



An impulsive predator–prey model with disease in the prey for integrated pest management

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ABSTRACT

In this paper, an impulsive predator–prey model with disease in the prey is investigated for the purpose of integrated pest management. In the first part of the main results, we get the sufficient condition for the global stability of the susceptible pest-eradication periodic solution. This means if the release amount of infective prey and predator satisfy the condition, then the pest will be doomed. In the second part of the main results, we also get the sufficient condition for the permanence of the system. This means if the release amount of infective prey and predator satisfy the condition, then the prey and the predator will coexist. In the last section, we interpret our mathematical results. We also point out some possible future work.

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1. Introduction

Pests outbreak often cause serious ecological and economic problems, and the warfare between human and pests has sustained for thousands of years. With the development of society and the progress of science and technology, man has adopted some advanced and modern weapons such as chemical pesticides, biological pesticides, remote sensing and measuring, and so on. Some brilliant achievements have been obtained. However, the warfare is not over, and will continue. A great deal of and a large variety of pesticides were used to control pests, because they can quickly kill a significant portion of a pest population and sometimes provide the only feasible method for preventing economic loss. However, pesticide pollution is also recognized as a major health hazard to human beings and beneficial insects. At present, more and more people are concerned about the effects of pesticide residues on human health and on the environment. In this regard, it has been observed that beneficial insects are often more susceptible to chemical pesticides than the target pests are. In the same time, the concentration of the pesticides in use tends to increase with time and usage, since many pests develop resistance to these chemicals (see [16,17] and the references therein).

An alternative to chemical control is biological control, including microbial control with pathogens, as diseases can be important natural controls of some pests. Insects, like humans and other animals, can be infected by disease-causing organisms such as bacteria, viruses and fungi. Under appropriate conditions, such as high humidity or high pest abundance, these

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naturally occurring organisms may multiply to cause disease outbreaks or epizootics that can decimate an insect population. Insect pathogens are used in two ways. In the first method, a small amount of pathogen is introduced in a pest population with the expectation that it will generate an epidemic which will persist at an endemic level. In the second method, an insect pathogen is used like biopesticides. In this case, the pathogen is applied whenever a pest population is at an economically significant level and there is no expectation that the pathogen will persist in the environment for a long time. There is a vast amount of literatures on the applications of entomopathogens to suppress pests [2,4,6,20]. One of the first successful cases of biological control in greenhouses was the use of the parasitoid *Encarsia formosa* against the greenhouse whitefly *Trialeurodes vaporariorum* on tomatoes and cucumbers [22,23]. There are also papers and books on mathematical models of the dynamics of microbial diseases in pest control [9,10,17,24,25].

People also use natural enemy to control pest or regulate it to densities below the threshold for economical damage. Often with augmentation or release, the natural enemy is applied like a pesticide after the pest has reached or exceed the economic threshold. There are many literatures concerning natural enemy for pest control [5,7,18,22].

Integrated pest management or IPM is a long term management strategy that uses a combination of biological, cultural, and chemical tactics to reduce pests to tolerable levels, with little cost to the grower and minimal effect on the environment (for more details, we can see [3,19]). IPM which has been proved by experiment [21] is more effective than classical one (such as biological control or chemical control).

Systems with impulsive effects describing evolution processes are characterized by the fact that at certain moments of time they abruptly experience a change of state. Processes of such type are studied in almost every domain of applied science. Impulsive equations [1,13] have been recently used in population dynamics in relation to impulsive vaccination [11], population ecology [14,15], the chemotherapeutic treatment of disease [12], the theory of the chemostat [8].

In this paper, we will consider the integrated pest management by using microbial control with pathogens and releasing natural enemy together. That is, we release infective pests and natural enemy to suppress the pest. The infective pests can be cultivated in the laboratory and the natural enemy can migrated from other regions. Once the susceptible pest meets with the infective pest, there is a chance to be infected. The infective pests have more possibility to death. We suppose the natural enemy eat the susceptible pest, but do not eat the infective pest. In our model, we call the pest and the natural enemy as prey and predator, respectively. For mathematically simple, we set several assumptions in the models. Details are shown in next section.

The main purpose of this paper is to construct an impulsive predator–prey model with biocontrol for pest management. The present paper is organized as follows. In Section 2, the model is constructed and the main biological assumptions are formulated. In Section 3, by using the Floquet's theory for impulsive differential equations, small-amplitude perturbation methods and comparison techniques, we investigate the global asymptotic stability of the susceptible pest-eradication periodic solution and the conditions for the permanence of the system. Finally, a brief discussion and some possible future work for pest management are provided in the last section.

2. Model formulation

In this paper, we study the following model for integrated pest management.

$$\left\{ \begin{array}{l} S'(t) = rS(t) \left(1 - \frac{S(t)+I(t)}{K} \right) - \beta S(t)I(t) - \frac{aS(t)y(t)}{1+\omega S(t)}, \\ I'(t) = \beta S(t)I(t) - d_1 I(t), \\ y'(t) = \frac{\delta a S(t)y(t)}{1+\omega S(t)} - d_2 y(t), \end{array} \right\} t \neq n\tau, \quad (1)$$

$$\left\{ \begin{array}{l} \Delta S(t) = 0, \\ \Delta I(t) = p, \\ \Delta y(t) = q, \end{array} \right\} t = n\tau, \quad n \in Z_+ = \{1, 2, 3, \dots\}.$$

Here $S(t)$, $I(t)$ represent the densities of the susceptible prey (pest) population and the infective prey (pest) population, respectively. $y(t)$ is the density of predator (natural enemy) population. The model is derived with the following assumptions.

- (H_1) The prey population grow with Logistic rate, $r > 0$ is the intrinsic growing rate, $K > 0$ is the carrying capacity.
- (H_2) There is a disease among the prey population, and the prey is divided into two classes, susceptible and infective. The incidence rate is the classic bilinear $\beta S(t)I(t)$, β is the contact number per unit time for every infective prey with susceptible prey.
- (H_3) The predator only catch susceptible prey, and the predation functional response is Holling type II. The parameters a, ω are positive constants, and δ is the conversing rate from the prey to the predator.
- (H_4) d_1, d_2 is the death rate for infective prey and predator, respectively.
- (H_5) At time $t = n\tau$, $n \in Z_+$, we release infective prey population and predator population periodically with releasing amount p ($p > 0$) and q ($q > 0$), respectively.

3. Preliminary

We give some definitions, notations and lemmas which will be useful for stating and proving our main results. Let $R_+ = [0, \infty)$, $R_+^3 = \{x = (x_1, x_2, x_3) \in R^3 : x_1, x_2, x_3 > 0\}$.

Denote $f = (f_1, f_2, f_3)$ the mapping defined by the right-hand side of system (1).

Let $V : R_+ \times R^3 \rightarrow R_+$. Then V is said to belong to class V_0 if

(i) V is continuous in $(n\tau, (n+1)\tau] \times R^3$ and for each $x \in R_+^3$, $n \in Z_+ = \{1, 2, 3, \dots\}$ and the limit

$$\lim_{(t,y) \rightarrow (n\tau^+, x)} V(t,y) = V(n\tau^+, x)$$

exists and is finite.

(ii) V is locally Lipschitzian in x .

Definition 3.1. For $V \in V_0$ and $(t, x) \in (n\tau, (n+1)\tau] \times R^3$, the upper right Dini derivative of $V(t, x)$ with respect to the impulsive differential system (1) is defined as

$$D^+V(t, x) = \lim_{h \rightarrow 0^+} \sup \frac{1}{h} [V(t+h, x+hf(t, x)) - V(t, x)].$$

Definition 3.2. System (1) is said to be permanent if there exists a compact region $D \in \text{int}R_+^3$ such that every solution of system (1) with positive initial values will eventually enter and remain in region D .

The solution of system (1), denoted by $X(t) = (S(t), I(t), y(t)) : R_+ \rightarrow R_+^3$, is continuously differentiable on $(n\tau, (n+1)\tau] \times R^3$, $n \in Z_+$ and the limit $X(n\tau^+) = \lim_{t \rightarrow n\tau^+} X(t)$ exists and is finite for $n \in Z_+$. Obviously, the global existence and the uniqueness of solution of system (1) are guaranteed by the smoothness properties of f (see [1,13] for details on fundamental properties of impulsive systems). The proofs of the following lemmas are obvious.

Lemma 3.3. Suppose that $X(t)$ is a solution of (1) with $X(0^+) \geq 0$. Then $X(t) \geq 0$ for all $t > 0$. Further, if $X(0^+) > 0$ then $X(t) > 0$ for all $t > 0$.

Lemma 3.4 [13]. Let $V : R_+ \times R^3 \rightarrow R$ and $V \in V_0$. Assume that

$$\begin{cases} D^+V(t, X) \leq g(t, V(t, X)), & t \neq n\tau, \\ V(t, X(t^+)) \leq \Psi_n(V(t, X(t))), & t = n\tau, \end{cases}$$

where $g : R_+ \times R_+ \rightarrow R$ is continuous in $(n\tau, (n+1)\tau] \times R_+$ and for each $v \in R_+^3$, $n \in Z_+$

$$\lim_{(t,y) \rightarrow (n\tau^+, v)} g(t,y) = g(n\tau^+, v)$$

exists and is finite. $\Psi_n : R_+ \rightarrow R_+$ is nondecreasing. Let $R(t)$ be the maximal solution of the scalar impulsive differential equation

$$\begin{cases} U'(t) = g(t, U), & t \neq n\tau, \\ U(t^+) = \Psi_n(U(t)), & t = n\tau, \\ U(0^+) = U_0, \end{cases}$$

defined on $[0, \infty)$. Then $V(0^+, X_0) \leq U_0$ implies that $V(t, X(t)) \leq R(t)$, $t \geq 0$, where $X(t)$ is any solution of system (1).

Lemma 3.5. There exists a positive constant M such that $S(t) \leq M$, $I(t) \leq M$, $y(t) \leq M$, for each solution $(S(t), I(t), y(t))$ of system (1) with positive initial values, where t is large enough.

Proof. Define a function V such that

$$V(t) = S(t) + I(t) + y(t).$$

By simple computation, we see that when $t \neq n\tau$,

$$D^+V|_{(1)} + dV = (r+d)S(t) - rS(t) \frac{S(t)+I(t)}{K} - (d_1-d)I(t) - (d_2-d)y(t) - \frac{(1-\delta)ay(t)}{1+\omega S(t)} \leq (r+d)S(t) - \frac{rS^2(t)}{K},$$

where $d = \min\{d_1, d_2\}$. Obviously, the right-hand side of the above equality is bounded from above for all $(S(t), I(t), y(t)) \in R_+^3$. Hence there exists a positive constant λ such that

$$D^+V|_{(1)} + dV < \lambda \quad \text{for } t \neq n\tau.$$

From the fourth, fifth, sixth equations of system (1) we see that,

$$V(n\tau^+) = V(n\tau) + \mu,$$

where $\mu = p + q$. According to Lemma 2.2, [1], we derive

$$V(t) = V(0)e^{-dt} + \int_0^t \lambda e^{-d(t-s)} ds + \sum_{0 < k\tau < t} \mu e^{-d(t-k\tau)} \rightarrow \frac{\lambda}{d} + \frac{\mu e^{d\tau}}{e^{d\tau} - 1}, \text{ as } t \rightarrow \infty.$$

Consequently, by the definition of $V(t)$ we obtain that each solution of (1) with positive initial values is uniformly ultimately bounded. This completes the proof. \square

Lemma 3.6. *System*

$$\begin{cases} u'(t) = -wu(t), & t \neq n\tau, \\ \Delta u(t) = \mu, & t = n\tau, \end{cases} \quad (2)$$

has a positive periodic solution $u^*(t)$, and for every solution $u(t)$ of this system with positive initial value $u(0^+)$, $|u(t) - u^*(t)| \rightarrow 0$ as $t \rightarrow \infty$, where

$$u^*(t) = \frac{\mu e^{-w(t-n\tau)}}{1 - e^{-w\tau}}$$

and

$$u^*(0^+) = \frac{\mu}{1 - e^{-w\tau}}.$$

Proof. The proof is obvious, in fact, since the solution of (2) is

$$u(t) = \left(u(0^+) - \frac{\mu}{1 - e^{-w\tau}}\right)e^{-wt} + u^*(t), \quad n\tau < t \leq (n+1)\tau. \quad \square$$

4. Main results

When $S(t) \equiv 0$ for all $t \geq 0$, we get the subsystem of system (1)

$$\begin{cases} \begin{cases} I'(t) = -d_1 I(t), \\ y'(t) = -d_2 y(t), \end{cases} & t \neq n\tau, \\ \begin{cases} \Delta I(t) = p, \\ \Delta y(t) = q, \end{cases} & t = n\tau. \end{cases} \quad (3)$$

In this system, we can see there is no relation between $I(t)$ and $y(t)$. Thus, we can solve them independently. By Lemma 3.6, we get the following result.

Theorem 4.1. *System (3) has a unique positive periodic solution*

$$I^*(t) = \frac{pe^{-d_1(t-n\tau)}}{1 - e^{-d_1\tau}}, \quad y^*(t) = \frac{qe^{-d_2(t-n\tau)}}{1 - e^{-d_2\tau}}, \quad \text{for } t \in (n\tau, (n+1)\tau],$$

where

$$I^*(0^+) = \frac{p}{1 - e^{-d_1\tau}}, \quad y^*(0^+) = \frac{q}{1 - e^{-d_2\tau}}.$$

In addition, for every solution of system (3) with initial values $I(0^+) > 0$, $y(0^+) > 0$, it follows that $I(t) \rightarrow I^*(t)$, $y(t) \rightarrow y^*(t)$ as $t \rightarrow \infty$.

Thus, the complete expression for the susceptible pest-eradication periodic solution of system (1) is obtained as $(0, I^*(t), y^*(t))$, $t \in (n\tau, (n+1)\tau]$, $n \in \mathbb{Z}_+$. The following theorems are results about the stability and attraction of the susceptible pest-eradication periodic solution $(0, I^*(t), y^*(t))$.

Theorem 4.2. *If $r\tau < (\frac{r}{K} + \beta)\frac{p}{d_1} + \frac{aq}{d_2}$, then the periodic solution $(0, I^*(t), y^*(t))$ is locally asymptotically stable for system (1).*

Proof. To prove the local stability of this periodic solution, we use small-amplitude perturbation methods. Let

$$S(t) = u(t), \quad I(t) = v(t) + I^*(t), \quad y(t) = w(t) + y^*(t),$$

where $u(t)$, $v(t)$, $w(t)$ are small perturbations. Then system (1) can be linearized by using Taylor expansions and after neglecting higher-order terms, the linearized equations read as

$$\left\{ \begin{array}{l} u'(t) = u(t)[r - (\frac{r}{K} + \beta)I^*(t) - ay^*(t)], \\ v'(t) = \beta I^*(t)u(t) - d_1 v(t), \\ w'(t) = \delta ay^*(t)u(t) - d_2 w(t) \end{array} \right\} \quad t \neq n\tau, \quad (4)$$

$$\left\{ \begin{array}{l} u(n\tau^+) = u(n\tau), \\ v(n\tau^+) = v(n\tau), \\ w(n\tau^+) = w(n\tau), \end{array} \right\} \quad t = n\tau.$$

Let $\Phi(t)$ be the fundamental solution matrix of (4). Then $\Phi(t)$ must satisfy

$$\frac{d\Phi(t)}{dt} = \begin{pmatrix} r - (\frac{r}{K} + \beta)I^*(t) - ay^*(t) & 0 & 0 \\ \beta I^*(t) & -d_1 & 0 \\ \delta ay^*(t) & 0 & -d_2 \end{pmatrix} \Phi(t),$$

$\Phi(0) = I_3$ is the identical matrix. Hence the fundamental solution matrix is

$$\Phi(t) = \begin{pmatrix} e^{\int_0^t [r - (\frac{r}{K} + \beta)I^*(t) - ay^*(t)]dt} & 0 & 0 \\ * & e^{-d_1 t} & 0 \\ * & 0 & e^{-d_2 t} \end{pmatrix}.$$

Also, the fourth, fifth and sixth equations in (4) read as

$$\begin{pmatrix} u(n\tau^+) \\ v(n\tau^+) \\ w(n\tau^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u(n\tau) \\ v(n\tau) \\ w(n\tau) \end{pmatrix}.$$

Hence, if all eigenvalues of

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Phi(T)$$

have absolute values less than one, then the periodic solution $(0, I^*(t), y^*(t))$ is locally asymptotically stable. Since the eigenvalues of M are

$$\lambda_1 = e^{-d_2 T} < 1, \quad \lambda_2 = e^{-d_1 T} < 1, \quad \lambda_3 = e^{\int_0^T [r - (\frac{r}{K} + \beta)I^*(t) - ay^*(t)]dt},$$

it follows that $|\lambda_3| < 1$ if and only if $r\tau < (\frac{r}{K} + \beta)\frac{p}{d_1} + \frac{aq}{d_2}$ holds. According to the Floquet theory of impulsive differential equations, in this situation, the susceptible pest-eradication periodic solution $(0, I^*(t), y^*(t))$ is locally asymptotically stable. The proof is complete. \square

Theorem 4.3. If $r\tau < (\frac{r}{K} + \beta)\frac{p}{d_1} + \frac{aq}{1+\omega K}\frac{q}{d_2}$, then the periodic solution $(0, I^*(t), y^*(t))$ is globally asymptotically stable for system (1).

Proof. By the given condition and Theorem 4.2, it is easy to know that $(0, I^*(t), y^*(t))$ is locally asymptotically stable. Therefore, we only need to prove its global attraction. Since $r\tau < (\frac{r}{K} + \beta)\frac{p}{d_1} + \frac{aq}{1+\omega K}\frac{q}{d_2}$, we can choose a ε_1 small enough such that

$$\int_0^\tau \left[r - \left(\frac{r}{K} + \beta \right) (I^*(t) - \varepsilon_1) - \frac{a}{1 + \omega K} (y^*(t) - \varepsilon_1) \right] dt = \sigma < 0.$$

Besides, we have

$$I'(t) = \beta S(t)I(t) - d_1 I(t) \geq -d_1 I(t).$$

From Lemmas 3.4 and 3.6, there exists a n_1 such that

$$I(t) \geq I^*(t) - \varepsilon_1, \quad \text{for } t \geq n_1 \tau. \quad (5)$$

Similarly, there exists a n_2 ($n_2 > n_1$) such that

$$y(t) \geq y^*(t) - \varepsilon_1, \quad \text{for } t \geq n_2 \tau. \quad (6)$$

Thus, for $t \geq n_2 \tau$, we have

$$\begin{aligned} S'(t) &= rS(t) \left(1 - \frac{S(t) + I(t)}{K} \right) - \beta S(t)I(t) - \frac{aS(t)y(t)}{1 + \omega S(t)} \leq rS(t) - \left(\frac{r}{K} + \beta \right) S(t)I(t) - \frac{aS(t)y(t)}{1 + \omega K} \\ &\leq S(t) \left[r - \left(\frac{r}{K} + \beta \right) (I^*(t) - \varepsilon_1) - \frac{a(y^*(t) - \varepsilon_1)}{1 + \omega K} \right]. \end{aligned}$$

From the above inequality, we get

$$S(t) \leq S(n_2\tau) e^{\int_{n_2\tau}^t \left[r - \left(\frac{r}{K} + \beta \right) (I^*(t) - \varepsilon_1) - \frac{a(y^*(t) - \varepsilon_1)}{1 + \omega K} \right] dt} \leq S(n_2\tau) e^{k\sigma},$$

where $t \in ((n_2 + k)\tau, (n_2 + k + 1)\tau]$, $k \in \mathbb{Z}_+$. Since $\sigma < 0$, we can easily see that $S(t) \rightarrow 0$ as $k \rightarrow +\infty$. Thus, for an arbitrary positive constant ε_2 small enough, there exists a n_3 ($n_3 > n_2$) such that $S(t) < \varepsilon_2$ for all $t \geq n_3\tau$. From which we get

$$I'(t) = \beta S(t)I(t) - d_1 I(t) \leq (\beta \varepsilon_2 - d_1)I(t).$$

From Lemmas 3.4 and 3.6, there exists a n_4 ($n_4 > n_3$) such that

$$I(t) \leq I_2^*(t) + \varepsilon_1, \quad \text{for } t \geq n_4\tau, \quad (7)$$

where $I_2^*(t) = \frac{pe^{-(d_1 - \beta \varepsilon_2)(t - k\tau)}}{1 - e^{-(d_1 - \beta \varepsilon_2)\tau}}$, for $t \in (k\tau, (k + 1)\tau]$, $k \in \mathbb{Z}_+$.

By similarly argument, there exists a n_5 ($n_5 > n_4$) such that

$$y(t) \leq y_2^*(t) + \varepsilon_1, \quad \text{for } t \geq n_5\tau, \quad (8)$$

where $y_2^*(t) = \frac{qe^{-(d_2 - \beta \varepsilon_2)(t - k\tau)}}{1 - e^{-(d_2 - \beta \varepsilon_2)\tau}}$, for $t \in (k\tau, (k + 1)\tau]$, $k \in \mathbb{Z}_+$. Note that $\varepsilon_1, \varepsilon_2$ are positive constants small enough, and $I_2^*(t) \rightarrow I^*(t)$, $y_2^*(t) \rightarrow y^*(t)$, as $\varepsilon_2 \rightarrow 0$, together with Eqs. (5)–(8), we get $I(t) \rightarrow I^*(t)$ and $y(t) \rightarrow y^*(t)$ as $t \rightarrow +\infty$. Therefore, the periodic solution $(0, I^*(t), y^*(t))$ is globally asymptotically stable. \square

Corollary 4.4. (1) If $p = 0$, then the condition of Theorem 4.3 becomes

$$q > \frac{r\tau d_2(1 + \omega K)}{a} \doteq q_0. \quad (9)$$

(2) If $q = 0$, then the condition of Theorem 4.3 becomes

$$p > \frac{r\tau d_1}{\beta + \frac{r}{K}} \doteq p_0. \quad (10)$$

Theorem 4.5. If $r\tau > \left(\frac{r}{K} + \beta\right) \frac{p}{d_1} + \frac{aq}{d_2}$, then system (1) is permanent.

Proof. We will prove the theorem by several steps. By Lemma 3.5, without loss of generality, we can suppose $S(t), I(t), y(t) \leq M$ for all $t \geq 0$, for simplifying the next proof process.

Step 1. We will prove $I(t)$ and $y(t)$ are ultimately positively bounded below.

Firstly, from Eq. (5), we get $I(t) \geq I^*(t) - \varepsilon_1 \geq \frac{pe^{-d_1\tau}}{1 - e^{-d_1\tau}} - \varepsilon_1 \doteq m_1 > 0$, for $t \geq n_1\tau$.

Secondly, from Eq. (6), we get $y(t) \geq y^*(t) - \varepsilon_1 \geq \frac{qe^{-d_2\tau}}{1 - e^{-d_2\tau}} - \varepsilon_1 \doteq m_2 > 0$, for $t \geq n_2\tau$.

Therefore, $I(t)$ and $y(t)$ are ultimately positively bounded below.

Step 2. We will prove $S(t)$ is ultimately positively bounded below.

Since $r\tau > \left(\frac{r}{K} + \beta\right) \frac{p}{d_1} + \frac{aq}{d_2}$, we can select positive constant ε and m_3 small enough such that

$$\left[r \left(1 - \frac{m_3}{K} \right) - \left(\frac{r}{K} + \beta \right) \varepsilon - a\varepsilon \right] \tau - \left[\left(\frac{r}{K} + \beta \right) \frac{p}{d_1 - \beta m_3} + a \frac{q}{d_2 - \delta a m_3} \right] \doteq \sigma_2 > 0.$$

We claim that for an arbitrary $N_1 \in \mathbb{Z}_+$, $S(t) < m_3$ cannot hold for all $t \geq N_1\tau$. Otherwise, there exists a $N_1 \in \mathbb{Z}_+$, such that $S(t) < m_3$ for all $t \geq N_1\tau$. Then, we have

$$I'(t) = \beta S(t)I(t) - d_1 I(t) \leq (\beta m_3 - d_1)I(t)$$

$$y'(t) = \frac{\delta a S(t)y(t)}{1 + \omega S(t)} - d_2 y(t) \leq (\delta a m_3 - d_2)y(t)$$

for all $t \geq N_1\tau$. By Lemmas 3.4 and 3.6, we know there exists a N_2 ($N_2 > N_1$) such that

$$I(t) \leq I_2^*(t) + \varepsilon, \quad y(t) \leq y_2^*(t) + \varepsilon,$$

for all $t \geq N_2\tau$, where

$$I_2^*(t) = \frac{pe^{-(d_1 - \beta m_2)(t - k\tau)}}{1 - e^{-(d_1 - \beta m_2)\tau}}, \quad y_2^*(t) = \frac{qe^{-(d_2 - \delta a m_2)(t - k\tau)}}{1 - e^{-(d_2 - \delta a m_2)\tau}}$$

are the unique positive periodic solutions of systems (11) and (12)

$$\begin{cases} I'(t) = (\beta m_3 - d_1)I(t), & t \neq n\tau, \\ \Delta I(t) = p, & t = n\tau, \end{cases} \quad (11)$$

$$\begin{cases} y'(t) = (\delta a m_3 - d_2)y(t), & t \neq n\tau, \\ \Delta y(t) = q, & t = n\tau, \end{cases} \quad (12)$$

respectively. From which, we get

$$S'(t) = rS(t) \left(1 - \frac{S(t) + I(t)}{K} \right) - \beta S(t)I(t) - \frac{aS(t)y(t)}{1 + \omega S(t)} \geq S(t) \left[r - \frac{rm_3}{K} - \left(\frac{r}{K} + \beta \right) (I_2^*(t) + \varepsilon) - a(y_2^*(t) + \varepsilon) \right].$$

Thus,

$$S((N_2 + k)\tau) \geq S(N_2\tau) e^{\int_{N_2\tau}^{(N_2+k)\tau} \left[r - \frac{rm_3}{K} - \left(\frac{r}{K} + \beta \right) (I_2^*(t) + \varepsilon) - a(y_2^*(t) + \varepsilon) \right] dt} \geq S(N_2\tau) e^{k\sigma_2}.$$

We easily get $S((N_2 + k)\tau) \rightarrow +\infty$ as $t \rightarrow +\infty$. This is a contradiction with the boundedness of $S(t)$. Thus our claim is true, and for an arbitrary $N_1 \in \mathbb{Z}_+$, there exists at least a $t_1 \geq N_1\tau$ such that $S(t_1) \geq m_3$. There are two cases:

Case 1. $S(t) \geq m_3$ for all $t \geq t_1$. Then our aim is obtained. Otherwise, we consider the next case.

Case 2. We consider those solutions which leave the region $\Gamma = \{(S(t), I(t), y(t)) \in \mathbb{R}_+^3 : S(t) < m_3\}$ and reenter it again. Let $t^* = \inf_{t \geq t_1} \{S(t) < m_3\}$. Then $S(t) \geq m_3$ for $t \in [t_1, t^*]$ and $S(t^*) = m_3$ since $S(t)$ is continuous. Suppose $t^* \in (N_3\tau, (N_3 + 1)\tau]$, $N_3 \in \mathbb{Z}_+$. Select $N_4, N_5 \in \mathbb{Z}_+$ such that

$$N_4\tau > \max \left\{ \frac{-\ln \frac{\varepsilon}{2M}}{d_1 - \beta m_3}, \frac{-\ln \frac{\varepsilon}{2M}}{d_2 - \delta a m_3} \right\},$$

$$N_5\sigma_2 > \left(\frac{r}{K} + \beta + a \right) M(N_4 + 1)\tau.$$

Denote $T = (N_4 + N_5)\tau$. We claim that $S(t) < m_3$ cannot hold for all $t \in [(N_3 + 1)\tau, (N_3 + 1)\tau + T]$. Otherwise, $S(t) < m_3$ for all $t \in [(N_3 + 1)\tau, (N_3 + 1)\tau + T]$. Then

$$I'(t) = \beta S(t)I(t) - d_1 I(t) \leq (\beta m_3 - d_1)I(t)$$

holds for all $t \in [(N_3 + 1)\tau, (N_3 + 1)\tau + T]$. Consider the following system

$$\begin{cases} u'(t) = (\beta m_3 - d_1)u(t), & t \neq n\tau, \quad n = N_3 + 1, N_3 + 2, \dots, \\ u(n\tau^+) = u(n\tau^-) + p, & t = n\tau, \quad n = N_3 + 1, N_3 + 2, \dots, \end{cases} \quad (13)$$

with initial value $u((N_3 + 1)\tau^+) = I((N_3 + 1)\tau^+)$. Obviously, the solution of system (13) is

$$u(t) = I_2^*(t) + (u((N_3 + 1)\tau^+) - I_2^*)e^{-(d_1 - \beta m_3)(t - (N_3 + 1)\tau)}.$$

And

$$|u(t) - I_2^*(t)| \leq 2Me^{-(d_1 - \beta m_3)(t - (N_3 + 1)\tau)} < \varepsilon$$

for all $t \in [(N_3 + N_4 + 1)\tau, (N_3 + 1)\tau + T]$. So, by Lemma 3.4, we get

$$I(t) \leq u(t) \leq I_2^*(t) + \varepsilon, \quad (14)$$

for all $t \in [(N_3 + N_4 + 1)\tau, (N_3 + 1)\tau + T]$. Similarly, we have

$$y(t) \leq y_2^*(t) + \varepsilon, \quad (15)$$

for all $t \in [(N_3 + N_4 + 1)\tau, (N_3 + 1)\tau + T]$. Thus, we have

$$S'(t) = rS(t) \left(1 - \frac{S(t) + I(t)}{K} \right) - \beta S(t)I(t) - \frac{aS(t)y(t)}{1 + \omega S(t)} \geq S(t) \left[r - \frac{rm_3}{K} - \left(\frac{r}{K} + \beta \right) (I_2^*(t) + \varepsilon) - a(y_2^*(t) + \varepsilon) \right].$$

for all $t \in [(N_3 + N_4 + 1)\tau, (N_3 + 1)\tau + T]$. And

$$S((N_3 + 1)\tau + T) \geq S((N_3 + N_4 + 1)\tau) e^{\int_{(N_3 + N_4 + 1)\tau}^{(N_3 + 1)\tau + T} \left[r - \frac{rm_3}{K} - \left(\frac{r}{K} + \beta \right) (I_2^*(t) + \varepsilon) - a(y_2^*(t) + \varepsilon) \right] dt} \geq S((N_3 + N_4 + 1)\tau) e^{N_5\sigma_2}. \quad (16)$$

In the interval $t \in [N_3\tau, (N_3 + N_4 + 1)\tau]$, we have

$$S'(t) = rS(t) \left(1 - \frac{S(t) + I(t)}{K} \right) - \beta S(t)I(t) - \frac{aS(t)y(t)}{1 + \omega S(t)} \geq - \left[\frac{r}{K} + \beta + a \right] MS(t) \doteq \rho S(t) \quad (17)$$

and

$$S((N_3 + N_4 + 1)\tau) \geq S(N_3\tau) e^{\int_{N_3\tau}^{(N_3 + N_4 + 1)\tau} \rho dt} \geq S(N_3\tau) e^{(N_4 + 1)\tau \rho}. \quad (18)$$

From Eqs. (16) and (18), we get

$$S((N_3 + 1)\tau + T) \geq S(N_3\tau)e^{(N_4+1)\tau\rho}e^{N_5\sigma_2}$$

By $N_5\sigma_2 > (\frac{r}{k} + \beta + a)M(N_4 + 1)\tau$, we get $S((N_3 + 1)\tau + T) \geq m_3$. This is a contradiction. Thus, there exists at least a $t_2 \in [(N_3 + 1)\tau, (N_3 + 1)\tau + T]$ such that $S(t_2) \geq m_3$. So, for $t \in [t_1, t_2]$, $S(t) \geq m_3e^{\rho(t_2 - N_3\tau)} = m_4$. For $t > t_2$, the same arguments can be continued since $S(t_2) \geq m_3$. This proves that $S(t)$ is ultimately positively bounded below.

Step 3. Denote $m = \min\{m_1, m_2, m_4\}$, $D = \{R_+^3 : m \leq S(t), I(t), Y(t) \leq M\}$. Combining step 1, step 2 and Lemma 3.5, we know that every solution of system (1) with positive initial values will eventually enter and remain in region D . By Definition 3.2, we know system (1) is permanent. The proof is complete. \square

Corollary 4.6.

(1) If $p = 0$, then the condition of Theorem 4.5 becomes

$$q < \frac{r\tau d_2}{a} =: q_1. \quad (19)$$

(2) If $q = 0$, then the condition of Theorem 4.5 becomes

$$p < \frac{r\tau d_1}{\beta + \frac{r}{k}} =: p_0. \quad (20)$$

5. Discussion

In this paper, an impulsive predator–prey model with disease in the prey is investigated for the purpose of integrated pest control. In Theorem 4.3, we get the sufficient condition for the global stability of the susceptible pest-eradication periodic solution, which means that if the release amount of infective pest and natural enemy are large enough, then the susceptible pest will be doomed. By the result of Theorem 4.5, we get the sufficient condition for the permanence of system (1), which means that the pest and the natural enemy will coexist for all time. Corollary 4.4 shows that if we only take one measure, either release infective pest or release natural enemy, then the release amount must satisfy Eqs. (9) or (10). Obviously, our results shows that integrated pest management strategy is superior to those who only release infective prey (pest) or only release predator (natural enemy). Therefore, our mathematical results present a more prior strategy for pest management. However, in our model, we suppose releasing infective pests and natural enemy at the same time, which is not always true in real situation. How about the result, if we release infective pest and natural enemy at different time? We leave it as a future work.

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