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Anomalous subharmonics excited by intensive ultrasonic pulses with a single frequency

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Anomalous subharmonics named as quasisubharmonics (QSHs) in plates excited by intensive ultrasonic pulses are first observed, in which the QSH frequencies are incommensurable with the fundamental frequency. The experimental results show that the low (such as 1/3) order subharmonics appear in the transient process of the ultrasonic excitation, and then several QSHs successively appear as the ultrasonic power increases and approaches steady state, in which the first QSH appears near the system eigenfrequency. A phenomenological model is presented and the theoretical simulations indicate that the QSHs are produced by the intermittent contacts between the transducer horn and the plate. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2937405]

Subharmonic (SH) phenomena excited by intensive ultrasonic pulses have been studied in crack detections, in which the frequencies of SH series are integer submultiples of the fundamental frequency (FF).[1–4] In our experiments on SH phenomena, anomalous SH phenomena with frequencies incommensurable with the FF are first observed in metal plates excited by intensive ultrasonic pulses with a single frequency. We call the phenomena quasi-SH (QSHs). The QSHs have not been described by classical nonlinear oscillation theories, so a phenomenological model is presented to simulate the QSHs theoretically.

The experimental system is schematically shown in Fig. 1.[5] A stainless steel plate with $200\times100\times2$ mm^3 is fixed with its two shorter edges to the bracket and excited by an ultrasonic transducer horn pushed on the central area of the plate. The ultrasonic pulse with the frequency of about 20 kHz and the width of 230 ms is produced by a generator with the power of about 900 W. The vibration velocity is detected about 10 mm nearby the epicenter by a laser vibrometer. The velocity record starts about 15 ms before the ultrasonic operation, and the sampling frequency is 128 kHz.

A typical waveform is shown in Fig. 2(a). A short time Fourier transform (STFT) is taken by selecting a duration of 8 ms in the steady state (at 200 ms) of the waveform, and a vibration velocity spectrum (0–19.94 kHz) is obtained, as shown in Fig. 2(b). Several QSHs can be observed in the spectrum. The QSH frequencies are not integer submultiples of the FF and two neighboring frequency intervals may be different from each other. However, multiple-frequency phenomena can be observed. For example, the frequencies 7.35 and 11.00 kHz are two and three times of the 3.67 kHz, respectively. Meanwhile, the QSHs are in symmetrical distributions with the central frequency 9.97 kHz. Besides, the spectrum can be repeatedly observed in the ranges of 19.94–39.88 and 39.88–59.82 kHz, etc., which can be regarded as the periodicity.

The time-frequency spectra of the QSHs obtained by the continuous STFT are shown in Fig. 3, in which the QSHs are successively produced and gradually approach steady state. Once the intensive ultrasonic excitation starts, the magnitude of the FF rapidly approaches saturation. Then the increasing ultrasonic power will excite a series of QSHs. The QSHs at 3.67 and 16.27 kHz are first excited and approach saturation. They are called the first QSHs (FQSHs), and one of them sometimes has the magnitude larger than that of the fundamental. When FQSHs approach saturation, other QSHs appear. The magnitudes of them gradually increase but cannot apparently approach saturation because the surplus ultrasonic energy may be insufficient. From the generation processes of the QSHs shown in Fig. 3(b), it can be deduced that the QSHs are produced successively by the FQSHs combining with the fundamental.

From Fig. 3, two additional phenomena should be mentioned.

First, the classical SH with 1/3 order at 6.66 kHz almost appears immediately as the ultrasonic excitation starts. However, it decreases greatly as the FQSHs approach to saturation. Considering the magnitude of the velocity in Fig. 2(a), it may indicate that the 1/3 order SH is excited by a weaker ultrasonic intensity, but the QSH phenomena gradually become strong and replace the SH vibration when the ultrasonic intensity increases.

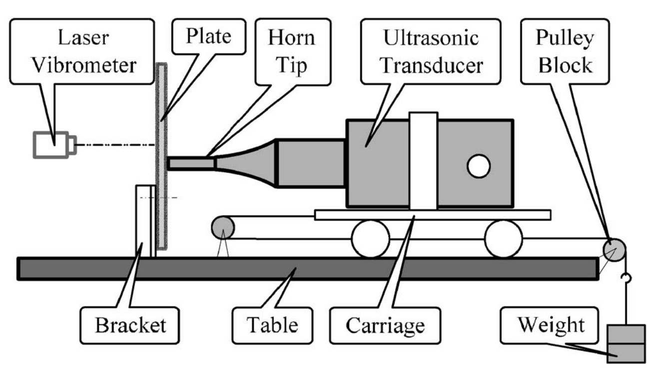

FIG. 1. Schematic diagram of experimental setup.

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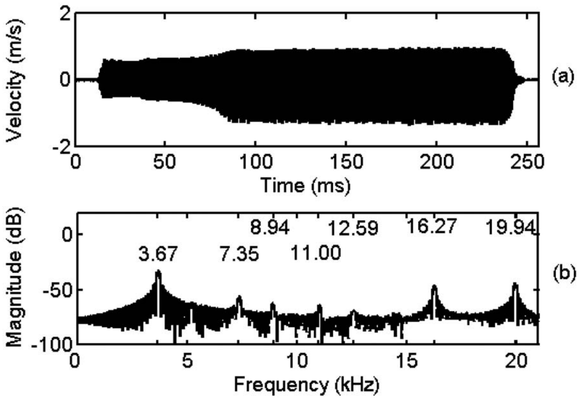

FIG. 2. Stainless steel plate excited by intensive ultrasonic pulse, (a) velocity waveform, and (b) QSH spectrum in 0–20 kHz.

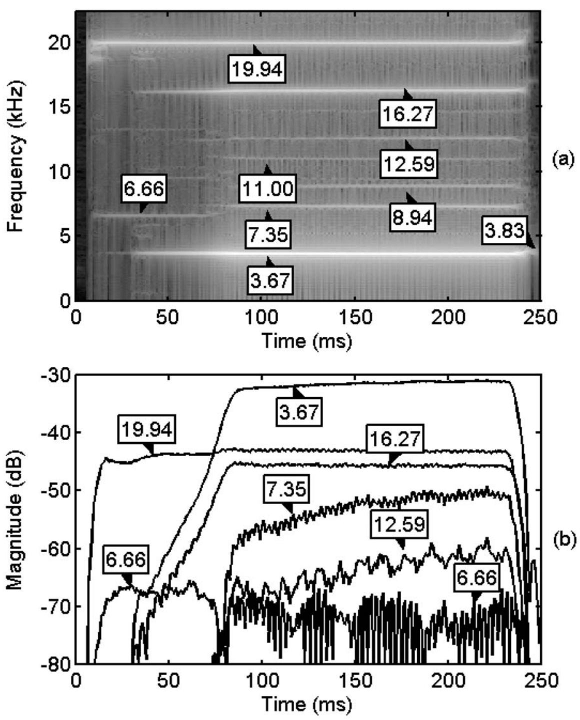

FIG. 3. Vibration spectra of plate excited by ultrasonic pulse: (a) spectrum distribution and (b) generation process and variation.

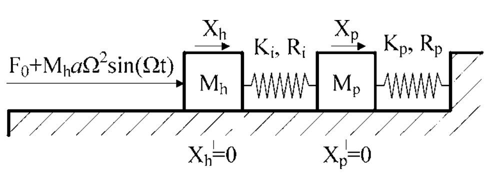

FIG. 4. Phenomenological model of plate vibration excited by intensive ultrasonic pulse.

Second, as the ultrasonic excitation stops at 245 ms, the fundamental and all QSHs disappear at once, but a vibration at 3.83 kHz appears and then attenuates slowly. Therefore the 3.83 kHz should be the eigenfrequency of the experimental system, and one of the FQSHs appears near the eigenfrequency.

In addition, as the ultrasonic coupling condition and the pushing force between the horn and the plate change, the frequencies of the FQSHs slightly change, so does the spectrum. If the FQSH occasionally appears at one of the fractions of the FF, the SH phenomenon can be observed.

The QSHs can also be observed in different stainless steel plates and aluminum alloy plates excited by the intensive ultrasonic pulses, but the QSH spectra will be different. However, if the sample shapes or experimental conditions change, for example, the sizes of the plate greatly change, a different mounting method is applied, or the coupled ultrasonic energy changes largely, the general SHs and/or chaos will be excited.[1]

To theoretically study the generation mechanism of QSHs, a phenomenological model is presented. The interaction of the transducer horn and the plate is simplified as impacts of two oscillators. The horn is regarded as an active oscillator with a mass M_h, a displacement x_h, and a harmonic vibration $a\sin(\Omega t)$. The plate is regarded as a passive oscillator with a mass M_p, a displacement x_p, a stiffness coefficient K_p, a damping coefficient R_p, and a nonlinearity coefficient ξ_p. A constant force F_0 pushes the horn toward the plate after each elastic impact. During the impact process, the interaction is simplified as an inextensible spring behavior with an equivalent stiffness coefficient K_i and a damping coefficient R_i. The theoretical model is shown in Fig. 4, in which the origins of x_p and x_h are their equilibrium positions.

When the transducer horn excites the plate, if $x_h - x_p \geqslant 0$, the plate and the horn are in contact. It is assumed that the interacting force is proportional to the distance $(x_h - x_p)$ with the stiffness coefficient K_i and the damping force is proportional to the velocity difference $(\dot{x}_h - \dot{x}_p)$ with the damping coefficient R_i. Since $(x_h - x_p)$ is much smaller, its high order behavior of is ignored. In this case, the motions of the horn and the plate can be described by

$$\begin{cases} M_h\ddot{x}_h = F_0 + M_h a\Omega^2\sin(\Omega t) - R_i(\dot{x}_h - \dot{x}_p) - K_i(x_h - x_p) \\ M_p\ddot{x}_p = R_i(\dot{x}_h - \dot{x}_p) + K_i(x_h - x_p) - R_p\dot{x}_p - K_p(x_p + \xi x_p^3) \end{cases} . \tag{1}$$

When $x_h - x_p < 0$, the plate and the horn move separately, and the motions can be described by

$$\begin{cases} M_h\ddot{x}_h = F_0 + M_h a\Omega^2\sin(\Omega t) \\ M_p\ddot{x}_p = -R_p\dot{x}_p - K_p(x_p + \xi x_p^3) \end{cases} . \tag{2}$$

The initial conditions are $x_h = x_p = \dot{x}_h = \dot{x}_p = 0$, at $t = 0$.

Based on the experimental conditions, the mass of the plate M_p is 0.314 kg, the amplitude $a\Omega$ of the vibration ve-

TABLE I. Parameter of theoretical model.

K_i (N/m)	R_i (N s/m)	M_h (kg)	F_0 (N)	K_p (N/m)	R_p (N s/m)	ξ_p (m^{-2})
2.1854×10^{11}	1.4114×10^{5}	2.8080	4.7456×10^{4}	1.4493×10^{9}	1.5683×10^{4}	4.7374×10^{7}

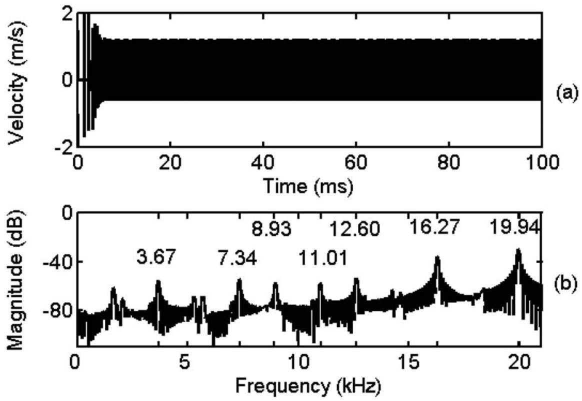

FIG. 5. Theoretical simulation of plate vibration under intensive ultrasonic pulse excitation: (a) velocity waveform and (b) QSH spectrum in 0–20 kHz.

locity and the frequency $\Omega/2\pi$ of the horn are 10 m/s and 19.94 kHz, respectively. The other parameters of the model can be simulated and are listed in Table I.

The velocity waveform of the plate excited by the horn is simulated, as shown in Fig. 5(a). The Fourier spectrum of the plate vibration velocity taken from the waveform (at 80 ms) is calculated, as shown in Fig. 5(b). The result shows that the calculated QSHs are in excellent agreement with those of the experiments shown in Fig. 2(b). Furthermore, the simulated displacement waveforms of the horn and the plate are shown in Fig. 6(a), which demonstrates that both oscillators experience alternately separating or contacting processes. When the horn contacts with the plate, the eigenfrequency of a double-oscillator system can be obtained by

$$\omega^2 = [(\mu\omega_i^2 + \omega_p^2 + \omega_i^2) \pm \sqrt{(\mu\omega_i^2 + \omega_p^2 + \omega_i^2)^2 - 4\omega_i^2\omega_p^2}]/2, \quad (3)$$

where $\omega_i^2 = K_i/M_h$, $\omega_p^2 = K_p/M_p$, $\mu = M_h/M_p$, so the calculated eigenfrequency is 4.83 kHz. If the damping and/or nonlinearity effects are considered, the eigenfrequency may be reduced and approach to 3.83 kHz, as observed in the experiment. When the plate and the horn are separated, the eigenfrequency of the plate oscillator is $f_p = (1/2\pi)\sqrt{K_p/M_p} = 15.34$ (kHz). It is close to 16.27 kHz obtained in the experiment and the sum (20.17 kHz) of both eigenfrequencies is close to the FF (19.94 kHz). Therefore, the QSHs are produced by the alternate contacts and separations between the horn and the plate under the excitation of intensive ultrasonic pulses.

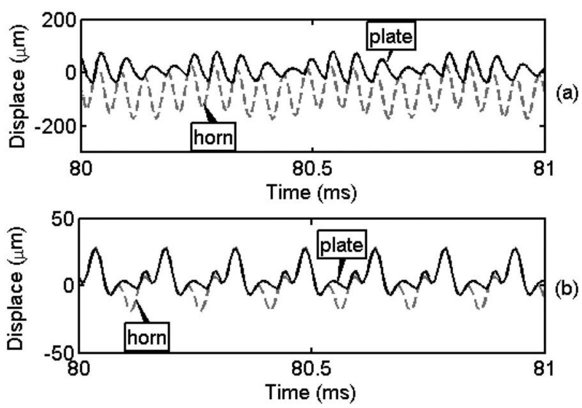

FIG. 6. Theoretical simulation of displacement waveform of horn and plate: (a) $a\Omega = 10$ m/s and (b) $a\Omega = 1.08$ m/s.

Besides, when the ultrasonic intensity is weaker (such as $a\Omega = 1-1.5$ m/s), the theoretical simulated displacement waveform is shown as Fig. 6(b). The impact processes repeatedly appear in every interval of three excitation periods. Thus, the 1/3 order SH appears. Figures 6(a) and 6(b) illustrate theoretically that the QSH phenomena will be excited and replace the SH vibrations as the ultrasonic intensity increases.

In conclusion, except that the low order SHs appear in the transient process of the excitation, the QSHs are observed firstly in the stable process of the excitation in the experiments of plate vibrations excited by the intensive ultrasonic pulses. The QSHs are successively produced and approach to saturation. Especially, one of the FQSHs has very high magnitude, even higher than that of the fundamental. The other QSHs are produced by the combinations of the fundamental with the FQSHs. The QSHs have several anomalous characteristics: the frequencies are incommensurable with the FF, and two neighboring frequency intervals may be different from each other, but the QSHs have multiple frequency, symmetry, and periodicity. Generally, QSHs can be described as two series: one is represented by multiples of the FQSH and the other by the multiples of the FQSH subtracted from the FF. In order to explain the phenomena, a phenomenological model is presented and the theoretical simulations indicate that the intermittent contacts between the plate and the transducer horn induce the FQSH near the egeinfrequency of the experimental system, and then the QSHs are produced by the combinations of the fundamental and the FQSHs.

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[1]X. Han, W. Li, Z. Zeng, L. D. Favro, and R. L. Thomas, Appl. Phys. Lett. **81**, 3188 (2002).
[2]A. Moussatov, V. Gusev, and B. Castagnede, Phys. Rev. Lett. **90**, 124301 (2003).
[3]I. Y. Solodov, J. Wackerl, K. Pfleiderer, and G. Busse, Appl. Phys. Lett. **84**, 5386 (2004).
[4]Y. Ohara, T. Mihara, R. Sasaki, T. Ogata, S. Yamamoto, Y. Kishimoto, and K. Yamanaka, Appl. Phys. Lett. **90**, 011902 (2007).
[5]L. D. Favro, X. Han, Z. Ouyang, G. Sun, and R. L. Thomas, Anal. Sci. **17** special issue of the 11th International Conference on Photoacoustic and Phtothermal Phenomena, s451 (2001).