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A ROBUST INTEGRAL TYPE BACKSTEPPING CONTROLLER DESIGN FOR CONTROL OF UNCERTAIN NONLINEAR SYSTEMS SUBJECT TO DISTURBANCE

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Abstract. *This paper considers the design of a robust controller for a class of nonlinear systems subject to both model uncertainties and unknown external disturbances. Using the concept of both backstepping design and sliding mode control theory, an integral type control algorithm is presented. The proposed controller not only stabilizes nonlinear systems in the presence of mismatched uncertainties, eliminates/reconstructs exogenous disturbances, but also provides smooth control effort. As a result, the developed method is adequate for practical implementation. A criterion of control gains setting for achieving closed-loop stability and disturbance rejection is addressed. Control system design for an unstable nonlinear system is used to illustrate the applicability of the proposed approach and experimental study is also provided to demonstrate the disturbance rejection/reconstruction capability.*

Keywords: Integral controller, Disturbance rejection, Sliding mode control, Chattering

1. **Introduction.** In the uncertain nonlinear control system design, model uncertainties and external disturbances are usually taken as the main issues when designing robust controllers. Model uncertainties affect stability of the closed-loop systems and exogenous disturbances degrade control precision, e.g., steady state accuracy. To analyze the stability of perturbed systems, many approaches have been illustrated in [1]. Backstepping design is a systematic recursive design procedure based on the choice of Lyapunov functions. This approach is suitable for the design of a large class of feedback linearizable systems in strict feedback form. The main concept of the backstepping design is to treat the system variable as an independent input for subsystems and each step results in a new virtual controller for the next step. The virtual control law for each step is adopted with satisfaction of selected Lyapunov functions such that the stability of each subsystem can be guaranteed. Owing to its systematic design concept, design of synthetic backstepping controller has been explored to wide class of nonlinear systems and servo mechanisms [2-12]. In [2,3], an integral function was integrated into the backstepping control design, which makes the motion system insensitive to model uncertainties, external disturbance and improves the closed-loop performance. Two robust adaptive backstepping controllers were developed in [4] for dealing with systems subject to unknown backlash-like hysteresis nonlinearities. A sign function is applied for achieving better tracking performance. Combinations of sliding mode control (SMC) and adaption laws have also been studied in [5-7]. In [5], the robust adaptive backstepping sliding controllers were developed for handling motion control with

parametric uncertainties and friction force, where an adaptive law was introduced to estimate the value of a lumped uncertainty in real time. A discontinuous sign function was modified as the sigmoid-like function to smooth the switching action, which was considered in [6]. For nonlinear systems with non-strict feedback form can refer to the work in [7].

On the critical demand of control precision, disturbance estimation and rejection should be taken into consideration. Owing to the robustness against to matched model uncertainties and disturbances, SMC has been widely used to different control systems [13-18]. One of the important issues in SMC is the design of fast discontinuous actions, that force the system to operate between two different dynamic structures, such that the desired system behaviour, called sliding mode, appears on a sliding manifold [17]. The SMC methodology has also been applied to the recovery of fault signals [18]. The basic idea behind the employment of SMC for fault detection is the selection of an invariant manifold or a sliding surface, where its dynamics involves the information of lumped fault signals. Then, the task of fault reconstruction is to design a robust control law that achieves zero output of the sliding dynamics in finite time. Since a pure switching control is utilized in the conventional SMC, the solution of the given sliding dynamic equation is understood in the sence of Filippov [19]. Actually, the solution not only represents the average control effort of switching control signals used to maintain the ideal sliding motion, but also stands for the profile of external faults as well. This average control effort is also referred to as equivalent control injection [18]. However, the main problem in practical applications is that the ideal solution is not available. In order to simultaneous alleviate control switching and extract the solution hidden in the discontinuous signals, boundary layer techniques are usually applied, at the cost of the resulting control performance.

In this decade, design of chattering-free SMC has attracted more and more attention. Many results have been presented based on the works [20,21]. Since the conventional approaching phase is no longer involved in the chattering-free SMC algorithms [20,21], proof of the finite time approaching turns into the main issue and the procedure to determine the control gains can not be carried out in a straightforward manner as that used in conventional SMC. A comparative study of the work [20] was recently presented in [22]. Without the information of system model, the method [22] is capable of offering precise estimates of state variables in the face of additive noises. Other alternatives to generate smooth SMC and preserve conventional design steps have been reported in [23,24]. However, the applied sliding surfaces involve unmeasurable states so that extra sensors are required for controller realizations. This drawback can be improved by considering the observer based control frameworks proposed in [25,27]. A common feature of the smooth sliding controllers is the use of an integral sign function. This control component provides extra robustness especially for external disturbance and is going to be applied in this study.

This paper considers the control design problem for a class of nonlinear systems subject to both mismatched model uncertainties and a matched exogenous unknown disturbance. The main structure of the integral controller is similar with those presented in [26-28]. However, in [28], the design of control gains requires the second time derivative of lumped uncertainties, which will cause difficulty for control gains determination and realization. This drawback will be eliminated in this study. Moreover, without the use of the so called sliding dynamics, we are going to address the closed-loop stability by the way of interconnected system. Under this control framework, the selection of control gains become systematic, where the switching gain used in the integral sign function is used to reject the external disturbance and the rest of control parameters will be determined to achieve system stability. A numerical example is given to demonstrate the design procedure and experimental studies are also conducted to illustrate the disturbance rejection capability of the proposed robust controller.

2. System Description and Control.

To provide a more general control framework, the following n-order uncertain nonlinear system with strict feedback form is considered for the controller design.

$$\begin{aligned}
\dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2 + \varsigma_1(x_1) + \varrho_1(x_2) \\
\dot{x}_2 &= f_2(x_1, x_2) + g_2(x_1, x_2)x_3 + \varsigma_2(x_1, x_2) + \varrho_2(x_3) \\
\dot{x}_3 &= f_3(x_1, x_2, x_3) + g_3(x_1, x_2, x_3)x_4 + \varsigma_3(x_1, x_2, x_3) + \varrho_3(x_4) \\
&\vdots \\
\dot{x}_{n-1} &= f_{n-1}(x_1, \cdots, x_{n-1}) + g_{n-1}(x_1, \cdots, x_{n-1})x_n + \varsigma_{n-1}(x_1, \cdots, x_{n-1}) + \varrho_{n-1}(x_n) \\
\dot{x}_n &= f_n(x_1, \cdots, x_n) + g_n(x_1, \cdots, x_n)u + \varsigma_n(x_1, \cdots, x_n) + d(t)
\end{aligned} \tag{1}$$

where $x = [x_1, \cdots, x_n] \in R^n$, $u \in R$. For the nonlinear systems, the following assumptions are imposed.

Assumption 1. The term $f_i(0) = 0$ and $g_i(x) \neq 0$ with $i = 1, \cdots, n$. $f_i(x)$ and $g_i(x)$ are smooth functions. Moreover, the functions $g_i(x)$ do not change sign during the whole control processes. Consequently, there must exist positive constants G_{mi} and G_{Mi} such that $G_{mi} \leq g_i(x) \leq G_{Mi}$.

Assumption 2. $\varsigma_i(x)$ and $\varrho_i(x)$ represents model uncertainty which satisfies $|\varsigma_i(x)| \leq \sum_{j=1}^{i} \ell_{ij}|x_j|$ with $\ell_{ij} \geq 0$ and $|\varrho_i(x)| \leq h_i|x_{i+1}|$ for a certain control region of interest, where $0 \leq h_i < G_{mi}$ and $i = 1, \cdots, n-1$.

Assumption 3. $d(t) \in C^1$ stands for a matched unknown exogenous disturbance.

Due to the presence of $\varrho_i(x)$, it definitely causes uncertain coupling terms that perturb system stability. Therefore, the determination of stabilizing control laws during backstepping design and selection of control gains become extremely important. This issue will be illustrated in the next section. In summary, two main objects are concerned in this study; namely, achieving system stability in the presence of mismatched model uncertainties and rejecting disturbance by a dynamic (nonlinear) integral controller.

2.1. Backstepping design based SMC revisited.

In the following, backstepping design is applied to the system (1) and the concept of system stabilization will be briefly illustrated. First, treat the system state x_2 as an independent input and then suppose that there exists a state feedback stabilizing control law $\phi_1(x_1)$ such that

$$\dot{x}_1 = f_1(x_1) + g_1(x_1)\phi_1(x_1) + \varsigma_1(x_1) + \varrho_1(\phi_1) \tag{2}$$

is asymptotically stable.

Let a Lyapunov function be V_1 for the subsystem x_1 which satisfies $V_1 > 0$ for $x_1 \neq 0$ and

$$\dot{V}_1 = \frac{\partial V_1}{\partial x_1}[f_1(x_1) + g_1(x_1)\phi_1(x_1) + \varsigma_1(x_1) + \varrho_1(\phi_1)] \leq -Q_1(x_1) \tag{3}$$

where $Q_1(x_1) > 0$ for $x_1 \neq 0$. Equation (3) indicates that the desired closed-loop subsystem is robust against the existence of model uncertain term $\varsigma_1(x)$ and perturbed virtual control input $\varrho_1(\phi_1)$. By adding and subtracting $g_1(x_1)\phi_1(x_1)$ (i.e., a virtual control law) to the subsystem x_1, let a new error variable be $z_1 = x_2 - \phi_1(x_1)$ and then the subsystem

(x_1, z_1) can be represented as

$$\begin{aligned}\dot{x}_1 =& \ f_1(x_1) + g_1(x_1)(\phi_1(x_1) + z_1) + \varsigma_1(x_1) + \varrho_1(\phi_1, z_1)\\ \dot{z}_1 =& \ f_2(x_1, z_1) + g_2(x_1, z_1)x_3 - \dot{\phi}_1(x_1) + \varsigma_2(x_1, z_1) + \varrho_2(x_3)\end{aligned} \tag{4}$$

In a similar manner, consider x_3 a virtual input and let the stabilizing control law be $\phi_2(x_1, z_1)$. Suppose that by applying the virtual control law, there exists a Lyapunov candidate V_2 such that $V_2 > 0$ for $x_1, z_1 \neq 0$ and its corresponding time derivative satisfies

$$\begin{aligned}\dot{V}_2 =& \frac{\partial V_2}{\partial x_1}\left[f_1(x_1) + g_1(x_1)(\phi_1(x_1) + z_1) + \varsigma_1(x_1) + \varrho_1(\phi_1, z_1)\right]\\ &+ \frac{\partial V_2}{\partial z_1}\left[f_2(x_1, x_2) + g_2(x_1, x_2)\phi_2(x_1, z_1) - \dot{\phi}_1(x_1) + \varsigma_2(x_1, z_1) + \varrho_2(\phi_2)\right]\\ \leq& - Q_2(x_1, z_1)\end{aligned} \tag{5}$$

where $Q_2(x_1, z_1)$ is a positive definite function.

Further define a new error variable as $z_2 = x_3 - \phi_2(x_1, z_1)$. From the control point of view, it is easily found that the origin of the subsystem (x_1, z_1) is a stable equilibrium point as long as the error variable z_2 is equal to zero. Note that the virtual control laws, $\phi_1(x_1)$ and $\phi_2(x_1, z_1)$, are by no means specific forms.

For example, consider the following subsystems

$$\begin{aligned}\dot{x}_1 &= x_2 + \varsigma_1(x_1) + \varrho_1(x_2)\\ \dot{x}_2 &= x_3 + \varsigma_2(x_1, x_2) + \varrho_2(x_3)\end{aligned} \tag{6}$$

Select the following stabilizing control laws

$$\begin{aligned}\phi_1(x_1) &= -k_1 x_1\\ \phi_2(x_1, z_1) &= -x_1 - k_2 z_1 + \frac{\partial \phi_1(x_1)}{\partial x_1}(z_1 + \phi_1(x_1))\end{aligned} \tag{7}$$

Based on the Assumption 2, the nonlinear uncertain terms satisfy $\varsigma_1(x_1) \leq \ell_{11}|x_1|$, $\varsigma_2(x_1, x_2) \leq \ell_{21}|x_1| + \ell_{22}|x_2|$, $\varrho_1(x_2) \leq h_1|x_2|$ and $\varrho_2(x_3) \leq h_2|x_3|$ with h_1, $h_2 < G_{m1} = G_{m2} = 1$ for an interesting domain. Considering $x_2 = \phi_1(x_1)$ and selecting a Lyapunov candidates as $V_1 = x_1^2/2$ follows

$$\dot{V}_1 \leq -\underbrace{(k_1(1 - h_1) - \ell_1)|x_1|^2}_{Q_1(x_1)} \leq 0 \tag{8}$$

In practice, the first subsystem should be modified to read $\dot{x}_1 = z_1 + \phi_1(x_1) + \varsigma_1(x_1) + \varrho_1(x_1, z_1)$ when $z_1 = x_2 - \phi_1(x_1) \neq 0$. Now, letting $x_3 = \phi_2(x_1, z_1)$ and considering $V_2 = V_1 + z_1^2/2$ yields

$$\dot{V}_2 \leq -\underbrace{(k_1(1 - h_1) - \ell_{11})\begin{pmatrix}|x_1|\\|z_1|\end{pmatrix}^T\begin{pmatrix}1 & P_2\\ P_2 & P_1\end{pmatrix}\begin{pmatrix}|x_1|\\|z_1|\end{pmatrix}}_{Q_2(x_1, z_1)} \leq 0 \tag{9}$$

where

$$\begin{aligned}P_1 &= \frac{k_2(1 - h_2) - \ell_{22} - k_1(h_1 + h_2)}{k_1(1 - h_1) - \ell_{11}}\\ P_2 &= -\frac{\ell_{21} + k_1(\ell_{11} + \ell_{22}) + (1 + k_1^2)(\hbar_1 + \hbar_2)}{2(k_1(1 - \hbar_1) - \ell_{11})}\end{aligned}$$

Equation (9) shows that the subsystems are locally robust stable providing the selected control gains satisfy

$$\begin{aligned} k_1 &> \frac{\ell_{11}}{1-h_1} \\ k_2 &> \frac{\ell_{22}+k_1(h_1+h_2)+(k_1(1-h_1)-\ell_{11})P_2^2}{1-h_2} \end{aligned} \tag{10}$$

In addition, the stability criterion reduces to

$$\begin{aligned} k_1 &> \ell_{11} \\ k_2 &> \ell_{22}+\frac{(\ell_{21}+k_1(\ell_{11}+\ell_{22}))^2}{4(k_1-\ell_{11})} \end{aligned} \tag{11}$$

when $h_1 = h_2 \equiv 0$. Therefore, the applied control gains must be carefully selected especially when $\varrho_i(x) \neq 0$.

From (4), by adding and subtracting $g_2(x_1, z_1)\phi_2(x_1, z_1)$ to the subsystem z_1, then the subsystem dynamics of (x_1, z_1, z_2) can be represented as

$$\begin{aligned} \dot{x}_1 &= f_1(x_1) + g_1(x_1)(\phi_1(x_1) + z_1) + \varsigma_1(x_1) + \varrho_1(\phi_1, z_1) \\ \dot{z}_1 &= f_2(x_1, z_1) + g_2(x_1, z_1)(\phi_2(x_1, z_1) + z_2) - \dot{\phi}_1(x_1) + \varsigma_2(x_1, z_1) + \varrho_2(\phi_2, z_2) \\ \dot{z}_2 &= f_3(x_1, z_1, z_2) + g_3(x_1, z_1, z_2)x_4 - \dot{\phi}_2(x_1, z_1) + \varsigma_3(x_1, z_1, z_2) + \varrho_3(x_4) \end{aligned} \tag{12}$$

Using the recursive steps until the final subsystem, one can derive the following transformed $n-1$ order system

$$\begin{aligned} \dot{x}_1 &= f_1(x_1) + g_1(x_1)(\phi_1(x_1) + z_1) + \varsigma_1(x_1) + \varrho_1(\phi_1, z_1) \\ \dot{z}_1 &= f_2(x_1, z_1) + g_2(x_1, z_1)(\phi_2(x_1, z_1) + z_2) - \dot{\phi}_1(x_1) + \varsigma_2(x_1, z_1) + \varrho_2(\phi_2, z_2) \\ &\vdots \\ \dot{z}_{n-2} &= f_{n-1}(x_1, \cdots, z_{n-2}) + g_{n-1}(x_1, \cdots, z_{n-2})(\phi_{n-1}(x_1, \cdots, z_{n-2}) + z_{n-1}) \\ &\quad - \dot{\phi}_{n-2}(x_1, \cdots, z_{n-3}) + \varsigma_{n-1}(x_1, \cdots, z_{n-2}) + \varrho_{n-1}(\phi_{n-1}, z_{n-1}) \end{aligned} \tag{13}$$

and a final subsystem in which the control input appears is

$$\dot{z}_{n-1} = f_N(x_1, \cdots, z_{n-1}) + g_n(x_1, \cdots, z_{n-1})u + H \tag{14}$$

where $f_N(x_1, \cdots, z_{n-1})$ stands for a known term and $H = f_U(x_1, \cdots, z_{n-1}) + d(t)$ denotes as a smooth lumped uncertain perturbation. Moreover, the smooth functions satisfy $f_N = 0$ and $f_U = 0$ at the origin.

For the transformed mismatched uncertainties in (13), considering Assumption 2 gives

$$\begin{aligned} |\varsigma_1(x_1)| &\le \ell_{11}|x_1| \\ |\varsigma_2(x_1, z_1)| &\le \ell_{21}|x_1| + \ell_{22}(|\phi_1| + |z_1|) \\ |\varsigma_3(x_1, z_1, z_2)| &\le \ell_{31}|x_1| + \ell_{32}(|\phi_1| + |z_1|) + \ell_{33}(|\phi_2| + |z_2|) \\ &\vdots \\ |\varsigma_{n-1}(x_1, \cdots, z_{n-1})| &\le \ell_{n-1,1}|x_1| + \cdots + \ell_{n-1,n-1}(|\phi_{n-2}| + |z_{n-2}|) \end{aligned} \tag{15}$$

and

$$\begin{aligned}|\varrho_1(\phi_1, z_1)| &\leq h_1(|\phi_1| + |z_1|)\\ |\varrho_2(\phi_2, z_2)| &\leq h_2(|\phi_2| + |z_2|)\\ &\vdots \\ |\varrho_{n-1}(\phi_{n-1}, z_{n-1})| &\leq h_{n-1}(|\phi_{n-1}| + |z_{n-1}|)\end{aligned} \tag{16}$$

Equations (15) and (16) are used to evaluate adequate control gains during backstepping design. For (13), choose a Lyapunov function that satisfies $\dot{V}_{n-1} \leq -Q_{n-1}(x_1, \cdots, z_{n-2})$ with $Q_{n-1}(x_1, \cdots, z_{n-2}) > 0$ when $z_{n-1} = x_n - \phi_{n-1}(x_1, \cdots, z_{n-2}) = 0$. However, for $z_{n-1} \neq 0$, there exists a function $f_L(x_1, z_1, \cdots, z_{n-2})z_{n-1}$ which leads to $\dot{V}_{n-1} \leq -Q_{n-1}(x_1, \cdots, z_{n-2}) + f_L(x_1, z_1, \cdots, z_{n-2})z_{n-1}$. Therefore, by the recursive design, the stability of the n-order system can be simplified as a regulation problem of a scalar system (14). In explicit words, the state $z_{n-1} = 0$ can be considered as a prescribed nonlinear constraint or a sliding surface applied in the conventional SMC theory.

Using the concept of SMC, for $g_n(x_1, \cdots, z_{n-1}) \neq 0$, first design

$$\begin{aligned}u &= \frac{1}{g_n(x_1, \cdots, z_{n-1})}\left(\nu - wsgn(z_{n-1})\right)\\ \nu &= -f_L(x_1, \cdots, z_{n-2}) - f_N(x_1, \cdots, z_{n-1}) - k_n z_{n-1}\end{aligned} \tag{17}$$

where $f_L(x_1, \cdots, z_{n-1})$ stands for a feedback term coming from the Lyapunov stability requirement and k_n and w are positive constants. Considering a Lyapunov function $V = V_{n-1} + V_{z_{n-1}}$ in which $V_{z_{n-1}} = z_{n-1}^2/2$ and applying (17) results in

$$\begin{aligned}\dot{V} =& - Q_{n-1}(x_1, \cdots, z_{n-2}) + f_L(x_1, \cdots, z_{n-2})z_{n-1}\\ &+ z_{n-1}\left(-f_L(x_1, \cdots, z_{n-2}) - k_n z_{n-1} - wsgn(z_{n-1} + H)\right)\\ \leq& \underbrace{-Q_{n-1}(x_1, \cdots, z_{n-2}) - k_n z_{n-1}^2 - z_{n-1} f_U(x_1, \cdots, z_{n-1})}_{\mathcal{M}(x_1, \cdots, z_{n-1})} - |z_{n-1}|(w - d(t))\end{aligned} \tag{18}$$

Consider Assumption 2, there might exist a proper k_n and a positive definite function $Q_n(x_1, \cdots, z_{n-1})$ such that $\mathcal{M}(x_1, \cdots, z_{n-1}) \leq -Q_n(x_1, \cdots, z_{n-1})$ is satisfied for a certain control domain of interest. Based on this condition, it is evident that by selecting $w \geq max|d(t)|$, Equation (18) reduces to

$$\dot{V} \leq -Q_n(x_1, \cdots, z_{n-1}) \tag{19}$$

As a result, the asymptotic stability can be achieved by applying (17). Note that the switching gain is selected to tackle the external disturbance only. However, (19) does not provide any information about the existence of approaching phase. In the following, we are going to address that whether the controller in (17) can be taken as a sliding controller depends on the size of the switching gain w.

Considering the Lyapunov function $V_{z_{n-1}}$ again and taking the time derivative gives

$$\begin{aligned}\dot{V}_{z_{n-1}} &= z_{n-1}(f_N(x_1, \cdots, z_{n-1}) + g_n(x_1, \cdots, z_{n-1})u + H)\\ &\leq z_{n-1}(-f_L(x_1, \cdots, z_{n-2}) - k_n z_{n-1} - wsgn(z_{n-1}) + |H|)\\ &\leq -k_n z_{n-1}^2 - |z_{n-1}|(w - \mathcal{F}(x_1, \cdots, z_{n-1}) - |d(t)|)\end{aligned} \tag{20}$$

where $\mathcal{F}(x_1, \cdots, z_{n-1}) = |f_L(x_1, \cdots, z_{n-2})| + |f_U(x_1, \cdots, z_{n-1})|$ is a nonnegative function. Three options regarding the magnitude of w are addressed:

$$w \geq \max|d(t)| \tag{21a}$$

$$w \geq w_o + \max|d(t)| \tag{21b}$$

$$w \geq w_o + \mathcal{F}(x_1, \cdots, z_{n-1}) + \max|d(t)| \tag{21c}$$

It is obvious that (19) can be achieved by substituting any one of (21) into (18). However, for (20), the use of (21a) achieves the asymptotic convergence of the scalar dynamics but no finite time arrival is guaranteed. On the contrary, (21c) provides the approaching condition immediately, i.e., $\dot{V}_{z_{n-1}} \leq -k_n z_{n-1}^2 - w_o|z_{n-1}|$. Once $z_{n-1} = 0$ is attained, the stability of reduced order dynamics can be understood by $\dot{V}_{n-1} \leq -Q_{n-1}(x_1, \cdots, z_{n-2})$, which is obtained by the preceding backstepping design.

Now, consider the case given in (21b). For a given (relatively small) value of w_o, the condition $w_o > \mathcal{F}(x_1, \cdots, z_{n-1})$ may not be satisfied at the control beginning. Fortunately, since (19) is also available by using (21b), it further reveals that the controlled state trajectories will enter a compact residual set Ω_a, described by

$$\Omega_a = \{x_1, \cdots, z_{n-1} | \mathcal{F}(x_1, \cdots, z_{n-1}) < w_o\} \tag{22}$$

Building on the condition of (22), it can be seen that the approaching phase occurs eventually.

According to the preceding analysis, the use of (21c) is able to fulfill the approaching phase immediately. Nevertheless, the magnitude of w utilized in (21c) depends on system state variables. For arbitrary given initial positions, it may lead to large value of w and thereby causes serious control chattering. Applying (21b) alleviates the size of control switching but the fulfillment of sliding mode is postponed. In conclusion, the controller (17) associated with the use of (21b) or (21c) can be referred to as approaching phase guaranteed sliding control laws. Although (17) together with the option (21a) cannot be taken as a sliding controller, it significant reduces the discontinuous control force and the asymptotic stability of the closed-loop system is still maintained.

Remark 2.1. *In practical implementations, a common possibility is that the information of external disturbances is totally known. Under this circumstance, it is difficult to confirm that the selected w satisfies (21a), (21b), (21c) or even none of them. Consequently, the magnitude of w is usually selected a large value but gives rise to serious control switching. On the other hand, the selection of switching gain can be made with less conservativeness if the upper bound of the disturbance is previously known. Nevertheless, no matter how large the w is applied, the discontinuous control signals are still inevitable and the switching magnitude depends on the size of unknown exogenous disturbance. As a result, a simple integral controller will be presented to remedy this issue and some properties are going to be discussed.*

3. **A Robust Integral Controller Design.**

Based on the work presented in [28], design a (nonlinear) integral control law as follows:

$$\begin{aligned} u &= \frac{1}{g_n(x_1, \cdots, z_{n-1})} \left(\nu_1 + \int_0^t \nu_2 \, d\tau \right) \\ \nu_1 &= -f_N(x_1, \cdots, z_{n-1}) - k_n z_{n-1} \\ \nu_2 &= -k_{n+1} z_{n-1} - \xi sgn(z_{n-1}) \end{aligned} \tag{23}$$

Proposition 3.1. *By using the control law (23) in which $\xi > \max|\dot{d}(t)|$ is applied, there may exist a proper pair (k_n, k_{n+1}) such that the controlled nonlinear uncertain system is semi-globally quadratic stable.*

Proof: Substituting (23) into (14) yields

$$\dot{z}_{n-1} = -k_n z_{n-1} + \int_0^t \nu_2 \, d\tau + H \tag{24}$$

where its time derivative can be represented by $\ddot{z}_{n-1} = -k_n \dot{z}_{n-1} + \nu_2 + \dot{H}$. Let $\dot{z}_{n-1} = z_n$, it leads to an augmented auxiliary 2[nd] order system as follows:

$$\begin{aligned} \dot{z}_{n-1} &= z_n \\ \dot{z}_n &= -\, k_{n+1} z_{n-1} - k_n z_n - \xi sgn(z_{n-1}) + \dot{H} \end{aligned} \tag{25}$$

Consider the $n-1$ order dynamics given in (13) together with the augmented 2[nd] order dynamics, the following object is to prove that the trajectories of the perturbed interconnected system converge towards the equilibrium point providing the triple (k_n, k_{n+1}, ξ) is properly selected. To this aim, the approach so called composite Lyapunov function [1] is used. The main idea is briefly stated as below: separately find Lyapunov candidates for the corresponding isolated status of systems (13) and (25) and then show that the origin is a stable equilibrium point. Then, combine the Lyapunov functions to be a new Lyapunov candidate for the interconnected system. The last step is to design the triple (k_n, k_{n+1}, ξ) and prove that the origin of the interconnected system is also a stable equilibrium point in the presence of uncertain coupling terms.

For (13), the corresponding isolated $n-1$ order subsystem can be described by

$$\begin{aligned} \dot{x}_1 &= f_1(x_1) + g_1(x_1)(\phi_1(x_1) + z_1) + \varsigma_1(x_1) + \varrho_1(\phi_1, z_1) \\ \dot{z}_1 &= f_2(x_1, z_1) + g_2(x_1, z_1)(\phi_2(x_1, z_1) + z_2) - \dot{\phi}_1(x_1) + \varsigma_2(x_1, z_1) + \varrho_2(\phi_2, z_2) \\ &\vdots \\ \dot{z}_{n-2} &= f_{n-1}(x_1, \cdots, z_{n-2}) + g_{n-1}(x_1, \cdots, z_{n-2})\phi_{n-1}(x_1, \cdots, z_{n-2}) \\ &\quad - \dot{\phi}_{n-2}(x_1, \cdots, z_{n-3}) + \varsigma_{n-1}(x_1, \cdots, z_{n-2}) + \varrho_{n-1}(\phi_{n-1}) \end{aligned} \tag{26}$$

It has been shown by backstepping design that (26) is asymptotic stable if $z_{n-1} \equiv 0$, i.e., $\dot{V}_{n-1} \leq -Q_{n-1}(x_1, \cdots, z_{n-2})$.

Now, for (25), rewrite $\dot{H}$ by

$$\dot{H} = H_{d1}(x_1, \cdots, z_{n-2}) + H_{d2}(z_{n-1}, z_n) + \dot{d}(t) \tag{27}$$

Therefore, the corresponding isolated subsystem of (25) can be represented by

$$\begin{aligned} \dot{z}_{n-1} &= z_n \\ \dot{z}_n &= -\, k_{n+1} z_{n-1} - k_n z_n - \xi sgn(z_{n-1}) + H_{d2}(z_{n-1}, z_n) + \dot{d}(t) \end{aligned} \tag{28}$$

Further consider $\xi > \max|\dot{d}(t)|$ and a change of state variable by $z_n = \sigma - \gamma z_{n-1}$, where $\gamma > 0$. The dynamics of (28) can be governed by

$$\begin{aligned} \dot{z}_{n-1} &= -\, \gamma z_{n-1} + \sigma \\ \dot{\sigma} &= -\, \gamma_1 z_{n-1} - \gamma_2 \sigma - \tilde{\xi} sgn(z_{n-1}) + H_{d2}(z_{n-1}, \sigma) \end{aligned} \tag{29}$$

where $\gamma_1 = \gamma^2 - k_n \gamma + k_{n+1} > 0$, $\gamma_2 = k_n - \gamma > 0$ and $\tilde{\xi} \in [\xi^+, \xi^-] > 0$.

Based on (15) and (16), there exist positive constants α_p, β_q and μ_p with $p = 1, \cdots, n-1$ and $q = 1, 2$ so that

$$\begin{aligned} |H_{d1}(x_1, \cdots, z_{n-2})| &\leq \alpha_1|x_1| + \alpha_2|z_1| + \cdots + \alpha_{n-1}|z_{n-2}| \\ |H_{d2}(z_{n-1}, \sigma)| &\leq \beta_1|z_{n-1}| + \beta_2|\sigma| \\ |f_L(x_1, \cdots, z_{n-2})| &\leq \mu_1|x_1| + \mu_2|z_1| + \cdots + \mu_{n-1}|z_{n-2}| \end{aligned} \tag{30}$$

in the domain of control interest.

For the isolated 2nd order system (29), choose as a Lyapunov function

$$V_\sigma = \frac{\gamma_1}{2} z_{n-1}^2 + \frac{1}{2}\sigma^2 + \tilde{\xi}|z_{n-1}| \tag{31}$$

Taking the time derivative follows

$$\begin{aligned} \dot{V}_\sigma =& \gamma_1 z_{n-1}(-k_{n+1}z_{n-1} + \sigma) + \tilde{\xi}(-k_{n+1}z_{n-1} + \sigma)sgn(z_{n-1}) \\ &+ \sigma(-\gamma_1 z_{n-1} - \gamma_2\sigma - \tilde{\xi}sgn(z_{n-1}) + H_{d2}(z_{n-1}, \sigma)) \\ \leq& -\gamma_1 k_{n+1} z_{n-1}^2 - \gamma_2\sigma^2 - \tilde{\xi}k_{n+1}|z_{n-1}| + |\sigma||H_{d2}(z_{n-1}, \sigma)| \\ \leq& -\gamma_1 k_{n+1}|z_{n-1}|^2 + \beta_1|z_{n-1}||\sigma| - (\gamma_2 - \beta_2)|\sigma|^2 - \tilde{\xi}k_{n+1}|z_{n-1}| \\ =& -\begin{pmatrix} |z_{n-1}| \\ |\sigma| \end{pmatrix}^T \underbrace{\begin{pmatrix} \gamma_1 k_{n+1} & -\beta_1/2 \\ -\beta_1/2 & (\gamma_2 - \beta_2) \end{pmatrix}}_{\mathcal{S}} \begin{pmatrix} |z_{n-1}| \\ |\sigma| \end{pmatrix} - \tilde{\xi}k_{n+1}|z_{n-1}| \end{aligned} \tag{32}$$

It can be seen that the isolated 2^{nd} order perturbed system is quadratic stable if $\mathcal{S} > 0$ is attained.

Consequently for the $n + 1$ order dynamics, a composite Lyapunov function is selected as follows

$$\mathcal{V}_\mathcal{C} = V_{n-1} + V_\sigma \tag{33}$$

Similar to those steps utilized to obtain (9), the positive function $Q_{n-1}(x_1, \cdots, z_{n-2})$ can also be represented by a quadratic form. Thus, taking the time derivative of (33) and considering (30) gives

$$\begin{aligned} \dot{\mathcal{V}}_\mathcal{C} \leq& -Q_{n-1}(x_1, \cdots, z_{n-2}) - \begin{pmatrix} |z_{n-1}| \\ |\sigma| \end{pmatrix}^T \mathcal{S} \begin{pmatrix} |z_{n-1}| \\ |\sigma| \end{pmatrix} - \tilde{\xi}k_{n+1}|z_{n-1}| \\ &\underbrace{+|f_L(x_1, \cdots, z_{n-2})||z_{n-1}| + |H_{d1}(x_1, \cdots, z_{n-2})||\sigma|}_{\mathcal{F}_C} \\ \leq& -\begin{pmatrix} |x_1| \\ \vdots \\ |z_{n-2}| \end{pmatrix}^T \mathcal{P} \begin{pmatrix} |x_1| \\ \vdots \\ |z_{n-2}| \end{pmatrix} - \begin{pmatrix} |z_{n-1}| \\ |\sigma| \end{pmatrix}^T \mathcal{S} \begin{pmatrix} |z_{n-1}| \\ |\sigma| \end{pmatrix} \\ &+ (\mu_1|x_1| + \cdots + \mu_{n-1}|z_{n-2}|)|z_{n-1}| + (\alpha_1|x_1| + \cdots + \alpha_{n-1}|z_{n-2}|)|\sigma| \\ \leq& -\begin{pmatrix} \mathcal{Z}_1 \\ \mathcal{Z}_2 \end{pmatrix}^T \underbrace{\begin{pmatrix} \mathcal{P} & -\mathcal{C}^T \\ -\mathcal{C} & \mathcal{S} \end{pmatrix}}_{\Xi} \begin{pmatrix} \mathcal{Z}_1 \\ \mathcal{Z}_2 \end{pmatrix} \end{aligned} \tag{34}$$

where $\mathcal{Z}_1 = (|x_1|, \cdots, |z_{n-2}|)$, $\mathcal{Z}_2 = (|z_{n-1}|, |\sigma|)$ and $\mathcal{C} = \begin{pmatrix} \mu_1/2 & \mu_2/2 & \cdots & \mu_{n-1}/2 \\ \alpha_1/2 & \alpha_2/2 & \cdots & \alpha_{n-1}/2 \end{pmatrix}$.

Equation (34) illustrates that the closed-loop stability is affected by the coupling term $\mathcal{F}_C$. However, building on the properly selected control gains, there may exist $\Xi = \Xi^T > 0$

such that (34) becomes

$$\dot{\mathcal{V}}_{\mathcal{C}} \leq -\mathcal{Z}^T \Xi \mathcal{Z} \tag{35}$$

where $\mathcal{Z} = (\mathcal{Z}_1, \mathcal{Z}_2)$.

Based on (35), it can be concluded that the controlled system trajectories converge towards the origin. Note that the Lyapunov stability criterion can be achieved by the way of high gain control. The larger the applied gain values, the larger the stabilizing domain, which leads to semi-globally quadratic stability.

The main design steps can be summarized as follows:

a). Apply the backstepping design the control law (23).
b). Analyze the first time derivative of the lumped perturbation term $\dot{H}$.
c). Select proper control gains based on the Lyapunov stability criterion (34).

Remark 3.1. *Suppose that $\mathcal{Z} \to 0$ has been achieved by applying (23), it follows from (24) that*

$$\int_0^\infty \nu_2 \, d\tau \to -H \tag{36}$$

Note that $H \to d(t)$ when $\mathcal{Z} \to 0$. Therefore, it implies that the matched external disturbance is eliminated asymptotically. In [18], it has been shown that external disturbances can be reconstructed by way of the so called equivalent injection. The equivalent control effort is extracted by using a boundary layer, where the size of the boundary layer dominates the estimate precision. That is, the larger the size of boundary layer, the smoother the control effort but the lower the estimate precision. This defect can be remedied by the proposed method.

Remark 3.2. *Note that the proposed integral type controller does not result the so called ideal sliding motion. However, it still guarantees the closed-loop stability, which is similar to the controller (17) together with the use of (21a). In regard to the selection of control parameters, the control gains used in the recursive backstepping design are selected to achieve the stability of each subsystem subject to mismatched model uncertainty. Based on the state transformation, the pair (k_n, k_{n+1}) is designed to guarantee the quadratic stability of the interconnected system. Finally, the parameter ξ is introduced to eliminate the effect of exogenous disturbance $d(t)$. The determination of the controller parameter corresponding to design objective in each stage is clearly addressed. Based on the properly selected triple (k_n, k_{n+1}, ξ), the stability issue turns into a stability problem of an auxiliary interconnected dynamics as summarized in Figure 1. The developed method not only provides robust stability, but removes serious control chattering as well. In addition, the 'shape' of external disturbance can be precisely reconstructed by measuring control signals as long as $|\dot{d}(t)|$ is finite, which is not a critical assumption. As argue in Remark 2.1, the determination of ξ may not be easy if the disturbance is totally unknown. However, different from the controller (17), ξ can be replaced by a sufficient large value without causing serious control chattering.*

4. **Numerical Study: Controlling of an Uncertain Nonlinear Systems.** In the following, an unstable nonlinear uncertain system is used to demonstrate the feasibility of the backstepping design based integral control algorithm.

$$\begin{aligned} \dot{x}_1 &= x_1 + x_1^2 + a x_2 \\ \dot{x}_2 &= x_1 - b x_2 + x_3 \\ \dot{x}_3 &= u + d(t) \end{aligned} \tag{37}$$

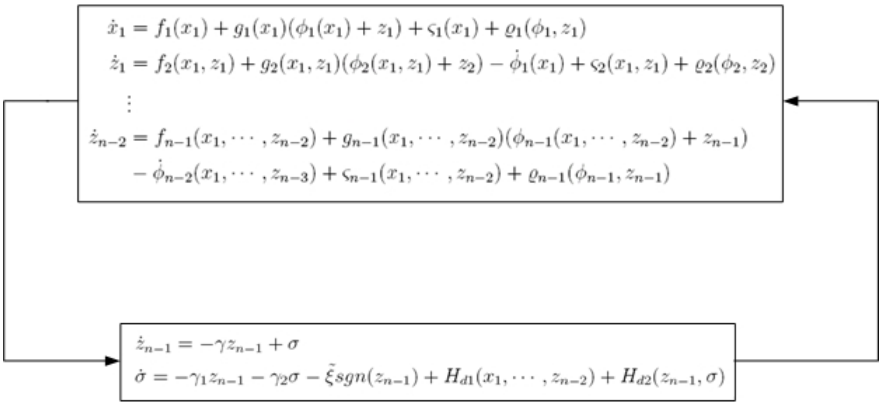

FIGURE 1. An auxiliary interconnected system

The control objective is to drive the system trajectories moving towards origin in the face of model uncertainties and an external perturbation. Suppose that system parameters are not exactly known. Equation (37) is represented by

$$\begin{aligned}
\dot{x}_1 &= x_1 + \hat{a}x_2 + \varsigma_1(x_1) + \varrho_1(x_2) \\
\dot{x}_2 &= x_1 + x_3 + \varsigma_2(x_1, x_2) + \varrho_2(x_3) \\
\dot{x}_3 &= u + d(t) + \varsigma_3(x_1, x_2, x_3)
\end{aligned} \tag{38}$$

Comparing (38) with (1) follows $f_1(x_1) = x_1$, $f_2(x_1, x_2) = x_1$, $f_3(x_1, x_2, x_3) = 0$, $g_1(x_1) = 1$, $g_2(x_1, x_2) = 1$, $g_3(x_1, x_2, x_3) = 1$, $\varsigma_1(x_1) = x_1^2$, $\varsigma_2(x_1, x_2) = -bx_2$, $\varsigma_3(x_1, x_2, x_3) = 0$, $\varrho_1(x_2) = \tilde{a}x_2$ and $\varrho_2(x_3) = 0$ in which

$$\begin{aligned}
|\varsigma_1(x_1)| &\leq \ell_{11}|x_1| \\
|\varsigma_2(x_1, x_2)| &\leq \ell_{22}|x_2| \\
|\varsigma_3(x_1, x_2, x_3)| &= 0 \\
|\varrho_1(x_2)| &\leq h_1|x_2| \\
|\varrho_2(x_3)| &= 0
\end{aligned} \tag{39}$$

for $|x_1| \leq \ell_{11}$, $\ell_{22} = |b|$ and $h_1 = |\tilde{a}|$. Following the Steps (2) – (14) results in

$$\begin{aligned}
\dot{x}_1 &= x_1 + \hat{a}(\phi_1(x_1) + z_1) + \varsigma_1(x_1) + \varrho_1(\phi_1(x_1), z_1) \\
\dot{z}_1 &= x_1 + \hat{a}^{-1}(1 + k_1)(x_1 + \hat{a}(\phi_1(x_1) + z_1)) + \phi_2(x_1, z_1) + z_2 \\
&\quad + \hat{a}^{-1}(1 + k_1)(\varsigma_1(x_1) + \varrho_1(\phi_1(x_1), z_1)) + \varsigma_2(x_1, z_1)
\end{aligned} \tag{40}$$

and the last subsystem can be represented in the form of (14) as follows

$$\dot{z}_2 = f_N(x_1, z_1, z_2) + u + H \tag{41}$$

in which $H = f_U(x_1, z_1, z_2) + d(t)$ and

$$\begin{aligned}
f_N(x_1, z_1, z_2) &= (\mathcal{K}_1 + \mathcal{K}_2)(x_1 + \hat{a}(\phi_1(x_1) + z_1)) + \mathcal{K}_3(x_1 + \phi_2(x_1, z_1) + z_2) \\
f_U(x_1, z_1, z_2) &= (\mathcal{K}_1 + \mathcal{K}_2)(\varsigma_1(x_1) + \varrho_1(\phi_1(x_1), z_1)) + \mathcal{K}_3\varsigma_2(x_1, z_1) \\
\mathcal{K}_1 &= 1 + \hat{a} + \hat{a}^{-1}(1 + k_1) - \hat{a}^{-1}(1 + k_1)^2 \\
\mathcal{K}_2 &= \hat{a}^{-1}(1 + k_1)(1 + k_1 + k_2) \\
\mathcal{K}_3 &= 1 + k_1 + k_2
\end{aligned}$$

The transformed variables are defined by $z_1 = x_2 - \phi_1(x_1)$, $z_2 = x_3 - \phi_2(x_1, z_1)$ and the applied stabilizing control laws used to obtain (40) and (41) are

$$\begin{aligned}\phi_1(x_1) &= -\hat{a}^{-1}(1 + k_1)x_1 \\ \phi_2(x_1, z_1) &= -(1 + \hat{a})x_1 - \hat{a}^{-1}(1 + k_1)(x_1 + \hat{a}(\phi_1(x_1) + z_1)) - k_2 z_1\end{aligned} \tag{42}$$

For (40), selecting a Lyapunov function $V_2 = x_1^2/2 + z_1^2/2$ and considering the corresponding isolated system, i.e., $z_2 \equiv 0$, gives

$$\dot{V}_2 \le -\begin{pmatrix}|x_1|\\|z_1|\end{pmatrix}^T \underbrace{\begin{pmatrix}P_1 & P_3\\P_3 & P_2\end{pmatrix}}_{\mathcal{P}} \begin{pmatrix}|x_1|\\|z_1|\end{pmatrix} \tag{43}$$

where

$$\begin{aligned}P_1 &= (1 - h_1\hat{a}^{-1})k_1 - \ell_{11} - h_1\hat{a}^{-1} \\ P_2 &= k_2 - \ell_{22} - h_1\hat{a}^{-1}(1 + k_1) \\ P_3 &= -\frac{h_1 + \hat{a}^{-1}(\ell_{11} + \ell_{22})(1 + k_1) + h_1\hat{a}^{-2}(1 + k_1)^2}{2}\end{aligned} \tag{44}$$

To make $\mathcal{P} > 0$, k_1 and k_2 should be selected to satisfy

$$\begin{aligned}k_1 &> \frac{\ell_{11} + h_1\hat{a}^{-1}}{1 - h_1\hat{a}^{-1}} \\ k_2 &> h_1\hat{a}^{-1}(1 + k_1) + \ell_{22} + P_1^{-1}P_3^2\end{aligned} \tag{45}$$

For (41), applying the control law (23) and considering the isolated 2$^{\text{nd}}$ order system follows the form of (29). The derivatives of the lumped perturbation satisfy

$$\begin{aligned}|H_{d1}(x_1, z_1)| &\le \alpha_1|x_1| + \alpha_2|z_1| \\ |H_{d2}(z_2, \sigma)| &\le \beta_1|z_2| + \beta_2|\sigma|\end{aligned} \tag{46}$$

where

$$\begin{aligned}\alpha_1 &= 2\ell_{11}|\mathcal{K}_1 + \mathcal{K}_2|(\ell_{11} + |1 - a\hat{a}^{-1} - a\hat{a}^{-1}k_1|) \\ &\quad + (h_1|\mathcal{K}_1 + \mathcal{K}_2| + \ell_{22}|\mathcal{K}_3|)|\hat{a} - \hat{a}^{-1} - 2\hat{a}^{-1}k_1 - \hat{a}^{-1}k_1^2| \\ \alpha_2 &= (h_1|\mathcal{K}_1 + \mathcal{K}_2| + \ell_{22}|\mathcal{K}_3|)(2 + k_1 + k_2) + 2\ell_{11}|a(\mathcal{K}_1 + \mathcal{K}_2)| \\ \beta_1 &= h_1|\mathcal{K}_1 + \mathcal{K}_2| + \ell_{22}|\mathcal{K}_3| \\ \beta_2 &= 0\end{aligned} \tag{47}$$

Applying the Steps (32) and (33) yields the form of (34), where $\mathcal{C} = \begin{pmatrix}0 & 1/2\\ \alpha_1/2 & \alpha_2/2\end{pmatrix}$ in this case. Consequently, the final step is to select a proper pair (k_3, k_4) used in (23) such that

$$\Xi = \begin{pmatrix}\mathcal{P} & -\mathcal{C}^T\\ -\mathcal{C} & \mathcal{S}\end{pmatrix} > 0 \tag{48}$$

In the following simulation, initial position of the unstable nonlinear system was set to be at $[x_1(0), x_2(0), x_3(0)] = [-0.1, 0.05, 0]$. The parameters $\ell_{11} = 0.125$, $a = 1$, $b = 0.1$ and $\tilde{a} = 0.2$ are considered. The exogenous disturbance is simulated by $d(t) = e^{(\sin\omega_1 t + \cos\omega_2 t)} - \sin\omega_3 t$ with $\omega_1 = 1.0$, $\omega_2 = 0.5$ and $\omega_3 = 2.5\,rad/s$, respectively. The control gains $\eta = 1$, $(k_1, k_2, k_3, k_4) = (0.3125, 1.4734, 125, 135)$ that meet (48) and $\xi = 25 > \max|\dot{d}(t)| = 24.7$ are applied.

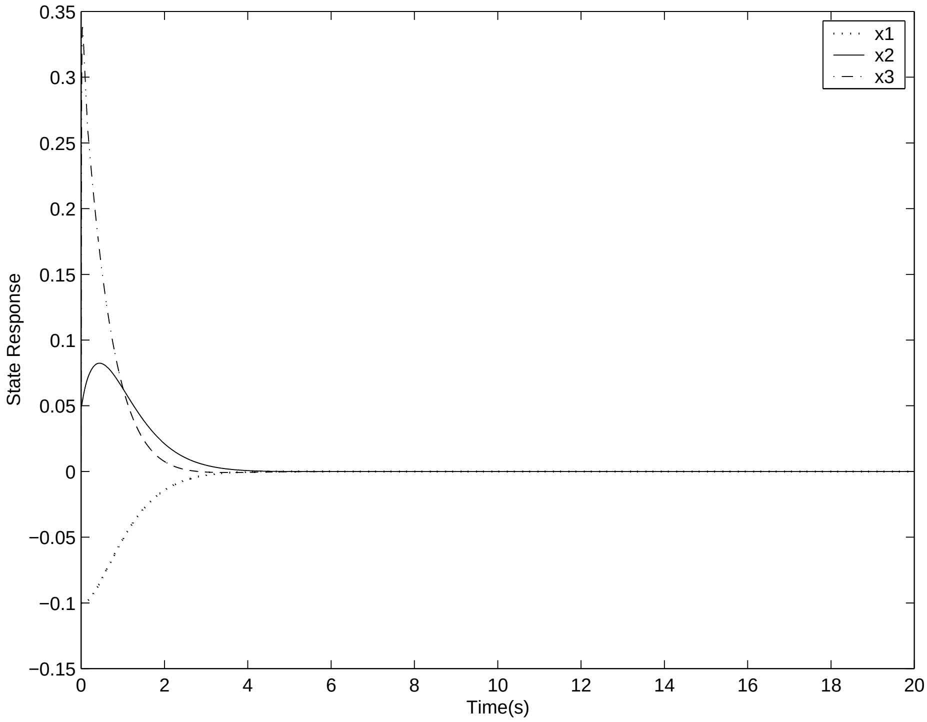

FIGURE 2. Closed-loop response

Figure 2 shows the closed-loop state response. By using the proposed backstepping integral controller, the system trajectories converge towards the origin even in the presence of model uncertainties and external disturbances. The resulting control effort is illustrated in Figure 3, which shows that the applied control signals are smooth. Since $\mathcal{Z} \to 0$, it follows that the resulting control effort represents the external disturbance with the same magnitude but opposite sign. During the finite control interval, the given disturbance seems to be irregular such that the corresponding magnitude and period are hard to observe. Under this situation, external model based disturbance rejection scheme [31,32] might not be easy for implementation. Nevertheless, simulations evidently demonstrate that the disturbance rejection and recovery task are simultaneously achieved by using a single integral controller.

5. **Experiments: Voltage Drifting Reconstruction.** In the previous example, it has been shown that the integral control law is capable of achieving disturbance rejection. To demonstrate this property further, we are going to design an experiment, which is a practical application of the proposed method, for voltage drifting reconstruction. It is well known that due to the long-term use of hardware, it unavoidably causes wear and tear in hardware devices. Here, we suppose that this fault induces voltage drifting. Thus, the following objective is to detect this fault and reconstruct its drifting values by the way of control manner.

To realize this experiment, a servo motor is used to be an experimental apparatus, where the dynamical equation can be simply represented by

$$\begin{aligned} \dot{\theta}_1 &= \theta_2 \\ \dot{\theta}_2 &= J^{-1}(v_{in} - v_f) \end{aligned} \tag{49}$$

where θ_1 and θ_2 are angular position and velocity, respectively. $J = 0.581$ is the moment of inertia. The applied control input is denoted by v_{in} and the fault voltage drifting is

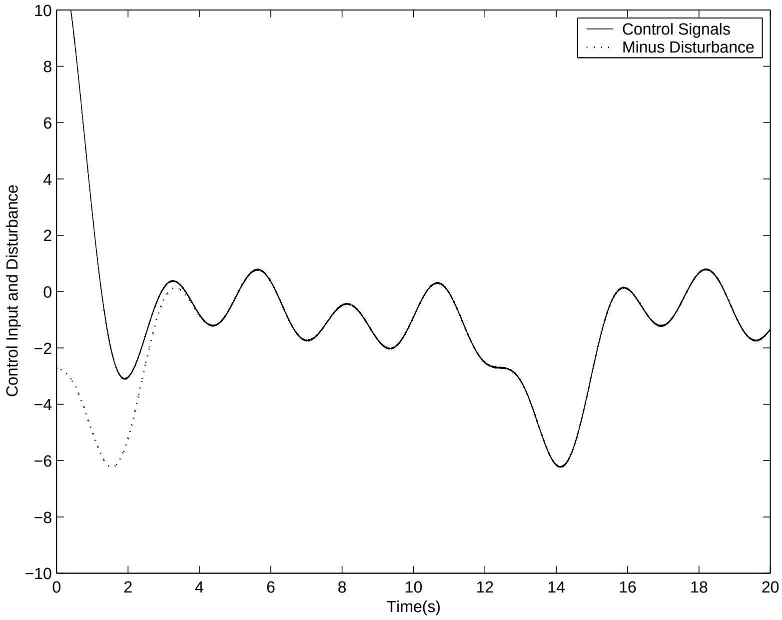

FIGURE 3. Control input and disturbance

given by v_f. Note that Equation (49) is the simplest form of (1) and thus it can be represented by

$$\begin{aligned}\dot{\theta}_1 &= \phi_1(\theta_1) + z_1 \\ \dot{z}_1 &= f_N(\theta_1, z_1) + J^{-1}(v_{in} - v_f)\end{aligned} \tag{50}$$

where $z_1 = \theta_2 - \phi_1(\theta_1)$, $\phi_1(\theta_1) = -k_1\theta_1$ and $f_N(\theta_1, z_1) = k_1(\phi_1(\theta_1) + z_1)$. The integral controller (23) is applied for (50) with $(k_1, k_2, k_3) = (10, 69, 344)$ and $\xi = 86$.

In the following experiments, the command position was set to be zero and four types of voltage drifting including sinusoidal, triangular, saw-tooth and composite waves, are purposely injected into the system through the channel v_f.

By applying the developed control algorithm, Figures 4 and 5 show that the resulting control effort, which are applied to hold the system at origin, precisely represents the voltage drifting. Note note the slope at the corners shown in Figure 5 can be understood by the so called generalized gradient; that is, the time derivative at the corners are bounded by the left and right derivatives. According to the result presented in [28], the dynamic controller is not capable of overcoming triangular waves because the waves are not twice time differentiable. However, as shown in Figure 5, the triangular fault can be precisely reconstructed. Consequently, the conservativeness in [28] is reduced by this work.

Figures 6 and 7 illustrate unsatisfied estimates at certain critical points in which the injected voltage changes abruptly. This phenomenon is caused by the fact that the time derivatives of the faults at these specific points are infinity. According to the developed method, disturbances can only be rejected in the case where the external faults are generalized first time differentiable. Consequently for the saw-tooth and composite waves, it is impossible to find a finite value of ξ to suppress these infinite fast switching. That is, $d(t) \notin C^1$ at these time instants. Except for these critical points, the computed control signals approach to the real faults closely.

Referring to the disturbance observer (DO) based approaches [29,30], the external disturbance is assumed to be slow varying relative to the observer dynamics. The estimation precision is affected by the varying rates of given disturbances. That is, the estimate

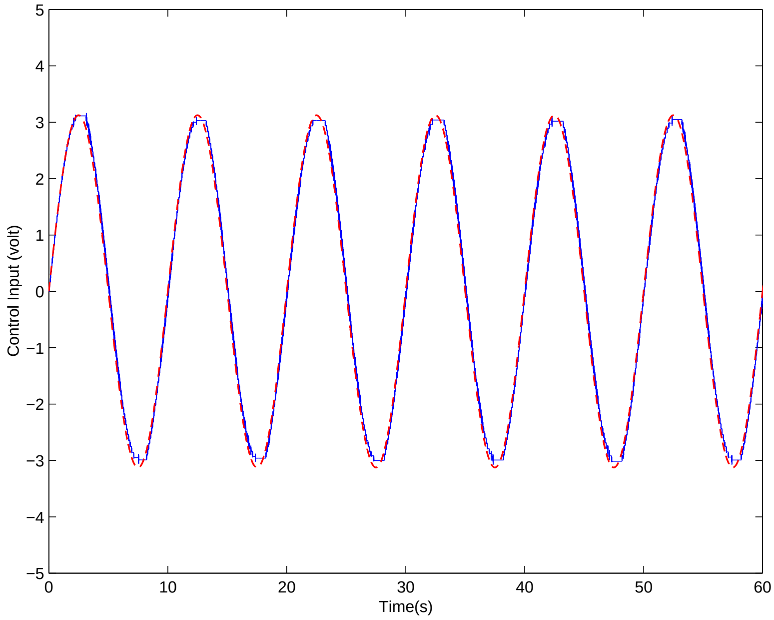

FIGURE 4. Sinusoidal waves

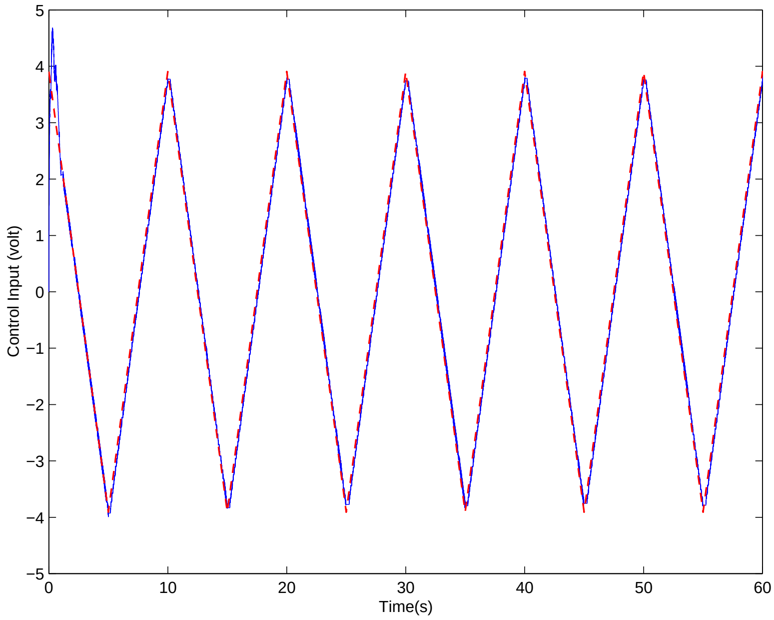

FIGURE 5. Triangular waves

performance will be degraded by a large varying rate. However, in the proposed method, the disturbance is allowed to be time varying and the asymptotic estimation level can be reached as long as $d(t) \in C^1$. In addition, only one control parameter ξ is needed to be determined for disturbance elimination, which makes the controller realization efficient.

Recently, a disturbance rejection scheme by utilizing half-period integration technique was proposed [31,32]. With assumptions that the functions of the external disturbances are previously known and periodic, these works are able to precisely estimate tricky types of disturbances where the derivatives are not bounded such as saw-tooth, square waves, etc. However, the functions of disturbances may not be easily known in advance such that it cause certain degree of difficulty to apply the model based disturbance rejection scheme.

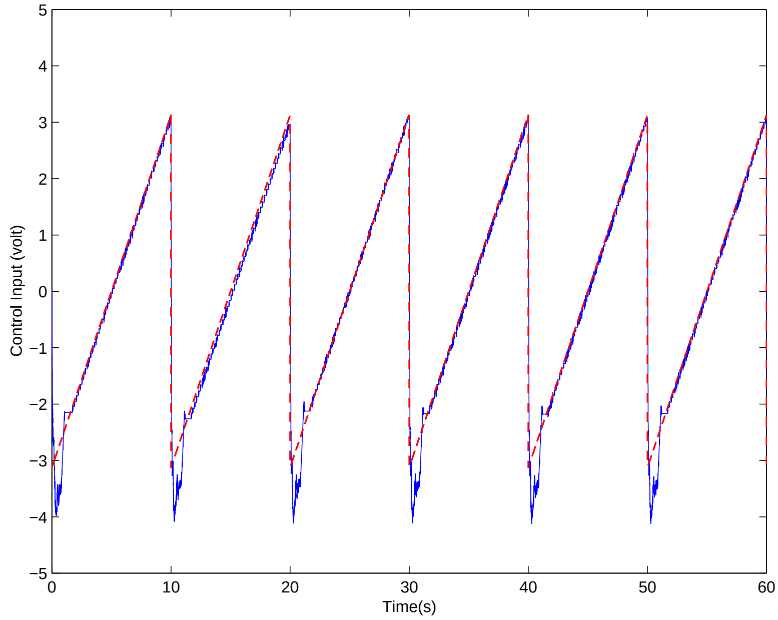

Figure 6. Saw-tooth waves

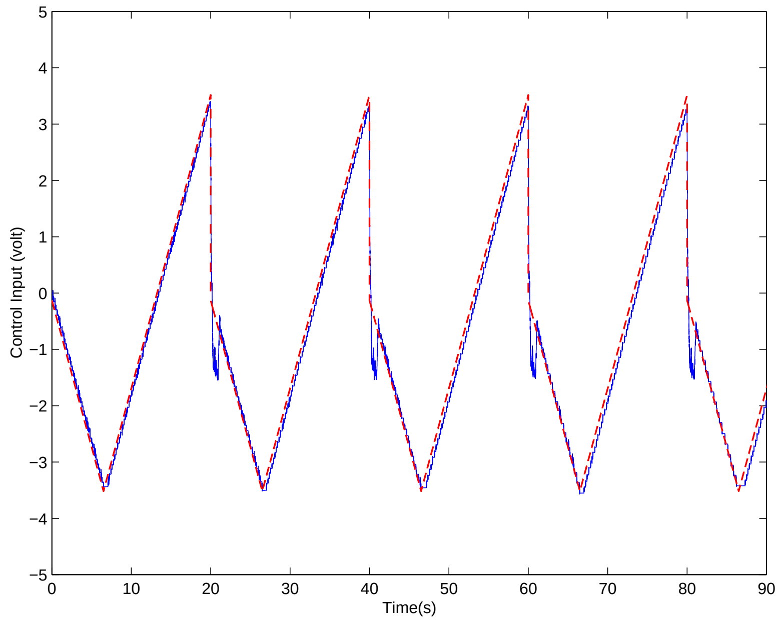

Figure 7. Composite waves

A more practical consideration has been argued in [32] that estimation of arbitrary disturbances causes infinite dimension external model, which is not realizable in practice. Thus, an adequate way is to estimate and compensate the main frequency components involved in the disturbances. Interested readers can refer to these studies for detail.

In the previous couple of experiments, it is worthy to point out that the applied integral controller together with control parameters are fixed during the estimate of all types of unknown voltage faults. As a result, the developed approach is suitable for the estimation of wide class of unknown faults. The proposed approach does not use any extra compensator or observer for the disturbance elimination and reconstruction. The controller involves the least information about external disturbance and therefore it saves lots of effort for control law realization. As a result, the proposed method is adequate for practical case studies.

6. **Conclusions.** In this work, a robust integral controller is developed, which is an extension of our previous result to a class of perturbed nonlinear systems. With the aid of systematic backstepping design, the controller can be applied to wide class of nonlinear systems subject to mismatched uncertainties and a matched external unknown disturbance. We also simplified the stability proof given in [28] and further proposed a new criterion for the closed-loop stability. The way of selection of control parameters for achieving system stability and disturbance rejection is given based on the new criterion. Different from [27], the developed method doesn't involved any extra observer (or dynamic compensator). Moreover, the assumption $d(t) \in C^2$ imposed on external disturbance [27,28] is also relaxed to be $d(t) \in C^1$ in the proposed approach. Briefly, the effectiveness and efficiency of the proposed method include: 1). No extra compensator is required in the control framework so that the time and cost for design work can be reduced. 2). The robust PI-type controller is capable of recovering external disturbance precisely as long as the disturbance is generalized first time differentiable. 3). System stabilization and disturbance rejection can be simultaneously achieved. 4). Different to the previous work [28], criteria to determine the applied control gains is addressed apparently. 5). The generated control effort is smooth and adequate for practical implementation. 6). The proposed approach can be extended to solve the problem of mismatched faults recovery [33,34]. 7). Finally, realization of the control framework is systematic and simple.

Numerical simulations and experiments are presented to show the effectiveness and applicability of the proposed method.

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