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A novel evolving model for power grids

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In this paper, a novel power grid evolving model, which can well describe the evolving property of power grids, is presented. Based on the BA model, motivated by the fact that in real power grids, connectivity of node not only depends on its degree, but also is influenced by many uncertain factors, so we introduce the subconnection factor K for each node. Using the mean-field theory, we get the analytical expression of power-law degree distribution with the exponent $\gamma \in (3,\infty)$. Finally, simulation results show that the new model can provide a satisfactory description for empirical characteristics of power network, and power network falls somewhere in between scale-free network and uncertain network.

power grid, evolving model, scale-free

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1 Introduction

In the past few years, a number of large power blackouts over the last decade have led to an increasing interest in the study of bulk power grid [1–5]. The topology of power grid can be simply modeled as a graph with m nodes and n edges. Nodes represent buses (power station or transformer) of power system, and edges represent transmission lines. The topography of power grid is critical to the vulnerability due to cascading failure [6–10], so it is very significant to do further research on modeling the evolving power grid. In the past few years, much work has been done on the modeling of complex networks [11–17]. One of the most important work was Barabási-Albert (BA) scale-free network model [18]. The BA model includes two parts: the growth of network and the preferential attachment mechanisms. It is founded that the degree of the vertices of many complex networks has no obvious characteristic scale, the degree distribution $P(k) \propto k^{-r}, r = 3,$ where γ is the degree exponent. As a type of complex networks many power grids show the characteristic of scale-free [18]. In power grids, there are always a few central buses with high economic or geographical significance, in the planning of power system, these buses are supposed to hold more links than other buses, the characteristic of evolution of power grid contributes to few nodes with high degree, which is called the scale-free power network. The scale-free characteristic of network makes power grid robust to random attack yet fragile to intentional attack.

Although the BA model captures the scale-free characteristic of power grids, there exist some differences between the model and power grids. In the planning of power system many other factors are considered, such as random factor, the distance between two stations, optimal power flow, network loss and so on, these factors are different from each other, so there exist some uncertain links in the evolution of power grid. Power network falls somewhere in between scale-free network and uncertain network, in order to give more satisfactory description of network characteristic of power system, uncertainty of evolution of power network must be considered. Based on the above analysis, we intro-

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duce the subconnectivity factor K, which includes all the uncertain factors of the evolution of power grid.

In this paper, we propose a new evolving power grid model. The rest of this paper is organized as follows. In Section.2, our novel model is introduced, followed by analytical calculation of degree distribution and analysis of network features in Sections 3 and 4. In Section.5, we compare the simulation result with the US West South Power Grid. Finally, some conclusions are given in Section 6.

2 Our model for power grid

Based on the BA model, we introduce the subconnectivity factor K, which includes all the uncertain factors of the evolution of power network. To simplify the analysis, we assume that the subconnection factor of all nodes is K. The model is generated by the following algorithm:

(1) Initial condition: The network consists of m_0 nodes and n_0 edges.

(2) Growth: At each time step, add a new node with m ($m< m_0$) edges.

(3) Preferential attachment: Each edge of the new node is attached to the existing node i. The probability Π that a new node will be connected to node i depends on the linear combination of the degree k_i of node i and the subconnection factor K, that is:

$$\Pi_i = \frac{\alpha k_i + (1-\alpha)K}{\sum_j \alpha k_j + (1-\alpha)K}, \tag{1}$$

where $0<\alpha<1$. After t time steps, the system develops to be a network with $N=m_0+t$ nodes and $L=mt$ edges.

3 Analytical expression of degree distribution

We use the mean-field theory [19] to analyze the degree distribution of the network. According to continuum theory, we assume that the degree k_i is continuous, thus the degree change rate of i in the new evolving model is

$$\frac{\partial k_i}{\partial t} = m\frac{\alpha k_i + (1-\alpha)K}{\sum_j \alpha k_j + (1-\alpha)K}. \tag{2}$$

After t steps involution, the total node number of the local community $N_G=m_0+t$ and mt edges.

So the summation of degree in the local community is

$$\sum_j k_j = 2m(t-1) \approx 2mt, \tag{3}$$

and the degree change rate of node i is

$$\frac{\partial k_i}{\partial t} = a\frac{k_i}{t} + b\frac{1}{t}, \tag{4}$$

where $a = \dfrac{m\alpha}{2m\alpha + (1-\alpha)K}$, $b = \dfrac{mK(1-\alpha)}{2m\alpha + (1-\alpha)K}$.

With the initial condition, $k_i(t_i) = m$, we have

$$k_i(t) = -\frac{b}{a} + \left(m + \frac{b}{a}\right)\left(\frac{t}{t_i}\right)^a. \tag{5}$$

Because of each step in the network evolution, we add one node to the network, the probability density of t_i is

$$P(t_i) = \frac{1}{m_0 + t} \tag{6}$$

We can obtain

$$P(k_i < k) = P\left[t_i > \left(\frac{m+b/a}{k+b/a}\right)^{1/a} t\right] = 1 - \frac{1}{m_0+t}\left(\frac{m+b/a}{k+b/a}\right)^{1/a} t. \tag{7}$$

Let

$$P(k) = \frac{\partial P[k_i(t) < k]}{\partial k}. \tag{8}$$

We get the probability density of degree as follows:

$$P(k) = \frac{t}{a(m_0+t)}\left(m + \frac{b}{a}\right)^{\frac{1}{a}}\left(k + \frac{b}{a}\right)^{-\gamma}. \tag{9}$$

For $m_0 \ll t$,

$$P(k) = \frac{1}{a}\left(m + \frac{b}{a}\right)^{\frac{1}{a}}\left(k + \frac{b}{a}\right)^{-\gamma}, \tag{10}$$

where $\gamma = 1 + \dfrac{1}{a}$.

Since $a = \dfrac{m\alpha}{2m\alpha + (1-\alpha)K}$,

$$\gamma = 3 + \frac{(1-\alpha)K}{m\alpha}. \tag{11}$$

Since $0<\alpha<1$, $\gamma \propto (3,\infty)$.

4 Analysis of network features

4.1 The clustering coefficient

The clustering coefficient C_i of a single node i describes the density of connections in the neighborhood of this node. It is given by the ratio of the number e_i of links between the nearest neighbors of i and the potential number of such links $e_{\max} = k_i(k_i - 1)/2$.

$$C_i = \frac{2e_i}{k_i(k_i - 1)}. \tag{12}$$

The clustering coefficient of the whole network is the average of all nodes clustering coefficient.

$$C = \frac{1}{N}\sum C_i. \tag{13}$$

Figure 1 reports the clustering coefficient of the network at various values of K. In the simulation, the size of network N=5000. Each data is averaged by 20 runs.

From Figure 1, we can see that the clustering coefficient of the model can be turned by the value of K, with the increase of K, the clustering coefficient gets smaller. The clustering coefficient of power grids describes the width of fault spreading, the smaller it is, the smaller the scale of fault spreading will be.

4.2 Average path length

The average distance is also one of the most important parameters to measure the efficiency of networks, which is defined as the mean distance over all pairs of nodes. We use d_{ij} to represent the shortest path between nodes i and j, the average path length of the network L is defined as eq. (14):

$$L = \frac{1}{n(n-1)} \sum_{i,j \in n, i \neq j} d_{ij}, \tag{14}$$

where n denotes the total node number of the network.

Figure 2 reports the average path length of the network at various values of K. In the simulation, the size of network N=5000. Each data is averaged by 20 runs.

Figure 2 shows that the average path length of the model can be also turned by the value of K, with the increase of K, the average path length gets bigger. This is good for the security of power grids, because the average path length of power grids describes the depth of fault spreading, the bigger it is, the smaller the scale of fault spreading will be.

4.3 Maximum degree

In the BA model, due to the preferential attachment mechanism, there exist few nodes with very high degree. But in real power grids, the maximum degree of nodes is not as high as the BA model. The West US Power Grid (WSPG) is composed of N=4941 nodes and L=6594 lines, but the maximum degree is just 19, and the average degree $\langle k \rangle$= 2.67.

Figure 3 compares the maximum degree of the network in our model with the BA model. In the simulation, we set the parameters of network: α=0.6, K=2, 10. The size of network n changes from 1000 to 5000. The parameter $k_{\max}$ denotes the maximum degree of the network.

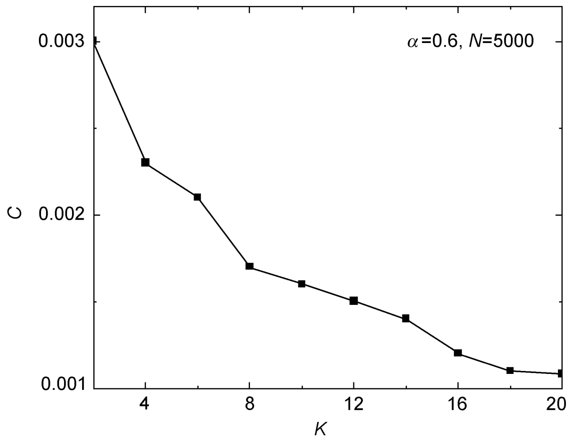

Figure 1 The clustering coefficient of the network at various values of K.

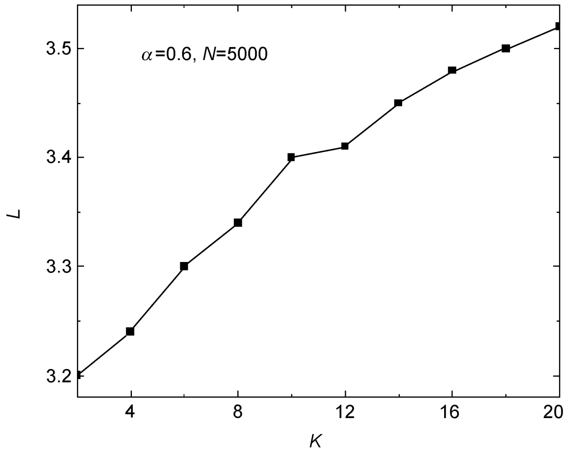

Figure 2 The average path length of the network at various values of K.

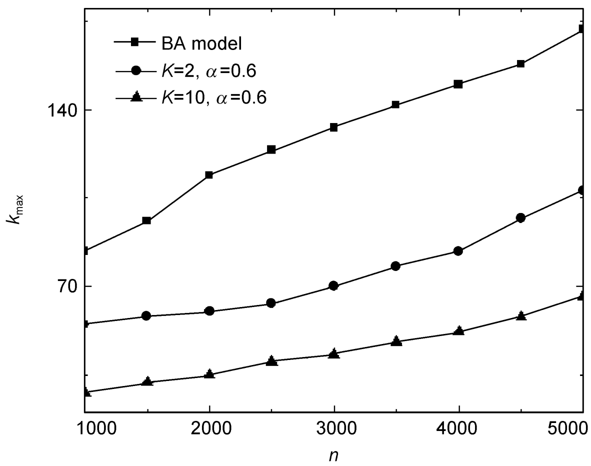

Figure 3 Curves of the maximum degree of the network.

From Figure 3, it is seen that the maximum degree of the network in our model is much smaller than that of the BA model with the same size. With K increasing, the maximum degree of the network gets smaller, namely K controls the few nodes with high degree, it degrades the tendency "the winner takes all" and makes the whole network more homogeneous. As we know, power grids are homogeneous networks.

5 Simulation result

To test our new model, we do the simulation to obtain the degree distribution of our model and compare it with the West South Power Grid (WSPG). WSPG is composed of 4941 nodes and 6594 lines. In the simulation, we set α =0.9, K=2 and the size of network N=4941.

In our model, the average degree of network is m.

$$\frac{L}{N} = \frac{mt}{m_0 + t} \approx m. \tag{15}$$

From eq. (15), it is seen that the edges count is m times of nodes count.

Table 1 reports the topography property of some Chinese Power Grids and the US West Power Grid (WSPG) [20]. From Table 1, we can see that power grids are highly sparse networks, the average degree is between 2 and 3, the ratio of L/N is between 1 and 2. Since L/N=m, in power grids 1<L/N<2, we assume m=2.

Figures 4 and 5 describe the degree distributions of our new model and the BA model. Power network falls somewhere in between scale-free network and uncertain network, because uncertainty of evolution of power network has been considered in our model, the degree distribution of our new model agrees better with WSPG. Therefore in order to give more satisfactory description of network characteristic of power system, uncertainty of evolution of power grid must be considered. The value of K represents the part of uncertainty of power network, the bigger it is, the network is more closed to uncertain network, the number of hub buses turns smaller, the scale-free characteristic is weakened.

6 Conclusion

This paper has proposed a new evolving model for power grid. Using the mean-field theory, we get the analytical expression of degree distribution with the exponent $\gamma \in (3, \infty)$. Simulation results show that power network is neither scale-free network nor totally uncertain network, but in between them, the new model can provide a more satisfactory description for empirical characteristics of power networks.

Table 1 Topography property of some Chinese power grids and the West US Power Grid

Item	Nodes count (N)	Edges count (L)	Average degree	L/N
North Chinese Power Grid	8092	9018	2.23	1.11
Northeast Chinese Power Grid	1144	1309	2.29	1.14
Northern Chinese Power Grid	3706	4045	2.18	1.09
Sichuan-Chongqing Power Grid	853	898	2.11	1.05
Center Chinese Power Grid	2379	2756	2.32	1.16
WSPG	4941	6594	2.67	1.33

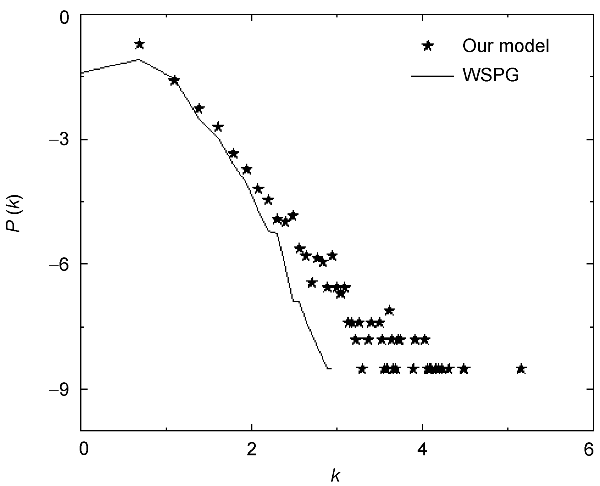

Figure 4 Distribution of degrees of our model and WSPG.

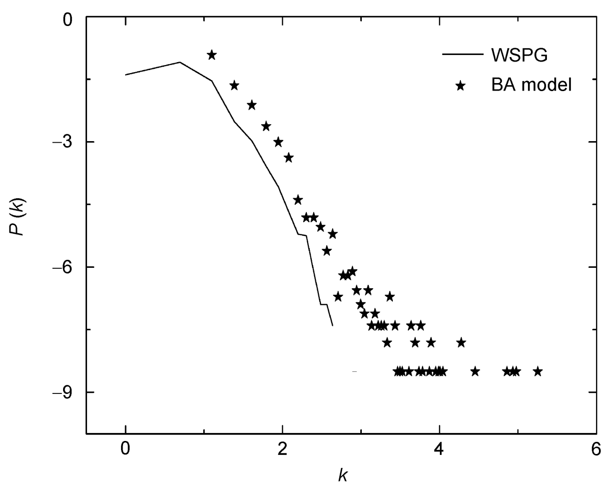

Figure 5 Distribution of degrees of BA model and WSPG.

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