

Design of Adaptive Interval Type-2 Fuzzy Control System and Its Stability Analysis

Jiangtao Cao, Xiaofei Ji, Ping Li, and Honghai Liu

Abstract

Due to the requirement of dealing with the uncertainty and random disturbance in the real-time applications, a framework of adaptive type-2 fuzzy control system is designed. Since most of existed stability analysis results can not be used for the proposed system, by transferring the interval membership degrees into the constrained typical membership degrees, the closed-loop stability of proposed control system is analyzed. The necessary stability conditions are deduced. All stability analysis results are summarized into a Theorem. Comparing with other stability analysis results, the proposed method need less constraints and is easy to be used for controller design issue.

Keywords: *Interval Type-2 fuzzy system, stability analysis, adaptive control.*

1. Introduction

Recently, fuzzy type-2 methods, which were introduced by Zadeh[1], have been further developed to improve the fuzzy logic controllers (FLCs) performance for handling the high level uncertainty [2, 3]. Fuzzy type-2 method is that its fuzzy set is further defined by a typical fuzzy membership function, i.e., the membership degree of belonging for each element of this set is a fuzzy set in $(0, 1)$, not a crisp number [4-6]. In comparison with the type-1 fuzzy logic system (FLS), a type-2 FLS has the two-fold advantages as follows. Firstly, it has the capability of directly handling the uncertain factors of fuzzy rules caused by expert experience or linguistic description. Secondly, it is efficient to employ a type-2 FLS to cope with scenarios in which it is difficult or impossible to determine an exact membership function and related measurement of uncertainties. These strengths have made researchers consider type-2 FLS as the preference for real-world applications [7-9].

About the type-2 FLS, some researches have been done to study the set theoretic operations, properties of membership grades and the uncertainty bounds of type-2 fuzzy sets [10]. From the viewpoint of real-time application, interval type-2 fuzzy logic system has been widely studied and utilized in many research fields, such as autonomous mobile robots control, adaptive control of non-linear system, noise cancellation, quality control and wireless communications, etc. That is mainly due to the interval type-2FLS's simple computing methods and less computational expense on type reduction which is still a bottleneck for other type-2 FLS to be used in real-time applications.

Because of the difficulties of building proper crisp membership functions from the uncertainty of expert knowledge or experience for some complex non-linear systems (i.e., active suspension system), inspired by the idea of type-2 fuzzy methods, an adaptive fuzzy logic controller with interval type-2 fuzzy membership functions was proposed in our previous research [11]. With the designed feedback structure, the optimization algorithms can be integrated to adaptively tune the interval reasoning results which have been successfully verified in the vehicle active suspension system. However, the closed-loop stability of proposed adaptive fuzzy control system need to be analyzed and the stability conditions need to be deduced for its practical applications.

Stability is one of the most important issues in analysis and design of control systems. Stability analysis of fuzzy control system has been more difficult because the system is essentially non-linear. Reviewing the existing stability analysis results of typical fuzzy control systems, T-S fuzzy-model-based control systems provided great development of systematic approaches to stability analysis and controller design of fuzzy control systems in view of powerful conventional control theory and techniques [12]. The major techniques that have been used include quadratic stabilization, LMI, Lyapunov stability theory, bilinear matrix inequalities. Inspired from the above stability analysis approaches, in this paper, for the previous proposed framework of adaptive interval type-2 FLC [11], its closed-loop stability is analyzed.

The rest of the paper is organized as follows. Firstly, with the proposed framework of control system, the interval type-2 FLC system is demonstrated in Section 2. Secondly, Section 3 represents the general formulation

Corresponding Author: Jiangtao Cao is with the School of Information and Control Engineering, Liaoning Shihua University, Liaoning Province, P.R.China, 113001.

E-mail: jtcao@lnpu.edu.cn

Manuscript received 8 Oct. 2011; revised 5 Nov. 2011; accepted 22 Dec. 2011.

of the adaptive FLC system. In Section 4, the stability of proposed fuzzy control system is analyzed and the sufficient stability conditions are obtained. Concluding remarks, perspectives and challenges are discussed in Section 5.

2. The Framework of Adaptive Interval Type-2 FLC

In this section, a framework for adaptive interval type-2 FLC is designed to deal with the uncertainty and imprecision in the real-time applications. With the designed feedback structure, optimal algorithms are used to adaptively tune the interval region and to obtain a crisp output which can bring the better control performance. The framework of proposed method is represented for control the non-linear and uncertain systems.

The framework of adaptive interval type-2 FLC is shown in Fig.1. In comparison with the conventional interval type-2 FLS [13], the proposed structure builds a more general framework to represent the type-reduction and defuzzification process. If an optimal goal of the proposed FLC can be described, the convergence of the optimization method is guaranteed, the proposed framework is shrunk to the same form as the conventional interval type-2 FLC.

Among the different kinds of type-reduction methods, Karnik-Mendel algorithms and Wu-Mendel algorithms are considered in this adaptive control system [5]. The first type of method calculate the exact solutions monotonically and super-exponentially fast with simple formula and they can be run in parallel, but the time delay caused by algorithmic iteration is the bottleneck for real-time applications. On the other hand, the second type of method replaces the type-reduction by four uncertainty bounds. These bounds only depend on the lower and upper firing levels of each rule and the centroid of each rule's consequent set. For the purpose of computational efficiency, the proposed method uses the second type-reduction method to calculate the end-points of reasoning results.

Furthermore, under the proposed structure, the crisp output of the proposed FLC represents twofold information. One is the fuzzy reasoning result which is based on fuzzy rules extracted from expert knowledge or industrial experience, the other is the further optimal goal which is required by practical issues (e.g., saving energy) or is impossible to be combined into the fuzzy rules.

Optimization algorithms can be selected in terms of domain-dependent goals and practical requirements. Here, for real-time control, two optimization algorithms are used, one is the least-mean-square (LMS) method which is a gradient-based method and the other is the particle swarm optimization (PSO) method which is a

recently invented high-performance non-linear optimizer and requires less computational cost in real-time applications.

3. The General Formulation of Proposed Control System

A brief introduction on typical T-S fuzzy control systems and the interval type-2 T-S fuzzy system is firstly presented in this section, then the type-2 reasoning methods and proposed adaptive structure are demonstrated, finally the section is concluded with the general formulation of proposed adaptive interval type-2 fuzzy control system.

A. Typical T-S Fuzzy Control Systems

T-S fuzzy model was proposed by Takagi and Sugeno [14]. This model is based on using a set of fuzzy rules to describe a global non-linear system in terms of a set of local linear models which are smoothly connected by fuzzy membership functions. A lot of theoretical results on function approximation, stability analysis, and controller synthesis have been developed for T-S fuzzy model during recent decades [15-21]. The research results have shown that T-S fuzzy model is able to approximate any smooth non-linear functions to any degree of accuracy in any convex compact region [22]. In this section, the T-S fuzzy model is represented and the T-S model-based fuzzy controller by parallel distributed compensation (PDC) method is rebuilt.

The T-S fuzzy dynamic model is described by fuzzy IF-THEN rules, which represent local linear input-output relations of non-linear systems. The i th rule of T-S fuzzy model is shown as follows.

$R^{(i)} : \text{IF } z_1 \text{ is } M_1^i \text{ and } z_2 \text{ is } M_2^i, \dots, \text{ and } z_g \text{ is } M_g^i \text{ THEN } x(t+1) = A_i x(t) + B_i u(t). (i=1,2,\dots,m)$

where M is the typical fuzzy set, $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T$ denotes the state vector, $u(t)$ denotes the input vector, m denotes the number of fuzzy rules, and $z_1, z_2, \dots, z_g$ denote measurable variables.

$$x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^p, A_i \in \mathbb{R}^{n \times n}, B_i \in \mathbb{R}^{n \times p} \quad (1)$$

Assume that, $g = n, z_1 = x_1(t), \dots, z_n = x_n(t)$. For a pair of inputs $(x(t), u(t))$, the output of T-S fuzzy system is present as follows:

$$\begin{aligned} x(t+1) &= \frac{\sum_{i=1}^m \omega_i [A_i x(t) + B_i u(t)]}{\sum_{i=1}^m \omega_i} \\ &= \sum_{i=1}^m h_i [A_i x(t) + B_i u(t)] \end{aligned} \quad (2)$$

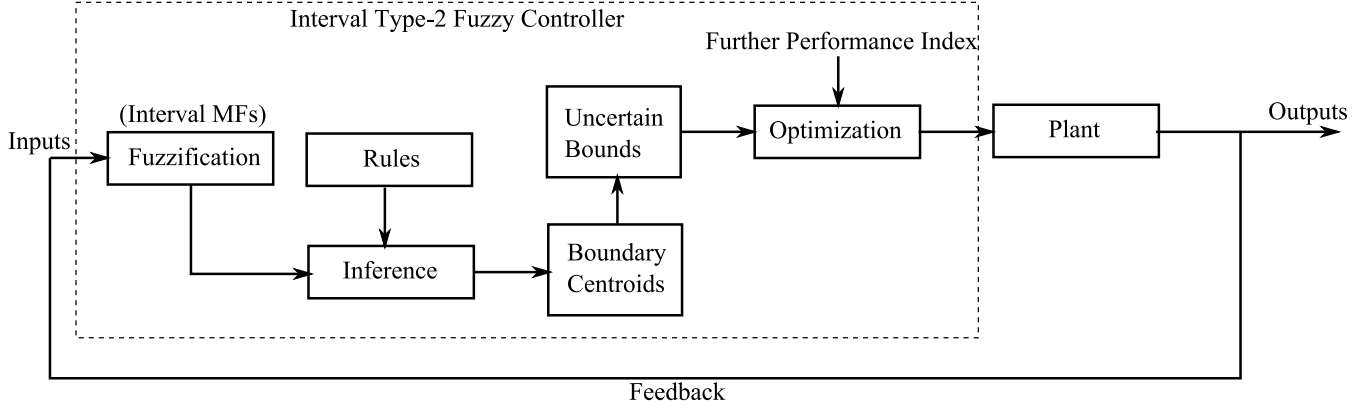


Fig.1. The framework of adaptive interval type-2 fuzzy control system.

where

$$\omega_i = \prod_{j=1}^n M_{ij}(x_j(t)) \quad (3)$$

$$h_i = \frac{\omega_i}{\sum_{i=1}^m \omega_i}$$

here, $M_{ij}(x_j(t))$ is the membership grade of $x_j(t)$ in the fuzzy set M_{ij} . For general, the normalized form of ω_i and h_i are defined as:

$$\begin{aligned} \omega_i &\geq 0, i = 1, 2, \dots, m \\ \sum_{i=1}^m \omega_i &\geq 0 \\ h_i &\geq 0, i = 1, 2, \dots, m \\ \sum_{i=1}^m h_i &= 1 \end{aligned} \quad (4)$$

The T-S fuzzy controller can be designed by the method PDC. The i th control rule is described as:

$R^{(i)}$: IF z_1 is M_1^i and z_2 is $M_2^i, \dots$, and z_n is M_n^i THEN $u(t) = K_i x(t), (i = 1, 2, \dots, m)$.

The control rules have linear state feedback control laws in its consequent parts. The final output of this fuzzy controller is:

$$u(t) = \sum_{i=1}^m h_i K_i x(t), i = 1, 2, \dots, m \quad (5)$$

Substitute equation 5 into equation 2, the closed-loop T-S fuzzy control system is presented as follows.

$$x(t+1) = \sum_{i=1}^m \sum_{j=1}^m h_i h_j (A_i - B_i K_j) x(t) \quad (6)$$

B. Interval Type-2 T-S Fuzzy System

Considering an interval type-2 T-S fuzzy model with m rules represented ad the general form:

$R^{(l)}$: IF z_1 is $\tilde{F}_1^l$ and z_2 is $\tilde{F}_2^l, \dots$, and z_v is $\tilde{F}_v^l$ THEN $x(t+1)$ is $g^l(X, U)$. ($l \in L := 1, 2, \dots, m$).

where $R^{(l)}$ denotes the l th fuzzy inference rule, m denotes the number of fuzzy rules, $\tilde{F}_j^l (j = 1, 2, \dots, v)$ denote the interval type-2 fuzzy sets, $z(t) := [z_1, z_2, \dots, z_v]$ denote measurable variables, $x(t) \in \mathfrak{R}^n$ denotes the state vector, $u(t) \in \mathfrak{R}^p$ denotes the input vector, and the T-S consequent terms g^l is defined in equation 7.

$$g^l(X, U; \theta^l) = A_l x(t) + B_l u(t) + a_l \quad (7)$$

$$l \in L := 1, 2, \dots, m$$

where A_l, B_l and a_l are the parameter matrices of the l th local model.

Its firing strength of the l th rule belongs to the following interval set:

$$\omega_l(x) \in [\underline{\omega}_l(x), \overline{\omega}_l(x)] l = 1, 2, \dots, m \quad (8)$$

where

$$\underline{\omega}_l(x) = \underline{\mu}_{\tilde{F}_1^l}(x) * \underline{\mu}_{\tilde{F}_2^l}(x) \cdots \underline{\mu}_{\tilde{F}_m^l}(x) \quad (9)$$

$$\overline{\omega}_l(x) = \overline{\mu}_{\tilde{F}_1^l}(x) * \overline{\mu}_{\tilde{F}_2^l}(x) \cdots \overline{\mu}_{\tilde{F}_m^l}(x) \quad (10)$$

in which, $\underline{\mu}_{\tilde{F}_l^l}(x)$ and $\overline{\mu}_{\tilde{F}_l^l}(x)$ denote the lower and upper membership grades, respectively. Then the inferred interval type-2 T-S fuzzy model is defined as

$$\begin{aligned} x(t+1) &= \sum_{l=1}^m (\alpha \cdot \underline{\omega}_l(x) + \beta \cdot \overline{\omega}_l(x)) (A_l x + B_l u) \\ &= \sum_{l=1}^m \tilde{\omega}_l(x) (A_l x + B_l u) \end{aligned} \quad (11)$$

where

$$\begin{aligned}\tilde{\omega}_l(x) &= \alpha \cdot \underline{\omega}_l(x) + \beta \overline{\omega}_l(x) \in [0,1] \\ \sum_{l=1}^m \tilde{\omega}_l(x) &= 1\end{aligned}\quad (12)$$

Herein, the values of α and β are both depend on the uncertainty which potentially existed in parameters and fuzzy rules.

C. Interval Type-2 T-S Fuzzy Control System

In order to control the non-linear system based on the T-S fuzzy model described by equation 11, and adaptive T-S fuzzy controller is represented and its fuzzy rules are given as below,

$R^{(r)}$: IF z_1 is $\tilde{F}_1^r$ and z_2 is $\tilde{F}_2^r, \dots$, and z_v is $\tilde{F}_v^r$, THEN $u(t)$ is $\tilde{K}_r x(t)$. ($l \in L := 1, 2, \dots, m$).

where $\tilde{K}_r$ stands for the r th local linear control gain. The output of this controller is defined as

$$u(t) = \sum_{r=1}^m f(\omega_r^L(x), \omega_r^U(x)) \tilde{K}_r \cdot x \quad (13)$$

here,

$$\omega_r^L(x) = \frac{\underline{\omega}_r(x)}{\sum_r (\underline{\omega}_r(x) + \overline{\omega}_r(x))} \quad (14)$$

$$\omega_r^U(x) = \frac{\overline{\omega}_r(x)}{\sum_r (\underline{\omega}_r(x) + \overline{\omega}_r(x))} \quad (15)$$

ω_r^L and ω_r^U are satisfied with

$$\sum_{r=1}^m (\omega_r^L(x) + \omega_r^U(x)) = 1 \quad (16)$$

and the value of $f(\omega_r^L(x), \omega_r^U(x))$ depends on the type reduction methods and belongs to an interval.

In the recent research of [23], with the normalized central method (i.e.,

$$f(\omega_r^L(x), \omega_r^U(x)) = (\omega_r^L(x) + \omega_r^U(x)) / 2,$$

the stability of interval type-2 T-S fuzzy control system was studied and the stability condition was conducted. However, it can not work on the proposed interval type-2 FLC in this paper because the type reduction method is different. That is, based on the proposed framework of adaptive interval type-2 FLC system, the interval fuzzy outputs were optimized by the optimization algorithms, such as LMS and PSO methods. Then related with the equation 13, a general formula for the control output of proposed method is written as:

$$u(t) = \sum_{r=1}^m (\tilde{\alpha} \omega_r^L(x) + \tilde{\beta} \omega_r^U(x)) \tilde{K}_r \cdot x \quad (17)$$

For general, the coefficients $\tilde{\alpha}$ and $\tilde{\beta}$ should be satisfied with the condition: $\tilde{\alpha} + \tilde{\beta} = 1$. Since the stability analysis methods for typical fuzzy systems will require the crisp or precise value of $\omega_l(x)$, these approaches cannot be directly used to analyze the stability of interval type-2 fuzzy control systems. So in next section, the stability analysis approach will be re-structured by integrating the lower and upper membership grades.

4. Stability Analysis of the Proposed Fuzzy Control System

In this section, the stability of the proposed closed-loop interval type-2 T-S fuzzy control system is analyzed. For easily understanding the stability theory of T-S fuzzy systems, the main existed approaches of stability analysis and their stability conditions for typical T-S fuzzy control systems are reviewed in Section 4.1. Then the stability analysis of proposed adaptive interval type-2 T-S fuzzy control system is deduced in Section 4.2.

A. Stability Conditions with Lyapunov Stability Theory

As mentioned by [12], based on the above T-S fuzzy control system in equation 6, the existing main methods for stability analysis include quadratic stabilization, linear matrix inequalities and bilinear matrix inequalities, and so on. Most of these methods will require a Lyapunov function $V(x) = x^T P x$ (e.g., common quadratic Lyapunov function, piecewise quadratic Lyapunov function and fuzzy Lyapunov function). The basic stability condition for the open-loop T-S fuzzy system can be presented by the following Lemma 1.

Lemma 1: The equilibrium of system in 2 (with $u = 0$) is asymptotically stable in the large if there exists a common positive definite matrix P such that

$$A_i^T P + P A_i < 0 \quad (18)$$

for all subsystems, that is, $i = 1, 2, \dots, m$.

A common Lyapunov function in equation 18 can be solved numerically by convex programming algorithms (e.g., LMI method). More details can be found in the book by [18]. For the further stability analysis of T-S fuzzy control system, several stability conditions were summarized by [24]. These stability conditions are re-arranged by using the general T-S fuzzy control system

in the equation 6.

Firstly, the T-S fuzzy control system 6 can be represented in a general form as follows.

$$\begin{aligned} x(t+1) &= G_0 x(t) + \sum_{i=1}^m \sum_{j=1}^m h_i h_j \Delta G_{ij} x(t) \\ &= G_0 x(t) + \sum_{i=1}^m h_i h_j \Delta G_{ii} x(t) + \sum_{i < j}^m h_i h_j \Delta F_{ij} x(t) \\ &= \{G_0 + W\Delta(t)Z\}x(t) \end{aligned} \quad (19)$$

here,

$$G_0 = \frac{1}{m} \sum_{i=1}^m (A_i + B_i K_i) \quad (20)$$

$$\begin{aligned} \Delta G_{ij} &= A_i + B_i K_j - G_0 \\ \Delta G_{ii} &= Q_{ii} \Phi_{ii} S_{ii}^T \end{aligned} \quad (21)$$

$$\Delta F_{ij} = \Delta G_{ij} + \Delta G_{ji} = Q_{ij} \Phi_{ij} S_{ij}^T < j$$

Q and S are unitary matrix, and $W \in \mathbb{R}^{n \times r}$, $\Delta(t) \in \mathbb{R}^{r \times r}$, $Z \in \mathbb{R}^{r \times n}$, $\gamma = [n \times m \times (m+1)]/2$, the matrices W, Z are as follows:

$$\begin{aligned} W &= [\bar{Q}_1 \quad \bar{Q}_2 \quad \cdots \quad \bar{Q}_m] \\ Z &= [\bar{S}_1 \quad \bar{S}_2 \quad \cdots \quad \bar{S}_m]^T \\ \Delta(t) &= \text{block-diag}[\bar{\Phi}_1^e \quad \bar{\Phi}_2^e \quad \cdots \quad \bar{\Phi}_m^e] \end{aligned} \quad (22)$$

where

$$\begin{aligned} \bar{Q}_i &= [Q_{ii} \quad Q_{ii+1} \quad \cdots \quad Q_{im}] \\ \bar{S}_i &= [S_{ii} \quad S_{ii+1} \quad \cdots \quad S_{im}] \\ \bar{\Phi}_i^e &= \text{block-diag}[e_{ii} \Phi_{ii} \quad e_{ii+1} \Phi_{ii+1} \quad \cdots \quad e_{im} \Phi_{im}] \\ e_{ii} &= h_i h_j \end{aligned} \quad (23)$$

Based on the equation 22, the matrices M and N are defined as follows:

$$M = N = \text{block-diag}[\bar{\Phi}_1^d \quad \bar{\Phi}_2^d \quad \cdots \quad \bar{\Phi}_m^d] \quad (24)$$

where

$$\begin{aligned} \bar{\Phi}_i^d &= \text{block-diag} \left[\frac{d_{ii}}{2} \Phi_{ii} \quad \frac{d_{ii+1}}{2} \Phi_{ii+1} \quad \cdots \quad \frac{d_{im}}{2} \Phi_{im} \right] \\ d_{ij} &= \max h_i h_j \end{aligned} \quad (25)$$

Lemma 2: The equilibrium of a general T-S fuzzy control system as given in equation 6 is quadratically stable in the large if and only if one of the following conditions is satisfied.

C1) There exists a positive definite matrix P such that

$$\begin{aligned} &P(G_0 + WMZ) + (G_0 + WMZ)^T P \\ &+ PWN^T W^T P + Z^T Z < 0 \end{aligned}$$

C2) $G_0 + WMZ$ is a stable matrix and

$$\|Z(sI - G_0 - WMZ)^{-1}WN\|_\infty < 1$$

C3) If defined

$$H = \begin{bmatrix} G_0 + WMZ & -WNN^T W^T \\ Z^T Z & -(G_0 + WMZ) \end{bmatrix}$$

the condition is

$$\text{Re } \lambda_i(H) \neq 0, i = 1, 2, \dots, 2 \times n.$$

C4) There exists a positive definite matrix P such that

$$\begin{bmatrix} P(G_0 + WMZ) + (G_0 + WMZ)^T P + Z^T Z & PWN \\ N^T W^T P & -I \end{bmatrix} < 0$$

C5) There exists a positive definite matrix P such that

$$\begin{bmatrix} P(G_0 + WMZ) + (G_0 + WMZ)^T P & PWN & Z^T \\ N^T W^T P & -I & 0 \\ Z & 0 & -I \end{bmatrix} < 0$$

Remark 1: The condition C1 is obtained by using the common quadratic method which finds the positive definite solution P from Riccati equation. The condition C2 connects the global stability with the H_∞ control performance. By the research of [25], the conditions C3-C5 are equivalent to C1 and C2, however they are described by LMI methods. There have been some efficient algorithms to check the global stability by LMI methods [18, 26].

Remark 2: All these conditions can be fitted for the deterministic T-S fuzzy systems, but not for the stochastic T-S fuzzy systems [12]. That is means, if the membership grades are not crisp values (e.g., uncertain or interval variables), these above conditions can not directly work. Several authors have made attempt to address these issues [27, 28]. However, the existing results are not enough for stability analysis of the proposed adaptive interval type-2 FLC in this paper.

B. Stability Conditions of the Proposed Fuzzy Control System

With the interval type-2 T-S fuzzy model in equation 11 and the proposed controller in equation 17, the closed-loop interval type-2 T-S fuzzy control system can be described as follows,

$$x(t+1) = \sum_{i=1}^m \sum_{j=1}^m G_{ij} (A_i + B_i K_j) x(t) \quad (26)$$

where, $G_{i,j}$ denotes the fixed membership grade from the interval type-2 antecedents and T-S consequent, it is described as,

$$G_{ij} = [\tilde{\alpha}\omega_r^L(x) + \tilde{\beta}\omega_r^U(x)]\tilde{\omega}_j = \omega_i\tilde{\omega}_j \quad (27)$$

where $\omega_r^L(x), \omega_r^U(x)$ and $\tilde{\omega}_j$ are defined in equation 14, 15 and 12.

With the closed-loop interval type-2 T-S fuzzy control system in equation 26, a Lyapunov function candidate is defined as:

$$V(t) = x(t)^T P x(t) \quad (28)$$

here, the matrix is positive definite, and this function satisfies the following properties: $V(0) = 0, V(x(t)) > 0$ for $x(t) \neq 0$ and $V(x(t))$ approaches infinity as $\|x(t)\| \rightarrow \infty$.

Then

$$\dot{V}(t) = \dot{x}(t)^T P x(t) + x(t)^T P \dot{x}(t) \quad (29)$$

Substituting the equation 26 into the equation 29, we have

$$\begin{aligned} \dot{V}(t) = & \left(\sum_{i=1}^m \sum_{j=1}^m [\tilde{\alpha}\omega_j^L(x) + \tilde{\beta}\omega_j^U(x)] \tilde{\omega}_i (A_i + B_i K_j) x(t) \right)^T P x(t) \\ & + x(t)^T P \left(\sum_{i=1}^m \sum_{j=1}^m [\tilde{\alpha}\omega_j^L(x) + \tilde{\beta}\omega_j^U(x)] \tilde{\omega}_i (A_i + B_i K_j) x(t) \right) \end{aligned} \quad (30)$$

For using the general formulations by [29, 23], the equation 30 can be rewritten as

$$\begin{aligned} \dot{V}(t) = & \sum_{i=1}^m \sum_{j=1}^m [\tilde{\alpha}\omega_j^L(x) + \tilde{\beta}\omega_j^U(x)] \tilde{\omega}_i (A_i + B_i K_j) Z(t)^T Q_{ij} Z(t) \\ = & Z(t)^T \Psi Z(t) \end{aligned}$$

where $Z(t) = P x(t)$,

$$Q_{ij} = A_i P^{-1} + P^{-1} A_i^T + B_i K_j P^{-1} + (K_j P^{-1})^T B_i^T,$$

$$\Psi = \sum_{i=1}^m \sum_{j=1}^m [\tilde{\alpha}\omega_j^L(x) + \tilde{\beta}\omega_j^U(x)] \tilde{\omega}_i Q_{ij}. \quad (31)$$

Based on the Lyapunov stability theory, if the condition $\dot{V}(t) \leq 0$ is satisfied, the related interval type-2 fuzzy control system is asymptotic stable. From equation 31, $\Psi \leq 0$ should be satisfied. Considering the well-known expression of stability analysis [14, 29, 30, 31], let $\Xi = -\Psi$ and $\tilde{Q}_{ij} = -Q_{ij}$. If the following condition is proved,

$$-\dot{V}(t) = -Z(t)^T \Psi Z(t) = Z(t)^T \Xi Z(t) \geq 0 \quad (32)$$

$\dot{V}(t) \leq 0$ can be obtained.

Most of existing stability analysis results for typical T-S fuzzy control system considered the $\tilde{Q}_{ij}$ in the case of same membership grades of fuzzy controller and fuzzy model (i.e., $\mu_i = \eta_i$ here, μ_i is one of membership grade of fuzzy model and η_i is one of membership grade of fuzzy controller). However, in the

proposed interval type-2 fuzzy control system, the membership grades of fuzzy model $\tilde{\omega}_i$ are not same as the membership grades of fuzzy controller $\tilde{\alpha}\omega_j^L(x) + \tilde{\beta}\omega_j^U(x)$. So some new conditions must be reconsidered.

Since the interval type-2 fuzzy membership grades belong to an interval, there will be some constraints between these membership grades of fuzzy model and fuzzy controller.

$$\tilde{\eta}_i - \frac{\omega_i^U}{\omega_i} \tilde{\mu}_i = \tilde{\eta}_i + a_i \tilde{\mu}_i \leq 0 \quad (33)$$

$$-\tilde{\eta}_i - \frac{\omega_i^L}{\omega_i} \tilde{\mu}_i = -\tilde{\eta}_i + b_i \tilde{\mu}_i \leq 0 \quad (34)$$

here, $\tilde{\eta}_i = \tilde{\alpha}\omega_i^L(x) + \tilde{\beta}\omega_i^U(x)$ and $\tilde{\mu}_i = \tilde{\omega}_i$.

The conditions in equation 33 and 34 can be written as,

$$\begin{aligned} \sum_{i=1}^m (\tilde{\eta}_i + a_i \tilde{\mu}_i) & \leq 0 \\ \sum_{i=1}^m (-\tilde{\eta}_i + b_i \tilde{\mu}_i) & \leq 0 \end{aligned} \quad (35)$$

The position definite matrices Γ_k ($k=1, 2$) are defined as:

$$\begin{aligned} \Gamma_1 &= \sum_{j=1}^m (\tilde{\mu}_j R_{j1} + \tilde{\eta}_j \hat{R}_{j1}) \geq 0 \\ \Gamma_2 &= \sum_{j=1}^m (\tilde{\mu}_j T_{j2} + \tilde{\eta}_j \hat{T}_{j2}) \geq 0 \end{aligned} \quad (36)$$

Multiplying the first condition in equation 35 by Γ_1 , we get a negative-semidefinite matrix:

$$H_1 = \sum_{i=1}^m \sum_{j=1}^m (\tilde{\eta}_i \tilde{\mu}_j R_{j1} + \tilde{\eta}_i \tilde{\eta}_j \hat{R}_{j1} + a_i \tilde{\mu}_i \tilde{\mu}_j R_{j1} + a_i \tilde{\mu}_i \tilde{\eta}_j \hat{R}_{j1}) \leq 0 \quad (37)$$

Multiplying the second condition in equation 35 by Γ_2 , we get another negative-semidefinite matrix:

$$H_2 = \sum_{i=1}^m \sum_{j=1}^m (-\tilde{\eta}_i \tilde{\mu}_j T_{j2} - \tilde{\eta}_i \tilde{\eta}_j \hat{T}_{j2} + b_i \tilde{\mu}_i \tilde{\mu}_j T_{j2} + b_i \tilde{\mu}_i \tilde{\eta}_j \hat{T}_{j2}) \leq 0 \quad (38)$$

Subsequently, it is evident that if $\Xi + H_1 \geq 0$ and $\Xi + H_2 \geq 0$ can be proved, then $\Xi \geq 0$ is satisfied. That is,

$$\begin{aligned}
\Xi + H_1 &= \sum_{i=1}^m \sum_{j=1}^m (\tilde{\mu}_i \tilde{\eta}_j \tilde{Q}_{ij} + \tilde{\eta}_i \tilde{\mu}_j R_{j1} \\
&+ \tilde{\eta}_i \tilde{\eta}_j \hat{R}_{j1} + a_i \tilde{\mu}_i \tilde{\mu}_j R_{j1} + a_i \tilde{\mu}_i \tilde{\eta}_j \hat{R}_{j1}) \\
&= \sum_{i=1}^m (a_i \tilde{\mu}_i^2 R_{i1} + \tilde{\eta}_i^2 \hat{R}_{i1}) + \sum_{i < j \leq m} (\tilde{\mu}_i \tilde{\mu}_j (a_i R_{j1} + a_j R_{i1}) \\
&+ (\tilde{\eta}_i \tilde{\eta}_j (\hat{R}_{i1} + \hat{R}_{j1}))) + \sum_{i=1}^m \sum_{j=1}^m \tilde{\mu}_i \tilde{\eta}_j (\tilde{Q}_{ij} + R_{i1} + a_i \hat{R}_{j1}) \\
&\quad (39)
\end{aligned}$$

$$\begin{aligned}
\Xi + H_2 &= \sum_{i=1}^m \sum_{j=1}^m (\tilde{\mu}_i \tilde{\eta}_j \tilde{Q}_{ij} - \tilde{\eta}_i \tilde{\mu}_j T_{j2} \\
&- \tilde{\eta}_i \tilde{\eta}_j \hat{T}_{j2} + b_i \tilde{\mu}_i \tilde{\mu}_j T_{j2} + b_i \tilde{\mu}_i \tilde{\eta}_j \hat{T}_{j2}) \\
&= \sum_{i=1}^m (b_i \tilde{\mu}_i^2 T_{i2} - \tilde{\eta}_i^2 \hat{T}_{i2}) + \sum_{i < j \leq m} (\tilde{\mu}_i \tilde{\mu}_j (b_i T_{j2} + b_j T_{i2}) \\
&- (\tilde{\eta}_i \tilde{\eta}_j (\hat{T}_{i2} + \hat{T}_{j2}))) + \sum_{i=1}^m \sum_{j=1}^m \tilde{\mu}_i \tilde{\eta}_j (\tilde{Q}_{ij} - T_{i2} + b_i \hat{T}_{j2}) \\
&\quad (40)
\end{aligned}$$

Defining the matrices $X_{ij} = X_{ji}^T, i, j = 1, \dots, m$, it is satisfied with the conditions as follows.

$$a_i R_{j1} + a_j R_{i1} \geq X_{ij} + X_{ji} \quad (41)$$

$$\hat{R}_{i1} + \hat{R}_{j1} \geq X_{(i+m)(j+m)} + X_{(j+m)(i+m)} \quad (42)$$

$$\tilde{Q}_{ij} + R_{i1} + a_i \hat{R}_{j1} \geq X_{i(j+m)} + X_{(j+m)i} \quad (43)$$

$$b_i T_{j2} + b_j T_{i2} \geq X_{ij} + X_{ji} \quad (44)$$

$$-\hat{T}_{i2} - \hat{T}_{j2} \geq X_{(i+m)(j+m)} + X_{(j+m)(i+m)} \quad (45)$$

$$\tilde{Q}_{ij} - T_{i2} + b_i \hat{T}_{j2} \geq X_{i(j+m)} + X_{(j+m)i} \quad (46)$$

$$\begin{pmatrix} X_{11} & \cdots & X_{1(2r)} \\ \vdots & \ddots & \vdots \\ X_{(2r)1} & \cdots & X_{(2r)(2r)} \end{pmatrix} > 0 \quad (47)$$

With the conditions in equation 41-43, the equation 39 can be extended

$$\begin{aligned}
\Xi + H_1 &\geq \sum_{i=1}^m (\tilde{\mu}_i^2 X_{ii} + \tilde{\eta}_i^2 X_{ii}) + \\
&\sum_{i=1}^j \sum_{i < j \leq m} (\tilde{\mu}_i \tilde{\mu}_j (X_{ij} + X_{ji}) + \tilde{\eta}_i \tilde{\eta}_j (X_{(i+m)(j+m)} + X_{(j+m)(i+m)})) \\
&+ \sum_{i=1}^m \sum_{j=1}^m (\tilde{\mu}_i \tilde{\eta}_j (X_{i(j+m)} + X_{(j+m)i})) \\
&\quad (48)
\end{aligned}$$

With the different conditions in equation 44-46, the equation 40 can be extended as the formula as equation 48.

Hence, with the equation 48, $\Xi + H \geq 0$. Then

$\Xi \geq 0$. And based on the condition 32, $\dot{V}(t) \leq 0$. The proposed interval type-2 fuzzy control system is asymptotic stable.

Furthermore, the conditions of equations 41-46 can be rewritten as:

$$a_i R_{j1} + a_j R_{i1} + b_i T_{j2} + b_j T_{i2} \geq X_{ij} + X_{ji} \quad (49)$$

$$\hat{R}_{i1} + \hat{R}_{j1} - \hat{T}_{i2} - \hat{T}_{j2} \geq X_{(i+m)(j+m)} + X_{(j+m)(i+m)} \quad (50)$$

$$\tilde{Q}_{ij} + R_{i1} + a_i \hat{R}_{j1} - T_{i2} + b_i \hat{T}_{j2} \geq X_{i(j+m)} + X_{(j+m)i} \quad (51)$$

The stability analysis result is summarized in the following theorem.

Theorem 1: With the known conditions of equations 33 and 34, if there exist matrices $X_{ij} = X_{ji}^T, i, j = 1, 2, \dots, 2m$ and symmetric definite positive matrices $R_{j1}, \hat{R}_{j1}, T_{j2}$ and $\hat{T}_{j2}$ for all $j = 1, \dots, r$, which satisfy the conditions in equations 49-51 and equation 47, the proposed interval type-2 fuzzy control system in equation 26 is asymptotic stable.

Remark 3: When all the membership functions in fuzzy control system are typical membership functions, the closed-loop system is reduced to a typical T-S fuzzy control system. The constraints of membership grades in equations 33 and 34 will be changed to $\tilde{\mu}_i = \tilde{\eta}_j$.

The process of stability analysis in this section will be shrunk to the general formulation for typical T-S fuzzy system. So this stability analysis method is an extension of existing typical stability results to consider the interval membership grades in type-2 fuzzy system. Furthermore, the stability analysis can not only be utilized for proposed adaptive interval type-2 fuzzy control system, but also for the general interval type-2 fuzzy systems.

Remark 4: Since the stability result in this section is deduced from Lyapunov stability theory, a common quadratic Lyapunov function need to be found for all the local subsystems in T-S fuzzy model. And this stability conditions are sufficient conditions. Less conservative methods about interval type-2 T-S fuzzy control system will be studied in future.

5. Summary

With the proposed framework of adaptive interval type-2 FLC in Section 2, a closed-loop model of interval type-2 fuzzy control system was presented. Its closed-loop stability has been investigated by quadratic Lyapunov stability theory. Under the constraints of interval membership grades in the T-S fuzzy model, the proposed control system has been proved to be asymptotic stable. And the stability conditions have been carried out to guarantee its stability.

According to the analysis results, since the proposed control method has been guaranteed to be stable with required conditions, it provides a theoretical foundation for further experimental study and industrial application development.

Acknowledgment

The authors wish to thank the financial support by the Program for Liaoning Innovative Research Team in University (LNIRT) (2009T062, LT2010058), Program for Liaoning Excellent Talents in University (LNET) (LJQ2011032), and the University of Portsmouth.

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Jiangtao Cao received his M.S. in Control Engineering from the Liaoning Shihua University, China, and his Ph.D. degree in Engineering from the University of Portsmouth, UK, in 2003 and 2009, respectively. He previously held research appointments at the University of Portsmouth and Tokyo Metropolitan University. He is currently a Full Professor in the School of Information and Control Engineering, Liaoning Shihua University.

His research interests include intelligent control, humanoid robot and biometric recognition methods. He is an active

member of IEEE Computational Intelligence Society.



Xiaofei Ji received her M.S. in Control Engineering from the Liaoning Shihua University, China, and his Ph.D. degree in Computer Science from the University of Portsmouth, UK, in 2003 and 2010, respectively. She is currently a Lecturer in the School of Automation, Shenyang Aerospace University.

Her research interests include human motion analysis, computer vision, pattern recognition and vision surveillance. She is an active member of IEEE Computational Intelligence Society.



Ping Li received his M.S. in Control Engineering from the Northwestern Polytechnical University, China, and his Ph.D. degree in Control Engineering from the Zhejiang University, China, in 1988 and 1995, respectively. He is currently a Full Professor in the School of Information and Control Engineering, Liaoning Shihua University

and the President of Liaoning Shihua University.

His research interests include advanced process control, intelligent control and optimization methods. He has published more than 150 refereed journal and conference papers in the above area. He is a Senior Member of IEEE.



Honghai Liu received his Ph.D. degree in Intelligent Robotics from Kings College, University of London, UK. He previously held research appointments at the Universities of London and Aberdeen, and project leader appointments in large-scale industrial control and system integration industry. He is currently a Full Professor and the Head

of Intelligent Systems and Biomedical Robotics Group, School of Creative Technologies, at the University of Portsmouth.

He is interested in approximate computation, pattern recognition and intelligent robotics and their practical applications in cognitive systems, intelligent video analytics and healthcare with an emphasis on approaches which could make contribution to the intelligent connection of perception to action using contextual information. He is also a Senior Member of IEEE and a Fellow of IET.