Accuracy Controllable Method for Oblique Incidence on Bodies of Revolution

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***Abstract*—In the methods, for solving problems of bodies of revolution (BOR-problems), Fourier series are used to convert an original three-dimensional problem illuminated by plane wave into a series of 2-D problems, which we call Fourier components or harmonic series, illuminated by cylindrical plane waves. When the plane wave illuminates obliquely, the number of Fourier components should be larger than one. The quantitative relationship between this number and the accuracy of the results has not been well established yet. In this paper, a simple and accuracy controllable method based on a partly iterative procedure is proposed, which can be used to determine the number of Fourier components accurately for a desired accuracy of the results with the most economic computational cost. Although this method is introduced through the method of moments, it can work equally well to other numerical methods for solving BOR-problems. The validity of this method is confirmed by several numerical examples.**

***Index Terms*—Accuracy control, bodies of revolution (BOR), Fourier analysis, numerical method.**

I. Introduction

IN THE methods for solving problems of bodies of revolution, Fourier series are used to convert an original three-dimensional problem illuminated by incident plane wave into a series of two-dimensional problems, which we call Fourier components or harmonic series, illuminated by cylindrical plane waves (i.e., $J_m(k\rho \sin\theta_i)e^{jkz\cos\theta_i}$, where J_m is the Bessel function of the first kind of order m; k is the wave number in free space; (ρ, z) is the cylindrical coordinate of any point on the generating profile; θ_i is the incident angle of the plane wave). All these two-dimensional problems are independent to each other and solved one after another, which is the so called mode-by-mode method. Without losing generality, in this paper, we discuss it by using the method of moments (MOM). For brevity, the currents with respect to each individual harmonic are named as the harmonic currents and denoted as $\boldsymbol{I}_m$ for the mth order.

To determine the number of Fourier components, several approximate formulations have been proposed. M. G. Andreasen in 1965 truncated the harmonic series at $m = k\rho_{\max}\sin\theta_i + 6 \triangleq \kappa_{\max} + 6$ ($\triangleq$ means definition; $\rho_{\max} = \max(\rho)$; $\kappa_{\max} = k\rho_{\max}\sin\theta_i$) based on the asymptotic behavior of Bessel function for small arguments [1]. Since this truncation criteria is approximate and holds only for $\kappa_{\max} \geq 3$, “in the computer program developed”, the number of harmonics “is actually determined by searching a table of the Bessel function” [1]. This is familiar with the method proposed by R. A. Shore *et al.* [2]. In [2], the needed number of harmonics is set as $N_m = \mathrm{int}[(1+\alpha)\kappa_{\max}] + M \triangleq L + M$, where $0 < \alpha \ll 1$; function $\mathrm{int}[\,]$ takes the integer part of its parameter; M is the minimum integer satisfying $J_{L+M}(\kappa_{\max})/J_L(\kappa_{\max}) \leq \epsilon_B$ by searching a table of the Bessel function, where ϵ_B is a small positive number (e.g., $\epsilon_B = 0.005$ [2]). Both the above two methods use the convergence of the Bessel functions instead of the induced harmonic currents as the truncation criterion of the harmonics; this is not exact. In addition, α is an experimental number and, the linear relationship between N_m and $\kappa_{\max}$ is approximate. R. D. Graglia *et al.* [3]thought that the number of Fourier components will never exceed $2\kappa_{\max}$ for the oblique incident cases, but as we will see in the following examples, the harmonic numbers obtained according to this criterion are always too large or too small. The convergence of harmonic currents is studied in [4], which shows that, as m increases, the amplitude of harmonic currents does not decay monotonically; a table is given to search the harmonic numbers for different incident angles.

In this paper, we will introduce a novel method to truncate the harmonic series, which is simple and accuracy controllable. This method divides the mode-by-mode method into two steps; the second step includes an iterative procedure, so we call it a partly iterative method (PIM). Its validity will be confirmed by several numerical examples.

II. Partly Iterative Method

Theoretically, the harmonic currents are independent of the integral equation type, so we choose the magnetic field integral equation (MFIE) method to investigate the truncation problem. If we neglect the integral term in the MFIE, this equation degenerates to the physical optics (PO) method, which can reflect the amplitude level of the harmonic currents and therefore is good enough to investigate the convergence property of the harmonic series. In the PO method, the induced currents are determined by the incident wave illuminated on the surface of the object locally. Since the mth cylindrical plane wave is dominated by $J_m(\kappa)$, where positive parameter $\kappa = k\rho\sin\theta_i$, a local harmonic current is dominated by a local $J_m(\kappa)$. This can be the origin of the approximate formulations presented in [1]and [2]though it is not pointed out in these two papers.

When $m < \kappa$, the absolute value of $J_m(\kappa)$ oscillates, but as m increases, $|J_m(\kappa)|$ decays monotonically. Accordingly, we call $m_t = \mathrm{ceil}[\kappa_{\max}]$ (function $\mathrm{ceil}[\,]$ rounds its parameter toward infinity), the maximum round integer of the κ's of the whole BOR, the turning point of the harmonic series and the region with $m > m_t$ the decaying region of the series. To obtain induced currents of a desired accuracy with an accurate number of Fourier components, one can take two steps: first, obtain all

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the harmonic currents with indices from 0 to m_t without additional operations; second, add harmonics mode by mode together with a convergence judgment until the maximum contribution of the newly added harmonic currents is less than a given tolerance. The problem is how to evaluate the contribution of the newly added harmonic currents.

According to the localization property of the PO method, each position on the generating profile has its own turning point, which is proportional to its electrical radial coordinate (ρ), so the contribution of the harmonic currents with large indices mostly lies in the positions with large ρ's. When the harmonic index goes into the decaying region, the maximum contribution of the newly added harmonic currents will locate closely to the positions with the maximum radial coordinates ($\rho_{\max}$'s). Accordingly, the stop criterion of the iterative procedure can be

$$\frac{\left|\boldsymbol{I}_{m,j}^{\beta\gamma}\right|_{j=i(\rho_{\max})}}{\left|\sum_{n=0}^{m-1}\boldsymbol{I}_{n,j}^{\beta\gamma}\right|} < \epsilon_c \tag{1}$$

where $\boldsymbol{I}_m^{\beta\gamma}$ represents the mth harmonic currents in β (t or ϕ, t is in the tangent direction of any point on the generating profile, ϕ is the azimuth angle in the cylindrical coordinate system) direction when the incident plane wave is γ-polarized (θ or ϕ). They are obtained by multiplying the current coefficients with the basis functions and each $\boldsymbol{I}_m^{\beta\gamma}$ includes $\pm m$ harmonics [4]. Subscripts i and j represent the indices of the basis functions along the generating profile. Expression $j = i(\rho_{\max})$ means the basis function at the position with maximum radial coordinate is selected. If there is a group of such points, choose one of them. Since the maximum amplitude values of the harmonic currents with respect to two polarization type (γ) locate at the co-polarization currents, $\boldsymbol{I}_m^{t\theta}$ and $\boldsymbol{I}_m^{\phi\phi}$, which include factors $\cos m\phi$ [5], the values of $\left|\boldsymbol{I}_m^{t\theta}\right|$ and $\left|\boldsymbol{I}_m^{\phi\phi}\right|$ at $\phi = 0$ are the maximum on the surface of the BOR and therefore the co-polarization currents are selected as the carrier of the judgment in (1). There are other reasons for excluding the cross-polarization currents to be served as the judgment carrier. Firstly, $\boldsymbol{I}_m^{\phi\theta}$ and $\boldsymbol{I}_m^{t\phi}$ include factors $\sin m\phi$ [5]; they will be zero as m increases. Secondly, the voltage (element of excitation vector) with respect to $\boldsymbol{I}_m^{\phi\theta}$ has a factor $\sin v$, where v is the angle between the symmetric axis z and the direction of t, so if a position with $\rho_{\max}$ has $\sin v = 0$, such as the middle point of the generating profile of a sphere, there will be a concave at the position with $\rho_{\max}$ in the amplitude curve of $\boldsymbol{I}_m^{\phi\theta}$. Thirdly, the voltage with respect to $\boldsymbol{I}_m^{t\phi}$ has a factor $\cos\theta_i$, which will be zero when the incident angle is 90°.

Because the above criterion is a relative error, which is not very sensitive to the exact position of $|\boldsymbol{I}_{m,j}^{\beta\gamma}|$ and the positions with maximum contribution of any harmonic in the iterative procedure for electrically small objects are still near to, though not exactly at, the positions with $\rho_{\max}$'s, (1), although derived based on the PO approximation, works equally well for electrically small objects. This has been verified by comparing with the results of an always exact, but more time consuming criterion which takes the maximum amplitude value of the harmonic currents on the generating profile through a searching procedure in many numerical experiments (an example is given in Table II).

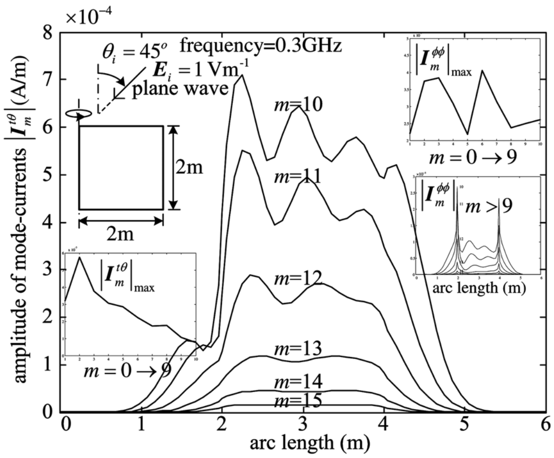

Fig. 1. Harmonic currents in the iterative procedure for the PEC cylinder.

Computed iteratively, the proposed criterion for judgement includes only two complex additions and two real divisions for each Fourier component.

III. Numerical Results

To verify the validity of the proposed method, let us consider several typical examples. Three-order Hermite basis function and a delta sampling procedure are applied [3]. The amplitude distributions of the $\boldsymbol{I}_m^{t\theta}$'s along the generating profile when $m > m_t$ are shown in the figures together with the geometry and excitation parameters, the change of the maximum amplitude of the $\boldsymbol{I}_m^{t\theta}$'s as m increases from 0 to m_t and the counterpoints of the other co-polarization harmonic currents $\boldsymbol{I}_m^{\phi\phi}$'s as inserts. The tolerance, ϵ_c in (1), can be chosen based on a required accuracy. Here we choose 0.01 as a benchmark, with which the contribution of any additional harmonic currents is hard to be recognized by naked eyes.

For the first example, we consider a PEC cylinder as shown in Fig. 1. According to its parameters, the turning point of the whole cylinder is $m_t = 9$. As we can see from Fig. 1, the amplitude of the harmonic currents decays monotonically when the harmonic index is larger than 9 and, most contribution of these harmonic currents (indices larger 9) really lies in the positions with the maximum radial coordinates. As m becomes larger, the shape of the amplitude of the harmonic currents with the largest amplitude will become more and more close to the shape of the generating profile with the largest radial coordinates.

Same phenomena can be found in other two examples: one is a two-joint PEC sphere; the other is a PEC sphere (see Figs. 2 and 3).

When we change the tolerance for each example, the harmonic numbers obtained by applying the PIM are listed in Table I together with those obtained according to [1], [2] and [3], which are denoted as $N_{m,1} = \mathrm{int}[\kappa_{\max}] + 7$, $N_{m,2} = \mathrm{int}[1.04\kappa_{\max}] + M$ and $N_{m,3} = \mathrm{int}[2\kappa_{\max}]$ respectively. In the expression of $N_{m,2}$, the parameter α follows the example in [2], i.e., $\alpha = 0.04$, and we set $\epsilon_B = 0.01$. If we keep ϵ_c as 0.01, two cases with small m_t, small $\rho_{\max}$ or small θ_i, for the PEC sphere are shown in Table II.

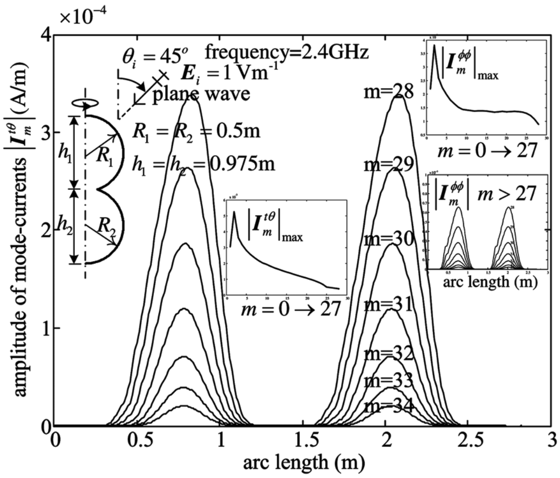

Fig. 2. Harmonic currents in the iterative procedure for a two-joint PEC sphere.

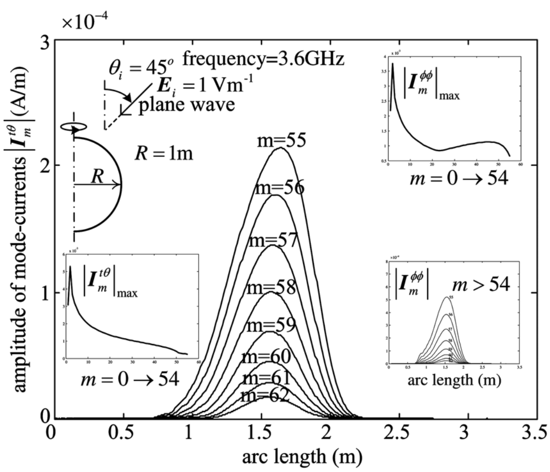

Fig. 3. Harmonic currents in the iterative procedure for the PEC sphere.

TABLE I
COMPARISON OF HARMONIC NUMBERS

	ϵ_c			according to [1], [2], [3]		
	0.1	0.01	0.001	$N_{m,1}$	$N_{m,2}$	$N_{m,3}$
cylinder	14	16	18	15	15	17
two-joint sphere	31	35	38	33	35	53
sphere	57	63	66	60	65	106

TABLE II
COMPARISON OF HARMONIC NUMBERS FOR THE PEC SPHERE WHEN $\kappa_{\max}$ IS SMALL

	$\epsilon_c = 0.01$	$N_{m,1}$	$N_{m,2}$	$N_{m,3}$
$f = 30$MHz, $\theta_i = (\pi/4)$rad	5	7	3	1
$f = 3.6$GHz, $\theta_i = 0.01$rad	5	7	3	1

From these tables, we can see that, when m_t is large, the harmonic numbers given according to [3] are too large; conversely, when m_t is very small, these numbers will produce incorrect results. The $N_{m,1}$'s and $N_{m,2}$'s are close to those obtained by using the PIM with $\epsilon_c = 0.01$. This indicates that the convergence of the Bessel functions is really a good approximation to that of the induced currents and, the required number of Fourier components is almost linear to $\kappa_{\max}$. However, when the generating profile of the BOR is electrically large, it will be a high cost to add just one Fourier component. The required CPU time [personal computer with CPU of Intel Pentium 4 (2.4GHz)] for the mth harmonic, T_m, will increase with m. Three typical examples are, $T_{13} \approx 16$ s for the PEC cylinder, $T_{30} \approx 14.3$ min for the two-joint PEC sphere and $T_{56} \approx 33.7$ min for the PEC sphere (the case in Fig. 3). When the harmonic number is not large enough, these two formulations will give undesired or even incorrect results. Change of α or ϵ_B can not make $N_{m,2}$ more suitable for all the cases.

By using the proposed method to obtain a desired accuracy of the results, one only has to add two complex additions and two real divisions for each harmonic in the iterative procedure. The number of harmonics can be determined accurately and the computational cost can be saved.

IV. CONCLUSION

This paper proposes a novel method based on a partly iterative procedure to determine the number of Fourier components for oblique incident cases in BOR-problems. In this method, the harmonics with the indices smaller than the turning point are calculated mode by mode without additional operations while the harmonics with the indices larger than the turning point are added one by one with an iterative judgment until the currents are convergent to a given tolerance. It is simple and accuracy controllable. No benefit can be found if the iterative judgment starts from the zero harmonic. Using the distribution property of the harmonic currents along the generating profile, the implementation of the truncation criterion is very costless. This method is effective not only in the MOM but also in other methods for solving the BOR-problems.

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