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A CRACK GROWTH MODEL BASED ON FATIGUE DAMAGE ACCUMULATION

BAOXIANG QIU[1,2], XIAOGUI WANG[1], ZENGLIANG GAO[1,*]

[1]*Zhejiang University of Technology, Institute of Process Equipment and Control Engineering,*
[2]*Wanxiang Group, Technical Research Center,*
Hangzhou, 310032, P .R. China
zlgao@zjut.edu.cn

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A novel model for predicting the fatigue crack growth had been developed based on the concept of the fatigue damage accumulation. Fatigue crack growth was considered as a process of continuous crack nucleation. The crack tip failed to form a fresh crack while the accumulative fatigue damage reached the critical damage. A simplified model of the general crack growth model was proposed with the assumption that the damage zone can be divided into many different zones and each zone had the same crack growth rate. The model was applied to predict the crack growth of the compact specimen made of 16MnR steel under the constant amplitude loading. The predicted crack growth rate was in excellent agreement with the experimental observations.

Keywords: Crack growth model; damage zone; fatigue damage accumulation.

1. Introduction

Most engineering components often experience the fatigue and fracture to the failure. An option to warren the long-term fatigue strength of a structure is through performing extensive experimental studies to understand the behavior of the fatigue initiation and the crack growth. However, the experimental studies consume large time and monetary expenditure. It has inevitably led to the development of some models which can easily predict the fatigue behavior, only requiring some basic material constants.

Many phenomenological models had been proposed to describe the fatigue crack growth process. These models basically may be classified into three types: (1) models[1] based on the geometries, which assumed that crack extension per cycle was directly related to the change in the opening displacement or the strain at the crack tip; and (2) models[2] based on the intermittent damage accumulation, which assumed that non-continuous incremental crack growth would occur when a cumulative damage parameter has reached to a critical vale within a variable or invariable crack length; and (3) models[3] based on cycle-by-cycle damage accumulation, which assumed that the crack tip extended the a length of a volume element after each cycle loading. A common characteristic of these models was that they provided insight into physical processes.

The aim of the current investigation was to propose a novel fatigue crack model based on the accumulative fatigue damage concept. A simplified model was further proposed based on an assumption. The ability of the model for predicting the crack growth rate was further assessed using the experimental observations on the crack growth of 16 MnR steel under the constant amplitude loading.

2. Fatigue Damage

A general fatigue damage model proposed by Jiang[4] was based on the cyclic plasticity, the critical plane and the plastic strain energy concept. The criterion adopted the following increment form,

$$dD = \left\langle \boldsymbol{\sigma}_{mr}/\boldsymbol{\sigma}_0 - 1 \right\rangle^m \left(1 + \boldsymbol{\sigma}/\boldsymbol{\sigma}_f\right) dY \tag{1}$$

where,

$$dY = b\boldsymbol{\sigma}\, d\boldsymbol{\varepsilon}^p + \frac{1-b}{2}\boldsymbol{\tau}\, d\boldsymbol{\gamma}^p. \tag{2}$$

D denoted fatigue damage. c_{mr} was a material memory parameter. Y represented the plastic strain energy (density) on the critical material plane. σ_f was the true fracture stress of the material. ε^p and γ^p represented plastic strains corresponding to the normal stress σ and the shear stress τ respectively. σ_0 was the endurance limit of the material. m and b were material constants. The braces $\langle\ \rangle$ denoted the Macauley bracket. In Jiang model, the critical plane was the material plane where the accumulated fatigue damage firstly reached the critical value. A point near the crack tip accumulated certain fatigue damage per loading cycle. The point would fail and a fatigue crack would initiate from this point when the accumulated damage of a point on the critical material plane reached firstly the critical damage value D_0[5].

3. A General Crack Growth Model

3.1. *General accumulative process of fatigue damage*

In the current investigation, the basic assumption was that crack growth was considered as a process of continuous crack nucleation of the crack tips in the crack propagating path. The crack nucleation was controlled by the accumulative fatigue damage. The crack tip would fail to form a fresh crack when the accumulative fatigue damage reached the critical damage.

The accumulative process of the fatigue damage for the representative point was shown in Fig. 1. The origin of the coordinate system was moved as the crack tip extended. In Fig. 1 (a), the crack had a crack length of a_0. There was a damage zone with a width of r_d ahead of the first crack tip. The boundary of the damage zone was just tangent to the representative point. During the propagation, all the points in the crack propagating path surrounded by the damage zone had the fatigue damage on the representative point. The first crack tip had the initial fatigue damage on the representative point. The points failed successively and the new crack formed successively from the first crack tip (Fig. 1(a)) to the n th crack tip (Fig. 1(d)). The n th crack tip coincided with the representative point.

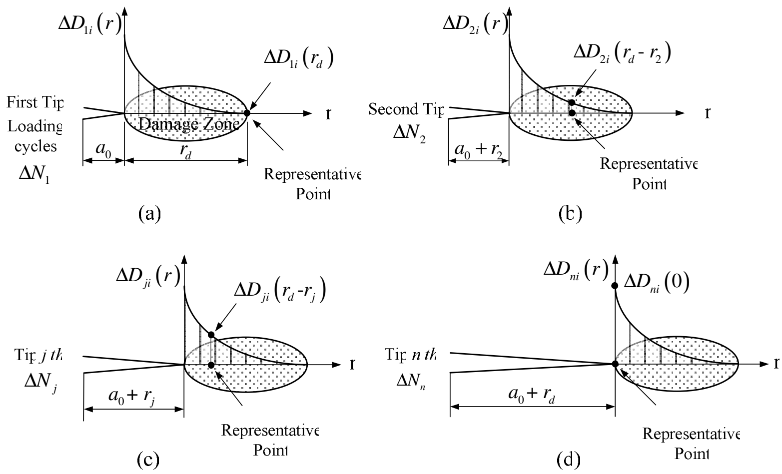

Fig. 1. Schematics of accumulative process of fatigue damage for representative point.

The j th crack tip in Fig. 1(c) was cited as an example to illuminate the accumulative fatigue damage. ΔN_j was the number of loading cycles at the j th crack tip from the moment that the crack reached the j th crack tip to the failure of the point at the j th crack tip. $\Delta D_{ji}(r)$ denoted the fatigue damage distribution at the i th loading cycle ahead of the j th crack tip. In the variable amplitude loading, the same crack tip may experience many different loading cases, so the fatigue damage distribution per cycle may be different. The fatigue damage per cycle for the point was $\Delta D_{ji}(r_d - r_j)$. In general, the accumulative fatigue damage for the representative point during the failure of the point at the j th crack tip was as follows:

$$\sum_{i=1}^{\Delta N_j} \Delta D_{ji}\left(r_d - r_j\right) = D_j. \tag{3}$$

As the crack advanced, the crack approached gradually the point. Finally, the total accumulative fatigue damage for the representative point was as follows:

$$\sum_{j=1}^{n} D_j = \sum_{j=1}^{n} \sum_{i=1}^{\Delta N_i} \Delta D_{ji}\left(r_d - r_j\right) = D_0 \tag{4}$$

3.2. *General model for crack growth under constant amplitude loading*

For constant-amplitude loading, the fatigue damage distribution at a crack tip was invariable in nature. So $\Delta D_{ji}\left(r_D - r_j\right)$ was a constant during the nucleation process of the j th crack tip. The damage distribution per cycle in Eq. (3) was converted as follows:

$$\Delta D_j(r_d - r_j) = \Delta D_{ji}(r_d - r_j). \tag{5}$$

Therefore, Eq. (4) can be rewritten for constant amplitude loading in the following form,

$$\sum_{j=1}^{n} \Delta N_j \Delta D_j (r_d - r_j) = D_0. \tag{6}$$

In Eq. (6), $\Delta D_j (r_d - r_j)$ was a function of the location, r_j (Fig. 1).

Schematic illustration of the history of the accumulative fatigue damage per loading cycle for the representative point was shown in Fig. 2. This figure had transformed the coordinate system to keep the origin of the coordinate system at the first crack tip. So the new form of Eq. (6) was as follows:

$$\sum_{j=1}^{n} \Delta N_j \Delta D(r_j) = D_0. \tag{7}$$

According to the definition of the integral, the equivalent equation of Eq. (7) was as follows:

$$\int_0^{r_d} \frac{\Delta D}{dr/\Delta N} dr = D_0. \tag{8}$$

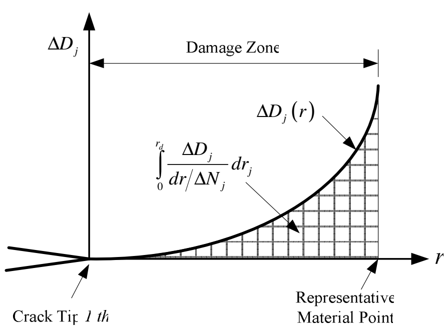

Fig. 2. History of accumulative fatigue damage.

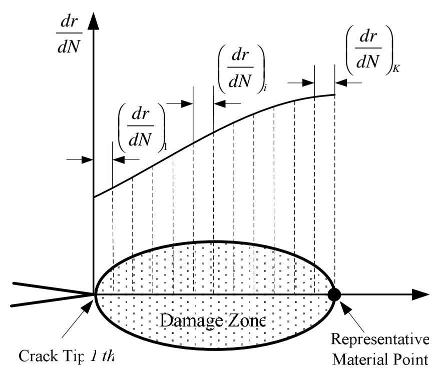

Fig. 3. Hypothesis of the crack growth.

In Eq. (8), ΔD, ΔN and $dr/\Delta N$ were the functions of the location *r*. The left of Eq.(8) was a complicated expression depending on the loading conditions, structural geometries and material properties. It was difficult to use a simple expression to describe the accumulative damage process. Assume that the damage zone can be divided into many different zones and each zone had the constant crack growth rate shown in Fig. 3. So the crack growth rate at the point was then calculated from Eq. (8) as:

$$\frac{da}{dN} = \left(\frac{dr}{dN}\right)_K = \frac{\int_{r(K-1)}^{r(K)} \Delta D(r) dr}{D_0 - \left(\frac{\int_0^{r(1)} \Delta D(r) dr}{(dr/dN)_1} + \ldots + \frac{\int_{r(i-1)}^{r(i)} \Delta D(r) dr}{(dr/dN)_i} + \ldots + \frac{\int_{r(K-2)}^{r(K-1)} \Delta D(r) dr}{(dr/dN)_{K-1}} \right)} \tag{9}$$

4. Example for Crack Growth under Constant Amplitude Loading

4.1. *Specimen geometries and loading conditions*

A compact specimen was employed for the investigations of the crack growth under constant amplitude loading. The compact specimen was made of 16MnR steel[6]. The dimensions and the boundary conditions of the compact specimen were shown in Fig. 4. The loading stress ratio was 0.1. The loading amplitude was equal to 2.25 kN.

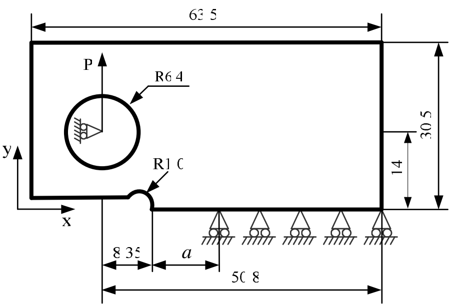

Fig. 4. Half model of compact specimen (mm).

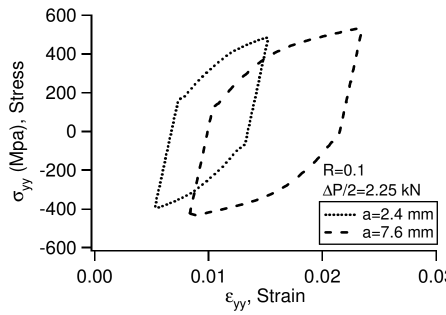

Fig. 5. Stress-strain hysteresis loops.

4.2. *Stress analysis*

The finite element method was adopted to implement the elastic-plastic stress analysis. The detailed modeling method can refer to the Ref. 7. The stress-strain results at two crack tips with different crack length were shown in Fig. 5.

4.3. *Damage analysis*

The accumulative history of fatigue damage for any point can be obtained by the fatigue damage model (Eq. (1)) with the detailed stress-strain history. The fatigue constants in the fatigue damage model can be found in another article[7]. The accumulative fatigue damage history of the representative point with $a = 3\,\text{mm}$ was shown in Fig. 6. The first crack tip which resulted in the initial fatigue damage on the representative point had a distance of $a = 0.5\,\text{mm}$ from the root of the notch.

4.4. *Results of fatigue crack growth simulation*

Assuming the crack growth rate in the damage zone can be considered as a constant, the fatigue crack growth rate was solved by Eq. (9). Such an assumption held true for small loading amplitude, which produced a small fatigue damage zone. For the larger amplitude loading, Eq. (9) was a better expression for crack growth rate. Fig. 7 compared predicted crack growth rates and experimental observations for the specimen. *a* denoted the crack length measured from the root of the notch shown in Fig. 1. In general, the model can predict well the crack growth rate under constant amplitude loading.

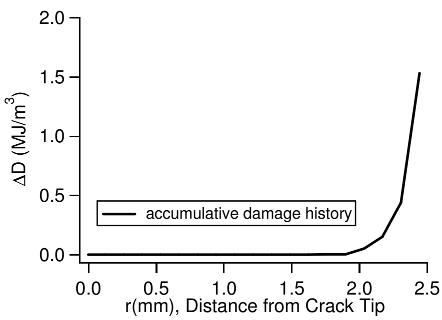

Fig. 6. Damage distribution for point.

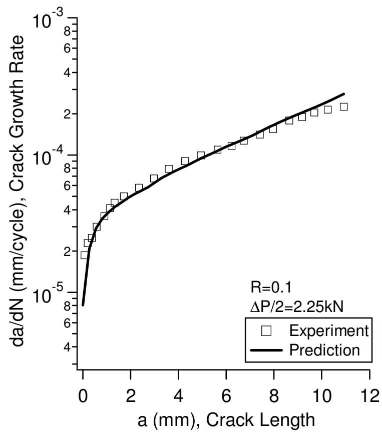

Fig. 7. Predicted crack growth rate.

5. Conclusion

Crack propagation was a process of continuous crack initiation. Fatigue damage induced the whole process of fatigue crack growth. Based on the accumulative fatigue damage concept, the general process of crack growth model was described in detail. A general crack growth model for the constant amplitude loading was proposed. A simplified model of the general crack growth model was further proposed with the assumption that the damage zone can be divided into many different zones and the crack growth rate was constant in each zone. The proposed model had the great capability for predicting the crack growth rate under constant amplitude loading.

Acknowledgments

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