Communications in Theoretical Physics

GENERAL

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To cite this article: Meng Hong-Juan *et al* 2012 *Commun. Theor. Phys.* **57** 1007

Commun. Theor. Phys. **57** (2012) 1007–1011 Vol. 57, No. 6, June 15, 2012

Nonlinear Ramsey Interferometry of Fermi Superfluid Gases in a Double-Well Potential*

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(Received November 14, 2011; revised manuscript received January 4, 2012)

Abstract *The nonlinear Ramsey interferometry of Fermi superfluid gases in a double-well potential is investigated in this paper. We found that the frequency of the Ramsey fringes exactly reflects the strength of nonlinearity, or the scattering length of the Fermi superfluid gases. The cases of sudden limit, the adiabatic limit and the general case are studied. The analytical result is in good agreement with the numerical ones. The adiabatic condition is proposed. In general situation, the zero-frequency point emerge. Finally the possible applications of the theory are discussed.*

PACS numbers: 03.75.Kk, 03.75.Lm, 37.10.Jk

Key words: Fermi superfluid gases, deep BEC regime, Ramsey interferometry

1 Introduction

The Bose–Einstein condensates (BECs) in double-well potential have been studied extensively in the past years to demonstrate quantum tunneling phenomena, such as Josephson oscillation and junction,[1−2] brag diffraction,[3] self-trapping,[4−8] interferometry,[9−15] phase transition,[16−17] Rosen–Zener transition,[18−19] as well as Landau-Zener transition[20−21] for both single species and multi-species systems.[22]

More recently, the Fermi superfluid gases in a double-well potential which provide the unique opportunity to study the Bardeen–Cooper–Schrieffer (BCS) to BEC crossover have been studied.[23−27] However, many questions remain unsolved.[24] One of the unsolved questions is the nonlinear Ramsey interferometry.

The technique of Ramsey interferometry is one of the most powerful tools in high-precision measurement and has versatile applications. For instance, Ramsey fringes between atoms and molecules in time domain have been observed by using trapped BEC of ^{85}Rb atoms,[28−29] which provides the basis of atomic fountain clocks that now serve as time standards[30−31] and stimulates the rapid advancement in the field of precision measurements in atomic physics. The atom interferometers with cold atoms have been used to measure rotation,[32] gravitational acceleration,[14,33] atomic fine-structure constant,[34] atomic recoil frequency,[35] and atomic scattering properties.[36] Moreover, nonlinear Ramsey interferometry and adiabatic Rosen–Zener (RZ) interferometry for the BECs have also been studied recently.[37−38]

In the present manuscript, we construct a Rosen–Zener interferometer to the superfluid Fermi gases in double well potential in deep BEC regime, We find that the frequency of the nonlinear Ramsey fringes reflects the strength of nonlinearity, or the scattering length $a_{\rm sc}$.

Our paper is organized as follows. In Sec. 2, basic equation and two-mode approximation for Fermi superfluid are developed. In Secs. 3 and 4, the Nonlinear Ramsey interferometry of superfluid Fermi gases in three limited cases were thoroughly investigated. In Sec. 5, the summary and the possible application are discussed.

2 Basic Equation

At zero temperature, for a large number of atoms, the statistical and dynamical collective properties of a one-dimensional (1D) trapped superfluid Fermi gases are expected to be properly described by DF GP equation including beyond mean-field corrections.[39−40] In deep BEC regime ($0 < a_{\rm sc} \ll 1$, where $a_{\rm sc}$ is the scattering length of the fermions), the chemical potential for this case has been studied by many researchers[24,41−42] which can be approximated by $\mu = C_0 n(1+C_1 n^{1/2})$ where $(C_0, C_1) = ((4\pi\hbar^2 a_{\rm sc}/m), (\alpha_0-\gamma_0)a_{\rm sc}^{3/2})$, $\alpha_0 = 32\nu/3\sqrt{\pi}$, $\gamma_0 = 32(\nu-1)/3\sqrt{\pi}$, $\nu = 1.05, 1.1, 1.15$, n is particle density and m is the mass of the dimer.

For a two-mode approximation, the order parameter of the system $\Psi(x,t)$ can be expressed as the superposition of individual wave functions in each well,[26,43−45] i.e.,

*Supported by the National Fundamental Research Program of China under Grant Nos. 2007CB814800 and 2011CB921503, the National Natural Science Foundation of China under Grant Nos. 10725521, 91021021, 10875098, the Natural Science Foundation of Northwest Normal University under Grant No. NWNU-KJCXGC-03-48

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$\Psi(x,t) = \psi_1(t)\phi_1(x) + \psi_2(t)\phi_2(x)$, where $\phi_1(x)$ and $\phi_2(x)$ are the spatial mode functions, $\psi_1(t)$ and $\psi_2(t)$ are the probability amplitude in two wells respectively. In general the functions $\psi_{1,2}(t)$ are complex and satisfy the condition: $|\psi_{1,2}(t)|^2 = N_{1,2}(t)$, then the total particle number is given by $|\psi_1(t)|^2 + |\psi_2(t)|^2 = N_1(t) + N_2(t) = N$. Then the following two-mode equations should be satisfied[46]

$$\mathrm{i}\dot{\psi}_1 = \frac{\gamma}{2}\psi_1 + (U_1 N_1 + V_1 N_1^{3/2})\psi_1 - k\psi_2\,, \tag{1}$$

$$\mathrm{i}\dot{\psi}_2 = -\frac{\gamma}{2}\psi_2 + (U_2 N_2 + V_2 N_2^{3/2})\psi_2 - k\psi_1\,, \tag{2}$$

where

$$\gamma = \int \phi_j(x)\Big[-\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + V(x)\Big]\phi_j(x)\,\mathrm{d}x$$

is the energy bias between two wells,

$$U_j = \frac{4\pi}{k_f}\frac{1}{y}\int \Phi_j^3\,\mathrm{d}x\,,$$

$$V_j = \frac{4\pi(\alpha_0 - \gamma_0)}{k_f^{5/2}}\frac{1}{y^{5/2}}\int \phi_j^{7/2}\,\mathrm{d}x, \quad (j = 1, 2)$$

are proportional to the atomic self-interaction energies which are related to the corresponding scattering length a_{sc}, or the dimensionless interaction parameter $y = 1/(k_f a_{\mathrm{sc}})$, where $k_f = (3\pi^2 n)^{1/3}$ and

$$k = \int \phi_1(x)\Big[-\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + V(x)\Big]\phi_2(x)\,\mathrm{d}x$$

describes the tunneling amplitude between two wells.

3 Nonlinear Ramsey Interference Patterns

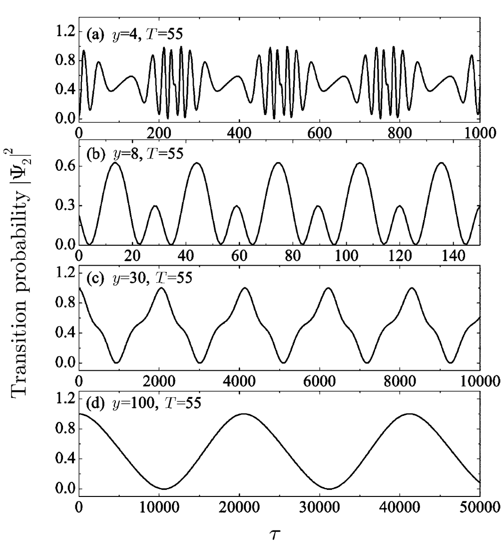

Fig. 1 Ramsey fringe patterns vs. different nonlinear parameter y, (a) $y = 4.0$, (b) $y = 8.0$, (c) $y = 30.0$, and (d) $y = 100.0$ where $T = 55$.

We now study the nonlinear Ramsey interferometer of Fermi superfluid gases. In order to realize it, we let $k(t)$ changes with time t as follows[19]

$$k(t) = \begin{cases} 0, & t < 0\,, \\ k_0 \sin^2(\pi t/T), & t \in [0, T]\,, \\ 0, & t \in [T, T+\tau]\,, \\ k_0 \sin^2[\pi(t - T - \tau)/T], & t \in [T+\tau,\ 2T+\tau]\,, \\ 0, & t > 2T + \tau\,, \end{cases} \tag{3}$$

where k_0 is the maximum strength of the coupling, T is the scanning period of Rosen–Zener pulse, and τ is a holding time between two pulses.

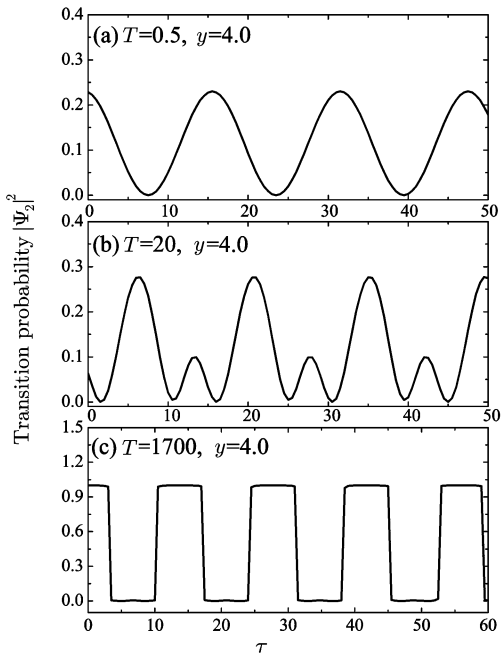

Fig. 2 Ramsey fringe patterns vs. different scanning period T, (a) $T = 0.5$, (b) $T = 20$, and (c) $T = 1700$ where $y = 4.0$.

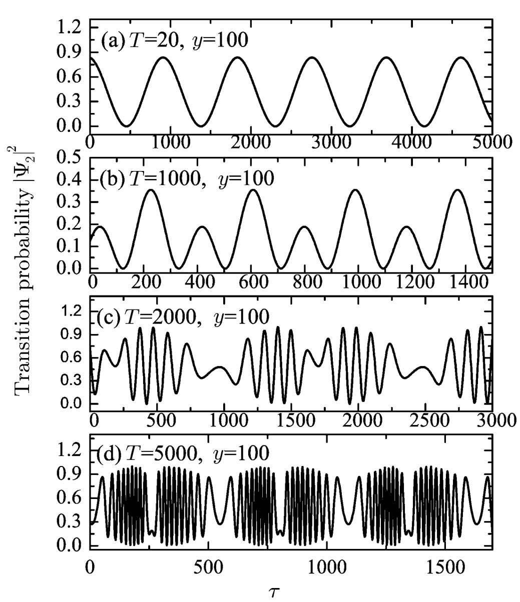

Fig. 3 Ramsey fringe patterns vs. different scanning period T, (a) $T = 20$, (b) $T = 1000$, (c) $T = 2000$, and (d) $T = 5000$ where $y = 100$.

Equations (1) and (2) have been solved numerically. We assume that the quantum state is initially prepared on one mode $|\psi_1(t=0)|^2 = 1$, $|\psi_2(t=0)|^2 = 0$ and observe the dependence of the final transition probability $|\psi_2(2T+\tau)|^2$ on the holding time τ. The numerical results have been displayed in Figs. 1, 2, and 3.

We find from Figs. 1, 2, and 3 that nonlinearity (y) and scanning period (T) can affect the pattern and the frequency of Ramsey fringes significantly. Moreover, we note that the Ramsey patterns include perfect sinusoidal oscillation, trigonometric oscillation with multiple period, and rectangular oscillation. Furthermore, the sinusoidal Ramsey pattern only exists in the cases of the smaller values of y and T. The rectangular oscillation only emerges in the slow scanning case or large value of T, for instance, ($T = 1700$). The rectangular oscillation pattern does not exist in the large values of y. In the following text we will explain these phenomena in two limited cases: one is the sudden limit case, the other is the adiabatic case.

4 Theoretical Analysis of the Angular Frequency

4.1 *Sudden Limit Case* ($T \ll 2\pi/k_0$)

The sudden limited case is the case that the scanning period of the pulse T is much smaller than the intrinsic period of the system $2\pi/k_0$. We choose $T = 0.5$ as an example which is shown in Fig. 4(a). The angular frequency ω of Ramsey patterns vs. the parameter y is shown in it. We find that as parameter y increases, the angular frequency ω decreases. When the y tends to infinite, the angular frequency ω tends to zero.

If the transition probability is small, an explicit analytic expression can be obtained by using perturbation technique. Defining $U_1 = U_2 = U$, $V_1 = V_2 = V$ and following the procedure of Li,[19] we obtain the transition probability as follows

$$|\psi_2(t)|^2 = \frac{2\pi^4 k_0^2\{1-\cos[(\gamma+U+V)T]\}}{(\gamma+U+V)^2[4\pi^2-(\gamma+U+V)^2T^2]^2}\,, \quad (4)$$

and the angular frequency of Ramsey patterns in the form

$$w = \Big|\frac{4(U+V){k_0}^2\pi^4[1-\cos((\gamma+U+V)T)]}{(\gamma+U+V)^2[4\pi^2-((\gamma+U+V)T)^2]^2} - (\gamma+U+V)\Big|\,. \quad (5)$$

The above analytical predictions are in good agreement with our numerical results (show in Fig. 4(a)).

4.2 *Adiabatic Limit Case* ($T \gg 2\pi/k_0$)

In the adiabatic limited case, the scanning period is large enough. Figure 4(b) shows the dependence of the angular frequency ω of Ramsey patterns on the nonlinear parameter y, in which we take $T = 1700$. We observe from Fig. 4(b) that there is a small amplitude oscillation in the region of $20 < y < 80$. The adiabatic condition in this region is violated. To confirm this conclusion, the dependence of s on y after the first RZ pulse are presented in Fig. 5.

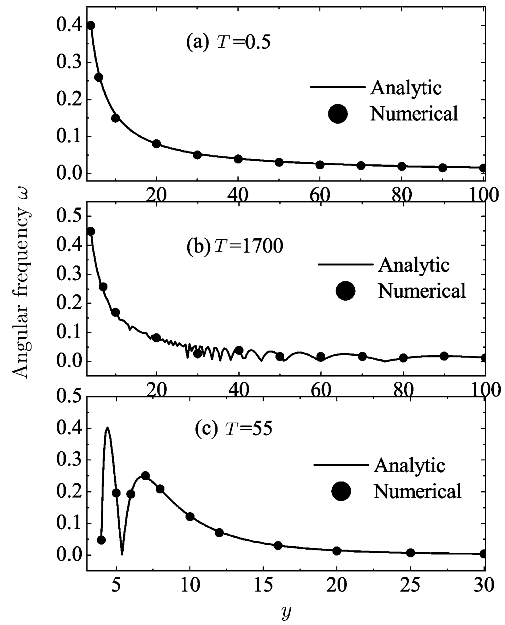

Fig. 4 Angular frequency ω of Ramsey fringes vs. the nonlinear parameter y with different T. (a) for sudden limit case; (b) for adiabatic limit case; and (c) for general situation. We can see the numerical results are good agreement with the analytical results.

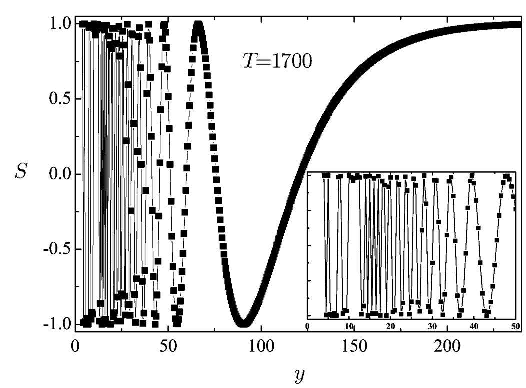

Fig. 5 The population imbalance s vs. the nonlinear parameter y with $T = 1700$.

We find that s jump between two points $+1$ and -1 as the parameter y increases from 4.0 to 20. However, irregular oscillation is observed in the region $20 < y < 80$. Furthermore, when the parameter $y > 100$ the oscillation does not appear. In this case, the frequency of oscillation is zero.

In order to explain the above phenomena, we write $\psi_{1,2} = \sqrt{N_{1,2}}\,\mathrm{e}^{\mathrm{i}\theta_{1,2}}$, and define $s = (N_2 - N_1)/N$, $\theta = \theta_2 - \theta_1$, where s denotes the population imbalance between the two wells and θ is the relative phase, we obtain the following equations

$$\frac{\mathrm{d}s}{\mathrm{d}t} = -2k\sqrt{1-s^2}\sin\theta\,, \quad (6)$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \gamma + \frac{d_0}{y}[(1+s)-(1-s)] + \frac{d_1}{y^{5/2}}[(1+s)^{3/2} - (1-s)^{3/2}] + \frac{2ks}{\sqrt{1-s^2}}\cos\theta\,, \tag{7}$$

where $d_0 = (2\pi N/k_F)\int \phi_1^3(x)\mathrm{d}x$,

$$d_1 = \frac{4\pi(\alpha_0-\gamma_0)}{k_F^{5/2}}\Big(\frac{N}{2}\Big)^{3/2}\int \phi_1^{7/2}(x)\mathrm{d}x\,.$$

The above two-mode equations can be cast into the canonical form:

$$\frac{\mathrm{d}s}{\mathrm{d}t} = -\frac{\partial H}{\partial\theta}, \quad \frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{\partial H}{\partial s}$$

with the classical Hamiltonian defined as

$$H = \gamma s + \frac{d_0}{2y}[(1+s)^2 + (1-s)^2] + \frac{2d_1}{5y^{5/2}}[(1+s)^{5/2} + (1-s)^{5/2}] - 2k\sqrt{1-s^2}\cos\theta\,. \tag{8}$$

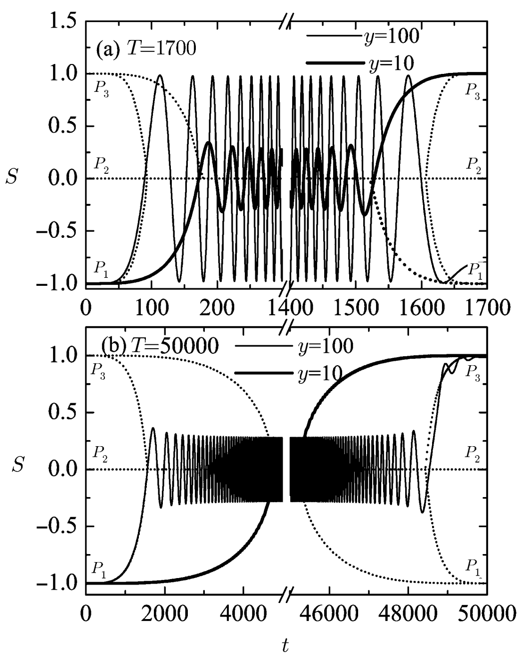

Fig. 6 The dynamical evolution contrasted with the adiabatic evolution of fixed points with different T, for (a) $T = 1700$, (b) $T = 50\,000$. Thin ($y = 100$) and thick ($y = 10$) solid line refer to the dynamical evolution respectively, thin ($y = 100$) and thick ($y = 10$) dash line refer to the adiabatic evolution respectively.

The classical Hamiltonian can describe completely the dynamic properties of system.[47] The adiabatic evolution of the quantum eigenstates can be evaluated by tracing the shift of the classical fixed points in phase space when the parameter k varies in time slowly.[48] Figure 6 shows the evolution of fixed point. Three fixed points are characterized by P_1, P_2, and P_3 respectively. In Figs. 6(a) and 6(b), we take the scanning period $T = 1700$ and $T = 50000$ respectively. It is shown a good agreement between dynamical evolution and adiabatic trajectory for the case of $y = 10$. However, for $y = 100$, the difference between the evolution of fixed point P_1 and dynamical evolution can be observed. In the case of $T = 1700$, it shows a clear deviation between two fixed points [see Fig. 6(a)], while it can follow the adiabatic evolution at $T = 50\,000$ [see Fig. 6(b)]. Therefore, we give the adiabatic condition as follows[18]

$$T \gg \mathrm{Max}\Big[\frac{2\pi}{U+V}, \frac{2\pi}{k_0}\Big]\,. \tag{9}$$

4.3 *General Situation*

For the general case, the scanning period of RZ pulse have the same order with $2\pi/k_0$, in this case, we take $T = 55$ as an example. We find that the population difference s can greatly affect the frequency of Ramsey fringes. The fundamental frequency of Ramsey patterns is shown in Fig. 4(c). The comparison between numerical results and theoretical prediction show a good agreement. There also have two zero-frequency points.

5 Discussions and Applications

In the present work, we have investigated nonlinear Ramsey interferometry of superfluid Fermi gases in a double-well potential in deep BEC regime. Three cases of the sudden limits, the adiabatic limits and the general situation have been studied respectively. We find that the frequency of the nonlinear Ramsey patterns exactly reflects the strength of nonlinearity. The adiabatic condition is given.

In the sudden limit, the approximate transition probabilities are obtained analytically and in good agreement with the numerical ones. In the adiabatic limit, the equivalent classical Hamiltonian, the fixed points are investigated and the adiabatic condition is obtained analytically. The analytical results are in good agreement with the numerical ones. In general situation, the zero-frequency point emerges.

In such a system, the wave function can be described by a superposition of two states that localize in each well separately. The double-well can be created, for example, by superimposing a blue-detuned laser beam upon the center of the magnetic trap.[49] In this case, γ denotes the difference of the zero-point energy between two wells. The scattering length a_{sc} can be adjusted flexibly by Feshbach resonance. The barrier height (k) can be effectively controlled by adjusting the intensity of the blue-detuned laser beam. Initially, we upload superfluid fermions pairs into one well, then ramp up and down the barrier slowly, the Nonlinear Ramsey fringes should be observed.

The results tell us that the scattering length a_{sc} or the dimensionless interaction parameter y plays an important role on the quantum interferometry probabilities. It suggests that quantum interferometry of the superfluid Fermi gases from one well to the other can be controlled by Feshbach resonance.

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