Limited Feedback Bit Allocation for Cooperative Multi-cell Systems with Multi-user MIMO

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Abstract: For the cooperative multi-cell systems with multi-user MIMO, a new limited feedback bit allocation scheme is proposed to minimize the rate loss caused by quantization error. In the proposed scheme, the Channel State Information (CSI) feedback of cell-edge user for the local service cell and the adjacent interference cell are separately quantized. Based on the upper bound of the rate loss of cell-edge user due to the limited feedback, the number of feedback bits for quantized CSI of the local service cell and the adjacent cell are optimized with the fixed total bits of the limited feedback. The simulation shows that our proposed scheme of feedback bits allocation efficiently decreases the interference and increases the rate of systems compared with that of equal bits allocation and those of other allocations.

Key words: limited feedback; Block Diagonalization (BD) precoding; multi-cell; multi-user MIMO

I. INTRODUCTION

Cellular systems are known as interference-limited in nature, especially when universal frequency reusing and patternsectoring are employed in Ref. [1]. These techniques, such as sectoring and spectrum spreading, are not efficient to mitigate Inter-Cell Interference (ICI). Recently, a number of studies have been done on the cooperative multi-cell to mitigate the Co-Channel Interference (CCI) and improve the sum rate [2-4].

One of the key strategies in cooperative multi-

cell systems is to design the precoding. With the perfect CSI at the base station in cooperative multi-cell systems, the intra-cell and inter-cell interference can be mitigated by BD precoding [5]. In the field of cooperative multi-cell with Multi-User (MU) MIMO transmission, some academic works mainly focused on the research of the maximal sum capacity with multi-cell joint precoding under some given constraints, e.g., per cell power constraint [6-7]. Thus, the above-mentioned works assumed that both the desired and the interfering signals from different base stations arrive at each user simultaneously. However, the interference is inherently asynchronous [8]. The asynchronous interference model was proposed in Ref. [4]. The MU linear precoding and signal leak noise ratio precoding based on this asynchronous interference model were derived in Refs. [8-9] respectively. Ref. [10] designed and analyzed the internetwork with cooperative multi-cell systems, and proposed a randomized MIMO-OFDM cooperative coding.

Most of the literatures on cooperative multi-cell assumed that the perfect CSI of all the active users areavailable at the transmitter side, which is not feasible in the practical systems. In the practical systems, the limited feedback scheme is used to quantize the perfect CSI into several bits, which can reduce the uplink control overhead. However, the limited feedback affects the system performance a lot, which is a key factor for precoding design. In Ref. [11], the performance of zero-forcing precoding based on limited feedback is studied.

Similar to Ref. [11], the performance of BD with limited feedback is also studied in Ref. [12]. It is proved that limited feedback BD is interference-limited, if the number of feedback bits is fixed. Based on these techniques, some robust precodings are proposed considering the practical use in Refs. [13-14]. For demand of practical systems, some new limited feedback technique schemes are proposed. Refs. [14-15] find the phase ambiguity that resulted in throughout loss in relay systems due to the channel quantization for the limited feedback processing. The literature extends it to the multi-cell systems [16].

In this paper, a new limited feedback scheme is proposed in the cooperative multi-cell systems, which allocates different quantized bits to coordinated cells for SCI feedback. The total feedback bits are separately divided into two parts, one is allocated for local service cell, and the other is used for adjacent interference cells. Then the optimal feedback bit for local service cell and adjacent interference cell is obtained. Considering the practical system with limited feedback, the closed-form expression that approximately maximizes the rate of edge users is derived.

II. SYSTEM MODEL

In the proposed scheme, A^H denotes the conjugate transpose of matrix A, and I_N is the $N \times N$ identity matrix. $E(\cdot)$ stands for the expectation operator. d(A, B) is the Euclidean distance matrix. Considering the downlink MU-MIMO cooperative multi-cell system, each cell has N_t transmission antennas and K active users with $N_r \ge 2$ receiver antennas. There are N adjacent interference cells around the local service cell. The users at the center of the cell suffer only intra-cell MU interference, but the ones at the cell-edge suffer both intra-cell MU interference and ICI from adjacent interference simultaneously. The quantized CSI is considered as practical using, and we assume that the CSI of active users can be shared with all coordinated cells. In this cell interference system, cooperative multi-cell precoding is adopted to mitigate these two kinds of interferences.

For the cell-edge users, the received powers from local service cell and adjacent interference cell are given by P and αP , respectively. $\alpha \in (0,1)$ is a power coefficient, i.e., the power of signal from local cell is stronger than that of interference signal from adjacent cell. The value of α is affected by the large scale fading. Therefore, the received signal by the user k with perfect CSI is given by:

$$y_{k}^{c} = PH_{k}T_{k}s_{k} + P\sum_{\substack{i=1\\i\neq k}}^{K}H_{k}T_{i}s_{i} + n_{k}$$
MU interference caused by local cell

(for the cell center users)

$$y_k^e = PH_kT_ks_k + P\sum_{\substack{i=1\\i\neq k}\\\text{MU-interference caused}}^K H_kT_is_i + P\alpha\sum_{\substack{j=1\\CCI\ \text{caused by adjacent}\\\text{cell}}}^N G_kT_js_j + n_k \tag{2}$$

(for the cell edge users)

Eqs. (1) and (2) are the received signals by the users at cell-center and cell-edge respectively. $H_k \in \mathbb{C}^{N_r \times N_t}$ and $G_k \in \mathbb{C}^{N_r \times N_t}$ are the channel matrices from serving cell and adjacent interference cell to user k respectively. S_i is the transmitted signal, where $E[|S_i|^2] = 1$. $n_k \in \mathbb{C}^{N_r \times 1}$ is a zero-mean complex Gaussian noise vector with unit variance. T_k is the BD precoding with perfect CSI. Considering the perfect CSI at the transmitter side, the interference in Eqs. (1) and (2) will be annulled completely.

For our strategy, the local channel matrix \boldsymbol{H}_k and adjacent interference channel matrix \boldsymbol{G}_k are quantized by two separate random vector quantization codebooks, \boldsymbol{W}_c and \boldsymbol{W}_e . The quantization codebook used by each user is fixed beforehand and is known to the transmitter. The size of \boldsymbol{W}_c and \boldsymbol{W}_e are 2^{Bc} and 2^{Be} , i.e., $\boldsymbol{W}_c = [\boldsymbol{W}_1, \boldsymbol{W}_2, ..., \boldsymbol{W}_{2^{Bc}}]$ and $\boldsymbol{W}_e = [\boldsymbol{W}_1, \boldsymbol{W}_2, ..., \boldsymbol{W}_{2^{Bc}}]$, respectively. \boldsymbol{W}_c and \boldsymbol{W}_e are the matrices in $\mathbb{C}^{N_c \times N_c}$. \boldsymbol{B}_c and \boldsymbol{B}_e are the feedback bits for transmitting index of quantization channel matrices $\hat{\boldsymbol{H}}_k$ and $\hat{\boldsymbol{G}}_k$, respectively. Let \boldsymbol{B}_{tot} denote the total feedback bits, then $\boldsymbol{B}_c + \boldsymbol{B}_e = \boldsymbol{B}_{tot}$ for each user.

Since quantization errors, the MU interference caused by local service cell and ICI caused by adjacent interference cell cannot be completely canceled. This leads to residual interference. So the received signals from Eqs. (1) and (2) are changed re-

spectively as follows:

$$y_k^c = P\boldsymbol{H}_k \hat{\boldsymbol{T}}_k s_k + P \sum_{\substack{i=1\\i \neq k}}^K \boldsymbol{H}_k \hat{\boldsymbol{T}}_i s_i + n_k$$
(3)

$$y_{k}^{e} = PH_{k}T_{k}s_{k} + P\sum_{i=1}^{K}H_{k}\hat{T}_{i}s_{i} + P\alpha\sum_{i=1}^{N}G_{k}\hat{T}_{j}s_{j} + n_{k}$$
 (4)

where \hat{T}_k is the BD precoding with limited feedback with RVQ codebook. Each user feeds back the quantization channel matrix indices to local service cell according to the following criterion:

$$D_c = E[\min d^2(\boldsymbol{H}_k, \hat{\boldsymbol{H}}_k)] = E[\min d^2(\boldsymbol{H}_k, \boldsymbol{W}_c)]$$
 (5)

$$D_e = E[\min d^2(\mathbf{G}_k, \mathbf{G}_k)] = E[\min d^2(\mathbf{G}_k, \mathbf{W}_e)]$$
 (6) where D_c/D_e is the expectation of minimal Euclidean distance matrix between practical channel $\mathbf{H}_k/\mathbf{G}_k$, and codebook channel matrice $\mathbf{W}_c/\mathbf{W}_e$. $\hat{\mathbf{H}}_k$ and $\hat{\mathbf{G}}_k$ are the quantizations of \mathbf{H}_k and \mathbf{G}_k respectively. Furthermore, D_c and D_e can be upper bounded tightly by $D_c \leq \overline{D}_c$ and $D_e \leq \overline{D}_e$ [18] respectively. Where

$$\overline{D}_{c} = \frac{\Gamma\left(\frac{1}{T}\right)}{T} (C_{N,N_{c}})^{-\frac{1}{T}} 2^{-\frac{Bc}{T}}$$
(7)

$$\overline{D}_{e} = \frac{\Gamma\left(\frac{1}{T}\right)}{T} (C_{N,N_{r}})^{-\frac{1}{T}} 2^{-\frac{Be}{T}}$$
(8)

Here, $T = N_r(N_t - N_r)$ and $\Gamma(\cdot)$ is gamma function. C_{N,N_r} is given by $\frac{1}{T!} \prod_{i=1}^{N_r} \frac{(N_t - i)!}{(N_r - i)!}$.

The quantization channel indices of $\hat{\boldsymbol{H}}_k$ and $\hat{\boldsymbol{G}}_k$ are transmitted back to local service cell, then local service cell transmits the index of $\hat{\boldsymbol{G}}_k$ to the adjacent interference cell. Since the codebook is known at the transmitter, then the local service and adjacent interference cells can find the indices of $\hat{\boldsymbol{H}}_k$ and $\hat{\boldsymbol{G}}_k$ in the codebook for precoding.

III. OPTIMAL FEEDBACK BITS ALLO-CATION FOR MINIMIZING RATE LOSS

In this section, taking into account the channel quantization errors, we derive upper bound of rate loss and the optimal feedback bits for minimizing the rate loss.

3.1 Upper bound of the rate loss due to limited feedback with BD precoding

In this paper, the total feedback bits $B_{\rm tot}$ is fixed for

cell-edge users. B_c and B_e are fed back for local service cell and adjacent interference cell respectively. From Eq. (4), each cell-edge user's data rate with limited feedback and BD precoding can be given by:

$$\hat{R}_{k}^{e} = E \log_{2} \frac{\left| P \sum_{\substack{i=1\\i\neq k}}^{K} \boldsymbol{H}_{k} \hat{\boldsymbol{T}}_{i}^{T} \hat{\boldsymbol{T}}_{i}^{H} \boldsymbol{H}_{k}^{H} + P \alpha \sum_{j=1}^{N} \boldsymbol{G}_{k} \hat{\boldsymbol{T}}_{j}^{T} \hat{\boldsymbol{T}}_{i}^{H} \boldsymbol{G}_{k}^{H} + \boldsymbol{I}_{N_{i}} + P \boldsymbol{H}_{k} \hat{\boldsymbol{T}}_{k}^{T} \hat{\boldsymbol{T}}_{k}^{H} \boldsymbol{H}_{k}^{H} \right|}{P \sum_{\substack{i=1\\i\neq k\\\text{Local cell MU interference}}^{K} \boldsymbol{H}_{k} \hat{\boldsymbol{T}}_{i}^{T} \hat{\boldsymbol{T}}_{i}^{H} \boldsymbol{H}_{k}^{H} + P \alpha \sum_{j=1}^{N} \boldsymbol{G}_{k} \hat{\boldsymbol{T}}_{j}^{T} \hat{\boldsymbol{T}}_{i}^{H} \boldsymbol{G}_{k}^{H} + \boldsymbol{I}_{N_{i}} \right|}$$

$$+ I_{N_{i}} \sum_{\substack{i=1\\i\neq k\\\text{Local cell MU interference}}^{K} \boldsymbol{H}_{k} \hat{\boldsymbol{T}}_{i}^{T} \hat{\boldsymbol{T}}_{i}^{H} \boldsymbol{H}_{k}^{H} + P \alpha \sum_{j=1}^{N} \boldsymbol{G}_{k} \hat{\boldsymbol{T}}_{j}^{T} \hat{\boldsymbol{T}}_{i}^{H} \boldsymbol{G}_{k}^{H} + \boldsymbol{I}_{N_{i}}$$

$$+ I_{N_{i}} \sum_{\substack{i=1\\i\neq k\\\text{Local cell MU interference}}^{K} \boldsymbol{H}_{k} \hat{\boldsymbol{T}}_{i}^{T} \hat{\boldsymbol{T}}_{i}^{H} \boldsymbol{H}_{k}^{H} + P \alpha \sum_{j=1}^{N} \boldsymbol{G}_{k} \hat{\boldsymbol{T}}_{j}^{T} \hat{\boldsymbol{T}}_{i}^{H} \boldsymbol{G}_{k}^{H} + \boldsymbol{I}_{N_{i}}$$

$$+ I_{N_{i}} \sum_{\substack{i=1\\i\neq k\\\text{Local cell MU interference}}^{K} \boldsymbol{H}_{k} \hat{\boldsymbol{T}}_{i}^{T} \hat{\boldsymbol{T}}_{i}^{H} \boldsymbol{H}_{k}^{H} + P \alpha \sum_{j=1}^{N} \boldsymbol{G}_{k} \hat{\boldsymbol{T}}_{j}^{T} \hat{\boldsymbol{T}}_{i}^{H} \boldsymbol{G}_{k}^{H} + \boldsymbol{I}_{N_{i}}$$

From Eq. (2), the data rate of the cell-edge user with perfect CSI can be written as:

$$R_k^e = E \log_2 |\boldsymbol{I}_N + P\boldsymbol{H}_k \boldsymbol{T}_k \boldsymbol{T}_k^H \boldsymbol{H}_k^H| \tag{10}$$

The rate loss due to quantization error can be expressed as:

$$\Delta R_k^e = R_k^e - \hat{R}_k^e \tag{11}$$

We derive an upper bound for ΔR_k^e since obtaining a closed form expression for Eq. (11) is complicated. The rate loss of the cell-edge user due to limited feedback with BD precoding can be bounded by:

$$\Delta R_k^e \leq N_r \log_2 \left| 1 + \frac{PN_t}{N_t - N_r} \left[(K - 1)D_c + \alpha ND_e \right] \right| \quad (12)$$

Proof: See Appendix A

This provides a bound on rate loss of each celledge user. From Eq. (12), it can be seen that the value of ΔR_k^e is determined by D_c and D_e . Furthermore, D_c and D_e can be bounded by \overline{D}_c and \overline{D}_e from Eqs. (7) and (8) respectively. Since $B_{\text{tot}} = B_c + B_e$ is fixed, it is impossible to increase both B_c and B_e simultaneously. For the cell-edge users, if have B_c increased to mitigate the MU interference in local service cells, then B_e will be decreased, i.e., the ICI from adjacent interference cell is increased, vice versa. So, the key is to find the optimal tradeoff between the value of B_c and B_e , i.e., optimizing the feedback bits allocation for local service cell and adjacent interference cell.

3.2 The optimal feedback bits for edge users

Substituting D_c and D_e by its upper bound \overline{D}_c and \overline{D}_e , Eq. (12) can be written as:

$$\Delta R_k^e \leq \underbrace{N_r \times \log_2 \left| 1 + PN_t \left(M_1 2^{-\frac{h_r}{T}} + M_2 2^{-\frac{h_r-h}{T}} \right) \right|}_{Q}$$
 (13)

where
$$M_1 = \frac{(K-1)\frac{\Gamma(1/T)}{T}(CN_tN_r)^{-\frac{1}{T}}}{N_t - N_r}$$
 and

$$M_2 = \frac{\alpha N \frac{\Gamma(1/T)}{T} (CN_t N_r)^{-\frac{1}{T}}}{N_t - N_r}.$$
 For convenience,

let Q denote the right hand side of Eq. (13). To find the optimal feedback bits by quantizing the local service cell channel H_k and adjacent interference cell channel G_k , the relation between B_c and B_e is adopted, i.e., $B_{\text{tot}} = B_c + B_e$. Q is convex in $B_c \in [0,B_{\text{tol}}]$. The partial derivative of Q in terms of B_c is given by:

$$\frac{\partial Q}{\partial B_c} = N_r \frac{PN_t \left[M_1 \ln 2 \times 2^{\frac{\rho}{T}} \left(-\frac{1}{T} \right) + M_2 \ln 2 \times 2^{\frac{\rho-\rho}{T}} \frac{1}{T} \right]}{1 + PN_t \left(M_1 2^{\frac{\rho}{T}} + M_2 2^{\frac{\rho-\rho}{T}} \right)}$$
(14)

Hence, the value of B_c , which is to minimize Q, is an optimal value and is obtained by Eq. (14) to zero. A closed-form expression for B_c is given by:

$$B_c = \frac{T \log_2 \frac{K - 1}{\alpha N} + B_{\text{tot}}}{2} \tag{15}$$

The feedback bits should be real integer, so the optimal B_c^{opt} is given by $\lceil B_c \rceil$ or $\lfloor B_c \rfloor$ ($\lceil A \rceil$ and $\lfloor A \rfloor$ denote the ceiling and floor of A). In Eq. (15), it can be seen that the value of B_c is determined by the value of K and the number of interference cells N. If $K-1=\alpha N$, we can obtain that $B_c=B_e=B_{\text{tol}}/2$, i.e., B_c equals B_e . This means that the power of MU interference in local service cell equals the ICI power from adjacent interference cell. If $K-1>\alpha N$, it is obtained that the optimal B_c is increased with K increasing.

IV. SIMULATION RESULTS

The Rayleigh fading channel is considered in the simulations. And the number of interference cells N is 2 in our simulations. Assume P=10 dB, $N_t=4$, $N_r=2$ and there are two users in each cell, i.e., K=2. The total feedback bit $B_{\rm tot}=14$.

The rate loss as a function of the feedback bits B_c , which is for local cell channel quantization, is depicted in Figure 1. It can be seen that the rate loss does not decrease with B_c monotonously. We

take $\alpha = 0.8$, for example, If $B_c < 6$, we can obtain that the MU interference in the local service cell is the major interference. B_c should be increased to mitigate MU interference in the local cell until B_c goes up to about 6 bits. When $B_c > 6$, we can see that MU interference in the local cell is no longer the major interference, but the adjacent cell interference should be. Under this condition, if we increase the value of B_c , the rate loss will be increased. Hence, more feedback bits should be allocated to estimate the adjacent interferer cell. Therefore, to decrease the B_e (or to increase B_e) is the good choice for our systems. Meanwhile, α is the power rate of signal from local service cell and adjacent interference cell. So the rate loss is decreased with the coefficient α decreasing.

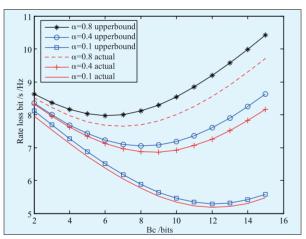


Fig.1 Rate loss with increasing of B_c . $P = 10 \text{ dB}, \alpha = \{0.1, 0.4, 0.8\}$

Figure 2 is the rate loss with the signal noise ra-

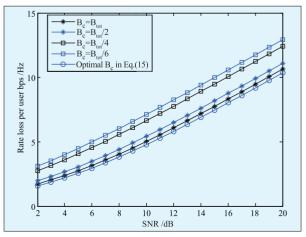


Fig.2 The rate loss as a function of SNR $\alpha = 0.1$

tio (SNR) with different values of feedback bits B_c . We can see that it is not the less rate to make the B_c lost more. Meanwhile, the rate loss is not decreased with B_c decreasing. It is seen that the rate loss of the proposed scheme has got the minimal rate loss. We also see that the rate loss of our proposed bits allocation is less than that of using equal bits allocation, i. e., $B_c = B_e = B_{\rm tot}/2$. This means that our feedback bit allocation strategy minimizes the rate loss in cooperative multi-cell systems with multi-user.

V. CONCLUSIONS

For multiuser MIMO in cooperative cell systems, the cell-edge user suffers not only MU interference caused by local service cell users, but also co-channel interference from adjacent cells. The BD precoding with limited feedback is adopted to mitigate these interferences. In this paper, the feedback bits of each cell-edge user are divided into two parts, one part is used for local service cell CSI feedback and the other part is fed back for adjacent interference cell CSI. The upper bound of the rate loss due to limited feedback and the optimal feedback bits allocation are derived. Our proposed bit allocation scheme efficiently mitigates the interference and decreases the rate loss due to limited feedback. The results obtained in this paper are significant for system design in practical systems 40 dt

Appendix

Proof of Eq. (12)

 $\Delta R_k^e = R_k^e - \hat{R}_k^e =$

Eq.(12) is proved as follows:

$$E \log_{2} \left| \boldsymbol{I}_{N_{r}} + P\boldsymbol{H}_{k}\boldsymbol{T}_{k}\boldsymbol{T}_{k}^{H}\boldsymbol{H}_{k}^{H} \right| - \left| P \sum_{j=1}^{K} \boldsymbol{H}_{k}\hat{\boldsymbol{T}}_{i}\hat{\boldsymbol{T}}_{i}^{H}\boldsymbol{H}_{k}^{H} + P\alpha \sum_{j=1}^{N} \boldsymbol{G}_{k}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}^{H}\boldsymbol{G}_{k}^{H} \right| + \left| P \sum_{i=1}^{K} \boldsymbol{H}_{k}\hat{\boldsymbol{T}}_{i}\hat{\boldsymbol{T}}_{i}^{H}\boldsymbol{H}_{k}^{H} + P\alpha \sum_{j=1}^{N} \boldsymbol{G}_{k}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}^{H}\boldsymbol{G}_{k}^{H} \right| + \left| P \sum_{i=1}^{K} \boldsymbol{H}_{k}\hat{\boldsymbol{T}}_{i}\hat{\boldsymbol{T}}_{i}^{H}\boldsymbol{H}_{k}^{H} + P\alpha \sum_{j=1}^{N} \boldsymbol{G}_{k}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}^{H}\boldsymbol{G}_{k}^{H} + \boldsymbol{I}_{N_{r}} \right| + \left| P \sum_{i=1}^{K} \boldsymbol{H}_{k}\hat{\boldsymbol{T}}_{i}\hat{\boldsymbol{T}}_{i}^{H}\boldsymbol{H}_{k}^{H} + P\alpha \sum_{j=1}^{N} \boldsymbol{G}_{k}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}^{H}\boldsymbol{G}_{k}^{H} + \boldsymbol{I}_{N_{r}} \right| + \left| P\alpha \sum_{j=1}^{N} \boldsymbol{G}_{k}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}^{H}\boldsymbol{T}_{j}^{H} + \boldsymbol{I}_{N_{r}} \right| + \left| P\alpha \sum_{j=1}^{N} \boldsymbol{G}_{k}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}^{H}\boldsymbol{T}_{j}^{H} + \boldsymbol{I}_{N_{r}} \right| + \left| P\alpha \sum_{j=1}^{N} \boldsymbol{G}_{k}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}^{H}\hat{\boldsymbol{T}}_{j}^{H} + \boldsymbol{I}_{N_{r}} \right| + \left| P\alpha \sum_{j=1}^{N} \boldsymbol{G}_{k}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}$$

$$\frac{\left|P\boldsymbol{H}_{k}\hat{\boldsymbol{T}}_{k}\hat{\boldsymbol{T}}_{k}^{H}\boldsymbol{H}_{k}^{H} + \boldsymbol{I}_{N_{r}}\right|}{P\sum_{\substack{i=1\\i\neq k\\\text{Local cell MU interference}}^{K}\boldsymbol{H}_{k}\hat{\boldsymbol{T}}_{i}\hat{\boldsymbol{T}}_{i}^{H}\boldsymbol{H}_{k}^{H} + P\alpha\sum_{\substack{j=1\\j=1\\\text{Adjacent cell co-channel interference}}^{N}\boldsymbol{G}_{k}\hat{\boldsymbol{T}}_{j}\hat{\boldsymbol{T}}_{j}^{H}\boldsymbol{G}_{k}^{H} + \boldsymbol{I}_{N_{r}}\right|}$$
(16)

(a)

$$\leq E \log_{2} \left| \boldsymbol{I}_{x_{r}} + P\boldsymbol{H}_{k} \boldsymbol{T}_{k}^{H} \boldsymbol{H}_{k}^{H} \right| - E \log_{2} \left| \boldsymbol{I}_{x_{r}} + P\boldsymbol{H}_{k} \boldsymbol{T}_{k}^{H} \boldsymbol{T}_{k}^{H} \boldsymbol{H}_{k}^{H} \right| + E \log_{2} \left| \boldsymbol{I}_{x_{r}} + P \sum_{i=1}^{K} \boldsymbol{H}_{k} \hat{\boldsymbol{T}}_{i}^{T} \hat{\boldsymbol{T}}_{i}^{H} \boldsymbol{H}_{k}^{H} + P \alpha \sum_{j=1}^{N} \boldsymbol{G}_{k} \hat{\boldsymbol{T}}_{j} \hat{\boldsymbol{T}}_{j}^{H} \boldsymbol{G}_{k}^{H} \right|$$

$$(17)$$

Since
$$P \sum_{\substack{i=1\\i\neq k}}^{K} \boldsymbol{H}_{k} \hat{\boldsymbol{T}}_{i} \hat{\boldsymbol{T}}_{i}^{H} \boldsymbol{H}_{k}^{H} + P \alpha \sum_{j=1}^{N} \boldsymbol{G}_{k} \hat{\boldsymbol{T}}_{j} \hat{\boldsymbol{T}}_{j}^{H} \boldsymbol{G}_{k}^{H}$$
 in numerator of

Eq. (16) has little effect on value of Eq. (16), it means that user is in high SNR. So, step (a) follows from neglecting it from the numerator in Eq. (16). Then (a) can be further expressed as follows:

(b)
$$=E \log_{2} \left| \boldsymbol{I}_{N_{r}} + P \sum_{\substack{i=1\\i\neq k}}^{K} \boldsymbol{H}_{k} \hat{\boldsymbol{T}}_{i} \hat{\boldsymbol{T}}_{i}^{H} \boldsymbol{H}_{k}^{H} + P \alpha \sum_{j=1}^{N} \boldsymbol{G}_{k} \hat{\boldsymbol{T}}_{j} \hat{\boldsymbol{T}}_{j}^{H} \boldsymbol{G}_{k}^{H} \right|$$
(18)

Considering BD procedure, T_k and \hat{T}_k are distributed isotropically, this means that the first two terms in Eq. (17) are identical, so (b) is being given.

The following demonstration mainly comes from Ref. [12]. The referenced content is at pages from

1 480 to 1 481. So (b) can be rewritten as:

$$= E \log_{2} \left| \boldsymbol{I}_{N_{r}} + P \tilde{\boldsymbol{H}}_{k} \sum_{\substack{i=1\\i\neq k}}^{K} \hat{\boldsymbol{T}}_{i}^{i} \hat{\boldsymbol{T}}_{i}^{H} \tilde{\boldsymbol{H}}_{k}^{H} \Lambda_{k} + P \alpha \tilde{\boldsymbol{G}}_{k} \sum_{j=1}^{N} \hat{\boldsymbol{T}}_{j}^{i} \hat{\boldsymbol{T}}_{j}^{H} \tilde{\boldsymbol{G}}_{k}^{H} N \Lambda_{k} \right|$$

$$\leq \log_{2} \left| \boldsymbol{I}_{N_{r}} + P \sum_{\substack{i=1\\i\neq k}}^{K} E \left[\tilde{\boldsymbol{H}}_{k} \hat{\boldsymbol{T}} \hat{\boldsymbol{T}}_{i}^{H} \tilde{\boldsymbol{H}}_{k}^{H} \right] N_{i} + P \alpha \sum_{j=1}^{N} E \left[\tilde{\boldsymbol{G}}_{k} \hat{\boldsymbol{T}}_{j}^{i} \hat{\boldsymbol{T}}_{j}^{H} \tilde{\boldsymbol{G}}_{k}^{H} \right] N_{i} \right|$$

$$(e)$$

$$= \log_{2} \left| \boldsymbol{I}_{N_{r}} + P(K - 1) \frac{D_{c}}{N_{i} - N_{r}} N_{i} + P \alpha N \frac{D_{c}}{N N_{i} - N_{r}} N_{i} \right|$$

$$= N_{r} \log_{2} \left| 1 + \frac{P N_{t}}{N_{t} - N_{r}} \left[(K - 1) D_{c} + \alpha N D_{c} \right] \right|$$

$$(19)$$

(c) follows BDcharacteristic precoding with limited feedback [13], where $\boldsymbol{H}_{k}\hat{\boldsymbol{T}}_{i}\hat{\boldsymbol{T}}_{i}^{H}\boldsymbol{H}_{k}^{H} = \tilde{\boldsymbol{H}}_{k}\hat{\boldsymbol{T}}_{i}\hat{\boldsymbol{T}}_{i}^{H}\tilde{\boldsymbol{H}}_{k}^{H}\boldsymbol{\Lambda}_{k}$; (d) follows Jensen's inequality and (e) comes from $E\left[\tilde{\boldsymbol{H}}_{k}\hat{\boldsymbol{T}}_{i}\hat{\boldsymbol{T}}_{i}^{H}\tilde{\boldsymbol{H}}_{k}^{H}\right] = \frac{D_{c}}{N_{t}-N_{r}}$ in Ref. [14].

Acknowledgements

The authors would like to thank the reviewers for their detailed reviews and constructive comments, which have helped to improve the quality of this paper. This work was supported partially by the National Natural Science Foundation of China under Grant No. 60772110; Fundamental Research Funds for the Central Universities.

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