# Double-control quantum interferences in a four-level atomic system

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**Abstract:** A new scheme is suggested to manipulate the probe transitions (and hence the optical properties of atomic vapors) via double-control destructive and constructive quantum interferences. The influence of phase coherence between the two control transitions on the probe transition is also studied. The most remarkable feature of the present scheme is that the optical properties (absorption, transparency and dispersion) of an atomic system can be manipulated using this double-control multi-pathway interferences (multiple routes to excitation). It is also shown that a four-level system will exhibit a two-level resonant absorption because the two control levels (driven by the two control fields) form a dark state (and hence a destructive quantum interference occurs between the two control transitions). However, the present four-level system will exhibit electromagnetically induced transparency to the probe field when the three lower levels (including the probe level and the two control levels) form a three-level dark state. The present scenario has potential applications in new devices (*e.g.* logic gates and sensitive optical switches) and new techniques (*e.g.* quantum coherent information storage).



**OCIS codes:** (270.0270) Quantum optics; (120.4530) Optical constants; (270.1670) Coherent optical effects

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## References and links

1. M. Fleischhauer and M. O. Scully, "Quantum sensitivity limits of an optical magnetometer based on atomic phase coherence," Phys. Rev. A **49,** 1973–1986 (1994).
2. S. E. Harris, "Electromagnetically induced transparency," Phys. Today **50**(7), 36–42 (1997) and references therein.
3. S. Y. Zhu and M. O. Scully, "Spectral line elimination and spontaneous emission cancellation via quantum interference," Phys. Rev. Lett. **76**, 388–391 (1996).
4. J. Q. Shen, "Quantum-vacuum geometric phases in the noncoplanarly curved fiber system," Eur. Phys. J. D **30,** 259–264 (2004).
5. J. Q. Shen, "Negative refractive index in gyrotropically magnetoelectric media," Phys. Rev. B **73,** 045113(1-5) (2006).
6. J. P. Marangos, "Electromagnetically induced transparency," J. Mod. Opt. **45,** 471–503 (1998).
7. J. L. Cohen and P. R. Berman, "Amplification without inversion: Understanding probability amplitudes, quantum interference, and Feynman rules in a strongly driven system," Phys. Rev. A **55,** 3900–3917 (1997).
8. L. V. Hau, S. E. Harris, Z. Dutton, and C. H. Behroozi, "Light speed reduction to 17 metres per second in an ultracold atomic gas," Nature **397,** 594–598 (1999) and references therein.
9. R. R. Moseley, S. Shepherd, D. J. Fulton, B. D. Sinclair, and M. H. Dunn, "Spatial consequences of electromagnetically induced transparency: Observation of electromagnetically induced focusing," Phys. Rev. Lett. **74,** 670–673 (1995).

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10. L. J. Wang, A. Kuzmich, and A. Dogariu, "Gain-assisted superluminal pulse propagation," Nature **406,** 277–279 (2000) and references therein.
11. P. Arve, P. Jänes, and Lars Thylén, "Propagation of two-dimensional pulses in electromagnetically induced transparency media," Phys. Rev. A **69,** 063809(1-8) (2004).
12. J. Q. Shen and S. He, "Dimension-sensitive optical responses of electromagnetically induced transparency vapor in a waveguide," Phys. Rev. A **74,** 063831(1-6) (2006).
13. Y. Q. Li and M. Xiao, "Transient properties of an electromagnetically induced transparency in three-level atoms," Opt. Lett. **20,** 1489–1491 (1995).
14. H. Schmidt and A. Imamoğlu, "Giant Kerr nonlinearities obtained by electromagnetically induced transparency," Opt. Lett. **21,** 1936–1938 (1996).
15. R.Y. Chiao, "Superluminal (but causal) propagation of wave packets in transparent media with inverted atomic populations," Phys. Rev. A **48**, R34–R37 (1993).
16. E. L. Bolda, J. C. Garrison, and R. Y. Chiao, "Optical pulse propagation at negative group velocities due to a nearby gain line," Phys. Rev. A **49**, 2938–2947 (1994).
17. S. Sangu, K. Kobayashi, A. Shojiguchi, and M. Ohtsu, "Logic and functional operations using a near-field optically coupled quantum-dot system," Phys. Rev. B **69**, 115334(1-13) (2004).
18. T. Kawazoe, K. Kobayashi, S. Sangu, and M. Ohtsu, "Demonstration of a nanophotonic switching operation by optical near-field energy transfer," Appl. Phys. Lett. **82**, 2957–2959 (2003).
19. T. Kawazoe, K. Kobayashi, and M. Ohtsu,"A nanophotonic NOT-gate using near-field optically coupled quantum dots," 2005 Conference on Lasers & Electro-Optics (CLEO), Baltimore, MD, USA, 728–730 (2005).
20. J. Q. Yao, H. B. Wu, and H. Wang, "The transient optical properties in four-level atomic medium induced by quantum interference effect,"Acta Sin. Quantum Opt. **9,** 121–125 (2003).

## 1. Introduction

During the past four decades, design and fabrication of artificial materials have attracted considerable attention in various scientific and technological areas [1, 2, 3, 4, 5]. The most remarkable feature of all these artificial materials is that the wave propagation (including the quantum optical properties [4]) could be manipulated by the materials. Recently, many theoretical and experimental investigations have shown that the control of phase coherence in multilevel atomic ensembles will give rise to many striking quantum optical phenomena in the wave propagation of near-resonant light [2, 3]. One of the most interesting quantum optical phenomena is electromagnetically induced transparency (EIT), in which one resonant laser beam propagating through the medium will get absorbed; but when two resonant laser beams propagate simultaneously through the same medium, neither will be absorbed due to the quantum interference between them, and thus the opaque medium is turned into a transparent one [2]. From the point of view of the physical mechanisms involved in the phenomenon, EIT results from destructive quantum interference and coherence effects in an atomic transition process from the ground state to the excited ones [2]. Besides the CPT (coherent population trapping) explanation (in which the concept of dark state is involved), EIT can also be interpreted in terms of the quantum interference between dressed states [6] and the quantum field theoretical explanation (using Feynman diagrams to represent the interfering process in EIT [7]). Apart from nearly zero absorption at resonance, the quantum coherence effect in EIT vapors will give rise to strong dispersion near resonance [8]. Since the optical properties of EIT vapors depend on the external control field intensities, EIT can also be used to realize the beam focusing (EIT lensing) [9]. As it can exhibit many intriguing effects, EIT could be applied to various areas of optics and enable us to achieve some novel results [2]. More recently, some unusual physical effects associated with EIT have been observed experimentally, including the ultraslow light pulse propagation [8], the superluminal light propagation [10], the light storage in an atomic vapor and semiconductor quantum-dot material [8, 11], the atomic ground state cooling and the sensitive EIT waveguide [12]. Some of them are believed to be useful for the development of new techniques in quantum optics, photonics and quantum electronics [2, 12, 13, 14].

In this paper, we suggest a new scheme for manipulating the light propagation by means of a double-control four-level system, where the four-level atomic system is coupled to the two

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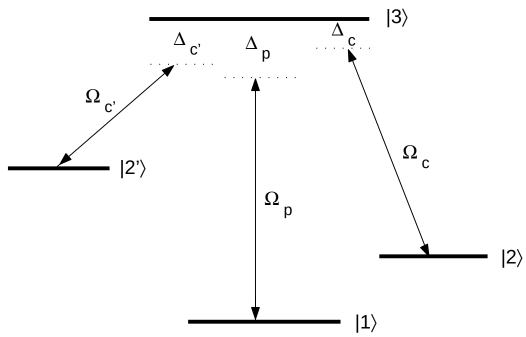


Fig. 1. The schematic diagram of a double-control four-level system. The two control laser beams, $\Omega_c$ and $\Omega_{c'}$, drive the $|2\rangle$-$|3\rangle$ and $|2'\rangle$-$|3\rangle$ transitions, respectively. The probe transition $|1\rangle$-$|3\rangle$ can be controllably manipulated via the destructive and constructive quantum interferences between the $|2\rangle$-$|3\rangle$ and $|2'\rangle$-$|3\rangle$ transitions. If levels $|1\rangle$, $|2\rangle$ and $|2'\rangle$ form a three-level dark state, then the atomic vapor is transparent to the probe field, whereas it is opaque to the probe field when levels $|2\rangle$ and $|2'\rangle$ form a two-level dark state.

control beams and one probe beam (see Fig. 1). Later we shall show how the three lower levels form a three-level dark state that can be viewed as a generalization of the two-level dark state (consisting of the probe and control levels) that appears in a conventional three-level EIT effect. As the destructive quantum interference occurs among the three optical fields, the population cannot be excited from the three-level dark state to the upper level ($|3\rangle$). This will lead the four-level system to exhibit an EIT effect. But under some conditions (including the control fields with proper intensities and frequency detunings), a destructive quantum interference would arise between the two control fields, *i.e.*, only the two control levels ($|2\rangle$ and $|2'\rangle$ interacting with the two control fields, respectively) form the dark state. This implies that the total contribution of transitions driven by the two control fields from the two control levels ($|2\rangle$ and $|2'\rangle$) to the upper level ($|3\rangle$) vanishes. Thus the four-level system is equivalent to a two-level system that can exhibit a two-level resonant absorption to the probe field. We study both the constructive and destructive quantum interferences between the two control transitions driven by the control fields, and show that such quantum interferences lead to the transparency and the absorption, respectively, to the probe field. In a conventional three-level EIT system, we have to change the (absolute) intensity of the control field in order to control the optical behaviors of the atomic vapor. However, the optical response of the present four-level atomic vapor can be tunable just by adjusting the relative intensities (the ratio of the intensities) of the two control fields. This means that the double-control scheme would be more convenient and efficient for manipulating the optical properties of the atomic vapors than the conventional three-level EIT scheme did.

## 2. Double-control four-level system and generalized dark state

Consider a four-level atomic ensemble with three lower levels $|1\rangle$, $|2\rangle$, $|2'\rangle$ and one upper level $|3\rangle$ (see Fig. 1). Such an atomic system interacts with three optical fields, *i.e.*, the two control laser beams and one probe laser beam, which couple the level pairs $|2\rangle$-$|3\rangle$, $|2'\rangle$-$|3\rangle$ and $|1\rangle$-$|3\rangle$, respectively. The three frequency detunings $\Delta_c$, $\Delta_{c'}$ and $\Delta_p$ are defined as follows: $\Delta_c = \omega_{32} - \omega_c$, $\Delta_{c'} = \omega_{32'} - \omega_{c'}$, and $\Delta_p = \omega_{31} - \omega_p$, where $\omega_{32}$, $\omega_{32'}$ and $\omega_{31}$ denote the atomic transition frequencies, and $\omega_c$, $\omega_{c'}$, $\omega_p$ represent the mode frequencies of the control and probe beams, respectively. For the present atomic system, the equation of motion of the probability amplitudes in accordance with the Schrödinger equation is

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$$\dot{a}_1 = \frac{i}{2}\Omega_{\rm p}^* a_3,$$
$$\dot{a}_2 = -\left[\frac{\gamma_2}{2} + i\left(\Delta_{\rm p} - \Delta_{\rm c}\right)\right] a_2 + \frac{i}{2}\Omega_{\rm c}^* a_3,$$
$$\dot{a}_{2'} = -\left[\frac{\gamma_2'}{2} + i\left(\Delta_{\rm p} - \Delta_{\rm c'}\right)\right] a_{2'} + \frac{i}{2}\Omega_{\rm c'}^* a_3,$$
$$\dot{a}_3 = -\left(\frac{\Gamma_3}{2} + i\Delta_{\rm p}\right) a_3 + \frac{i}{2}\left(\Omega_{\rm p} a_1 + \Omega_{\rm c} a_2 + \Omega_{\rm c'} a_{2'}\right), \tag{1}$$

where the Rabi frequencies of the probe beam and the two control beams are defined through $\Omega_{\rm p} = \wp_{31}\mathscr{E}_{\rm p}/\hbar$, $\Omega_{\rm c} = \wp_{32}\mathscr{E}_{\rm c}/\hbar$, and $\Omega_{\rm c'} = \wp_{32'}\mathscr{E}_{\rm c'}/\hbar$, respectively. Here $\mathscr{E}_{\rm p}$, $\mathscr{E}_{\rm c}$, and $\mathscr{E}_{\rm c'}$ stand for the probe and control field envelopes (slowly-varying amplitudes). The decay rates $\gamma_2$, $\gamma_2'$ and $\Gamma_3$ are defined by $\gamma_2 = \gamma_{23} + \gamma_{2'2} - \gamma_{2'3}$, $\gamma_2' = \gamma_{2'3} + \gamma_{2'2} - \gamma_{23}$ and $\Gamma_3 = \gamma_{23} + \gamma_{2'3} - \gamma_{2'2}$, where $\gamma_{ij}$'s denote the decay rates (including the contribution of the collisional dephasing and the spontaneous emission decay) of the density matrix elements $\rho_{ij}$.

In what follows, we will study the dark state in the present four-level system. Eq. (1) can be rewritten in the matrix form

$$\frac{\partial}{\partial t}\begin{pmatrix} a_1(t) \\ a_2(t) \\ a_{2'}(t) \\ a_3(t) \end{pmatrix} = \mathscr{A}\begin{pmatrix} a_1(t) \\ a_2(t) \\ a_{2'}(t) \\ a_3(t) \end{pmatrix}, \tag{2}$$

where the coefficient matrix $\mathscr{A}$ is defined by

$$\mathscr{A} = \begin{pmatrix} 0 & 0 & 0 & \frac{i}{2}\Omega_{\rm p}^* \\ 0 & 0 & 0 & \frac{i}{2}\Omega_{\rm c}^* \\ 0 & 0 & 0 & \frac{i}{2}\Omega_{\rm c'}^* \\ \frac{i}{2}\Omega_{\rm p} & \frac{i}{2}\Omega_{\rm c} & \frac{i}{2}\Omega_{\rm c'} & 0 \end{pmatrix}. \tag{3}$$

In order to see the essential physical meanings of the dark state, we ignore the frequency detunings and the decay rates in the equation of motion of probability amplitudes. Obviously, the solution to Eq. (2) can be of the form

$$\begin{pmatrix} a_1(t) \\ a_2(t) \\ a_{2'}(t) \\ a_3(t) \end{pmatrix} = \begin{pmatrix} a_1(0) \\ a_2(0) \\ a_{2'}(0) \\ a_3(0) \end{pmatrix} e^{\lambda t}. \tag{4}$$

Substitution of solution (4) into Eq. (2) yields

$$(\mathscr{A} - \lambda\mathscr{I})\begin{pmatrix} a_1(0) \\ a_2(0) \\ a_{2'}(0) \\ a_3(0) \end{pmatrix} \equiv \begin{pmatrix} -\lambda & 0 & 0 & \frac{i}{2}\Omega_{\rm p}^* \\ 0 & -\lambda & 0 & \frac{i}{2}\Omega_{\rm c}^* \\ 0 & 0 & -\lambda & \frac{i}{2}\Omega_{\rm c'}^* \\ \frac{i}{2}\Omega_{\rm p} & \frac{i}{2}\Omega_{\rm c} & \frac{i}{2}\Omega_{\rm c'} & -\lambda \end{pmatrix}\begin{pmatrix} a_1(0) \\ a_2(0) \\ a_{2'}(0) \\ a_3(0) \end{pmatrix} = 0, \tag{5}$$

where $\mathscr{I}$ denotes the identity matrix. This means that the determinant $\det(\mathscr{A} - \lambda\mathscr{I}) = 0$. Thus one can obtain a quartic equation

$$\lambda^2\left[\lambda^2 + \frac{\Omega_{\rm c}^*\Omega_{\rm c} + \Omega_{\rm c'}^*\Omega_{\rm c'} + \Omega_{\rm p}^*\Omega_{\rm p}}{4}\right] = 0. \tag{6}$$

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The four eigenvalues of Eq. (6) are given by

$$\lambda_1 = 0, \quad \lambda_2 = 0, \quad \lambda_\pm = \pm i \frac{\sqrt{\Omega_c^* \Omega_c + \Omega_{c'}^* \Omega_{c'} + \Omega_p^* \Omega_p}}{2}. \tag{7}$$

The dark state corresponds to the zero roots. According to Eqs. (2) and (4), one can arrive at

$$\Omega_p a_1 + \Omega_c a_2 + \Omega_{c'} a_{2'} = 0. \tag{8}$$

This means that the three lower levels ($|1\rangle$, $|2\rangle$ and $|2'\rangle$) form a three-level dark state. The probability amplitudes $a_1, a_2, a_{2'}$ of the atomic levels in the present three-level dark state are restricted by this relation. In the meanwhile, the probability amplitude of level $|3\rangle$ is zero (*i.e.* $a_3 = 0$) according to Eqs. (2) and (4). Here we can define a concept called "driving contribution" that is the product of the Rabi frequency (coupling coefficient) and the probability amplitude of a lower level. For instance, the driving contribution of the probe field is $\Omega_p a_1$. It follows from Eq. (8) that the total driving contribution of the probe and control fields is zero for the dark state (this can be viewed as a quantum destructive interference among the three optical fields). It seems that there is no net interaction between the three lower levels and the three optical fields, and that no population would be excited from the lower levels to the upper level. This leads to the EIT phenomenon.

## 3. Dispersion of atomic electric susceptibility

In the preceding section, we studied a three-level dark state in the double-control four-level system. In this section, we consider the dispersion of optical 'constants' of the four-level atomic vapor. We assume that the intensity of the probe beam is sufficiently weak and therefore nearly all the atoms remain in the ground state, *i.e.*, the atomic population at level $|1\rangle$ is unity. Under this assumption, Eq. (1) can be reduced to the following form

$$\begin{aligned}
\dot{a}_2 &= -\left[\frac{\gamma_2}{2} + i\left(\Delta_p - \Delta_c\right)\right] a_2 + \frac{i}{2}\Omega_c^* a_3, \\
\dot{a}_{2'} &= -\left[\frac{\gamma_2'}{2} + i\left(\Delta_p - \Delta_{c'}\right)\right] a_{2'} + \frac{i}{2}\Omega_{c'}^* a_3, \\
\dot{a}_3 &= -\left(\frac{\Gamma_3}{2} + i\Delta_p\right) a_3 + \frac{i}{2}\left(\Omega_c a_2 + \Omega_{c'} a_{2'}\right) + \frac{i}{2}\Omega_p.
\end{aligned} \tag{9}$$

The steady solution to Eq. (9) is given by

$$\begin{aligned}
a_2 &= -\frac{1}{4\mathscr{D}}\Omega_p \Omega_c^* \left[\frac{\gamma_2'}{2} + i\left(\Delta_p - \Delta_{c'}\right)\right], \\
a_{2'} &= -\frac{1}{4\mathscr{D}}\Omega_p \Omega_{c'}^* \left[\frac{\gamma_2}{2} + i\left(\Delta_p - \Delta_c\right)\right], \\
a_3 &= \frac{i}{2\mathscr{D}}\Omega_p \left[\frac{\gamma_2}{2} + i\left(\Delta_p - \Delta_c\right)\right]\left[\frac{\gamma_2'}{2} + i\left(\Delta_p - \Delta_{c'}\right)\right],
\end{aligned} \tag{10}$$

where the parameter

$$\begin{aligned}
\mathscr{D} \quad = \quad & \left(\frac{\Gamma_3}{2} + i\Delta_p\right)\left[\frac{\gamma_2}{2} + i\left(\Delta_p - \Delta_c\right)\right]\left[\frac{\gamma_2'}{2} + i\left(\Delta_p - \Delta_{c'}\right)\right] \\
& + \frac{1}{4}\Omega_{c'}^* \Omega_{c'} \left[\frac{\gamma_2}{2} + i\left(\Delta_p - \Delta_c\right)\right] + \frac{1}{4}\Omega_c^* \Omega_c \left[\frac{\gamma_2'}{2} + i\left(\Delta_p - \Delta_{c'}\right)\right].
\end{aligned} \tag{11}$$

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Note that the atomic electric polarizability of the probe transition $|1\rangle$-$|3\rangle$ is $\beta(\Delta_p) = 2\wp_{13}\rho_{31}/(\varepsilon_0\mathscr{E}_p)$ with the density matrix element $\rho_{31} = a_1^* a_3 \simeq a_3$. Substituting the above results into $\beta(\Delta_p)$, one can obtain the explicit expression for the electric polarizability

$$\beta(\Delta_p) = \frac{|\wp_{13}|^2}{\varepsilon_0 \hbar}\frac{i}{\mathscr{D}}\left[\frac{\gamma_2}{2} + i\left(\Delta_p - \Delta_c\right)\right]\left[\frac{\gamma_2'}{2} + i\left(\Delta_p - \Delta_{c'}\right)\right]. \tag{12}$$

The relative electric susceptibility is $\chi(\Delta_p) = N\beta(\Delta_p)$, where $N$ denotes the atomic concentration of the EIT vapor. The dispersive behavior of the real and imaginary parts of the electric susceptibility is plotted in Fig. 2, where the typical parameters of the atomic system are chosen as: $\Gamma_3 = 1.0\times10^8$ s$^{-1}$, $\gamma_2 = 1.0\times10^5$ s$^{-1}$, $\gamma_2' = 2.0\times10^5$ s$^{-1}$, $\wp_{13} = 1.0\times10^{-29}$ C·m, $\Omega_c = 1.0\times10^8$ s$^{-1}$, $\Omega_{c'} = 2.0\times10^8$ s$^{-1}$, $\Delta_c = 3.0\times10^7$ s$^{-1}$, $\Delta_{c'} = 8.0\times10^7$ s$^{-1}$, and $N = 5.0\times10^{20}$ m$^{-3}$. The absorption coefficient $\alpha$ (defined as $2\pi\mathrm{Im}\{n_r\}/\mathrm{Re}\{n_r\}$, *i.e.*, the loss in the medium per wavelength) is shown in Fig. 3 as a function of the frequency detuning of the probe beam. Note that in a conventional three-level EIT system, there is only one resonant frequency for the atomic system to exhibit zero absorption (see Fig. 3, where the absorption coefficient of the conventional three-level EIT vapor is plotted under the condition that the control field $\Omega_{c'}$ is absent). For the double-control system, however, there are two resonant frequencies, where the four-level vapor is transparent to the probe beam (zero absorption), *i.e.*, $\Delta_p \to \Delta_c$ or $\Delta_p \to \Delta_{c'}$. This can be called "double-control electromagnetically induced transparency".

It should be noted that the coherent population trapping in the double-control scheme occurs twice (*i.e.*, when $\Delta_p = \Delta_c$ and $\Delta_p = \Delta_{c'}$). In other words, the two resonant frequencies corresponding to the probe zero absorption are in fact caused by the usual two-level dark states formed by the levels $|1\rangle, |2\rangle$ and $|1\rangle, |2'\rangle$, respectively. Thus the so-called three-level dark state composed of all lower levels ($|1\rangle, |2\rangle, |2'\rangle$) satisfying relation (8) derived using $\Delta_p = \Delta_c = \Delta_{c'}$ is actually a state of a special case (*i.e.* completely resonant). However, in general cases (*e.g.* $\Delta_p = \Delta_c$ but $\Delta_p \neq \Delta_{c'}$, or $\Delta_p = \Delta_{c'}$ but $\Delta_p \neq \Delta_c$), such a three-level dark state would be reduced to the two-level dark state composed of the levels $|1\rangle, |2\rangle$ or $|1\rangle, |2'\rangle$.

## 4. Destructive and constructive quantum interferences

We have shown that the present four-level atomic vapor can exhibit a double-control EIT effect. But under certain conditions, the four-level vapor becomes opaque to the probe field. This question is closely related to the destructive and constructive interferences between the two transitions ($|2\rangle$-$|3\rangle$ and $|2'\rangle$-$|3\rangle$) driven by the two control fields. In the following discussions, we analyze the quantum interferences between the two control transitions. It follows from (10) that the ratio of the probability amplitudes of the two control levels ($|2\rangle$ and $|2'\rangle$) is

$$\frac{a_2}{a_{2'}} = \frac{\Omega_c^*}{\Omega_{c'}^*}\frac{\frac{\gamma_2'}{2} + i\left(\Delta_p - \Delta_{c'}\right)}{\frac{\gamma_2}{2} + i\left(\Delta_p - \Delta_c\right)}. \tag{13}$$

In the meanwhile, it can be readily verified from (12) that the atomic electric polarizability can exhibit a two-level resonant absorption, *i.e.*,

$$\beta(\Delta_p) \to i\frac{|\wp_{13}|^2}{\varepsilon_0 \hbar}\frac{1}{\frac{\Gamma_3}{2} + i\Delta_p}, \tag{14}$$

when the intensities of the two control fields agree with the following relation

$$\frac{\Omega_{c'}^*\Omega_{c'}}{\Omega_c^*\Omega_c} \simeq -\frac{\frac{\gamma_2'}{2} + i\left(\Delta_p - \Delta_{c'}\right)}{\frac{\gamma_2}{2} + i\left(\Delta_p - \Delta_c\right)}. \tag{15}$$

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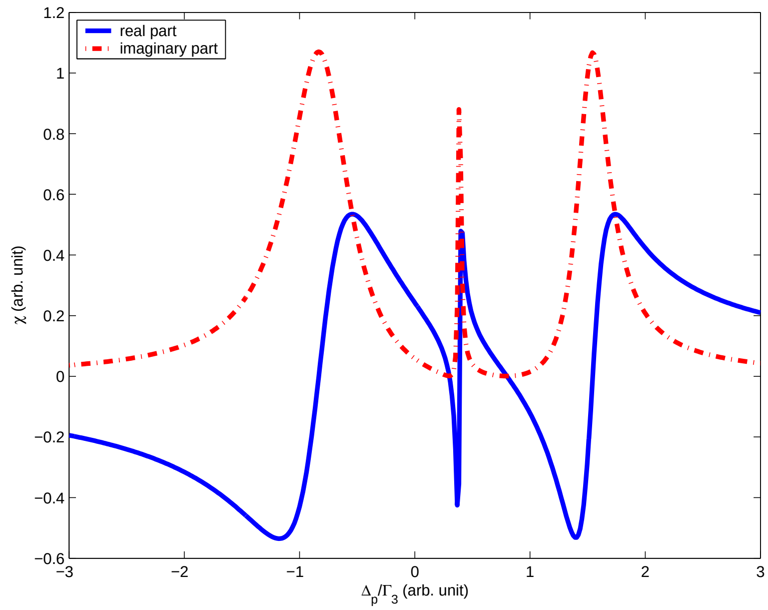


Fig. 2. The dispersive behavior of the real and imaginary parts of the electric susceptibility as the probe frequency detuning varies. Both the real and imaginary parts of $\chi(\Delta_p)$ tend to zero at probe frequency detunings $\Delta_p = 3.0 \times 10^7\ s^{-1}$ and $\Delta_p = 8.0 \times 10^7\ s^{-1}$.

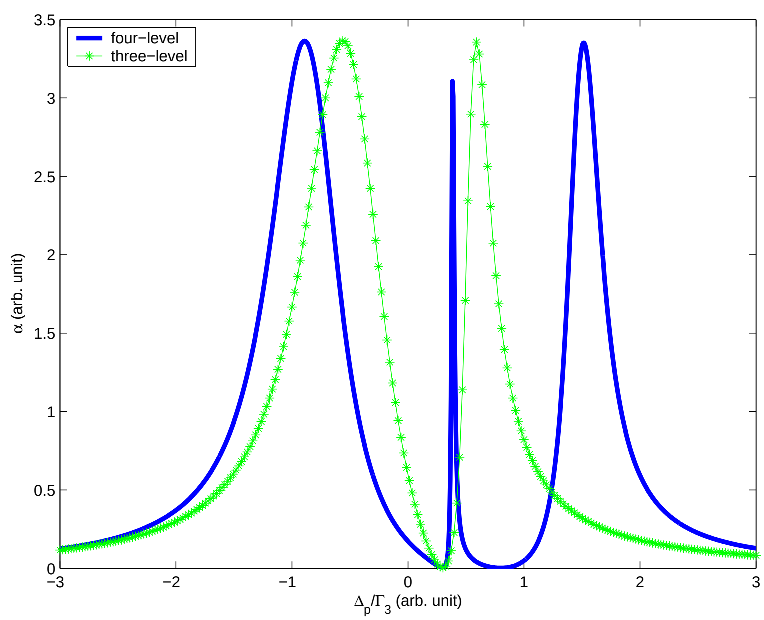


Fig. 3. The dispersive behavior of the absorption coefficients of both the three- and the four-level atomic vapors as the probe frequency detuning varies. There are two resonant frequencies ($\Delta_p = 3.0 \times 10^7\ s^{-1}$ and $\Delta_p = 8.0 \times 10^7\ s^{-1}$), where the four-level system exhibits zero absorption.

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According to expressions (13) and (15), one can obtain

$$\frac{a_2}{a_{2'}} \simeq -\frac{\Omega_{c'}}{\Omega_c} \qquad \text{or} \qquad \Omega_c a_2 + \Omega_{c'} a_{2'} \simeq 0. \tag{16}$$

This means that the two control levels ($|2\rangle$ and $|2'\rangle$) form a two-level dark state. As the destructive quantum interference occurs between the two control fields, no population is excited from levels $|2\rangle$, $|2'\rangle$ to level $|3\rangle$ though the two control fields are present. Different from the previous three-level dark state, where the three-level destructive interference takes place (which makes the atomic vapor transparent to the probe field), the two-level dark state wipes off the total contributions of the two control fields and the atomic vapor becomes an opaque medium. In other words, here the four-level system is equivalent to a two-level system (composed of $|1\rangle$ and $|3\rangle$) that can exhibit large resonant absorption.

In order to consider the general quantum interferences between the two control transitions ($|2\rangle$-$|3\rangle$ and $|2'\rangle$-$|3\rangle$ transitions) excited by the control fields, we extend (16) to a generalized form

$$\Omega_c a_2 - \mathscr{C} e^{i\theta} \Omega_{c'} a_{2'} = 0, \tag{17}$$

where $\mathscr{C}$ is a positive number and $\theta$ a parameter that characterizes the double-control atomic phase coherence. The effects of the double-control destructive and constructive quantum interferences can be investigated by analyzing these parameters, particularly the phase parameter $\theta$. Clearly, Eq. (17) will be reduced to (16) if we take $\mathscr{C} = 1$, $\theta = \pi$ (destructive interference). For the general cases, the value of $\mathscr{C} e^{i\theta}$ can be obtained by relation (13), *i.e.*

$$\mathscr{C} e^{i\theta} = \frac{\Omega_c^* \Omega_c}{\Omega_{c'}^* \Omega_{c'}} \cdot \frac{\frac{\gamma_2'}{2} + i\left(\Delta_p - \Delta_{c'}\right)}{\frac{\gamma_2}{2} + i\left(\Delta_p - \Delta_c\right)}. \tag{18}$$

Thus, the atomic electric polarizability can be rewritten as

$$\beta(\Delta_p) = i\frac{|\wp_{13}|^2}{\varepsilon_0 \hbar} \frac{\frac{\gamma_2}{2} + i\left(\Delta_p - \Delta_c\right)}{\left(\frac{\Gamma_3}{2} + i\Delta_p\right)\left[\frac{\gamma_2}{2} + i\left(\Delta_p - \Delta_c\right)\right] + \frac{1}{4}\Omega_c^*\Omega_c\left(1 + \frac{1}{\mathscr{C}}e^{-i\theta}\right)}. \tag{19}$$

Alternatively, it can be rewritten as

$$\beta(\Delta_p) = i\frac{|\wp_{13}|^2}{\varepsilon_0 \hbar} \frac{\frac{\gamma_2'}{2} + i\left(\Delta_p - \Delta_{c'}\right)}{\left(\frac{\Gamma_3}{2} + i\Delta_p\right)\left[\frac{\gamma_2'}{2} + i\left(\Delta_p - \Delta_{c'}\right)\right] + \frac{1}{4}\Omega_{c'}^*\Omega_{c'}\left(1 + \mathscr{C}e^{i\theta}\right)}. \tag{20}$$

This can also be obtained by using the permutation procedure: $\gamma_2 \to \gamma_2'$, $\Delta_c \to \Delta_{c'}$, $\mathscr{C} \to 1/\mathscr{C}$, $\theta \to -\theta$, $\Omega_c^*\Omega_c \to \Omega_{c'}^*\Omega_{c'}$. Obviously, the electric susceptibility at the probe frequency depends on the atomic phase-coherence parameter $\theta$. In general, the dephasing rates $\gamma_2$, $\gamma_2'$ are negligibly small (*e.g.*, only one part in 1000 of the spontaneous decay rates in the vapor). Thus, Eq. (18) can be simplified to the form

$$\mathscr{C} e^{i\theta} = \frac{\Omega_c^* \Omega_c \left(\Delta_p - \Delta_{c'}\right)}{\Omega_{c'}^* \Omega_{c'} \left(\Delta_p - \Delta_c\right)}. \tag{21}$$

This, therefore, implies that $\mathscr{C} e^{i\theta}$ is a real number, namely, the phase parameter can only be chosen $\theta = 0$ or $\pi$. In order to see how the double-control destructive and constructive quantum interferences influence the optical properties of the atomic vapor, we here consider a simple case with the module $\mathscr{C} = 1$. If the phase parameter $\theta = 0$, then from expressions (19) and (20) for the electric polarizability, the present atomic vapor is transparent to the probe field. However,

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the atomic vapor would be opaque if the phase parameter $\theta = \pi$. Then expressions (19) and (20) will be reduced to (14) that characterizes the two-level resonant absorption. Thus, the four-level atomic vapor would exhibit the transparency effect once the three levels $|1\rangle$, $|2\rangle$, and $|2'\rangle$ form a *three-level* dark state, and it would exhibit the resonant absorption once the two levels $|2\rangle$ and $|2'\rangle$ form a *two-level* dark state. On the other hand, the present $\beta(\Delta_p)$ will be reduced to the atomic electric polarizability of a typical three-level Lambda-configuration EIT system, *i.e.*,

$$\beta(\Delta_p) = i\frac{|\wp_{13}|^2}{\varepsilon_0\hbar}\frac{\frac{\gamma_2}{2} + i\left(\Delta_p - \Delta_c\right)}{\left(\frac{\Gamma_3}{2} + i\Delta_p\right)\left[\frac{\gamma_2}{2} + i\left(\Delta_p - \Delta_c\right)\right] + \frac{1}{4}\Omega_c^*\Omega_c}, \tag{22}$$

when $\mathscr{C} \gg 1$, *e.g.*, the intensity of the control field $\Omega_c$ is much larger than that of the control field $\Omega_{c'}$ (*i.e.* $\Omega_c^*\Omega_c \gg \Omega_{c'}^*\Omega_{c'}$), or the control field $\Omega_c$ drives the atomic system at resonance ($\Delta_p - \Delta_c \to 0$, $\Delta_p - \Delta_{c'} \neq 0$).

As the phase coherence and quantum interference in three-level systems have been demonstrated experimentally in both the atomic vapors [8, 10, 15, 16] and the quantum-dot materials [17, 18, 19], the present double-control scheme could in principle be realized in experiments in the near future. Here we suggest some ideas to connect our scheme to the experimental work:

1) *negative group velocity*: It follows from the dispersive behavior of the absorption coefficient in Fig. 3 that the double-control four-level EIT system experiences a dramatic absorption enhancement (a very sharp increase of $\alpha$ between the two resonant frequencies) as compared with a usual three-level EIT system. This means that the property (particularly the real part of the susceptibility between the two EIT transparency windows) of the double-control four-level atomic vapor is more sensitive to the probe frequency. This may lead to a dramatic change of the speed of the light in the four-level medium. In the literature, the ultraslow light and the superluminal propagation (negative group velocity) in the three-level EIT media have received attention from many researchers [8, 10, 15, 16]. As the dispersion in both the real and imaginary parts of optical ‘constants’ is stronger than that in a three-level EIT vapor, the ultraslow and superluminal propagations of light also deserve consideration in the case of double-control four-level vapor.

2) *logic gates*: Recently, ideas of realizing logic gates by using new optoelectronic materials have captured attention of some researchers [17, 18, 19]. We believe that the double-control interference effects can also be used to realize some logic and functional operations (*e.g.* the operations of NOT and AND gates). Here we give an example to show how a NOT gate works based on the double-control interference effects: choose the proper Rabi frequencies of the two control fields that satisfy the relation (*i.e.* $\Omega_c a_2 + \Omega_{c'} a_{2'} = 0$) for the destructive quantum interference between the two control levels ($|2\rangle$ and $|2'\rangle$). Once the control field $\Omega_{c'}$ is switched off (logic operation IN$=0$), the present scheme will exhibit a three-level EIT effect (logic operation OUT$=1$). But when the control field $\Omega_{c'}$ is switched on (logic operation IN$=1$), the present double-control scheme will exhibit a two-level resonant absorption to the probe field (logic operation OUT$=0$). This is the working mechanism of the double-control NOT gate.

3) *optical switches*: The double-control destructive and constructive interference effects can be applicable to some quantum optical and photonic devices. For example, such a coherent effect can be utilized for designing optical switches. This switches might be a useful technique for future photonic microcircuits on silicon, in which light replaces electrons. At present, the all-optical switch on silicon where one controls light with light on chip has been increasingly developed. We hope the present optical switches based on double-control interference would have potential applications in this field and other related areas, *e.g.* integrated optical circuits.

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## 5. Concluding remarks

As the total driving contribution of all the laser beams (the control and probe fields) to the atomic population excited from the three-level dark state to the upper level is zero due to the destructive quantum interference among the three optical fields, the present atomic vapor is transparent to the probe field. But when we choose certain intensities of the control fields, the three-level dark state will be reduced to a two-level dark state that is a linear combination of the two control levels ($|2\rangle$ and $|2'\rangle$). Thus, the destructive quantum interference takes place between the two transitions ($|2\rangle$-$|3\rangle$ and $|2'\rangle$-$|3\rangle$) driven by the two control fields, and the four-level system would be equivalent to a two-level system that can exhibit a large resonant absorption for the probe field. All these processes can be called "double-control multi-pathway interferences" (multiple routes to excitation). Therefore, the optical properties (transparent or opaque) of the present double-control four-level system can be controlled by adjusting the coherence parameter (phase $\theta$) that characterizes the quantum interference between the two control fields. As the phase $\theta$ depends on the control frequency detunings and the control Rabi frequencies, we can manipulate the present four-level atomic vapor through changing the intensities of the control fields or by using the Zeeman effect (level shifted by an external magnetic field).

There are two resonant frequencies ($\Delta_{\rm p} = \Delta_{\rm c}$, $\Delta_{\rm p} = \Delta_{\rm c'}$), where the four-level system can exhibit the EIT effect. This is a new feature that is different from the conventional three-level EIT system, where there is only one resonant frequency. As the double-control four-level EIT vapor exhibits a large dispersion in both the real and imaginary parts of optical 'constants', the optical properties would be more sensitive to the probe frequency as compared with a three-level EIT vapor. The present double-control quantum interference scheme can hence be applicable to many new techniques such as sensitive optical switches, optical magnetometers and wavelength sensors. For example, the optical magnetometers could be used to detect magnetic fields with very high sensitivity because of the strong dispersion caused by the double-control atomic phase coherence, and the double-control EIT-based wavelength sensor can be utilized to measure the probe wavelength. Such a device can be applied to some practical areas like color matching and sorting, where precise measurements of light wavelengths and frequencies are needed.

In the present paper, in order to demonstrate the novel effects of the destructive and constructive quantum interferences, we considered the steady optical properties of a double-control four-level system. But in fact, the system will experience a transient evolution once the control fields are switched on or off [11, 13]. The transient evolution is a very important physical process when one considers the mechanism of storage and readout of pulses in the future technology of *quantum coherent information storage*. In order to see how fast the optical behaviors respond to the switching on of the control fields, in the literature, Yao *et al.* first studied the transient optical properties of the four-level N-configuration system under certain approximation conditions [20]. As there are two resonant frequencies and large dispersion in the double-control EIT, the scheme can also be applied to the technique of coherent information storage. Thus, the transient evolutional behavior also deserves consideration for the present double-control four-level system.

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