A modified vector mode solution of step index fiber

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ABSTRACT

For the sake of clarifying the characteristics of cylindrical travelling wave and step index fiber, the modified zero order Hankel function, which is an approximate harmonic complex function without singularity, is recommended as the eigen function of cylindrical travelling wave in homogeneous dielectric. As the modified zero order Neumann function avoids the singularity in the origin of coordinate, and the concept of the essence of mode field of fiber is engaged, both of zero order Bessel function and modified zero order Neumann function are recommend as the eigen functions of axial electric or magnetic component in the core layer of step index fiber. Then, the analysing method of vector mode of fiber is used for reference, a new eigen equation of step index fiber is recommended. These may provide a new method for analyzing the characteristics of cylindrical travelling wave and step index fiber.

Keywords: cylindrical travelling wave; step index fiber; vector mode; eigen function; eigen equation; modified zero order Hankel function; harmonic function

1. INTRODUCTION

The classical theory of mode field of step index fiber is established creatively by E. Snitzer [1], and its simple description in weakly guiding approximation is described by D. Gloge [2], a verdict of mode field theory for step index fiber appears to be formed. The Bessel function and imaginary parameter Hankel function are engaged as the eigen functions of core layer and cladding layer of step index fiber respectively, the Neumann function is excluded from the eigen function of core layer of step index fiber on account of the singularity in the origin of coordinate [1-4]. However, some puzzles of the mode theory stayed in people's mind, such as the physical meaning of eigen equation and the total mode number of multimode fiber, and several new ideas are presented for explaining the essence of fiber [5-9].

For clarifying the eigen mode field characteristics of step index fiber, the eigen function of the cylindrical travelling wave in homogeneous dielectric is discussed firstly. The classical expression, that is derivated from the conservation law of energy for radiation source [10], isn't the solution of the scalar Helmholtz's equation. As the Helmholtz's equation and the method of separation of variables are engaged, zero order Hankel function is suggested as the describing expression of cylindrical travelling wave [11, 12], the product of half order Hankel function and harmonic angular function is proposed as a describing function too [9]. However, the singularities of above-mentioned describing functions in the origin of coordinate are still in puzzles, these describing functions need to be modified in the small independent variable. Then, the modified zero order Hankel function without singularity, which is built by zero order Bessel function and modified zero order Neumann function, is recommended to act as the eigen function of the cylindrical travelling wave in homogeneous dielectric.

As the modified zero order Neumann function relieves the puzzle of singularity of zero order Neumann function in the origin of coordinate, both of zero order Bessel function and modified zero order Neumann function are suggested as the eigen functions of core layer in step index fiber. As the electric or magnetic components in x and y coordinates coexist in the circular fiber, the precondition of linear polarization mode doesn't come into existence, only the vector mode method is logical analysing method for step index fiber. Then, the analysing method of vector mode of fiber is used for reference, a new eigen equation of step index fiber and some novel description of the characteristics of step index fiber are recommended.

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2. THE DESCRIBING FUNCTION OF FIELD COMPONENT

In general, the describing function of cylindrical travelling wave in homogeneous dielectric is expressed by the wave number k, radial coordinate r, angular coordinate φ, angular frequency ω and time t, and the direction of wave vector coincides with the radial coordinate r. As the scalar Helmholtz's equation [3] and the method of separation of variables are engaged, the solution of electric or magnetic components in one of the rectangular coordinate axes of the Cartesian coordinates system is given as follows,

$$\Psi(r,\varphi,z,t) = C_0 H_m^{(1,2)}(k_r r)\exp(\pm \mathrm{i} m\varphi)\exp(\pm \mathrm{i} k_z z - \mathrm{i}\omega t)\,, \tag{1}$$

where C_0 is a constant, $H_m^{(1,2)}(k_r r)$ is the m order Hankel function, the superscripts 1 and 2 are the labels of the kinds of Hankel function, the subscript m is the order number of angular parameter, it is the integer, the sign i is the imaginary quantity, it equals to the square root of minus one, k_r and k_z are the radial component and axial component of wave number of light wave respectively, and $k_z=(k^2-k_r^2)^{1/2}$, the operators "+" and "–" of "±" and superscripts 1 and 2 of the Hankel function are used to express divergent and convergent light wave respectively.

When $k_z=0$, $k_r=k$, the direction of wave vector coincides with the radial coordinate r, Eq. (1) is an imaginable expression of cylindrical travelling wave, thereinto, when $m=0$ and $k_z=0$, Eq. (1) is suggested as one of the describing expression of cylindrical travelling wave [11, 12].

In mathematics, $H_m^{(1,2)}(\xi)=J_m(\xi)\pm \mathrm{i}Y_m(\xi)$, where ξ is the independent variable, $J_m(\xi)$ is the m order Bessel function, $Y_m(\xi)$ is the m order Neumann function.

Whereas, the origin of coordinate is a singularity of m order Neumann function, namely, the Neumann function $Y_m(\xi)$ diverges [3,4] at $\xi=0$, if the zero order Hankel function is engaged to describe the field distribution of cylindrical travelling wave, the electric and magnetic component $\Psi(r,t)$ tends to infinity for the radial coordinate r closed to zero. So, the zero order Hankel function $H_0^{(1,2)}(\xi)$ is not a perfect describing function of cylindrical travelling wave in homogeneous dielectric, if it is employed to describe the cylindrical travelling wave in homogeneous dielectric, it needs to be modified in the small independent variable.

Now, for avoiding the singularity of describing function of cylindrical travelling wave in homogeneous dielectric, the approximate expression of cylindrical travelling wave in homogeneous dielectric is suggested as,

$$\Psi(r,t) = C_0 M H_0^{(1,2)}(kr)\exp(-\mathrm{i}\omega t)\,, \tag{2}$$

where $MH_0^{(1,2)}(\xi)$ is the modified zero order Hankel function of first kind or second kind, it is suggested as following expression,

$$MH_0^{(1,2)}(\xi) = J_0(\xi) \pm \mathrm{i} MY_0(\xi)\,, \tag{3}$$

where $MY_0(\xi)$ is the modified zero order Neumann function.

If the modified zero order Hankel function $MH_0^{(1,2)}(\xi)$ is an approximate harmonic function, and the original zero order Neumann function is modified near the origin of coordinate merely, then, the modified zero order Neumann function $MY_0(\xi)$ is suggested as follow expression approximately,

$$MY_0(\xi) = J_1(\xi) - \frac{\xi}{2(\xi^2+c^2)} J_0(\xi)\,, \tag{4}$$

where c is a constant in the selected release.

In mathematics, the value of first order derivative of zero order Neumann function is the negative first order Neumann function, i.e., $Y_1(\xi)=-\mathrm{d}Y_0(\xi)/\mathrm{d}\xi$. Following above formula as a model, lets $MY_1(\xi)=-\mathrm{d}MY_0(\xi)/\mathrm{d}\xi$, namely, the value of modified first order Neumann function equals to the negative first order derivative of modified zero order Neumann function, then, $MY_1(\xi)$ is expressed by,

$$MY_1(\xi) = -J_0(\xi) - \frac{\xi^2-c^2}{2(\xi^2+c^2)^2} J_0(\xi) + \frac{\xi^2+2c^2}{2\xi(\xi^2+c^2)} J_1(\xi)\,. \tag{5}$$

For the sake of giving relative reasonable modified zero order Neumann function $MY_0(\xi)$ and modified first order Neumann function $MY_1(\xi)$, although the value π is not the optimum choice for the constant c in the Eq. (4) and Eq. (5), it may be a rational choice.

According to Eq. (4) and c=π, the characteristics of modified zero order Neumann function $MY_0(\xi)$ is shown as solid line in Fig. 1, it shows that the modified zero order Neumann function $MY_0(\xi)$ is close to zero order Neumann function $Y_0(\xi)$ except for the small independent variable ξ. In particular, the value $MY_0(\xi)$ is zero for ξ=0, it relieves the puzzle of singularity of original zero order Neumann function $Y_0(\xi)$ in the origin of coordinate. And as a control curve, the characteristic of zero order Neumann function $Y_0(\xi)$ is shown as dotted line in Fig. 1 also.

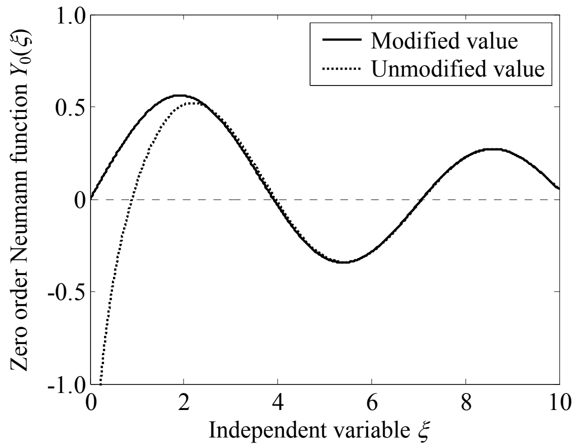

Fig. 1. Characteristics of modified and unmodified zero order Neumann function

The solutions of equation $MY_0(\xi)$=0 are ξ_1=0, ξ_2=3.9100, ξ_3=7.0748, ξ_4=10.2182, ξ_5=13.3592, ξ_6=16.4999 and so on, the solutions of equation $Y_0(\xi)$=0 are ξ_1=0.8936, ξ_2=3.9577, ξ_3=7.0861, ξ_4=10.2223, ξ_5=13.3611, ξ_6=16.5009 and so on. The differences between solutions of equation $MY_0(\xi)$=0 and equation $Y_0(\xi)$=0 are $\delta\xi_1$=-0.8936, $\delta\xi_2$=-0.0477, $\delta\xi_3$=-0.0113, $\delta\xi_4$=-0.0041, $\delta\xi_5$=-0.0019, $\delta\xi_6$=-0.0010 and so on, the absolute values of these differences show a trend of monotonic decreasing.

According to Eq. (5) and c=π, the characteristics of modified first order Neumann function $MY_1(\xi)$ is shown as solid line in Fig. 2, it shows that modified first order Neumann function $MY_1(\xi)$ is close to first order Neumann function $Y_1(\xi)$ except for the small independent variable ξ. In particular, the value $MY_1(\xi)$ is -0.4493 for ξ=0, it relieves the puzzle of singularity of the original first order Neumann function $Y_1(\xi)$ in the origin of coordinate. And as a control curve, the characteristic of first order Neumann function $Y_1(\xi)$ is shown as dotted line in Fig. 2 also.

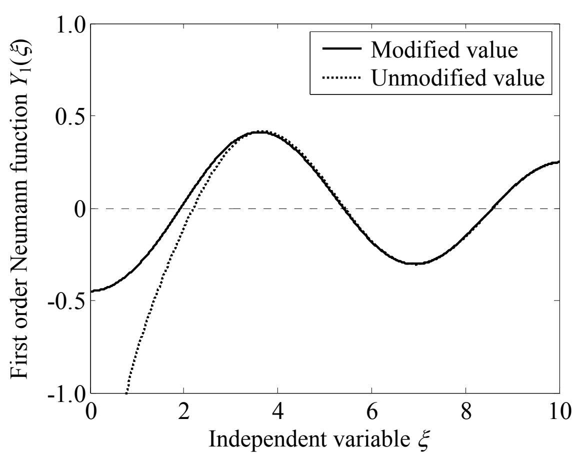

Fig. 2. Characteristics of modified and unmodified first order Neumann function

The solutions of equation $MY_1(\xi)$=0 are ξ_1=1.9314, ξ_2=5.4042, ξ_3=8.5888, ξ_4=11.7462, ξ_5=14.8960, ξ_6=18.0426 and so on, the solutions of equation $Y_1(\xi)$=0 are ξ_1=2.1971, ξ_2=5.4297, ξ_3=8.5960, ξ_4=11.7492, ξ_5=14.8974, ξ_6=18.0434 and so on. The differences between the solutions of equation $MY_1(\xi)$=0 and equation $Y_1(\xi)$=0 are $\delta\xi_1$=-0.2657, $\delta\xi_2$=-0.0255, $\delta\xi_3$=-0.0072, $\delta\xi_4$=-0.0030, $\delta\xi_5$=-0.0014, $\delta\xi_6$=-0.0008 and so on, the absolute values of these differences show a trend of monotonic decreasing too.

As no singularity appears in the modified zero order Neumann function $MY_0(\xi)$, the modified zero order Hankel function $MH_0^{(1,2)}(\xi)$ is an approximate harmonic function without singularity, it may be a desirable eigen function of cylindrical travelling wave.

Then, the approximate expression of electric or magnetic components in rectangular coordinate axes of the Cartesian coordinates system is suggested as follows,

$$\Psi(r,z,t) = C_0 MH_0^{(1,2)}(k_r r)\exp(\pm \mathrm{i}k_z z - \mathrm{i}\omega t)\,. \tag{6}$$

According to the essence of mode field of fiber, the field of core layer of fiber is standing wave field, both of $J_0(\xi)$ and $MY_0(\xi)$ are the imaginable choice for the eigen function of electric or magnetic components in rectangular coordinate axes of core layer of fiber.

3. A MODIFIED VECTOR MODE METHOD FOR THE FIBER

The analysing method of linear polarization (LP) mode of weakly guiding fiber based on the hypothesis [2, 13] as following, the transverse field of step index fiber essentially polarized in one direction, namely, one of the transverse electric components E_x and E_y or the transverse magnetic components H_x and H_y in step index fiber is zero. Whereas, the electric or magnetic components in x and y coordinates coexist in the circular step index fiber, the precondition of linear polarization mode may not come into existence in step index fiber, then, the vector mode method may be only logical analysing method for step index fiber.

As the classical analysing method [1, 13] is used for reference, the eigen functions of electric or magnetic component in z coordinate axis of step index fiber are recommended as follows,

$$\Psi_{1,z}(r,z,t) = C_1\left[pJ_0(\frac{Ur}{a}) + qMY_0(\frac{Ur}{a})\right]\exp(\mathrm{i}\beta z - \mathrm{i}\omega t)\,, \tag{7}$$

$$\Psi_{2,z}(r,z,t) = C_2 K_0(\frac{Wr}{a})\exp(\mathrm{i}\beta z - \mathrm{i}\omega t)\,, \tag{8}$$

where $K_0(\xi)$ is the zero order imaginary parameter Hankel function, C_1 and C_2 are constants, subscripts 1 and 2 label the parameters of core layer and cladding layer of step index fiber, p and q are constants of zero or unit, there are only two case, p=1 for q=0, or p=0 for q=1, U and W are the normalized standing wave parameter of core layer and normalized evanescent wave parameter of cladding layer in step index fiber respectively, $U=(k_1^2-\beta^2)^{1/2}a$, $W=(\beta^2-k_2^2)^{1/2}a$, β is the propagation constant of step index fiber, a is the radius of core layer of step index fiber, k_1 and k_2 are the wave numbers of core layer and cladding layer of step index fiber respectively, $k_1=k_0n_1$, and $k_2=k_0n_2$, here, k_0 is the wave number of light wave in vacuum, $k_0=2\pi/\lambda$, λ is the wavelength of light wave in vacuum, n_1 and n_2 are the refractive indexes of core layer and cladding layer of step index fiber respectively.

According to the boundary condition of Maxwell's equations on the interface between two dielectric [13], the axial electric or magnetic components $\Psi_{1,z}(r,z,t)=\Psi_{2,z}(r,z,t)$ for $r=a$ in the step index fiber, then, the relationship between the constants C_1 and C_2 is the following equation,

$$C_2 = C_1 p\frac{J_0(U)}{K_0(W)} + C_1 q\frac{MY_0(U)}{K_0(W)}\,. \tag{9}$$

Based on the Maxwell's equations, only the angular electric component $E_{j,\varphi}$, radial magnetic component $H_{j,r}$ and axial magnetic component $H_{j,z}$ appear in the transverse electric (TE) mode of fiber, and the values of $E_{j,\varphi}$ and $H_{j,r}$ can be computed by $H_{j,z}$, only the angular magnetic component $H_{j,\varphi}$, radial electric component $E_{j,r}$ and axial electronic component $E_{j,z}$ appear in the transverse magnetic (TM) of fiber, and the values of $H_{j,\varphi}$ and $E_{j,r}$ can be computed by $E_{j,z}$, where j=1 and 2, they label the parameters of core layer and cladding layer of step index fiber respectively. And, the relationships between above mentioned parameters are expressed as follows [13],

$$E_{j,\varphi} = -\frac{\mathrm{i}\omega\mu_0}{k_j^2-\beta^2}\frac{\partial H_{j,z}}{\partial r}, \tag{10}$$

$$H_{j,r} = -\frac{\beta}{\omega\mu_0}E_{j,\varphi}, \tag{11}$$

$$H_{j,\varphi} = \frac{\mathrm{i}\omega\varepsilon_j}{k_j^2-\beta^2}\frac{\partial E_{j,z}}{\partial r}, \tag{12}$$

$$E_{j,r} = \frac{\beta}{\omega\varepsilon_j}H_{j,\varphi}, \tag{13}$$

where ε_j is the permittivity of core layer or cladding layer of step index fiber, μ_0 is the magnetic permeability in vacuum, and they satisfy the following equation, $k_j^2=\omega^2\varepsilon_j\mu_0$.

For TE mode, Eq. (7) and Eq. (8) express the longitudinal magnetic components of core and cladding layer in step index fiber, $H_{1,z}=\Psi_1(r,z,t)$, $H_{2,z}=\Psi_2(r,z,t)$, for TM mode, Eq. (7) and Eq. (8) express the longitudinal electric components of core and cladding layer in step index fiber, $E_{1,z}=\Psi_1(r,z,t)$, $E_{2,z}=\Psi_2(r,z,t)$, namely, the zero order Bessel function $J_0(Ur/a)$ and modified zero order Neumann function $MY_0(Ur/a)$ are the describing functions of longitudinal electric or magnetic component of core layer in step index fiber, the zero order imaginary parameter Hankel function $K_0(Wr/a)$ is the describing function of longitudinal electric or magnetic component of cladding layer in step index fiber. According to Eq (10) to Eq (13), the first order Bessel function $J_1(Ur/a)$ and modified first order Neumann function $MY_1(Ur/a)$ are the describing function of radial or angular electric or magnetic component of core layer in step index fiber, the first order imaginary parameter Hankel function $K_1(Wr/a)$ is the describing function of radial or angular electric or magnetic component of cladding layer in step index fiber.

As the method of analyzing vector mode of step index fiber [1, 13] is used for reference, the eigen equation of step index fiber is suggested as follows,

$$\eta\frac{1}{U}\left[p\frac{J_0'(U)}{J_0(U)}+q\frac{MY_0'(U)}{MY_0(U)}\right]+\frac{1}{W}\frac{K_0'(W)}{K_0(W)}=0, \tag{14}$$

where sign $'$ is the differential coefficient operator, namely, $f'(\xi)=\mathrm{d}f(\xi)/\mathrm{d}\xi$, η is a coefficient, η=1 for TE mode of step index fiber, and $\eta=(n_1/n_2)^2$ for TM mode of step index fiber.

As the relational expressions $\mathrm{d}J_0(\xi)/\mathrm{d}\xi=-J_1(\xi)$, $\mathrm{d}MY_0(\xi)/\mathrm{d}\xi=-MY_1(\xi)$ and $\mathrm{d}K_0(\xi)/\mathrm{d}\xi=-K_1(\xi)$ are engaged, Eq. (14) becomes as follows,

$$\eta\frac{1}{U}\left[p\frac{J_1(U)}{J_0(U)}+q\frac{MY_1(U)}{MY_0(U)}\right]+\frac{1}{W}\frac{K_1(W)}{K_0(W)}=0, \tag{15}$$

where $K_1(\xi)$ is the first order imaginary parameter Hankel function.

According to the names of the eigen functions of core layer of step index fiber, the names of fiber mode are called as the n sequence TE or TM Bessel mode and the n sequence TE or TM modified Nuemann mode, their short form are $\mathrm{TEB_n}$ mode, $\mathrm{TMB_n}$ mode, $\mathrm{TEMN_n}$ mode and $\mathrm{TMMN_n}$ mode, where subscript n is the sequence of mode, namely, it is the sequence of the solution of equation $J_0(V)=0$ or equation $MY_0(V)=0$, it is the positive integer, n=1, 2, 3, ···, V is the normalized frequency of step index fiber, $V=k_0a(n_1^2-n_2^2)^{1/2}$, it is called as the normalized structural parameter of step index fiber too.

Comparing with the classical theory of step index fiber [1, 13], the $\mathrm{TEB_n}$ mode and $\mathrm{TMB_n}$ mode are no other than the $\mathrm{TE_{0n}}$ mode and $\mathrm{TM_{0n}}$ mode.

In general, the characteristic of fiber is distinguished by the normalized frequency V. For TEB_n mode and TMB_n mode, the cut off normalized frequencies of step index fiber are determined by [1, 13] the solutions of equation $J_0(V)=0$, their numerical values are 2.4048, 5.5201, 8.6537, 11.7915, 14.9309 and so on. For $TEMN_n$ mode and $TMMN_n$ mode, the cut off normalized frequencies of step index fiber are determined by the solution of equation $MY_0(V)=0$, their numerical values are 0, 3,9100, 7.0748, 10.2182, 13.3592, 16,4999 and so on, these numerical values are very closed to the solutions of equation $Y_0(V)=0$ except for the first solution, the value of first solution of equation $Y_0(V)=0$ is 0.8936.

The fundamental mode of the step index fiber is the $TEMN_1$ mode and $TMMN_1$. According to what mentioned above, the condition of single mode appearing in step index fiber still the classical expression [1, 13], namely, the scale of normalized frequency of single mode fiber is between zero and 2.4048.

According to Eq. (15), the characteristics of normalized standing wave parameter U versus normalized frequency V of TE mode in step index fiber are shown as solid lines in Fig. 3. According to Eq. (15), the cut off normalized frequencies of TE modes and TM modes in the step index fiber are the same values, they are shown as dots in Fig. 3.

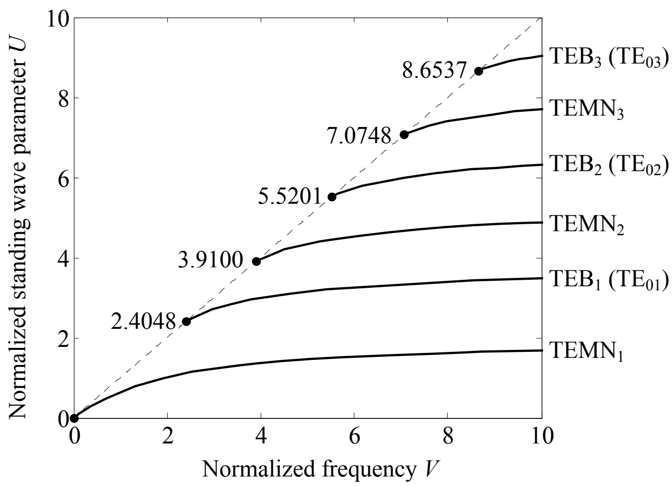

Fig. 3. Characteristics of normalized parameters U versus V

For the common step index fiber, the value of refractive index n_2 is very close to n_1, i.e., the value of coefficient η for TM modes is very near the unit. Then, the characteristics of TM modes of step index fiber are very close to the characteristics of TE modes of step index fiber.

If the value of normalized frequency of step index fiber is large enough, and the normalized parameters U and W are far larger than the unit, the asymptotic forms of $J_0(U)$, $MY_0(U)$ and $K_0(W)$ for large independent variable are engaged, then, Eq. (15) becomes as follows expression approximately [9],

$$W = U \tan\left[U - \frac{(2\chi-1)\pi}{4}\right], \tag{16}$$

where χ is the positive integer, i.e., χ=1, 2, 3,···. The odd positive integer $\chi=2n-1$ for $TEMN_n$ mode and $TMMN_n$ mode of step index fiber, the even positive integer $\chi=2n$ for TEB_n mode and TMB_n mode of step index fiber.

According to Eq. (16), the cut off normalized frequency of n sequence modified Nuemann mode or n sequence Bessel mode is expressed by the following expression approximately,

$$V_{a,c} = \frac{(2\chi-1)\pi}{4}. \tag{17}$$

According to Eq. (17), the approximate value of cut off normalized frequency of n sequence TE or TM modified Nuemann mode are 0.7854, 3.9270, 7.0686, 10.2102, 13.3518, 16.4934 and so on, these numerical values are very closed to the solutions of equation $MY_0(V)=0$ except for the first solution, the first solution of equation $MY_0(V)=0$ is zero. the approximate value of cut off normalized frequency of n sequence TE or TM Bessel mode are 2.3562, 5.4978, 8.6394, 11.7810, 14.9226, 18.0642 and so on, these numerical values are very closed to the solutions of equation $J_0(V)=0$.

If the difference between the approximate computing value of cut off normalized frequency $V_{a,c}$ and the computing value V_c from the solution of Eq. (15) is called the computing error value of cut off normalized frequency, i.e., $\Delta V_c = V_{a,c} - V_c$. Then, the characteristics of computing error values of cut off normalized frequencies of the modified Neumann modes are shown as asterisks in Fig. 4, where the independent variable χ is the odd positive integer $2n$-1, the characteristics of computing error values of cut off normalized frequencies of the Bessel modes are shown as points in Fig. 4, where the independent variable χ is the even positive integer $2n$.

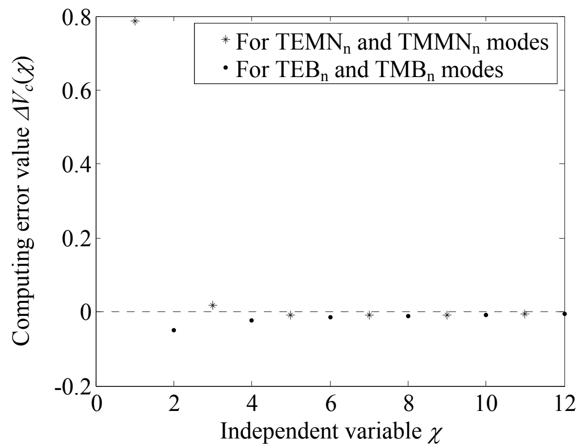

Fig. 4. Characteristics of computing error value of cut off normalized frequency versus variable χ

Fig.4 shows that, the computing error values of cut off normalized frequencies are very small except the first one, it means that, Eq. (17) is a reasonable approximate computational formula for the cut off normalized frequency of high order mode in step index fiber.

As the TE mode and TM mode coexist in the step index fiber, and Eq. (17) is engaged, the total mode number of multimode fiber [8] can be calculated by the following equation approximately,

$$M = 2\mathrm{fix}\left(\frac{2V}{\pi} + \frac{1}{2}\right), \tag{18}$$

where fix(ξ) rounds the elements of ξ toward zero, namely, it rounds the elements of nonnegative number ξ to the nearest integers less than or equal to ξ.

4. CONCLUSIONS

As the novel expression of modified zero order Neumann function is suggested, the puzzle of singularity of Neumann function or Hankel function in the origin of coordinate is relieved. The modified zero order Hankel function is suggested as the eigen function of cylindrical travelling wave in homogeneous dielectric. Both of the zero order Bessel function and modified zero order Neumann function are suggested as the eigen function of axial electric or magnetic components in core layer of step index fiber. Both of the first order Bessel function and modified first order Neumann function are suggested as the eigen function of radial and angular electric or magnetic components in core layer of step index fiber. A new eigen equation of step index fiber is recommended. Although these novel ideas may not correspond with the actual characteristics of the cylindrical travelling wave and step index fiber mode completely, they may afford some new mathematical models for further discussion.

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REFERENCES

[1] Snitzer E., “Cylindrical dielectric waveguide modes,” J. Opt. Soc. Am., 51(5), 491-498 (1961)

[2] Gloge D., “Weakly guiding fibers,” Appl. Opt., 10(10), 2252-2258 (1971)

[3] Kawano K. and Kitoh T., [Introduction to Optical Waveguide Analysis: Solving Maxwell’s Equation and Schrodinger Equation], John Wiley & Sons, Inc., New York, 36-57 (2001)

[4] Marcuse D., [Theory of dielectric optical waveguides, Second Edition], Academic Press Inc., Boston, 60-96 (1991)

[5] Yu S. M. and Yu T., “Covariant form of Sommerfeld’s formula for spherical wave and application in fiber optics,” Acta Physica Sinica, 50(6), 1097-1102 (2001)

[6] Yu S. M. and Yu T., “Electromagnetic adjoint transformation and mode analysis for guided waves in an optical fiber,” Acta Physica Sinica, 50(11), 2179-2184 (2001)

[7] Yu S. M. and Yu T., “Resonance modes in optical fibres,” Chin. Phys., 11(10), 981-987 (2002)

[8] Guo F. Y., Chen Y. Q., Lin B., Cao X. Q. and Shangguan C. M. S., “A sort of novel expression for the eigenfunctions of step index fiber,” Acta Photonica Sinica, 32(9), 1074-1078 (2003)

[9] Guo F. Y., Li L. H., Zheng H., Wang Y., Dai T. G. and Ke J. R., “Harmonic expressions for the eigen mode fields of step and graded index fibers,” J. Phys.: Conf. Ser., 276, 012058-1-012058-8 (2011)

[10] Iizuka K., [Engineering Optics, Third Edition], Springer, New York, 27-28 (2008)

[11] Yao D. Z. and Ruan Y. Z., “Complex ray expansion of inhomogenous cylindrical wave”, Bulletin of Science and Technology, 10(1), 17-20 (1994)

[12] Fu J. M. and Hong E. X., [Advanced Electromagnet Theory], Xi’an Jiaotong University Press, Xi’an, 45-46 (2000)

[13] Wu C. Q., [Theory of Optical Waveguide, Second Edition], Tsinghua University Press, Beijing, 27-60 (2005)