Physica A 389 (2010) 3828–3836

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Are developed and emerging agricultural futures markets multifractal? A comparative perspective

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A R T I C L E   I N F O

Article history:
Received 31 October 2009
Received in revised form 6 March 2010
Available online 1 June 2010

Keywords:
Agricultural futures markets
MF-DFA
Multifractal properties
Nonlinear temporal correlation
Non-Gaussian probability distribution
The strength of multifractality

A B S T R A C T

Although there are many reports on the empirical evidence of the existence of multifractality in various financial or commodity markets in current literature, few can be found to compare the multifractal properties of emerging and developed economies, especially for agricultural futures markets in those countries (regions). We therefore chose China as the representative of the transition and emerging economies, and USA as the representative of developed ones. We attempt to find the answers to the following questions: (1) Are all those different markets multifractal? (2) What are the dynamical causes for multifractality in those markets (if any)? (3) Are the multifractality strengths in those markets of the transition and emerging economies weaker (or stronger) than those of the developed ones? To answer these questions, Multifractal Detrended Fluctuation Analysis (MF-DFA) are applied to study some of the representative agricultural futures markets in China and USA, namely, wheat, soy meal, soybean and corn. Our results suggest that all the markets of China and USA exhibit multifractal properties except US soybean market, which is much closer to mono-fractal comparing with China's soybean market. To investigate the sources of multifractality, shuffling and phase randomization procedures are applied to destroy the temporal correlations and non-Gaussian distributions respectively. We found that multifractality can be mainly attributed to the non-Gaussian probability distribution and secondarily to the nonlinear temporal correlation mechanism for all the markets, except US soybean and soy meal, which derives from some other unknown factors. Furthermore, the average of $\tau(q)$ are applied to obtain the multifractal spectra of the two markets as a whole. The results show that the width of the multifractal spectrum of US agricultural futures markets is significantly narrower than that of China's. Based on our findings, we proposed a hypothesis that the strength of multifractality in developed economies may be weaker than that in emerging and transition ones.

1. Introduction

As an important representative of transition and emerging economies, China's economy is rapidly growing and getting more globally influential in recent decades. At the same time, although USA is still enduring the ongoing financial tsunami, it is still the most important developed economy in the world. As for the two economic giants, the following questions are waiting to be answered: (1) Are all those completely different economies multifractal? (2) What are the causes for the multifractality in the economies (if any)? (3) Are the multifractality strengths in the transition and emerging economies stronger (or weaker) than those in the developed ones? Focusing on agricultural futures markets, we try to answer those questions from a comparative perspective.

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0378-4371/$ – see front matter
doi:10.1016/j.physa.2010.05.021

The study of financial or commodity prices is largely based on current main stream literature, whose fundamental assumption is that stock price (or returns) follows a normal distribution and price behavior obeys 'random-walk' hypothesis (RWH), which was first introduced by Bachelier in 1900 [1], since then it has been adopted as the essence of many asset pricing models. However, some important results in econophysics suggest that price (or returns) in financial or commodity markets have fundamentally different properties that contradict or reject RWH. These ubiquitous properties identified are: fat tails [2], long-term correlation [3], volatility clustering [4], fractals and multifractals [5–8], chaos [9], etc. Nowadays, RWH has been widely criticized in the finance and econophysics literatures as this hypothesis fails to explain the market phenomena.

After investigating the prices of cotton, wheat and so on, Mandelbrot provided earliest empirical evidence that agricultural commodity spot prices do not obey RWH by means of fractal geometry [10,11]. Since then, fractal geometry has been widely applied in finance and market research domains. Peters introduced fractal theory into the capital market research, and provided empirical evidence of the mono-fractal properties in many financial markets by means of R/S analysis [12,13]. In order to study the mono-fractal properties of nonstationary series, Detrended Fluctuation Analysis (DFA) [14] and Detrended Moving Average Analysis (DMA) proposed by Carbone et al. [15–20] were introduced for the analysis of average and time-dependent long-range correlation. As Mono-fractals cannot describe the multiscale and subtle substructures of fractals in complex systems, many measures are applied to investigate the multifractality, such as height–height correlation function [21], Multifractal Detrended Fluctuation Analysis (MF-DFA) [22–31], the partition function method [32–34], etc. Empirical evidence shows that many financial markets are multifractal. Norouzzadeh et al. found the multifractal properties and scaling behaviors of the exchange rate variations of the Iranian rial against the US dollar, and found that the contributions of two major sources of multifractality are fat-tailed probability distributions and nonlinear temporal correlations [26]; Kumar and Deo studied the multifractal properties of the logarithmic returns of the Indian financial indices, and found that the multifractality is due to the contributions of nonlinear temporal correlations as well as the broad probability density function [31]; Oświęcimka et al. investigated the different multifractal properties between the time series of logarithmic price increments and the inter-trade intervals of time by high-frequency tick-by-tick data, and found that the multifractals come from the nonlinear temporal correlations as well as the non-Gaussian distributions of the fluctuations [24]. Similar results are found in commodity markets. Alvarez-Ramirez et al. investigated the multifractal properties of international crude oil prices and their dynamical properties [6]; Matia et al. analyzed daily price of 29 commodities (and 2449 stocks as well), and found that the price returns for commodities have a significantly broader multifractal spectrum than for stocks, and both of the multifractal properties can be attributed mainly to the broad probability distribution of price fluctuations and secondly to their temporal organization [23]; Lim et al. investigated the multifractal properties of price increments in the cases of derivative and spot markets, and found that multifractality due to a fat-tailed distribution is significant [27].

In agricultural futures markets domain, Chatrath et al. studied four futures as the representatives of US agricultural futures and found low-dimensional chaotic structures in the markets [35]; Corazza et al. studied six main US agricultural futures and found the existence of mono-fractals [36]. As for China's markets, although there are some results on multifractal properties in Shenzhen and Shanghai stock markets [29,32,33], few empirical evidence in current literature can offer the answer to the problem whether China's agricultural futures markets are multifractal or not.

Many scholars have compared many different markets and investigated their multifractal properties. K. Matia et al. investigated daily prices of 29 commodities and 2449 stocks, and found that the price returns for commodities have a significantly broader multifractal spectrum than for stocks [23]; L. Zunino et al. investigated the multifractality degree of developed and emerging stock market indices, and found that higher multifractality is associated with a less developed market [30]; Zhi-Qiang Jiang, Wei-Xing Zhou also investigated the emerging and developed stock markets, and found that there are not multifractality in the original series of the two markets [34], but their results on China's stock indices shows that there are multifractality properties in those markets [33]. Matos et al. use a new method of studying the Hurst exponent with time and scale dependency to recover the major events affecting worldwide markets which can measure and compare the behaviors in emergent/established markets [37]. Current studies focused on the commodity market, stock market and some other fields, but there is no report on comparative multifractal study between the emerging and developed agricultural futures markets.

Therefore, we chose wheat, soy meal, soybean and corn futures contracts from US and China's agricultural markets as the representatives of the emerging and developed markets, and applied MF-DFA to study the multifractal properties. Our results suggest that there are multifractal features in the two markets except US soybean market, which is much closer to mono-fractal comparing with its counterpart in China; furthermore, the dynamical resources of multifractality are investigated by means of shuffling and phase randomization procedures; finally, the average of $\tau(q)$ are applied to obtain the multifractal spectrum of whole markets, and the multifractal strengths of emerging and developed agricultural futures markets are compared.

2. Model

To keep our description as self-contained as possible, let us review briefly the model [38–40].
Let us suppose $P(i),\ i = 1, 2, \dots, L$, to be a price series, where L stands for the length of the analyzed series. Let us define the logarithmic returns as:

$$r(i) = |\ln (P(i + \Delta t)/P(i))| \quad (i \le L - \Delta t) \tag{2.1}$$

where $\Delta t = 1$, and the length of $r(i)$ is written as $N(N = L - \Delta t)$. Then determine the "profile":

$$Y(i) = \sum_{i=1}^{N} (r(i) - \bar{r}) . \tag{2.2}$$

Divide the profile $Y(i)$ into $N_s = \text{int}(N/s)$ non-overlapping segments of equal length s. Since the length N of the series is often not a multiple of the considered time scale s, a short part at the end of the profile may remain unused. In order not to disregard this part of the series, the same procedure is repeated starting from the opposite end. Thereby, $2N_s$ segments are obtained altogether. And then calculate the local trends for each of the $2N_s$ segments by mth order polynomial fit. Then the variance is given by

$$F^2(s, v) = \frac{1}{s} \sum_{i=1}^{s} \left[Y_v(i) - \tilde{Y}_v(i)\right]^2 . \tag{2.3}$$

Here, $\tilde{Y}_v(i)$ is the fitting polynomial in segment v. Let us then average over all segments to obtain the qth order fluctuation function:

$$F_q(s) = \left\{ \frac{1}{2N_s} \sum_{v=1}^{2N_s} \left[F^2(s, v)\right]^{\frac{q}{2}} \right\}^{\frac{1}{q}} . \tag{2.4}$$

When $q = 0$, we calculated its limit:

$$F_0(s) = \exp \left\{ \frac{1}{4N_s} \sum_{v=1}^{2N_s} \ln \left[F^2(s, v)\right] \right\} . \tag{2.5}$$

If the series is power-law correlated, with the increasing of s, it should obey the power-law:

$$F_q(s) \propto s^{h(q)}. \tag{2.6}$$

Through the least-square fit, the generalized Hurst exponent $h(q)$ can be estimated by the slope of $\ln F_q(s)$ and $\ln s$. If $h(q)$ is a constant, the series is mono-fractal; otherwise it is multifractal. When $q = 2$, MF-DFA becomes DFA, and $h(2)$ is the well-known Hurst exponent.

There is an analytical relationship between the generalized Hurst exponent $h(q)$ and the scaling exponent $\tau(q)$ defined by the standard partition function multifractal formalism [22]:

$$\tau(q) = qh(q) - 1. \tag{2.7}$$

Another way to characterize a multifractal series is the singularity spectrum $f(\alpha)$, that is related to $\tau(q)$ via a Legendre transform:

$$\alpha = \frac{\mathrm{d}\tau(q)}{\mathrm{d}q} \quad \text{and} \quad f(\alpha) = q\alpha - \tau(q). \tag{2.8}$$

Through Eqs. (2.7) and (2.8), it is straightforward to relate α and $f(\alpha)$ to $h(q)$:

$$\alpha = h(q) + qh'(q) \quad \text{and} \quad f(\alpha) = q(\alpha - h(q)) + 1. \tag{2.9}$$

If there exist multifractal properties, the generalized Hurst exponent $h(q)$ can be fitted by the following function [41]:

$$h(q) = \frac{1}{q} - \frac{\ln(a^q + b^q)}{q \ln 2} \quad (a > b) \tag{2.10}$$

where a and b stand for fitting parameters. $\tau(q)$ can be fitted by the function $\tau(q) = -\frac{\ln(a^q+b^q)}{\ln 2}$. Then we obtain $\tau'(q)$; thus singularity exponent α and singularity spectrum $f(\alpha)$ can be estimated by means of Eq. (2.8).

3. Data analysis and discussions

3.1. Data

The data used in this paper are the daily closing prices of hard winter wheat futures market from Dec. 28th, 1993 to Sep. 18th, 2009 ($L = 3183$) market from China's Zhengzhou Commodity Exchange, and soy meal futures from Jul. 17th, 2000 to Sep. 18th, 2009 ($L = 2200$), No. 1 soybean futures from Mar. 15th, 2002 to Sep. 18th, 2009 ($L = 1815$), corn futures from Sep. 22nd, 2004 to Sep. 18th, 2009 ($L = 1216$) market from China's Dalian Commodity Exchange. To compare the difference,

Table 1
The summary statistics of wheat, soy meal, soybean and corn.

		Mean	Std. dev.	Skewness	Kurtosis	Jarque–Bera
China	Wheat	1477.0	228.25	0.0693	2.1697	93.970*
	Soy meal	2424.2	602.70	0.7134	2.9972	186.60*
	Soybean	3146.9	753.56	1.0707	3.5444	369.18*
	Corn	1506.8	207.71	−0.2054	1.7834	83.544*
USA	Wheat	409.39	169.74	1.8992	6.1287	3211.7*
	Soy meal	222.02	70.084	1.0952	3.0946	440.60*
	Soybean	787.16	256.44	1.1632	3.5556	432.62*
	Corn	365.08	129.72	0.8749	3.3191	160.30*

* Means reject the null hypothesis that the sample comes from a normal distribution at the significance of 0.01.

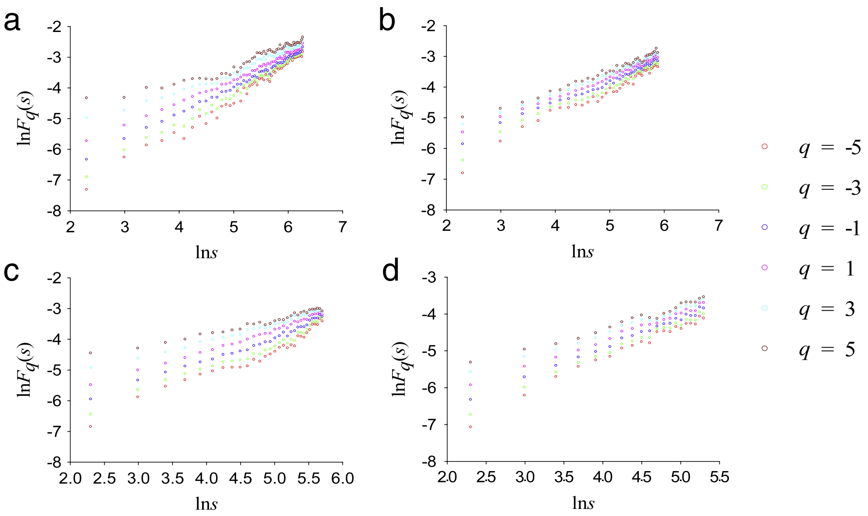

Fig. 1. The relationships between $\ln s$ and $\ln F_q(s)$ in China's agricultural futures markets ((a) wheat, (b) soy meal, (c) soybean and (d) corn, respectively) when $m = 3$.

we chose the same lengths of daily closing prices of wheat futures ($L = 3183$), soy meal futures ($L = 2200$), soybean futures ($L = 1815$) and corn futures ($L = 1216$) from CBOT, covering almost the same time periods as their counterparts in China. All our data are taken from Reuter©database. In the following discussions, the size s ranges from 10 to $[N/6]$ with the computation step 10; the degree of polynomial $m = 1, 2, 3$; because the theoretical moments seems divergent for the larger q values and one faces the so called "freezing" phenomenon which results in linearization of $\tau(q)$ [42], the range of q is restricted from −5 to 5 with the step 0.1.

To get a better understanding of our data sets, summary statistics of the four futures are provided (see Table 1), from which one can see clear departure from a normal distribution.

3.2. Multifractal spectrum analysis

By means of the above-mentioned model, first of all we obtained $\ln F_q(s)$ vs. $\ln s$ relationships of China's and US agricultural futures markets (see Figs. 1 and 2). From the figures one can find that they seem have crossovers, for example, in the US wheat case. These may be explained by the reason that the crossovers result from the competition between the scaling of the noise and the "apparent" scaling of the trend [43], because the MF-DFA method only can remove the polynomial trends. Then the relationships between q and $\tau(q)$ are obtained in Fig. 3, which show that the relationships between $\tau(q)$ and q are nonlinear except US soybean market, whose relevant curve is seemingly linear. It is also obvious that $h(q)$ is nonlinearly dependent on q, and decreases while q increases except US soybean market, which seems more close to a constant (see Fig. 4). Fig. 5 presents the nontrivial multifractal spectra. All these pieces of empirical evidence imply that multifractality properties can be found both in China's and US agricultural futures markets except US soybean market, which is much closer to mono-fractal comparing with its counterpart in China. Especially, when $q = 2$, all the Hurst exponents of agricultural futures markets are greater than 0.5 (see Table 2), for example, $h(2) = 0.7619 \pm 0.0300$ for China's wheat, $h(2) = 0.6515 \pm 0.0741$ for US wheat, which imply that all the markets do not obey random walk and show persistent properties. The results also indicate that the Hurst exponents of China's agricultural futures markets are greater than US, which suggests that agricultural futures markets in China are more persistent than those in US and that the agricultural futures markets in US are more efficient than those in China [13].

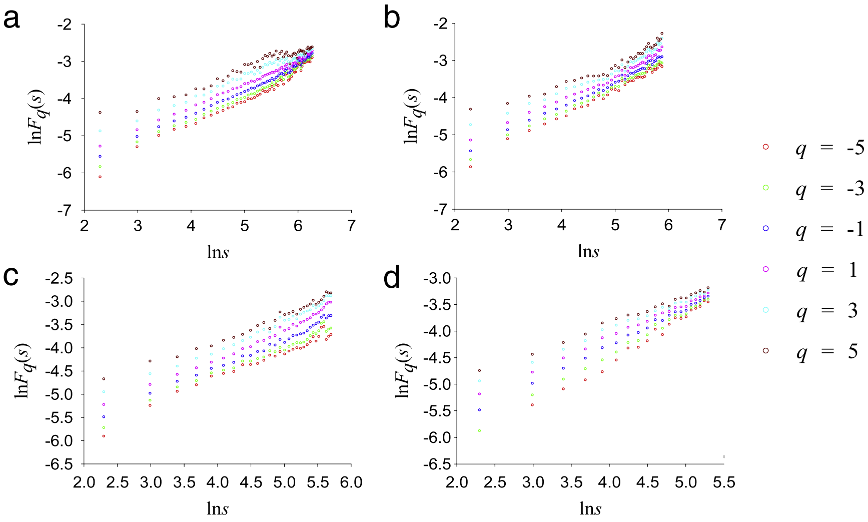

Fig. 2. The relationships between $\ln s$ and $\ln F_q(s)$ in US agricultural futures markets ((a) wheat, (b) soy meal, (c) soybean and (d) corn, respectively) when $m = 3$.

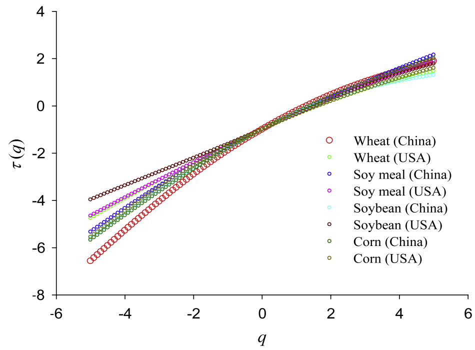

Fig. 3. The $q \sim \tau(q)$ curves when $m = 3$ and $-5 \leq q \leq 5$ with the step 0.1, from which we can find nonlinear relationships between $\tau(q)$ and q in both China's and US agricultural futures markets.

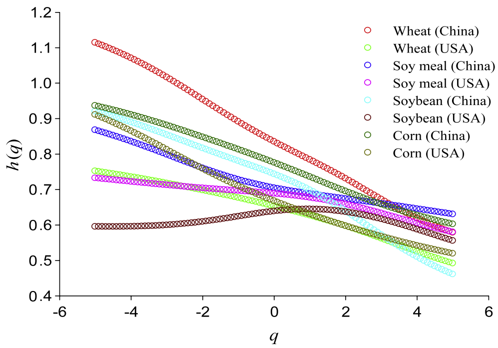

Fig. 4. The $q \sim h(q)$ relationships where $m = 3$ and $-5 \leq q \leq 5$ with the step 0.1.

4. Sources of multifractality

In current literature, two major sources of multifractality are widely acknowledged which can be found in various time series. One is long-range temporal correlation for small and large fluctuations, the other is non-Gaussian probability distribution of increments [23,44]. Usually, two procedures can be applied to identify the contributions of two sources and to indicate the multifractality strength, that is, shuffling and phase randomization [23]. In order to investigate the dynamical causes of multifractality in the markets, both of the two methods are used in this article. The Shuffling procedure will destroys any temporal correlations, aka, long-range or short-range memories in the markets, but the distributions

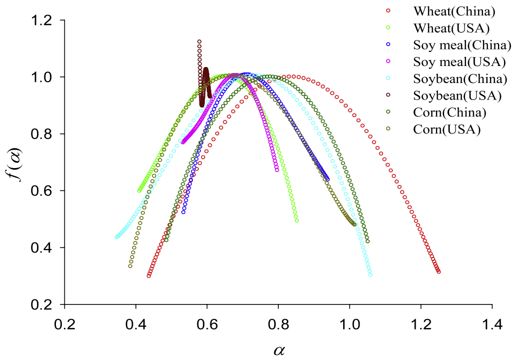

Fig. 5. The relationships between α and $f(\alpha)$, where $-5 \leq q \leq 5$ with the step 0.1.

Table 2
Generalized Hurst exponents and the width of multifractal spectrum.

		Original				Shuffled				Surrogate			
		Wheat	Soy meal	Soybean	Corn	Wheat	Soy meal	Soybean	Corn	Wheat	Soy meal	Soybean	Corn
$m = 1$													
China	$h(2)$	0.7919	0.7978	0.8049	0.7641	0.4859	0.5197	0.4671	0.4439	0.7550	0.7392	0.7198	0.7177
	$h(-5)$	1.1777	0.8881	1.0293	0.9614	0.7121	0.6242	0.6857	0.6526	0.7640	0.7106	0.7531	0.7734
	$h(5)$	0.6396	0.7430	0.6598	0.6383	0.2783	0.5064	0.3422	0.3646	0.7552	0.7430	0.7389	0.7117
	$\Delta\alpha$	0.8121	0.2425	0.5989	0.5526	0.6721	0.2195	0.5660	0.4994	0.0156	0.0702	0.0277	0.1183
USA	$h(2)$	0.7256	0.7388	0.7813	0.7602	0.4802	0.5136	0.5095	0.5117	0.6796	0.7042	0.7200	0.7044
	$h(-5)$	0.7939	0.7625	0.7287	1.0715	0.6013	0.6582	0.6261	0.6540	0.7017	0.6413	0.7042	0.7218
	$h(5)$	0.6685	0.6889	0.6890	0.7151	0.3849	0.4323	0.4188	0.4811	0.6794	0.7158	0.7228	0.6949
	$\Delta\alpha$	0.2029	0.1113	0.0293	0.5797	0.3723	0.3785	0.3596	0.3135	0.0439	0.1594	0.0412	0.0514
$m = 2$													
China	$h(2)$	0.7632	0.7011	0.6721	0.7148	0.5065	0.5150	0.4853	0.5025	0.6999	0.6621	0.6590	0.6899
	$H(-5)$	1.1424	0.8592	0.9258	0.9450	0.7273	0.6160	0.6947	0.7139	0.7260	0.6890	0.6903	0.7566
	$h(5)$	0.6287	0.6658	0.4943	0.5948	0.2786	0.4944	0.3591	0.4224	0.6860	0.6544	0.6635	0.6799
	$\Delta\alpha$	0.7855	0.3320	0.6824	0.5884	0.6897	0.2288	0.5507	0.5064	0.0787	0.0677	0.0487	0.1525
USA	$h(2)$	0.6323	0.7051	0.6985	0.6445	0.4856	0.4989	0.5230	0.5344	0.6061	0.6688	0.6317	0.6303
	$h(-5)$	0.7714	0.7423	0.6450	0.9363	0.6069	0.6365	0.6484	0.6521	0.6170	0.6573	0.6452	0.6719
	$h(5)$	0.5237	0.6451	0.6121	0.6021	0.3834	0.4056	0.4434	0.5058	0.6103	0.6739	0.6395	0.6402
	$\Delta\alpha$	0.4243	0.1592	0.0180	0.5483	0.3754	0.3889	0.3529	0.2705	0.0101	0.0374	0.0089	0.0613
$m = 3$													
China	$h(2)$	0.7306	0.6733	0.6390	0.6948	0.5303	0.5024	0.5004	0.5404	0.6862	0.6451	0.6250	0.6675
	$h(-5)$	1.1137	0.8672	0.9199	0.9358	0.7486	0.6118	0.7163	0.7861	0.7059	0.6945	0.6819	0.7455
	$h(5)$	0.5777	0.6299	0.4603	0.6018	0.3111	0.4807	0.3596	0.4627	0.6821	0.6378	0.6198	0.6539
	$\Delta\alpha$	0.8138	0.4050	0.7121	0.5650	0.6736	0.2458	0.5775	0.5408	0.0459	0.1129	0.1219	0.1829
USA	$h(2)$	0.5966	0.6581	0.6361	0.5967	0.5064	0.4936	0.5057	0.5270	0.6006	0.6321	0.6028	0.5953
	$h(-5)$	0.7514	0.7316	0.5946	0.9105	0.6262	0.6739	0.6611	0.6990	0.6406	0.6799	0.6309	0.6674
	$h(5)$	0.4913	0.5791	0.5549	0.5188	0.3936	0.3862	0.4312	0.4880	0.5954	0.6212	0.6037	0.5683
	$\Delta\alpha$	0.4418	0.2647	0.0293	0.6291	0.3883	0.4710	0.3889	0.3680	0.0889	0.1155	0.0520	0.1988

still remain exactly the same; while the surrogate data created by phase randomization will weaken the non-Gaussian distribution but still preserves the linear properties of the returns. If the multifractality derives from non-Gaussian distribution, the generalized Hurst exponent $h(q)$ obtained by the surrogate data should be a constant 0.5; if the temporal correlation is the only reason for the multifractal features, after the series is phase-randomized, $h(q)$ should be independent of q; nevertheless, if both of the two source are the reasons, the multifractality should remain but its strength should be weaker.

The shuffling procedure consists of the following steps [23]:

Step 1: generating pairs (m, n) of random integer numbers, which satisfies $m, n \leq N$, where N is the length of the time series to be shuffled;

Step 2: interchanging entries m and n of the time series;

Step 3: repeating the first and second steps for $20N$ times. It is critical to ensure that ordering of entries in the time series is fully shuffled, thus the long-range or short-range memories, if any, will be destroyed. The shuffling is repeated with different random seeds to avoid the systematic errors caused by random number generators.

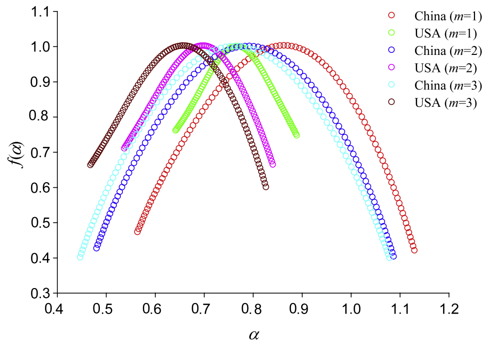

Fig. 6. The $\alpha_{av} \sim f_{av}(\alpha)$ curves with different orders of polynomial m, where $-5 \le q \le 5$ with the step 0.1.

And the algorithm of phase randomization [28,45]:

Step 1: taking the discrete Fourier transform of the time series.

Step 2: shuffling the phases of the complex conjugate pairs; please also note that the phases of the complex numbers must be shuffled pairwise to preserve the realness of the inverse Fourier transformation.

Step 3: taking the inverse Fourier transformation.

From the results for the shuffled and surrogate cases in Table 2, we can find that all the spectra widths become significantly narrower after phase randomization procedure but only decrease slightly after the shuffling procedure except that those of US soy meal and soybean markets, that is, $\Delta\alpha$ from 0.1784 ± 0.0863 (original) to 0.4128 ± 0.0343 (shuffled) and 0.1041 ± 0.0667 (surrogate) for US soy meal market, and from 0.0255 ± 0.0038 (original) to 0.3671 ± 0.0142 (shuffled) and 0.0340 ± 0.0251 (surrogate) for US soybean market. Thereby, non-Gaussian distribution constitutes the major contributions in the multifractality formation in all markets except US soy meal and soybean markets, which may be influenced by some unidentified dynamical causes.

Especially, as for the shuffled returns, when $q = 2$, all the Hurst exponents of China's and US agricultural futures markets are around 0.5 (see Table 2), i.e., $h(2) = 0.5076 \pm 0.0217$ (China's wheat) and 0.4907 ± 0.0105 (US wheat), $h(2) = 0.5124 \pm 0.0100$ (China's soy meal) and 0.5020 ± 0.0084 (US soy meal), $h(2) = 0.4843 \pm 0.0172$ (China's soybean) and 0.5127 ± 0.0070 (US soybean), $h(2) = 0.4956 \pm 0.0517$ (China's corn) and 0.5244 ± 0.0127 (US corn). These results clearly indicate that the shuffled series obey random walk [12,13].

Compared with the results for original cases, we can find that except for US soy meal and soybean markets, after the surrogate procedure successfully weakens the non-Gaussian distribution, the multifractality also becomes significantly weaker while at the same time preserves the correlation between the data; thereby, the non-Gaussian probability distribution is by no means the plausible main explanation for market multifractality formation, in China's agricultural futures markets. As for US soy meal markets, the multifractality is mainly due to non-Gaussian distribution, and for US soy meal and soybean markets, there might be other unknown factors which determine the multifractality formation in these markets.

5. Comparative analysis

Although we shed light on the plausible causes for multifractality formation in those markets, the following question is still waiting to be answered: Are the multifractal strengths in those markets of the transition and emerging economies weaker (or stronger) than those of the developed ones?

In order to compare China's and US agricultural futures markets as a whole, we applied the average of $\tau(q)$ proposed in Ref. [23]:

$$\tau_{av}(q) = \frac{1}{N}\sum_{i=1}^{4}\tau_i(q). \tag{5.1}$$

Therefore, we can obtain

$$\alpha_{av} = \frac{\mathrm{d}\tau_{av}(q)}{\mathrm{d}q} = \frac{1}{N}\frac{\mathrm{d}\sum_{i=1}^{4}\tau_i(q)}{\mathrm{d}q} = \frac{1}{N}\sum_{i=1}^{4}\alpha_i(q). \tag{5.2}$$

Then we can obtain $f_{av}(\alpha)$ by means of Eq. (2.8), thereby we can get the multifractal spectra of China's and US markets as two whole markets (see Fig. 6). We estimate widths of spectra by $\Delta\alpha_{av} \approx \alpha_{av}(-60) - \alpha_{av}(60)$. The numerical results in Table 3 tell us that in general the spectrum widths of US markets as a whole are significantly narrower than those of China's with different orders of polynomial m. A plausible explanation for this effect is that the developed markets (e.g. US agricultural markets) are much more efficient than developing (transition) ones (e.g. China's agricultural markets).

Table 3
The estimated width of multifractal spectrum $\Delta\alpha_{av}$.

The order of polynomial m	China	America
$m = 1$	0.5655	0.2475
$m = 2$	0.6065	0.3033
$m = 3$	0.6319	0.3585

The nontrivial findings also inspire us with a further hypothesis: the multifractal strengths in the developed economies may be weaker than those in the transition or emerging ones. At least, the answer is positive for the special and representative cases of China's and US agricultural futures markets. Of course, many other efforts and empirical or theoretical results from other peers may be called for in this field to accept or reject this hypothesis. No matter the final answer is positive or negative; the findings on this issue will definitely offer us better understandings on the dynamics of financial and commodity markets.

6. Conclusions

In this article, we investigated the multifractal properties in China's and US agricultural futures markets from the comparative perspective. Our nontrivial empirical findings can be summarized as follows:

First of all, multifractality is found in all those markets except the US soybean market, which is much closer to mono-fractal.

Secondly, non-Gaussian distribution constitutes the major contribution in multifractality formation and nonlinear temporal correlation also has impact on the markets, expect US soybean and soy meal market, which may be influenced by some other unknown factors.

Thirdly, the width of multifractal spectrum of US agricultural futures markets as a whole is significantly narrower than that of China's.

Of course, there are still some questions waiting to be answered in our future works: are there any more causes of multifractality in the analyzed markets along with nonlinear temporal correlation and non-Gaussian distribution, especially for the case of US soy meal market? Is our hypothesis valid for other cases? Many other pieces of further empirical evidence and theoretical proofs are needed from other commodity or financial markets in more emerging or transition economies.

Acknowledgements

The authors would like to thank the anonymous referees for valuable comments and suggestions. This work is supported by Chinese Universities Scientific Fund (No. 2009-1-23), XU GUOZHI Postdoctoral Foundation, and China Postdoctoral Science Foundation (No. 20090450602).

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