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APPLICATION OF SURROGATE BASED PARTICLE SWARM OPTIMIZATION TO THE RELIABILITY-BASED ROBUST DESIGN OF COMPOSITE PRESSURE VESSELS**

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ABSTRACT A surrogate based particle swarm optimization (SBPSO) algorithm which combines the surrogate modeling technique and particle swarm optimization is applied to the reliability-based robust design (RBRD) of composite pressure vessels. The algorithm and efficiency of SBPSO are displayed through numerical examples. A model for filament-wound composite pressure vessels with metallic liner is then studied by netting analysis and its responses are analyzed by using Finite element method (performed by software ANSYS). An optimization problem for maximizing the performance factor is formulated by choosing the winding orientation of the helical plies in the cylindrical portion, the thickness of metal liner and the drop off region size as the design variables. Strength constraints for composite layers and the metal liner are constructed by using Tsai-Wu failure criterion and Mises failure criterion respectively. Numerical examples show that the method proposed can effectively solve the RBRD problem, and the optimal results of the proposed model can satisfy certain reliability requirement and have the robustness to the fluctuation of design variables.

KEY WORDS structural optimization, reliability based robust design, composite pressure vessel, surrogate based particle swarm optimization, sequential algorithm

I. INTRODUCTION

Filament wound composite pressure vessels, which consist of a cylindrical drum and dome part, are widely applied in commercial and aerospace industries due to their high strength and lightweight. To prevent contained fluid from leaking out, a liner is usually required. Most investigators have used net theory to design the end closure and predict stresses in a fiber reinforced composite by neglecting the contribution of the resin system[1,2].

In engineering applications, performance of composite vessels may be influenced by many factors, such as component materials, interfacial properties, loading and complex environment. For dealing with random uncertainty factors which are likely to appear in practice, reliability-based design (RBD) optimization method is generally applied. By using RBD, one can obtain a solution with satisfied performance and required reliability index. Various reliability-based design optimization methods have

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been used to design and optimize composite laminations and cylinders[3–6], but few of them have focused on the composite pressure vessel[6].

Other types of uncertainty except for the random uncertainty may also exist and affect the structural performance. For example, a kind of epistemic uncertainty, i.e., a difference between the realized values and the optimum ones of parameters caused by manufacturing errors, will result in either a poorer performance than the expected one or violations of some constraint conditions. In this case, robust design is an effective way to make the solution still feasible as design variables or design parameters undergo variations around the nominal values to a certain degree[7,8]. When considering both the random and epistemic uncertainties, reliability-based robust design (RBRD) methodology can be utilized. In RBRD, one tries to get a solution with required reliability and is robust with respect to the variation of design variables or parameters. Since the RBRD is a nested optimization problem, computational burden is a key problem we should face in solving it.

In this paper, the reliability-based robust design for composite pressure vessels with metal liner is formulated and studied with the efficient optimization algorithm SBPSO (surrogate-based particle swarm optimization). First, we outline the (SBPSO) algorithm which combines the efficiency of the traditional optimization algorithm and the global search capability of PSO. Examples are followed to illustrate the feasibility and efficiency of the proposed SBPSO. Then, modeling of a composite pressure vessel is introduced and its reliability-based robust design (RBRD) is formulated. The optimum design is solved by SBPSO and discussion is provided along with the results.

II. SURROGATE-BASED PARTICLE SWARM OPTIMIZATION METHOD

2.1. Basic Particle Swarm Optimization

The basic PSO is a heuristic global optimization search technique based on the simulation of birds flocking for food. Owing to its simple concept, easy implementation and quick convergence, PSO has gained much attention and has been successfully applied in different fields[9–11] (Kennedy and Eberhart 2001, Chen et al. 2008, Tang et al. 2009). The search procedure of PSO can be described as follows. First, a group of random particles (individuals) is generated. Then each particle adjusts its flying pattern according to its own flying experience and that of its companions. Each particle represents a potential solution for the optimization design problem.

2.2. Description of SBPSO Algorithm

If basic particle swarm optimization (BPSO) is used directly in solving optimization problems with expensive black box functions, the computational cost may make the simulation impossible owing to a large number of function evaluations. To improve the computation efficiency, the SBPSO method is recently developed by the authors.

When solving an N-dimensional problem, SBPSO[12] generates Np particles at the beginning [$Np \geq (N+1)$ is recommended]. During iterations, each particle updates its velocity and position according to BPSO and its fitness is calculated through function evaluations. These positions and the corresponding function values are stored in a sample database. In the meantime, a hybrid surrogate model (HSM) is constructed to approximate the true response surface, and the well-established traditional optimization algorithms (e.g., sequential quadratic programming) can be used to solve the approximate optimization problem. After the approximate global position has been determined, response analysis is implemented at this position. In the kth iteration, there are three types of global position: the global position found by the particles, denoted as g_P^k, the approximate global position determined from HSM (g_H^k), and the global position g^k found by SBPSO algorithm which is the best among g_P^k, g_H^k and g^{k-1}. In the next iteration, each particle updates its velocity and position based on the PSO rule, and a new HSM is constructed using the sample data including all the particles' positions searched so far and all the approximate global positions obtained previously.

For an N-dimensional optimization problem, the proposed algorithm consists of following detailed steps.

Step 1: Generate a population of size Np using Latin hypercube sampling. Evaluate the function value of each particle and find the global optimum position. Store the positions searched by particles and the corresponding function values in a sample database.

Step 2: If the number of total samples $Ns \geq (N+1)(N+2)/2$, construct the surrogate model by HSM and obtain the approximate global position, otherwise, go to Step 4.

Step 3: Evaluate the function value at the approximate global position by performing the simulation analysis and add this point to the sample database.

Step 4: Apply BPSO to update particles' positions and velocities. Update the global position g^k and store the obtained results in the sample database.

Step 5: If a stopping criterion is satisfied, the algorithm terminates, otherwise, go to Step 2. In the following, when an assigned maximum iteration number is reached, the algorithm is terminated.

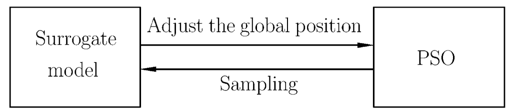

Fig. 1 Relation between the surrogate model and PSO in the proposed SBPSO.

The proposed SBPSO combines the advantages of traditional optimization algorithms and PSO, and can converge after several iterations with a small number of particles. Figure 1 shows the relation between the surrogate model and PSO in the proposed SBPSO. The approximate optimal solution obtained by solving the approximate optimization problem in terms of the surrogate model is used to adjust the global position. The PSO provides the rational sample distribution to improve the quality of HSM step by step. It is mentioned that the accuracy of HSM can be measured by the root mean square error (RMSE) and the normalized root mean square error (NRMSE) as indicated in Ref.[12]. It was demonstrated that almost perfect fit can be obtained through several examples.

The advantages of the proposed algorithms are listed below.

(1) For optimization problems with expensive black box functions, a global optimal solution is usually found after several iterations with a small number of particles. The computation cost required by SBPSO is reduced considerably as compared with BPSO or GA (genetic algorithm).

(2) In BPSO, after obtaining the global position and the particle's best positions, all the other information including particles' positions and the corresponding function values are discarded. However, in SBPSO, all the information about the particles is employed to establish the current surrogate model.

(3) A surrogate model generates approximate optimum points progressively during the iteration, and more and more samples are generated towards the global optimum, i.e. quality of HSM gets better and better especially in the interested area, which helps to improve the convergence of SBPSO and the accuracy of the solution.

2.3. SBPSO for Structural Design Optimization

A typical structural optimization problem as below is considered.

$$\begin{aligned} &\min_x f(x) \\ &\text{s.t. } g_i(x) \geq 0,\ i \in E \\ &\qquad h_j(x) = 0,\ j \in I \end{aligned} \tag{1}$$

with $f(x)$ being the objective function, $g_i(x)$ and $h_j(x)$ the inequality and equality constraint functions, respectively. The points satisfying above Equations are said to be feasible.

The penalty function method can be used to transform Eq.(1) into an unconstrained problem with a new objective function:

$$F(x) = f(x) + r\left\{\sum_i \left[\min\left(0,\ g_i(x)\right)\right]^2\right\} + r\left\{\sum_j \left[h_j(x)\right]^2\right\} \tag{2}$$

where the second and third terms of the right-hand side are penalty terms and the penalty factor r is greater than zero. An infinite penalty factor means that the constraints are satisfied rigorously. If the constraint conditions are violated at x, the value of the penalty terms becomes large such that the solution is pushed back towards to the feasible region. Otherwise, the constraint conditions are satisfied and the value of penalty terms is equal to zero.

For problems with expensive black box functions, by constructing the surrogate models for the objective function and constraint functions, an approximate optimization problem is formulated as

$$\begin{aligned} &\min_x \tilde{f}(x) \\ &\text{s.t. } \tilde{g}_i(x) \ge 0,\ i \in E \\ &\quad\ \ \tilde{h}_j(x) = 0,\ j \in I \end{aligned} \tag{3}$$

The approximate optimum can be easily solved by any well-established algorithm such as sequential linear programming (SLP) or sequential quadratic programming (SQP).

The expensive functions are then called at this point. If any of the constraints is violated, the strategy of constraint correction (CC) algorithm or correction at constant cost (CCC) is adapted to search the nearest feasible point around the infeasible approximate optimal point[13] (Arora 2004). The flowchart of the SBPSO is shown in Fig.2.

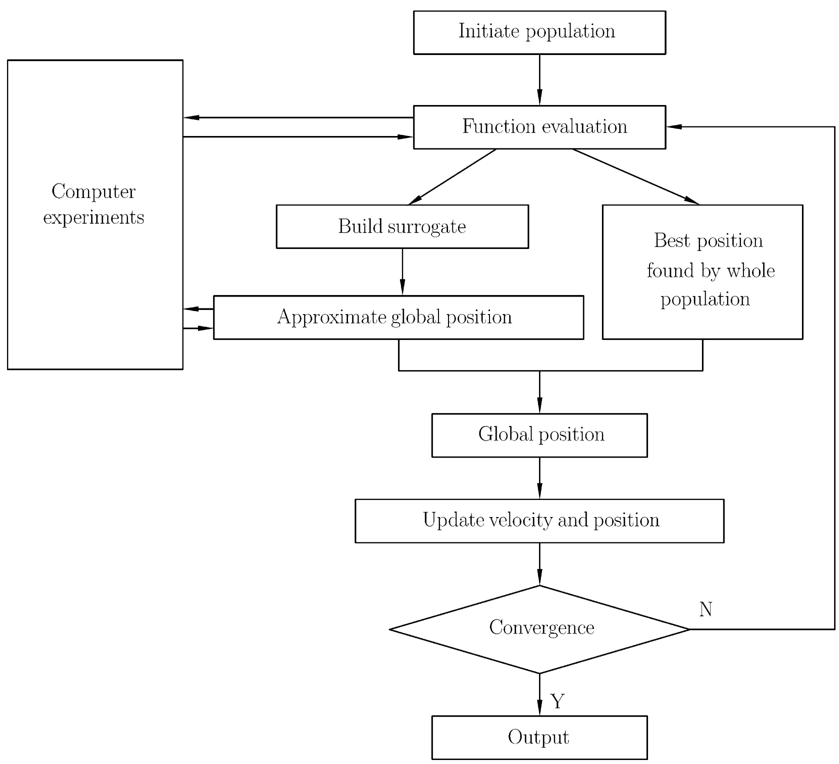

Fig. 2. Flowchart of SBPSO algorithm.

Example 1 Consider an optimization problem for six-hump camel back (SC) function:

$$\min_x f(x) = 4x_1^2 - 2.1x_1^4 + \left(\frac{1}{3}\right) x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4,\ \ (x_1,\ x_2) \in (-2,\ 2)$$

There are two global optimum points: $(0.08983, -0.7126)$ and $(-0.08983, 0.7126)$ with $f_{\min} = -1.0316$. For the purpose of illustration, the objective function is regarded as a black box function and is approximated by HSM method.

Figure 3 shows the initial particles' positions (left) and the positions searched from beginning to the end. The particle number is four and the maximum iteration number is eight. From the second iteration, HSM is constructed and the approximate optimum point is evaluated, leading to a total function evaluation number: $4 \times 8 + 7 = 39$. It is found that the particles concentrative flying to the global position area. So the constructed HSM model using these points gets more and more precise in the area. In later stage, the approximate optimum position is usually better than that obtained from PSO updating itself. Table 1 shows the results of this example from different algorithms. In the table, GA denotes the genetic algorithm[14], and MPS means the mode pursing sampling method[15]. We found that SBPSO can give a satisfied solution with the least computation effort.

Table 1. **Results of SC**

	x_1	x_2	f	nfe
SBPSO	-0.0899	0.7126	-1.0316	39
BPSO	-0.0903	0.7123	-1.0316	200
GA	-0.8915	0.6996	-1.0303	330
MPS	-0.0900	0.7130	-1.0316	48

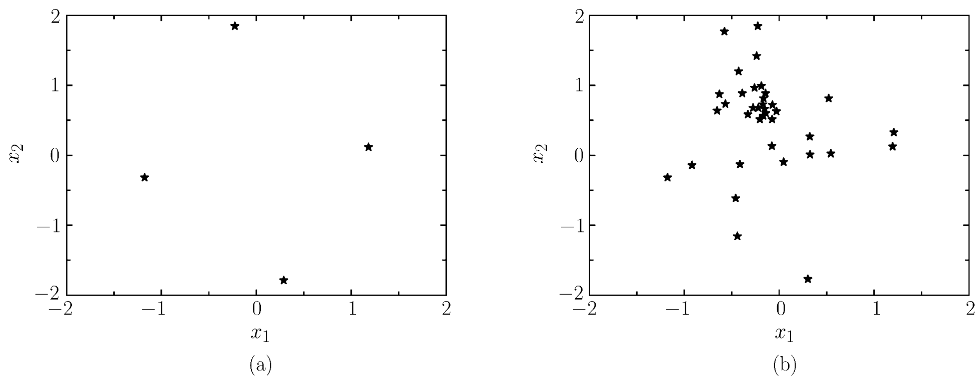

Fig. 3. Sample positions searched by SBPSO: (a) Initial positions; (b) All positions searched.

2.4. SBPSO for Reliability Based Design (RBD) of Structures

A RBD problem as below is studied.

$$\begin{aligned} &\min_{\boldsymbol{d}} f(\boldsymbol{d}) \\ &\text{s.t. } g_j^* \geq 0,\ j = 1, 2, \cdots, q \end{aligned} \tag{4}$$

with

$$\begin{aligned} &g_j^* = \min_{\boldsymbol{d},\boldsymbol{u}} g_j(\boldsymbol{d}, \boldsymbol{u}) \\ &\text{s.t. } \|\boldsymbol{u}\| = \Phi^{-1}(R_j) \end{aligned} \tag{5}$$

in which u is the standard random variable and R_j is the j-th target reliability. If BPSO together with FEA is used for this kind of problem, there is usually a burden in computation cost. The proposed SBPSO is suitable for complex RBD problems. Further, a sequential optimization approach is adopted to decompose the RBD into sequential cycles each includes a deterministic optimization followed by an inverse reliability analysis. The algorithm is as follows:

(1) Set $x = \mu_x$, solve the deterministic problem $\min\limits_{\boldsymbol{d}} f(\boldsymbol{d})$; s.t. $g_j(\boldsymbol{d}, \mu_X) \geq 0$ to get the solution d^1

(2) Solve $z^1 = \min\limits_{\boldsymbol{u}} g_j(d^1, \boldsymbol{u})$, s.t. $\|\boldsymbol{u}\| = \Phi^{-1}(R_j)$ to get $u^1_{\rm MPPIR}$. If $z^{1*} = g_j^*(d^1, u^1_{\rm MPPIR}) \geq 0$ and the minimum error requirement is satisfied, output the results, otherwise go to (3)

(3) $\min\limits_{\boldsymbol{d}} f(\boldsymbol{d})$, s.t. $g_j^*(\boldsymbol{d}, u^1_{\rm MPPIR}) \geq 0$ to obtain d^2

(4) $k = k + 1$ go to (2)

Note that step (2) and step (3) each requires solving an optimization problem. Whenever the expensive functions are involved, SBPSO will be utilized.

Example 2 A cantilever beam as shown in Fig.4 is considered. The loadings X and Y, together with the yielding strength R and the Young's modulus E are all normal random variables: $X \sim N$ (500, 100) lb, $Y \sim N\,(1000, 100)$ lb, $R \sim N\,(40000, 2000)$ psi, $E \sim N$ $(29 \times 10^6, 1.45 \times 10^6)$ psi. The cross section is a rectangle section with width w and height t. The allowed end deflection is $D_0 = 2.2535$ in (1 psi=6.895 kPa, 1 in =25.4 mm, 1 lb= 0.454 kg). Denote $(X_1, X_2, X_3, X_4) = (X, Y, R, E)$ and $(d_1, d_2) = (w, t)$. The design problem is stated as: Find w and t to minimize the beam weight so that the strength and deflection conditions should be satisfied with probability 0.9987 (i.e., the target reliability index is 3.0 for both the strength and the displacement constraints).

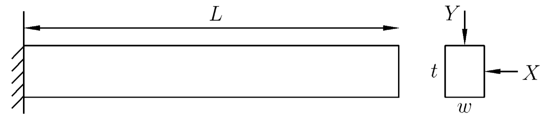

Fig. 4 A cantilever beam.

$$
\begin{aligned}
&\min_{\boldsymbol{d}} f(\boldsymbol{d}) = d_1 d_2 \\
&\text{s.t. } \ g_j^* \geq 0 \quad (j = 1, 2)
\end{aligned}
$$

where

$$
\begin{aligned}
&g_j^* = \min_{\boldsymbol{d},\boldsymbol{u}} g_j(\boldsymbol{d}, \boldsymbol{u}) \\
&\text{s.t. } \ \|\boldsymbol{u}\| = \Phi^{-1}(R_j) = 3.0 \\
&g_1(\boldsymbol{d}, \boldsymbol{X}) = \frac{X_3}{\mu_3} - \frac{6L}{\mu_3}\left(\frac{X_2}{d_1 d_2^2} + \frac{X_1}{d_1^2 d_2}\right) \\
&g_2(\boldsymbol{d}, \boldsymbol{X}) = D_0 - \frac{4L^3}{X_4 d_1 d_2}\sqrt{\left(\frac{X_2}{d_2^2}\right)^2 + \left(\frac{X_1}{d_1^2}\right)^2}
\end{aligned}
$$

in which μ_3 is a normalizing parameter.

For the illustrative purpose, SBPSO is applied to solve the problem, i.e., surrogated models are constructed for the objective function and the constraint functions although they are explicit cheap functions. The results are shown in Table 2. It is found that SBPSO, BPSO and SQP give almost the same results which are better than that from GA.

Since this problem is an explicit optimization, SQP has the advantage in computation efficiency. At the same time, it is confirmed that SBPSO shows a good ability for getting the global optimum with good precision and efficiency.

Table 2. **Solutions from different algorithms**

	W (in)	t(in)	f(in×in)	β_1	β_2	nfe
SBPSO	2.4162	3.9414	9.5231	3.0000	3.0000	774
BPSO	2.5179	3.7930	9.5507	3.0000	3.0000	3960
GA	2.9998	3.2423	9.7262	3.0000	3.0000	6220
SQP	2.4460	3.8922	9.5202	3.0000	3.0000	558

III. RELIABILITY BASED ROBUST DESIGN FOR COMPOSITE PRESSURE VESSELS WITH METAL LINER

3.1. Overview of Reliability Based Robust Design

Taking into consideration of a possible disparity between the nominal solution $\boldsymbol{d}'$ and the realized one $\boldsymbol{d}$, a reliability-based robust design (RBRD) problem, which ensures a certain level of robustness and reliability, is formulated as follows[16]:

$$\begin{aligned} & \min_{\boldsymbol{d}'} f(\boldsymbol{d}') \\ & \text{s.t. } g_j^* \geq 0, \ \boldsymbol{d}' \in [\boldsymbol{d}'_l, \boldsymbol{d}'_u] \end{aligned} \tag{6}$$

$$\begin{aligned} & g_j^* = \min_{\boldsymbol{d},\boldsymbol{u}} g_j(\boldsymbol{d}, \boldsymbol{u}) \\ & \text{s.t. } \|\boldsymbol{u}\| = \Phi^{-1}(R_j), \ \boldsymbol{d} \in \wp(\boldsymbol{d}', \alpha_t) \\ & \quad \wp(\boldsymbol{d}', \alpha_t) = \left\{ \boldsymbol{d} : \ \frac{\|\boldsymbol{d} - \boldsymbol{d}'\|}{\|\boldsymbol{d}'\|} \leq \alpha_t \right\} \end{aligned}$$

A sequential algorithm method using shifting factors (SFRBRD) is adopted to solve the above problem. For more details, readers can refer to Ref.[16]. For expensive functions, SBPSO is utilized.

3.2. Modeling of a Composite Pressure Vessel

Planar winding and geodesic winding are two typical winding methods. The former involves putting windings in one plane, and may be structural unstable. Meanwhile, geodesic winding involves geodesic path which is the shortest distance between two points on the winding surface to ensure the best winding stability. This study considers the geodesic condition in which Clairaut's equation holds

$$r \cdot \sin \alpha_n = r_0 \tag{7}$$

where α_n is the winding angle between a filament and a meridian line in a point on the surface, r is the radial distance to the axis of rotation z, and r_0 is the opening radius.

Let R denote the cylindrical internal radius and $y = z/R$, $x = r/R$. By using the netting theory and geodesic winding condition, the following meridian equation of dome can be obtained[17]

$$y = -\int \frac{x^3 \mathrm{d}x}{\left[(1 - x^2)(x^2 - a_1)(x^2 - a_2)\right]^{1/2}} + C \tag{8}$$

$$a_{1,2} = \frac{1}{2}\left[\pm\left(\frac{1 + 4X_0^2}{1 - X_0^2}\right)^{1/2} - 1\right] \tag{9}$$

where $X_0 = r_0/R$. The constant of integration is evaluated by the boundary condition that y=0 when x=1.

Additionally, it is assumed that the number of fibers crossing the equator and all other parallel circles in the dome is the same. Then the local composite layer's thickness is given by

$$t_n = \frac{R \cos \alpha}{x \cos \alpha_n} t \tag{10}$$

where α and t denote the winding angle and the total thickness of the helical layers in the cylindrical portion respectively, and the following relation holds.

$$t = \frac{PR}{2\sigma_u \cos^2 \alpha} \tag{11}$$

in which P is the internal pressure, σ_u the ultimate tensile strength of the composite in the fiber direction. The minimum number of helical layer and circular layers in the cylindrical portion are respectively

$$N_h = \frac{PR}{2t\sigma_u \cos^2 \alpha} \tag{12}$$

$$N_c = \frac{PR}{2t\sigma_u}(2 - \tan^2 \alpha) \tag{13}$$

In order to avoid the stress concentration caused by the sudden change of the thickness at the dome cylinder-interface, a transition region is introduced as shown in Fig.5. The size of the transition region can be adjusted. The stacking sequence of helical plies is $[+\alpha/-\alpha]$.

The general commercial software ANSYS is used to perform stress analysis for the pressure vessel. The composite plies are modeled by shell element 91 and metal liner by shell 99. It is assumed that the inner liner and the composite layer in the contact surface share the same displacement.

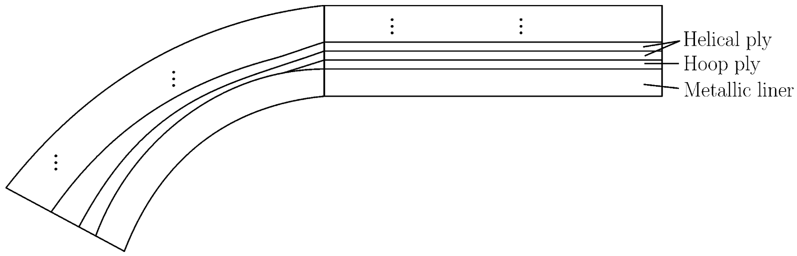

Fig. 5. Stacking sequence.

3.3. Formulation of RBRD for Composite Pressure Vessels with Metal Liner

Researchers have used a so called performance factor, defined as pressure times volume divided by weight, as an index for rating pressure vessel, the greater its value, the better the pressure vessel. Therefore, the objective function can be expressed as

$$\begin{aligned}
&f(\boldsymbol{d}') = \frac{1}{K} \to \min \\
&K = \frac{PV}{W} = \frac{PV}{W_c + W_m} \\
&V = \pi \int_0^{z_0} r(z)^2 \mathrm{d}z \\
&W_c = 2\pi\rho_c \int_0^{z_0} t_c(z) r(z) \sqrt{1 + \left(\frac{\mathrm{d}r}{\mathrm{d}z}\right)^2} \mathrm{d}z \\
&W_m = 2\pi\rho_m \int_0^{z_0} t_m(z)\, r(z) \sqrt{1 + \left(\frac{\mathrm{d}r}{\mathrm{d}z}\right)^2} \mathrm{d}z
\end{aligned} \tag{14}$$

where V and W denote the internal volume and the dome weight, respectively, W_c and ρ_c are the weight and density of composite plies, W_m and ρ_m are the weight and density of the metal liner, t_c is the total thickness of composite plies, t_m is the thickness of metal liner, and $r(z)$ follows the median equation (8).

The winding orientation for the helical plies in the cylindrical portion, the thickness of metal liner and the length of hoop ply drop off region are taken as design variables, that is, $\boldsymbol{d}' = (\alpha, t_m, d_0)$. Only the strength conditions are considered. For composite layers, by using Tsai-Wu criterion, the limit state function is represented as

$$\begin{aligned}
&g_1 = 1 - FI_c \geq 0 \\
&FI_c = F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + 2F_{12}\sigma_1\sigma_2 + 2F_{23}\sigma_2\sigma_3 + 2F_{13}\sigma_1\sigma_3 \\
&\qquad + F_{44}\tau_{12}^2 + F_{55}\tau_{23}^2 + F_{66}\tau_{13}^2 + F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3
\end{aligned} \tag{15}$$

For the metal liner, the Mises criterion is applied

$$\begin{aligned}
&g_2 = 1 - FI_M \geq 0 \\
&FI_M = \frac{1}{Y}\left[\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - (\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z) + 3(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2)\right]^{1/2}
\end{aligned} \tag{16}$$

Therefore, formulation of RBRD for a composite pressure vessel with metal liner can be represented as

$$\begin{aligned}
&\min_{\boldsymbol{d}'} \left[f(\boldsymbol{d}') = \frac{1}{K} \right] \\
&\text{s.t. } g_j^* \geq 0 \quad (j = 1, 2) \\
&g_j^* = \min_{\boldsymbol{d},\boldsymbol{u}} g_j(\boldsymbol{d}, \boldsymbol{u}) \\
&\text{s.t. } \|\boldsymbol{u}\| = \Phi^{-1}(R_j) = \beta_t, \ \ \boldsymbol{d} \in \wp(\boldsymbol{d}', \alpha_t)
\end{aligned} \tag{17}$$

3.4. Calculation Example

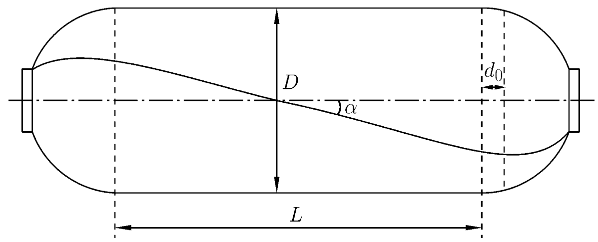

Fig. 6 The structure of a composite pressure.

A composite pressure vessel comprised of an aluminum liner and T-300/Epoxy composites is shown in Fig.6 The length of the cylindrical portion L is 700 mm, and the inside diameter $D = 150$ mm. The strength parameters of composite are $X_t = 1500$ MPa, $X_c = 1500$ MPa, $Y_t = 40$ MPa, $Y_c = 26$ MPa, $S = 68$ MPa. The thickness of each ply is 0.1 mm. The yield strength and ultimate strength of aluminum are 381 MPa and 496.2 MPa, The density of composite and aluminum are 1540 kg/m^2 and 2720 kg/m^2. The internal pressure is $p = 20$ MPa.

(1) Deterministic optimization (DO)

It is assumed that there exists no uncertainty in the parameters. The elastic constant of aluminum liner are $E_{Al} = 71$ GPa, $v_{Al} = 0.33$. The elastic constant of composite material are $E_X = 181$ GPa, $E_Y = 10.3$ GPa, $G_{XY} = 7.17$ GPa, $\nu_{xy} = 0.28$. Deterministic optimization problem is represented as

$$\begin{aligned} &\min_{\boldsymbol{d}'} \left[f(\boldsymbol{d}') = \frac{1}{K} \right] \\ &\text{s.t. } g_1(\boldsymbol{d}', \mu_X) \geq 0 \\ &\qquad g_2(\boldsymbol{d}', \mu_X) \geq 0 \end{aligned} \tag{18}$$

(2) Reliability based design (RBD)

It is assumed that the elastic constants follow standard normal distributions as shown in Table 3. The corresponding reliability based design (RBD) optimization with index $\beta_t = 3.0$ is expressed as

$$\begin{aligned} &\min_{\boldsymbol{d}'} \left[f(\boldsymbol{d}') = \frac{1}{K} \right] \\ &\text{s.t. } \Pr\left(g_1(\boldsymbol{d}', X) \geq 0\right) \geq 0.9987 \\ &\qquad \Pr\left(g_2(\boldsymbol{d}', X) \geq 0\right) \geq 0.9987 \end{aligned} \tag{19}$$

Table 3. **Statistical property of elastic constants**

	unit	Mean	COV	Distribution type
E_1	GPa	181	0.10	Normal
E_2	GPa	10.3	0.08	Normal
G_{12}	GPa	7.17	0.08	Normal
v	—	0.28	0.05	Normal
E_{Al}	GPa	71	0.10	Normal
v_{Al}	—	0.33	0.05	Normal

(3) Reliability based robust design (RBRD)

In engineering applications, due to the manufacturing errors or non-probabilistic uncertainties, there exists a disparity between the real value and the nominal optimal solutions of design variables. The disparity may cause the violation of constraints and result in catastrophic outcomes. The reliability-based robust design (RBRD) optimization provides the solution for these cases. Let the target feasible robustness level α_t be 0.05. It means that the constraints will be still satisfied when design variables undergo a variation not greater than 5% around the nominal optimal points. The corresponding RBRD problem can be represented by:

$$\begin{aligned} &\min_{\boldsymbol{d}'} \left[f(\boldsymbol{d}') = \frac{1}{K} \right] \\ &\text{s.t. } g_j^* \geq 0 \quad (j = 1,\ 2) \\ &g_j^* = \min_{\boldsymbol{d},\boldsymbol{u}} g_j\left(\boldsymbol{d}, \boldsymbol{u}\right) \\ &\text{s.t. } \|\boldsymbol{u}\| = 3.0,\ \ \boldsymbol{d} \in \wp\left(\boldsymbol{d}', 0.05\right) \end{aligned} \tag{20}$$

3.5. Results and Discussion

Table 4 gives the comparison of deterministic optimization (DO), reliability based design optimization (RBD) and reliability based robust design optimization (RBRD) models. Since the two failure criterions are implicit functions of design variables and stress analyses involve finite element analysis, the proposed SBPSO is used to carry out the optimization process. For RBD problems, a sequential algorithm which decouples the deterministic optimization and reliability analysis is used[18]. For RBRD problems, the shifting factor method SFRBRD is employed. The algorithm consists of a sequence of cycles and each cycle contains a deterministic optimization followed by an inverse robustness calculation together with the inverse reliability evaluation. The deterministic optimization step and the inverse robustness and reliability evaluations are all solved by SBPSO.

Table 4. **Comparisons of DO, RBD and RBRD models**

	DO	RBD	RBRD
D.V	α, t_m, d_0	α, t_m, d_0	α, t_m, d_0
Objective	$1/K$	$1/K$	$1/K$
Constraints	$g_j(\boldsymbol{d}', \mu_X) \geq 0$	$\Pr(g_j(\boldsymbol{d}', X) \geq 0) \geq 0.9987$	$g_j^* \geq 0(\beta_t = 3.0,\ \alpha_t = 0.05)$
Uncertainty	Deterministic	Random uncertainty	Random + Epistemic uncertainties
Methods	SBPSO	SBPSO+ sequential algorithm	SBPSO+SFRBRD

Table 5 compares DO, RBD and RBRD results. It can be seen that the objective function value increases from DO to RBD and then to RBRD. Due to the absence of the uncertainty, the objective function value is the smallest, but the actual feasible robustness indices $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are very small, implying that the strength constraints will be violated if little disparity between the realized values and the nominal optimum ones arise. The objective function value under RBD is smaller (better) than that of RBRD, but the actual feasible robustness index of RBD is also very small. Little disparity between real values and the optimum ones will cause the violation of strength or displacement constraint in RBD. In RBRD, a more conservative result is obtained. Both the reliability requirement and the feasible robustness requirement are guaranteed in RBRD. A larger target robustness index will result in a more conservative solution, i.e., robustness is achieved at the expense of functional performance. Solutions based on both DO and RBD can be viewed as the ones with 0 target robustness.

Table 5. **Comparison of DO, RBD and RBRD results**

Models	α (degree)	t_m (m)	d_0 (m)	f (m^{-1})	$\hat{\alpha}_1$	$\hat{\alpha}_2$
DO	34.3823	6.0871×10^{-3}	3.9017×10^{-3}	1.2702×10^{3}	1.4682×10^{-5}	6.9157×10^{-4}
RBD	34.3864	9.1099×10^{-3}	5.5740×10^{-3}	1.5462×10^{3}	2.3511×10^{-4}	3.3510×10^{-5}
RBRD	32.6063	9.7323×10^{-3}	1.8964×10^{-3}	1.6399×10^{3}	6.1634×10^{-2}	5.2461×10^{-2}

The convergence history of the sequential algorithm using shifting factors for RBRD (SFRBRD) is shown in Table 6 with (s_1, s_2) being the shifting factors. The algorithm converges after 5 cycles. At the 1^{th} and 2^{th} cycle, there is no movement of constraint boundary. It is pointed out that different values of α_t may result in different optimal solutions, the designers can choose a reasonable value of target robustness according to their experience, expert opinion, or engineering practice. This paper only shows results at target robustness index $\alpha_t = 0.05$ for the sake of illustration.

IV. CONCLUSIONS

A RBRD for composite pressure vessels with metal liner is presented. The optimal result based on the proposed model satisfies certain reliability requirement and has the feasible robustness to the epistemic uncertainty of design variables. FEA is used to accurately predict the behavior of filament wound composite vessels, and the proposed SBPSO is applied to solve the design problem for its

Table 6. **Convergence history of the SFRBRD method at** $\alpha_t = 0.05$

Iter.	α (degree)	t_m (m)	d_0 (m)	f (m^{-1})	g^*	(s_1, s_2)
1	34.3823	6.0871×10^{-3}	3.9017×10^{-3}	1.2702×10^{3}	$(-1.0639, -1.1394)$	(0, 0)
2	34.6436	10.0000×10^{-3}	1.0000×10^{-3}	1.6223×10^{3}	(0.1269, 0.1120)	(0, 0)
3	34.3351	8.7286×10^{-3}	4.0311×10^{-3}	1.5123×10^{3}	$(-0.1643, -0.0796)$	$(-0.0179, -0.0796)$
4	29.0377	9.4878×10^{-3}	1.1681×10^{-3}	1.7373×10^{3}	$(0.1521, -0.1726)$	$(-0.1391, -0.2093)$
5	32.6063	9.7323×10^{-3}	1.8964×10^{-3}	1.6399×10^{3}	(0.0577, 0.0193)	$(-0.1391, -0.2093)$

computational efficiency in obtaining global optimal solution. The proposed procedure can serve as an effective tool to optimize other complex structures under uncertainty.

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