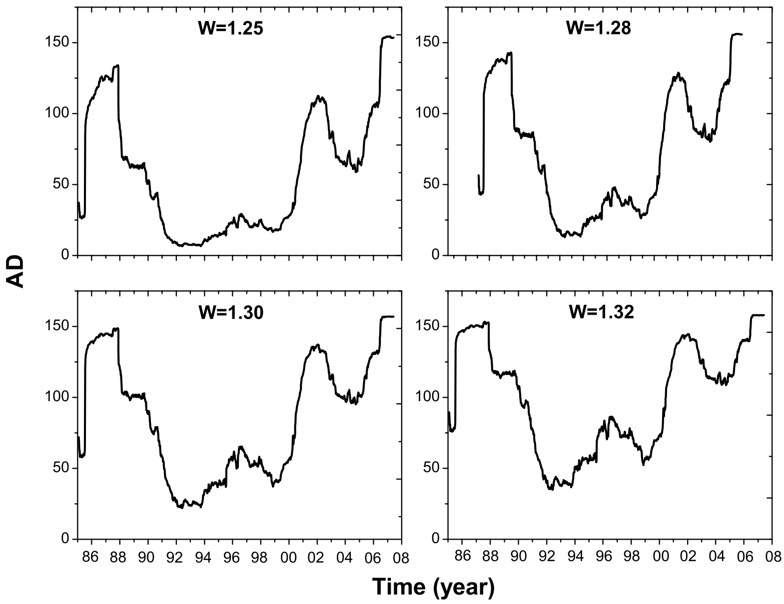
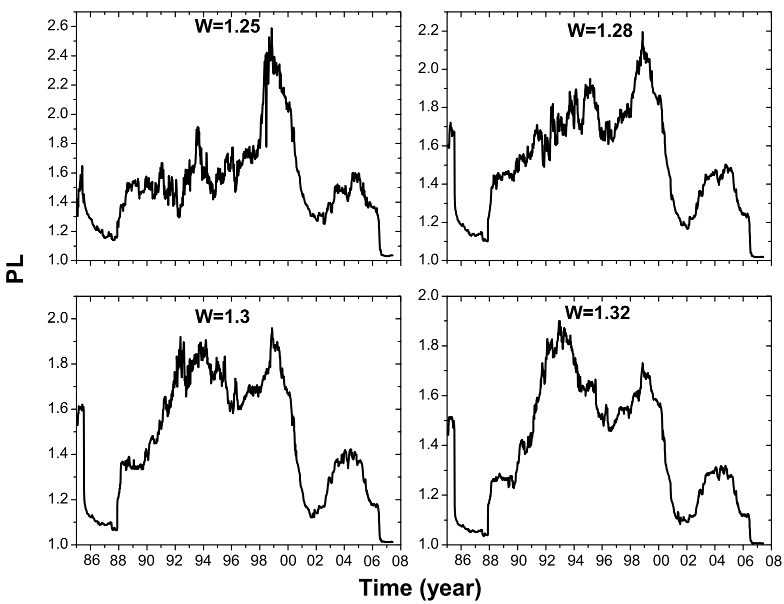


Fig. 3. The discrete curvatures of AD (upper panel), PL (middle panel) and CC (lower panel) evolve with time, which show obviously clustering behavior. The clusters corresponding to the financial crashes have larger magnitudes.

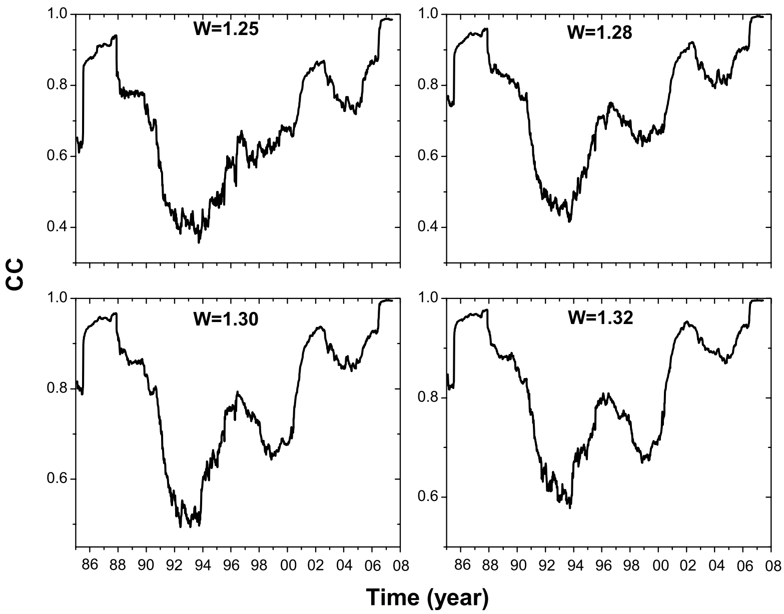


(a)



(b)

Fig. 4. The dynamic evolutions of ADs, PLs and CCs when the financial networks are constructed with the static threshold values $w = 1.25$, 1.28, 1.30 and 1.32, respectively. They show the similar trends as a function of time. Take the ADs as example, the large financial crises, such as the Black Monday, are significant for the lower static threshold value, while the general ones, such as the Asian financial crisis, become obvious when the financial networks are constructed with larger static threshold value.



(c)

Fig. 4. (*Continued*)

the other hand, the large financial crash becomes much more significant when the static threshold values decrease because they highly affect the collective dynamic of stocks with large capitals, and we can find the characteristic peak, such as the Black Monday, even the financial network is locally connected [see in Fig. 4(a)]. In addition, it should be noted that the constructed financial network based on the static threshold values also have robust small-world property.

4. Conclusion

In conclusion, we have constructed the correlation-based financial networks by the sliding window technology with both dynamic and static threshold values. The dynamic evolution of financial networks as a function of time are comprehensively studied to observe the dynamic topological properties and their relations to economic crises. By analyzing the three global parameters, AD, PL and CC evolving in a 14-year period, we find that the financial networks show the robust small-world property regardless of the choice of threshold value. Most importantly, the irregularities of curves indicating the dynamic evolution of financial network highly associate with the financial crashes, of which the discrete curvatures form a number of clusters and their locations corresponding to larger magnitude are consistent with the famous economic crises. Therefore, these interesting results may provide a novel view of complex network science to deeply understand the origin of economic crisis.

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