Average BER Analysis for Binary Signallings in Decode-and-Forward Dissimilar Cooperative Diversity Networks

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***Abstract*—In this letter, the average bit-error rate (BER) performance is analyzed for uncoded *decode-and-forward* (DF) cooperative diversity networks. We consider two typical networks: a single-relay cooperative network with the direct source-destination link and a two-relay cooperative network with the direct source-destination link, under *dissimilar* network settings, i.e., the fading channels of different relay branches may have different variances. We first derive a closed-form approximate average BER expression of binary signallings including noncoherent binary frequency shift keying (BFSK), coherent BFSK, and coherent binary phase shift keying (BPSK), for the single-relay network. We then generalize our analysis to the two-relay network, and a closed-form approximate average BER expression for binary signallings is derived. We also show that our BER expressions can be considered as generalizations of previously reported results in the literature. Throughout our analysis, only one approximation, so-called the piecewise-linear approximation, is made. Simulation results are in excellent agreement with the theoretical analysis, which validates our proposed BER expressions.**

***Index Terms*—Binary signallings, bit-error rate (BER), decode-and-forward (DF), dissimilar networks, piecewise-linear (PL) approximation.**

I. INTRODUCTION

DIVERSITY has been acknowledged as one of the most effective techniques to combat fading effects in wireless communications [1]. Recently, a new kind of diversity, cooperative diversity [2]–[5], has been proposed as a means of attaining broader cell coverage, mitigating channel impairments, and increasing the channel capacity without using multiple antennas at each terminal. One of the most well-known cooperative protocols is the decode-and-forward (DF) protocol, in which each relay receives and decodes the signal transmitted by the source and then forwards the decoded signal to the destination. There has been a lot of works carried out on the DF protocol incorporated with channel codes, for which the name *coded cooperation* is given [6]–[8]. Some bit-error rate (BER) and frame-error rate bounds have been derived in [6]–[8]. On the other hand, many works have focused on *uncoded DF cooperation* where no channel codes are used [9]–[13].

Recently, many researchers have started to analyze the performance of ML detection for uncoded DF cooperation [10]–[13]. However, the *exact* closed-form average BER analysis is extremely difficult even for *single*-relay cooperative networks with *binary* modulations [9], [10].[1] Therefore, most of the works have been devoted to the *approximate* average BER analysis. Nonetheless, still very limited results have been reported in the literature. In [10], Chen and Laneman have proposed an accurate approximation, which was referred to as the piecewise-linear (PL) approximation, and they obtained an accurate approximate closed-form average BER expression for single-relay systems with noncoherent binary frequency shift keying (BFSK). For single-relay systems with coherent BPSK, two approximate average BER expressions have been derived in [12] and [13] based on the PL approximation;[2] however, the BER expressions are expressed in *double-integral* forms, which are not truly closed-form in a strict sense and are quite difficult to use in practice. For single-relay systems with coherent BFSK, no average BER expression has been reported, and it has been acknowledged that it is hard to analyze the average BER performance of the ML detection in closed-form for coherent BFSK even for *single*-relay cooperative networks [10].[3] For *two*-relay cooperative networks, only one BER expression has been reported in the literature [11, eq. (4.14)]. However, this expression is only applicable for noncoherent BFSK and for *symmetric* networks where the channel variances of the first hop for different branches are equal. However, the symmetric settings is not practical as the author acknowledged [11].

To the best of our knowledge, no closed-form average BER expressions of coherent binary signallings for single-relay uncoded DF cooperative networks have been found, and no closed-form BER expressions of binary signallings for two-relay uncoded DF *dissimilar* cooperative networks have been found either. This motivated our work.

In this letter, we analyze the average BER performance of the ML detection for uncoded DF cooperative networks with dissimilar settings. Specifically, we consider two typical cooperative diversity networks: the single-relay cooperative diversity network [10]–[13] and the two-relay cooperative diversity network [11], [14], [15]. First, we derive the probability density functions (PDFs) and cumulative distribution functions

Manuscript received December 22, 2008; revised March 18, 2009; accepted May 17, 2009. The associate editor coordinating the review of this letter and approving it for publication was Y.-J. Zhang.

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This research was supported in part by the Natural Sciences and Engineering Research Council of Canada (NSERC) and in part by Ubiquitous Computing and Network (UCN) Project, Knowledge and Economy Frontier R&D Program of the Ministry of Knowledge Economy(MKE) in Korea as a result of UCN's subproject 09C1-C2-12T.

Digital Object Identifier 10.1109/TWC.2009.081680

[1]In [10], the authors commented "Analysis of DF diversity transmission is challenging since it must treat the nonlinear behavior of (7), which significantly complicates the effort of obtaining a closed-form solution for BER.".

[2]In [12] and [13], the ML receivers are termed optimum and sub-optimum, respectively, depending on whether the instantaneous source-relay channel coefficients are available at the destination.

[3]In [10], the authors commented "Note that, at present, we have no simple, closed-form expression for the BER of coherent DF . . . ".

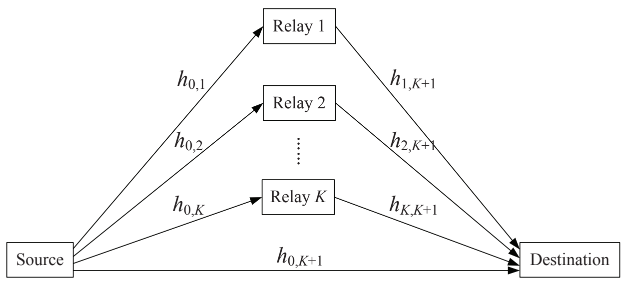

Fig. 1. Cooperative diversity network with K relays and the direct source-destination link.

(CDFs) of the sufficient statistics for the ML decision-making at the destination. Then we apply the accurate PL approximation and derive closed-form approximate average BER expressions with the help of the obtained PDFs and CDFs. Our BER expressions are shown to be valid for the general dissimilar uncoded DF networks adopting both coherent and noncoherent binary signallings. We also show that our BER expressions can be considered as generalizations of the previously reported results in the literature. The numerical results demonstrate that our BER expressions are extremely accurate.

The remainder of this letter is organized as follows. In Section II, we describe the system model and review the ML detection for uncoded DF networks. In Section III, we derive a closed-form approximate average BER expression for single-relay uncoded DF networks. In Section IV, a closed-form approximate average BER expression for two-relay uncoded DF networks is proposed. Section V presents some numerical results and Section VI concludes this letter.

Notation: For a real-valued random variable X, $X \sim \mathcal{N}(\mu_x, \Omega_x)$ indicates that X is a real-valued Gaussian random variable with mean μ_x and variance Ω_x. For a complex-valued random variable Z, $Z \sim \mathcal{CN}(\mu_z, \Omega_z)$ indicates that Z is a circularly symmetric complex-valued Gaussian random variable with mean μ_z and variance Ω_z.

II. System Model

Consider a cooperative diversity network consisting of a source, K relays, and a destination as shown in Fig. 1. The source and the destination are denoted by terminal 0 and terminal $K+1$, respectively, and relays are denoted by terminal i, $i = 1, 2, \cdots, K$. In this letter, we consider the *uncoded cooperation* with DF protocol, i.e., no channel codes are used. Each terminal in the network is equipped with a single antenna working in the half-duplex mode. We consider an *orthogonal* transmission scheme in which only one terminal is allowed to transmit at each time slot [4], [9], [10]. Therefore, the data transmission consists of two phases. In the first phase, the source broadcasts the signal, and each relay attempts to decode the signal. In the second phase, different relay terminals transmit their own remodulated signals in different time slots.

The channel coefficients $h_{i,j}$ for the link between terminal i and terminal j are modeled as mutually independent complex Gaussian random variables with $h_{i,j} \sim \mathcal{CN}(0, \sigma_{i,j}^2)$, $i, j \in \{0, 1, 2, \cdots, K+1\}$. The additive noise associated with $h_{i,j}$ is denoted by $n_{i,j}$ for BPSK and denoted by $n_{i,j,k}$, $k \in \{0, 1\}$, for BFSK, where the third subscript k is the index of the two frequency subbands for BFSK signalling. We model the noise terms as mutually independent additive white Gaussian noise (AWGN) with zero mean and variance N_0. Consequently, the instantaneous signal-to-noise ratio (SNR) $\gamma_{i,j}$ of the channel from terminal i to terminal j is given by $\gamma_{i,j} = E_i|h_{i,j}|^2/N_0$ and the average SNR is given by $\bar{\gamma}_{i,j} = E_i\sigma_{i,j}^2/N_0$, where E_i is the average transmission power at terminal i.

For BFSK signalling, the signals received by terminal i, $i = 1, 2 \cdots, K+1$, which were transmitted from the source, through the first frequency subband $y_{0,i,0}$ and through the second frequency subband $y_{0,i,1}$ are given by

$$\begin{aligned} y_{0,i,0} &= (1 - x_0)\sqrt{E_0}h_{0,i} + n_{0,i,0}, \\ y_{0,i,1} &= x_0\sqrt{E_0}h_{0,i} + n_{0,i,1}, \end{aligned} \tag{1}$$

where $x_0 = 0$ if the first frequency subband is used and $x_0 = 1$ if the second frequency subband is used.

For BPSK signalling, the signal $y_{0,i}$ received by terminal i, $i = 1, 2 \cdots, K+1$, which was transmitted from the source, is given by

$$y_{0,i} = (1 - 2x_0)\sqrt{E_0}h_{0,i} + n_{0,i}, \tag{2}$$

where $x_0 \in \{0, 1\}$.

In this letter, we will refer to x_0 as the transmitted signal at the source for BFSK and BPSK modulations, because x_0 can represent the two possibilities of the binary transmission. For DF, we suppose that, at relay i, the signal x_0 is decoded into x_i, $x_i \in \{0, 1\}$. Similarly, we will refer to x_i as the transmitted signal at relay i. Then for BFSK signalling, the signals received by the destination, which were transmitted from relay i, through the two frequency subbands, are given by

$$\begin{aligned} y_{i,K+1,0} &= (1 - x_i)\sqrt{E_i}h_{i,K+1} + n_{i,K+1,0}, \\ y_{i,K+1,1} &= x_i\sqrt{E_i}h_{i,K+1} + n_{i,K+1,1}. \end{aligned} \tag{3}$$

For BPSK signalling, the signal $y_{i,K+1}$ received by the destination, which was transmitted from relay i, is given by

$$y_{i,K+1} = (1 - 2x_i)\sqrt{E_i}h_{i,K+1} + n_{i,K+1}. \tag{4}$$

The log-likelihood ratio (LLR) of the ML detection for the uncoded DF cooperation with binary modulations has been shown to be given by [9], [10]:

$$\text{LLR} = t_0 + \sum_{i=1}^{K} \psi(t_i), \tag{5}$$

where

$$\psi(t_i) = \ln \frac{(1 - \epsilon_i)e^{t_i} + \epsilon_i}{\epsilon_i e^{t_i} + (1 - \epsilon_i)}. \tag{6}$$

In this equation, ϵ_i represents the average BER[4] at relay i,

[4]In this letter, we apply the same channel state information (CSI) assumption as in [10], [11] and [13] for coherent detection. Specifically, we assume that the destination only has the statistical CSI rather than the instantaneous CSI of the source-relay channels. Therefore, the destination only knows the average BER at the relays.

and it is given by

$$\epsilon_i = \begin{cases} \frac{1}{2}\left(1-\sqrt{\frac{\bar{\gamma}_{0,i}/2}{1+\bar{\gamma}_{0,i}/2}}\right), & \text{for coherent BFSK,} \\ \frac{1}{2}\left(1-\sqrt{\frac{\bar{\gamma}_{0,i}}{1+\bar{\gamma}_{0,i}}}\right), & \text{for coherent BPSK,} \\ \frac{1}{2+\bar{\gamma}_{0,i}}, & \text{for noncoherent BFSK,} \end{cases} \tag{7}$$

for $i = 1, 2, \cdots, K$. The sufficient statistics t_i, $i = 0, 1, \cdots, K+1$, are given by

$$t_i = \begin{cases} \frac{2\sqrt{E_i}\Re\{h^*_{i,K+1}(y_{i,K+1,0}-y_{i,K+1,1})\}}{N_0}, & \text{for coherent BFSK,} \\ \frac{4\sqrt{E_i}\Re\{h^*_{i,K+1}y_{i,K+1}\}}{N_0}, & \text{for coherent BPSK,} \\ \frac{\bar{\gamma}_{i,K+1}}{(1+\bar{\gamma}_{i,K+1})N_0}(|y_{i,K+1,0}|^2 - |y_{i,K+1,1}|^2), & \\ & \text{for noncoherent BFSK.} \end{cases} \tag{8}$$

It has been acknowledged that the exact closed-form BER analysis is very challenging even for single-relay systems with binary signallings due to the nonlinear behavior (6) of the ML detection [10]. Therefore, to facilitate the BER analysis, the PL approximation has been proposed [10]:

$$\psi(t_i) \approx \psi_{PL}(t_i) = \begin{cases} T_i, & \text{for } t_i \geq T_i, \\ t_i, & \text{for } -T_i < t_i < T_i, \\ -T_i, & \text{for } t_i \leq -T_i, \end{cases} \tag{9}$$

where $T_i = \ln\frac{1-\epsilon_i}{\epsilon_i}$, and $T_i > 0$ since $\epsilon_i < 1/2$. It has been demonstrated that the above PL approximation is very accurate [10]. For a symmetric network with identical channel variance of the first hop for each branch, T_i, $i = 1, 2, \cdots, K$, are identical. This simplified case was considered in [11] for a two-relay network. In this letter, we consider a general dissimilar network with different T_i for different branches.

In this letter, the PL approximation (9) which has been extensively used in many papers, e.g., [10]–[13], is also applied, and it is the only approximation we adopt in our analysis.

III. Closed-Form Approximate Average BER for Single-Relay Networks

In this section, we derive a closed-form approximate average BER expression for the single-relay DF cooperative diversity network. Note that t_i, $i = 0, 1, \cdots, K+1$, represent the sufficient statistics for ML detection. Hence, in order to assess the ML detection performance, it is needed to find the distribution of t_i. To this end, we first develop the following lemma.

Lemma 1: Consider a random variable $Y = |Z|X + |Z|^2$, where $X \sim \mathcal{N}(0, \sigma_x^2)$ and $Z \sim \mathcal{CN}(0, \sigma_z^2)$ are independent random variables. Then the PDF $f(a, b, c, y)$ of random variable Y is given by

$$f(a, b, c, y) = \begin{cases} a\exp(cy), & y \leq 0, \\ a\exp(by), & y > 0, \end{cases} \tag{10}$$

and the CDF $F(a, b, c, y)$ of random variable Y is given by

$$F(a, b, c, y) = \begin{cases} \frac{a}{c}\exp(cy), & y \leq 0, \\ \frac{a(b-c)}{bc} + \frac{a}{b}\exp(by), & y > 0, \end{cases} \tag{11}$$

where the parameters a, b, and c are given by

$$a = \frac{1}{\sigma_z\sqrt{\sigma_z^2 + 2\sigma_x^2}}, \tag{12}$$

$$b = \frac{1}{\sigma_x^2}\left(1 - \frac{\sqrt{\sigma_z^2 + 2\sigma_x^2}}{\sigma_z}\right), \tag{13}$$

$$c = \frac{1}{\sigma_x^2}\left(1 + \frac{\sqrt{\sigma_z^2 + 2\sigma_x^2}}{\sigma_z}\right). \tag{14}$$

Proof: From the standard probabilistic analysis, PDF of Y can be derived by taking expectation over the conditional PDF of Y given X, and CDF of Y can be derived by integrating the PDF of Y. □

Corollary 1: Suppose $Y = |Z|X - |Z|^2$, where $X \sim \mathcal{N}(0, \sigma_x^2)$ and $Z \sim \mathcal{CN}(0, \sigma_z^2)$ are independent random variables. Then the PDF and CDF of random variable Y are given by $f(a, b, c, -y)$ and $1 - F(a, b, c, -y)$, respectively.

Proof: Rewrite $Y = |Z|X - |Z|^2 = -(|Z|^2 - |Z|X)$, and denote $X^{'} = -X$, then $Y = -(|Z|^2 + |Z|X^{'})$. Since $X^{'}$ has the same distribution as X, by applying Lemma 1, the PDF of the term $|Z|^2 + |Z|X^{'}$ is given by $f(a, b, c, y)$. Therefore, it can be easily shown that the PDF of Y is $f(a, b, c, -y)$, and CDF is $1 - F(a, b, c, -y)$. □

Using Lemma 1 and Corollary 1, in the following theorem, we now derive the distribution of t_i, $i = 0, 1, \cdots, K$, given the transmitted signal x_i.

Theorem 1: The PDFs of t_i conditioned on $x_i = 0$ and $x_i = 1$ are given by $f(a_i, b_i, c_i, t_i)$ and $f(a_i, b_i, c_i, -t_i)$, respectively. The CDFs of t_i conditioned on $x_i = 0$ and $x_i = 1$ are given by $F(a_i, b_i, c_i, t_i)$ and $1 - F(a_i, b_i, c_i, -t_i)$, respectively. For coherent binary modulations, the parameters a_i, b_i, and c_i are given by (12), (13), and (14), respectively, with σ_z and σ_x replaced by $\sigma_{z_i} = 2\sqrt{\bar{\gamma}_{i,K+1}}$ and $\sigma_{x_i} = \sqrt{2}$ for coherent BPSK; by $\sigma_{z_i} = \sqrt{2\bar{\gamma}_{i,K+1}}$ and $\sigma_{x_i} = \sqrt{2}$ for coherent BFSK. For noncoherent BFSK, a_i, b_i, and c_i are given by $a_i = \frac{\lambda_i\lambda_i^{'}}{\lambda_i+\lambda_i^{'}}$, $b_i = -\lambda_i$, $c_i = \lambda_i^{'}$, where $\lambda_i = 1/\bar{\gamma}_{i,K+1}$ and $\lambda_i^{'} = 1 + \lambda_i$.

Proof: For coherent BFSK and BPSK, we can rewrite the expression of t_i in the same form as Y in Lemma 1. Therefore, the PDFs and CDFs for coherent BFSK and BPSK can be obtained by applying Lemma 1. For non-coherent BFSK, t_i is the subtraction of two exponential random variables. In this case, the distribution of t_i given x_i has been derived by Chen and Laneman in [10]. By comparing eq. (17) of [10] with our expression of (11), we observed that our obtained CDF expression of (11) reduce to eq. (17) of [10] when replacing a, b, and c with $a_i = \frac{\lambda_i\lambda_i^{'}}{\lambda_i+\lambda_i^{'}}$, $b_i = -\lambda_i$, $c_i = \lambda_i^{'}$, respectively, where $\lambda_i = 1/\bar{\gamma}_{i,K+1}$ and $\lambda_i^{'} = 1 + \lambda_i$. □

It can be seen from (5) that the ML detection for cooperative diversity networks with K relays involves $K+1$ sufficient statistics $t_0, t_1, \cdots, t_K$. In order to derive the average BER for single-relay cooperative networks, we develop the following lemma which deals with the probability involving two different statistics t_i and t_j.

Lemma 2: Define $\Pr(t_i + t_j < S_1, -S_2 < t_j < S_2 | x_i = 0, x_j) = g_{x_j}(a_i, b_i, c_i, a_j, b_j, c_j, S_1, S_2)$, where $x_j \in \{0, 1\}$, S_1 is any arbitrary real number, and $S_2 > 0$. Also, t_i and

t_j are two different statistics taken from (8) for a common modulation scheme with $i, j \in \{0, 1, \cdots, K\}$ and $i \neq j$. Define $\hat{b}_j = (1 - x_j)b_j - x_j c_j$ and $\hat{c}_j = (1 - x_j)c_j - x_j b_j$. Then $g_{x_j}(\cdot)$ is given as follows: If $b_i \neq \hat{b}_j$ and $c_i \neq \hat{c}_j$,

$$
\begin{aligned}
&g_{x_j}(a_i, b_i, c_i, a_j, b_j, c_j, S_1, S_2)\\
&= \begin{cases}
\frac{a_i(b_i-c_i)}{b_i c_i}\Big[F(a_j,\hat{b}_j,\hat{c}_j,S_2)-F(a_j,\hat{b}_j,\hat{c}_j,-S_2)\Big] \\
\quad +\frac{a_i a_j \exp(b_i S_1)}{b_i(b_i-\hat{c}_j)}\Big\{\exp\big[(b_i-\hat{c}_j)S_2\big]-1\Big\} \\
\quad +\frac{a_i a_j \exp(b_i S_1)}{b_i(b_i-\hat{b}_j)}\Big\{1-\exp\big[(\hat{b}_j-b_i)S_2\big]\Big\}, & \text{for } S_1 \geq S_2 > 0,\\
\frac{a_i(b_i-c_i)}{b_i c_i}\Big[F(a_j,\hat{b}_j,\hat{c}_j,S_1)-F(a_j,\hat{b}_j,\hat{c}_j,-S_2)\Big] \\
\quad +\frac{a_i a_j \exp(b_i S_1)}{b_i(b_i-\hat{c}_j)}\Big\{\exp\big[(b_i-\hat{c}_j)S_2\big]-1\Big\} \\
\quad +\frac{a_i a_j \exp(c_i S_1)}{c_i(c_i-\hat{b}_j)}\Big\{\exp\big[(\hat{b}_j-c_i)S_1\big]-\exp\big[(\hat{b}_j-c_i)S_2\big]\Big\} \\
\quad +\frac{a_i a_j \exp(b_i S_1)}{b_i(b_i-\hat{b}_j)}\Big\{1-\exp\big[(\hat{b}_j-b_i)S_1\big]\Big\}, & \text{for } 0 \leq S_1 < S_2,\\
\frac{a_i(b_i-c_i)}{b_i c_i}\Big[F(a_j,\hat{b}_j,\hat{c}_j,S_1)-F(a_j,\hat{b}_j,\hat{c}_j,-S_2)\Big] \\
\quad +\frac{a_i a_j \exp(c_i S_1)}{c_i(c_i-\hat{b}_j)}\Big\{1-\exp\big[(\hat{b}_j-c_i)S_2\big]\Big\} \\
\quad +\frac{a_i a_j \exp(b_i S_1)}{b_i(b_i-\hat{c}_j)}\Big\{\exp\big[(b_i-\hat{c}_j)S_2\big]-\exp\big[(\hat{c}_j-b_i)S_1\big]\Big\} \\
\quad +\frac{a_i a_j \exp(c_i S_1)}{c_i(c_i-\hat{c}_j)}\Big\{\exp\big[(\hat{c}_j-c_i)S_1\big]-1\Big\}, & \text{for } -S_2 \leq S_1 < 0,\\
\frac{a_i a_j \exp(c_i S_1)}{c_i(c_i-\hat{b}_j)}\Big\{1-\exp\big[(\hat{b}_j-c_i)S_2\big]\Big\} \\
\quad +\frac{a_i a_j \exp(c_i S_1)}{c_i(c_i-\hat{c}_j)}\Big\{\exp\big[(c_i-\hat{c}_j)S_2\big]-1\Big\}, & \text{for } S_1 < -S_2 < 0.
\end{cases}
\end{aligned}
\tag{15}
$$

If $b_i = \hat{b}_j$, the last terms in the first and second cases of (15) are replaced by $\frac{a_i a_j S_2 \exp(b_i S_1)}{b_i}$ and $\frac{a_i a_j S_1 \exp(b_i S_1)}{b_i}$, respectively. If $c_i = \hat{c}_j$, the last terms in the third and fourth cases of (15) are replaced by $-\frac{a_i a_j S_1 \exp(c_i S_1)}{c_i}$ and $\frac{a_i a_j S_2 \exp(c_i S_1)}{c_i}$, respectively.

Proof: By Theorem 1 and performing some mathematical manipulations, it is not hard to prove this lemma. □

From the expressions of a_i, b_i, and c_i given in Theorem 1, it can be easily verified that $a_i > 0$, $b_i < 0$, and $c_i > 0$. By some simple manipulations, it can be easily verified that $a_i \neq c_i$, $b_i + c_i \neq 0$, $b_i + b_j \neq 0$, $b_i - c_j \neq 0$, and $c_i - b_j \neq 0$ for any $i, j \in \{0, 1, \cdots, K\}$; but it is possible that $b_i = b_j$, $c_i = c_j$, or $b_i + c_j = 0$ for $i, j \in \{0, 1, \cdots, K\}$ and $i \neq j$. Therefore, the possibilities $b_i = \hat{b}_j$ and $c_i = \hat{c}_j$ need to be considered, and the possibilities $b_i - \hat{c}_j = 0$ and $c_i - \hat{b}_j = 0$ need not be considered in Lemma 2.

With Theorem 1 and Lemma 2, in the following theorem, we now derive an average BER expression of single-relay systems.

Theorem 2: For binary modulations, a closed-form approximate average BER $P_{B,1}$ of the single-relay ($K = 1$) cooperative diversity network is

$$
\begin{aligned}
P_{B,1} =& (1-\epsilon_1)\Big\{g_0(a_0,b_0,c_0,a_1,b_1,c_1,0,T_1) \\
&+ F(a_0,b_0,c_0,-T_1)\big[1-F(a_1,b_1,c_1,T_1)\big] \\
&+ F(a_0,b_0,c_0,T_1)F(a_1,b_1,c_1,-T_1)\Big\} \\
&+ \epsilon_1\Big\{g_1(a_0,b_0,c_0,a_1,b_1,c_1,0,T_1) \\
&+ F(a_0,b_0,c_0,-T_1)F(a_1,b_1,c_1,-T_1) \\
&+ F(a_0,b_0,c_0,T_1)\big[1-F(a_1,b_1,c_1,T_1)\big]\Big\},
\end{aligned}
\tag{16}
$$

where $\epsilon_1, T_1, a_i, b_i$, and c_i, $i = 0, 1$, are modulation-dependent parameters as described before.

Proof: Applying the total probability theorem, Theorem 1, and Lemma 2 yields this theorem. □

Note that (16) is truly a *closed-form* approximate average BER expression, which does not involve any numerical computations. In the expression of (16), by choosing a_i, b_i, and c_i as described in Theorem 1, ϵ_1 as given by (7), and $T_1 = \ln\frac{1-\epsilon_1}{\epsilon_1}$, we obtain the closed-form approximate average BERs of single-relay cooperative diversity networks for different binary modulation schemes including noncoherent BFSK, coherent BFSK, and coherent BPSK. In [10, eq. (14)], a closed-form approximate average BER expression was reported for the single-relay network with noncoherent BFSK. By simple manipulations, it can be shown that our BER expression of (16) reduces to eq. (14) of [10]. In this sense, therefore, our BER expression of (16) can be considered as a generalization of the expression of [10]. For coherent BPSK with a single relay, two *approximate* average BER expressions have been derived based on the PL approximation [12], [13]. However, these expressions involve numerical integrations, and thus, they are not truly closed-form. On the other hand, with the same PL approximation, our BER expression provides a truly closed-form approximate average BER expression for coherent BPSK. For coherent BFSK with a single relay, no average BER expression has been reported in the literature (see footnote 3). Therefore, our expression of (16) is the closed-form approximate average BER expression reported in the literature for the first time for coherent BFSK. Numerical results in Section V will demonstrate that (16) provides extremely accurate error probabilities for the ML detection in single-relay cooperative diversity networks adopting both coherent and noncoherent binary signallings.

In the next section, we will extend our analysis to the two-relay cooperative diversity networks.

IV. Closed-Form Approximate Average BER for Two-Relay Networks

In this section, we consider the two-relay *dissimilar* cooperative diversity network in which the channel variances of different relay branches are different in general. We derive a closed-form approximate average BER expression for the two-relay DF cooperative network again based on the PL approximation. Recall that the BER performance analysis of the single-relay case involved the computation of the probability for two statistics t_0 and t_1. For the two-relay case, one more statistic t_2 is involved, and therefore, the BER performance

TABLE I
COEFFICIENTS OF $A_{i,(x_j,x_k)}$ AND $B_{i,(x_j,x_k)}$.

$x_j=0$, $x_k=0$	$A_{1,(0,0)}=\big[1-F(a_j,b_j,c_j,T_j)\big]\big[1-F(a_k,b_k,c_k,T_k)\big]$ $B_{1,(0,0)}=\big[1-F(a_k,b_k,c_k,T_k)\big]$
	$A_{2,(0,0)}=\big[1-F(a_j,b_j,c_j,T_j)\big]F(a_k,b_k,c_k,-T_k)$ $B_{2,(0,0)}=F(a_k,b_k,c_k,-T_k)$
	$A_{3,(0,0)}=F(a_j,b_j,c_j,-T_j)\big[1-F(a_k,b_k,c_k,T_k)\big]$ $B_{3,(0,0)}=\big[1-F(a_j,b_j,c_j,T_j)\big]$
	$A_{4,(0,0)}=F(a_j,b_j,c_j,-T_j)F(a_k,b_k,c_k,-T_k)$ $B_{4,(0,0)}=F(a_j,b_j,c_j,-T_j)$
$x_j=0$, $x_k=1$	$A_{1,(0,1)}=A_{2,(0,0)}$, $B_{1,(0,1)}=B_{2,(0,0)}$
	$A_{2,(0,1)}=A_{1,(0,0)}$, $B_{2,(0,1)}=B_{1,(0,0)}$
	$A_{3,(0,1)}=A_{4,(0,0)}$, $B_{3,(0,1)}=B_{3,(0,0)}$
	$A_{4,(0,1)}=A_{3,(0,0)}$, $B_{4,(0,1)}=B_{4,(0,0)}$
$x_j=1$, $x_k=0$	$A_{1,(1,0)}=A_{3,(0,0)}$, $B_{1,(1,0)}=B_{1,(0,0)}$
	$A_{2,(1,0)}=A_{4,(0,0)}$, $B_{2,(1,0)}=B_{2,(0,0)}$
	$A_{3,(1,0)}=A_{1,(0,0)}$, $B_{3,(1,0)}=B_{4,(0,0)}$
	$A_{4,(1,0)}=A_{2,(0,0)}$, $B_{4,(1,0)}=B_{3,(0,0)}$
$x_j=1$, $x_k=1$	$A_{1,(1,1)}=A_{4,(0,0)}$, $B_{1,(1,1)}=B_{2,(0,0)}$
	$A_{2,(1,1)}=A_{3,(0,0)}$, $B_{2,(1,1)}=B_{1,(0,0)}$
	$A_{3,(1,1)}=A_{2,(0,0)}$, $B_{3,(1,1)}=B_{4,(0,0)}$
	$A_{4,(1,1)}=A_{1,(0,0)}$, $B_{4,(1,1)}=B_{3,(0,0)}$

analysis becomes even harder. In this analysis, Theorem 1 and Lemma 2 of Section III are still needed. In the following, we first derive two lemmas which deal with three sufficient statistics.

Lemma 3: Let t_i, t_j, and t_k be three different statistics taken from (8) for a common modulation scheme with $i,j,k \in \{0,1,\cdots,K\}$ and $i \neq j \neq k$, $x_j, x_k \in \{0,1\}$. Define $\Pr(t_i+t_j+t_k<0, -T_j<t_j<T_j, -T_k<t_k<T_k \big| x_i=0, x_j, x_k) = W_{x_j,x_k}(a_i,b_i,c_i,a_j,b_j,c_j,a_k,b_k,c_k,T_j,T_k)$ assuming $0 \leq T_j \leq T_k$. Define $\hat{b}_j=(1-x_j)b_j-x_jc_j$, $\hat{c}_j=(1-x_j)c_j-x_jb_j$, $\hat{b}_k=(1-x_k)b_k-x_kc_k$, and $\hat{c}_k=(1-x_k)c_k-x_kb_k$. Then $W_{x_j,x_k}(\cdot)$ is given as follows:

$$\begin{aligned}
&W_{x_j,x_k}(a_i,b_i,c_i,a_j,b_j,c_j,a_k,b_k,c_k,T_j,T_k)\\
&\quad=\frac{(-1)^{x_k}a_i(b_i-c_i)}{b_ic_i}\Big\{g_m(a_k,b_k,c_k,a_j,b_j,c_j,0,T_j)\\
&\quad-F(a_k,b_k,c_k,(-1)^{x_k+1}T_k)\big[F(a_j,b_j,c_j,T_j)\\
&\quad-F(a_j,b_j,c_j,-T_j)\big]\Big\}\\
&\quad+\eta_{x_j,x_k}(a_i,b_i,c_i,a_j,b_j,c_j,a_k,b_k,c_k,T_j,T_k)\\
&\quad+\varphi_{x_j,x_k}(a_i,b_i,c_i,a_j,b_j,c_j,a_k,b_k,c_k,T_j,T_k).
\end{aligned}\tag{17}$$

In this equation, $m=x_j \oplus x_k$, where $\oplus$ represents the modulo 2 addition. Function $\eta_{x_j,x_k}(\cdot)$ is defined as follows: If $b_i \neq \hat{b}_j$ and $b_i \neq \hat{b}_k$, $\eta_{x_j,x_k}(\cdot)$ is given on top of the next page; If $b_i=\hat{b}_j$, the second term of (18) is replaced by $\frac{a_ia_ja_k}{b_i(\hat{c}_k-b_i)}\Big[\frac{\exp(b_i-\hat{c}_k)T_j)-1}{b_i-\hat{c}_k}-T_j\exp((b_i-\hat{c}_k)T_k)\Big]$. If $b_i=\hat{b}_k$, the last term of (18) is replaced by $\frac{a_ia_ja_k}{b_i(b_i-\hat{c}_j)^2}\Big\{1+\big[(b_i-\hat{c}_j)T_j-1\big]\exp((b_i-\hat{c}_j)T_j)\Big\}$. Also, function $\varphi_{x_j,x_k}(\cdot)$ is defined as follows: If $c_i \neq \hat{c}_j$, $c_i \neq \hat{c}_k$, $\varphi_{x_j,x_k}(\cdot)$ is given on top of the next page; If $c_i=\hat{c}_j$, the second term of (19) is replaced by $\frac{a_ia_ja_k}{c_i(\hat{b}_k-c_i)}\Big[T_j\exp((\hat{b}_k-c_i)T_k)+\frac{1-\exp((\hat{b}_k-c_i)T_j)}{\hat{b}_k-c_i}\Big]$. If $c_i=\hat{c}_k$, the last term of (19) is replaced by $\frac{a_ia_ja_k}{c_i(\hat{b}_j-c_i)^2}\Big\{1+\big[(\hat{b}_j-c_i)T_j-1\big]\exp((\hat{b}_j-c_i)T_j)\Big\}$.

Proof: We can prove this lemma with the help of Theorem 1 and Lemma 2; but the proof is very long. Thus, we will just present the main ideas for the proof. Since three random variables t_i, t_j, and t_k are involved in the definition of $W_{x_j,x_k}(\cdot)$, a triple-integral needs to be solved. Because of the asymmetry of the PDF of t_i given x_i, it is needed to divide the integral region for each integral such that a fixed PDF function is determined in each integrand. After dividing the integral regions according to this requirement, the expression of $W_{x_j,x_k}(\cdot)$ can be derived through some simple manipulations. □

For the same reason as in Lemma 2, only the possibilities $b_i=\hat{b}_j$ and $b_i=\hat{b}_k$ are considered for (18), and $c_i=\hat{c}_j$ and $c_i=\hat{c}_k$ are considered for (19) in Lemma 3. Using Lemma 3, we can prove the following lemma.

Lemma 4: Let t_i, t_j, and t_k be three different statistics taken from (8) for a common modulation scheme with $i,j,k \in \{0,1,\cdots,K\}$ and $i \neq j \neq k$, $x_j,x_k \in \{0,1\}$. Define $\Pr(t_i+\psi_{PL}(t_j)+\psi_{PL}(t_k)<0 \big| x_i=0,x_j,x_k) = U_{x_j,x_k}(a_i,b_i,c_i,a_j,b_j,c_j,a_k,b_k,c_k,T_j,T_k)$ assuming $0 \leq T_j \leq T_k$. Then

$$\begin{aligned}
&U_{x_j,x_k}(a_i,b_i,c_i,a_j,b_j,c_j,a_k,b_k,c_k,T_j,T_k)\\
&\quad=A_{1,(x_j,x_k)}F\big[a_i,b_i,c_i,-(T_j+T_k)\big]\\
&\qquad+A_{2,(x_j,x_k)}F(a_i,b_i,c_i,T_k-T_j)\\
&\qquad+A_{3,(x_j,x_k)}F(a_i,b_i,c_i,T_j-T_k)\\
&\qquad+A_{4,(x_j,x_k)}F(a_i,b_i,c_i,T_j+T_k)\\
&\qquad+B_{1,(x_j,x_k)}g_{x_j}(a_i,b_i,c_i,a_j,b_j,c_j,-T_k,T_j)\\
&\qquad+B_{2,(x_j,x_k)}g_{x_j}(a_i,b_i,c_i,a_j,b_j,c_j,T_k,T_j)\\
&\qquad+B_{3,(x_j,x_k)}g_{x_k}(a_i,b_i,c_i,a_k,b_k,c_k,-T_j,T_k)\\
&\qquad+B_{4,(x_j,x_k)}g_{x_k}(a_i,b_i,c_i,a_k,b_k,c_k,T_j,T_k)\\
&\qquad+W_{x_j,x_k}(a_i,b_i,c_i,a_j,b_j,c_j,a_k,b_k,c_k,T_j,T_k),
\end{aligned}\tag{20}$$

and the coefficients $A_{i,(x_j,x_k)}$ and $B_{i,(x_j,x_k)}$, $i=1,2,3,4$, are presented in Table I.

Proof: Applying the total probability theorem according to the possibilities of t_j and t_k, $U_{x_j,x_k}(\cdot)$ can be written as a combination of nine terms. We observed that the nine terms can be classified as three types: the first four terms including the probability of a single random variable, the following four terms including the joint probability of two random variables, and the last including the joint probability of three random variables. Therefore, the first four terms can be determined by the individual CDFs (11) of t_i, t_j, and t_k, given x_i, x_j, and x_k, respectively. The following four terms can be determined through Lemma 2. The last term can be determined through Lemma 3. Finally, the expression of (20) can be derived. □

Using Lemma 4, we now derive an average BER expression for the two-relay cooperative networks in the following theorem.

Theorem 3: For binary modulations, a closed-form approximate average BER $P_{B,2}$ of the two-relay ($K=2$) cooperative

$$\begin{aligned}
&\eta_{x_j,x_k}(a_i,b_i,c_i,a_j,b_j,c_j,a_k,b_k,c_k,T_j,T_k)\\
&\quad=\frac{a_ia_ja_k}{b_i(\hat{c}_j-b_i)(\hat{c}_k-b_i)}\Big\{1-\exp\big[(b_i-\hat{c}_j)T_j\big]\Big\}\Big\{1-\exp\big[(b_i-\hat{c}_k)T_k\big]\Big\}\\
&\qquad+\frac{a_ia_ja_k}{b_i(\hat{c}_k-b_i)}\left\{\frac{\exp\big[(\hat{b}_j-\hat{c}_k)T_j\big]-1}{\hat{b}_j-\hat{c}_k}-\frac{\exp\big[(b_i-\hat{c}_k)T_k\big]\Big\{\exp\big[(\hat{b}_j-b_i)T_j\big]-1\Big\}}{\hat{b}_j-b_i}\right\}\\
&\qquad+\frac{a_ia_ja_k}{b_i(\hat{b}_k-b_i)}\left\{\frac{1-\exp\big[(b_i-\hat{c}_j)T_j\big]}{b_i-\hat{c}_j}-\frac{1-\exp\big[(\hat{b}_k-\hat{c}_j)T_j\big]}{\hat{b}_k-\hat{c}_j}\right\}.
\end{aligned}\tag{18}$$

$$\begin{aligned}
&\varphi_{x_j,x_k}(a_i,b_i,c_i,a_j,b_j,c_j,a_k,b_k,c_k,T_j,T_k)\\
&\quad=\frac{a_ia_ja_k}{c_i(\hat{b}_j-c_i)(\hat{b}_k-c_i)}\Big\{\exp\big[(\hat{b}_j-c_i)T_j\big]-1\Big\}\Big\{\exp\big[(\hat{b}_k-c_i)T_k\big]-1\Big\}\\
&\qquad+\frac{a_ia_ja_k}{c_i(\hat{b}_k-c_i)}\left\{\frac{\exp\big[(\hat{b}_k-c_i)T_k\big]\Big\{1-\exp\big[(c_i-\hat{c}_j)T_j\big]\Big\}}{\hat{c}_j-c_i}-\frac{1-\exp\big[(\hat{b}_k-\hat{c}_j)T_j\big]}{\hat{c}_j-\hat{b}_k}\right\}\\
&\qquad+\frac{a_ia_ja_k}{c_i(\hat{c}_k-c_i)}\left\{\frac{\exp\big[(\hat{b}_j-c_i)T_j\big]-1}{\hat{b}_j-c_i}-\frac{\exp\big[(\hat{b}_j-\hat{c}_k)T_j\big]-1}{\hat{b}_j-\hat{c}_k}\right\}.
\end{aligned}\tag{19}$$

diversity network is

$$\begin{aligned}
P_{B,2}=&\sum_{x_{n_1}\in\{0,1\}}\sum_{x_{n_2}\in\{0,1\}}\Big\{\big[(1-x_{n_1})(1-\epsilon_{n_1})+x_{n_1}\epsilon_{n_1}\big]\\
&\times\big[(1-x_{n_2})(1-\epsilon_{n_2})+x_{n_2}\epsilon_{n_2}\big]\\
&\times U_{x_{n_1},x_{n_2}}(a_0,b_0,c_0,a_{n_1},b_{n_1},c_{n_1},a_{n_2},b_{n_2},c_{n_2},T_{n_1},T_{n_2})\Big\}.
\end{aligned}\tag{21}$$

where n_1 and n_2 denote the two relays with $n_1=1$ and $n_2=2$ if $0\le T_1\le T_2$, and $n_1=2$ and $n_2=1$ if $0<T_2<T_1$; x_{n_1} and x_{n_2} are the possible transmitted signals at relay n_1 and relay n_2, respectively; The modulation-dependent parameters ϵ_i, T_i, $i=1,2$, a_j, b_j, and c_j, $j=0,1,2$, are described as before.

Proof: For binary modulations with equal priors (the signals 0 and 1 are transmitted by the source with equal probability), the average BER equals the probability of error when either 0 or 1 is transmitted [1]. Without loss of generality, $P_{B,2}$ can be written as follows:

$$\begin{aligned}
P_{B,2}&=\Pr(t_0+\psi_{PL}(t_1)+\psi_{PL}(t_2)<0\big|x_0=0)\\
&=\Pr(t_0+\psi_{PL}(t_{n_1})+\psi_{PL}(t_{n_2})<0\big|x_0=0),
\end{aligned}\tag{22}$$

where n_1 and n_2 are described as in Theorem 3. We have rewritten the expression of (22) in the second equality to give $0\le T_{n_1}\le T_{n_2}$ so that Lemma 4 can be applied. Then applying the total probability theorem based on possibilities of x_1 and x_2 yields the an expression of (23), which is given on top of next page. Finally, applying Lemma 4 to (23) gives the expression of (21). □

Note that (21) is truly a closed-form approximate average BER expression for dissimilar two-relay cooperative networks. Again, by choosing a_j, b_j, and c_j as described in Theorem 1, ϵ_i as given by (7), and $T_i=\ln\frac{1-\epsilon_i}{\epsilon_i}$, we obtain the average BERs for two-relay cooperative diversity networks with different binary modulation schemes including coherent BPSK, coherent BFSK, and noncoherent BFSK. For two-relay systems, the only BER expression reported in the literature was the closed-form approximate average BER expression in [11, eq. (4.14)]; however, it is valid only for symmetric networks with noncoherent BFSK. On the other hand, our expression of (21) is the closed-form approximate average BER expression for the general dissimilar two-relay networks with noncoherent BFSK. In this sense, therefore, our BER expression of (21) can be considered as a generalization of the BER expression of [11]. For coherent BFSK and coherent BPSK, no average BER expressions have been reported for two-relay systems. Therefore, our expression of (21) is the closed-form approximate average BER expression reported in the literature for the first time for coherent BFSK and coherent BPSK. Numerical results in Section V will demonstrate that (21) provides extremely accurate error probabilities for the ML detection in two-relay dissimilar cooperative diversity networks adopting both coherent and noncoherent binary modulations.

V. Numerical Results

In this section, we compare the proposed approximate BER expressions with the exact BER obtained by Monte-Carlo simulations of the exact ML detection of (5) along with (6). We use the alphabetic indices $\{s,r,d\}$ to characterize the single-relay network, and the numeric indices $\{0,1,2,3\}$ as in Section II to characterize the two-relay network. We consider an equal power allocation with total transmission power of the whole network normalized to 1. Specifically, $E_s=E_r=1/2$ for single-relay systems and $E_0=E_1=E_2=1/3$ for two-relay systems. For ease of exposition, we assume that the source, relays, and the destination are located in a straight

$$\begin{aligned}P_{B,2} = &(1-\epsilon_{n_1})(1-\epsilon_{n_2})\Pr(t_0+\psi_{PL}(t_{n_1})+\psi_{PL}(t_{n_2})<0\big|x_0=0,x_{n_1}=0,x_{n_2}=0)\\&+(1-\epsilon_{n_1})\epsilon_{n_2}\Pr(t_0+\psi_{PL}(t_{n_1})+\psi_{PL}(t_{n_2})<0\big|x_0=0,x_{n_1}=0,x_{n_2}=1)\\&+\epsilon_{n_1}(1-\epsilon_{n_2})\Pr(t_0+\psi_{PL}(t_{n_1})+\psi_{PL}(t_{n_2})<0\big|x_0=0,x_{n_1}=1,x_{n_2}=0)\\&+\epsilon_{n_1}\epsilon_{n_2}\Pr(t_0+\psi_{PL}(t_{n_1})+\psi_{PL}(t_{n_2})<0\big|x_0=0,x_{n_1}=1,x_{n_2}=1).\end{aligned} \tag{23}$$

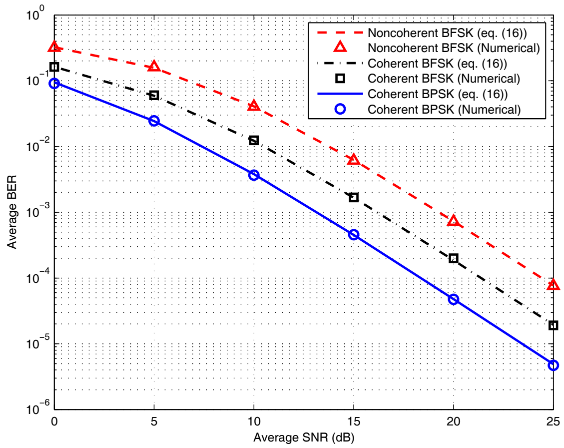

Fig. 2. Average BER of the single-relay cooperative network with $d_{s,r} = 0.1$.

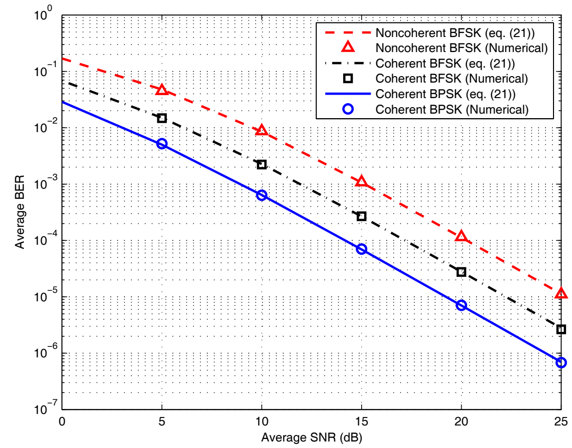

Fig. 4. Average BER of the two-relay symmetric network with $d_{0,1} = 0.5$ and $d_{0,2} = 0.5$.

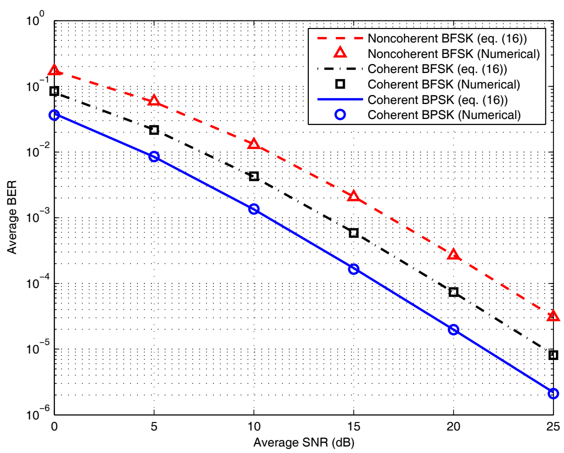

Fig. 3. Average BER of the single-relay cooperative network with $d_{s,r} = 0.5$.

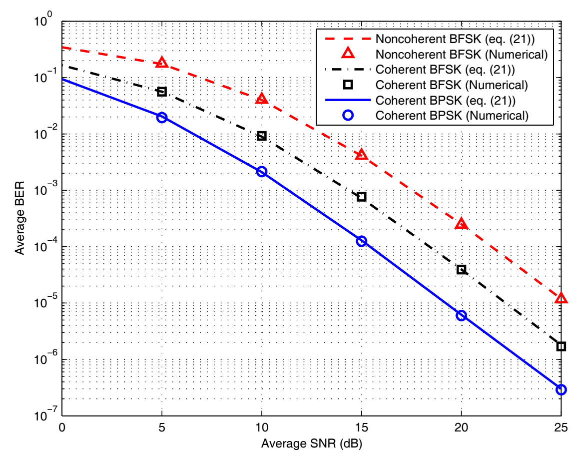

Fig. 5. Average BER of the two-relay dissimilar network with $d_{0,1} = 0.1$ and $d_{0,2} = 0.9$.

line.[5] The distance $d_{i,j}$ from node i to node j is normalized by the distance between the source node and the destination node, and hence, $d_{s,r}+d_{r,d} = 1$ for $K = 1$, and $d_{0,i}+d_{i,3} = 1$, $i = 1, 2$, for $K = 2$. The channel variances are modeled as $\sigma_{i,j}^2 = d_{i,j}^{-4}$. We plot the average BER with respect to the ratio of total power over the noise variance, which is $1/N_0$.

Firstly, cooperative diversity networks with a single relay are considered. Figs. 2 and 3 show the results for cooperative networks with $d_{s,r} = 0.1$ and $d_{s,r} = 0.5$, respectively. Secondly, we consider cooperative diversity networks with two relays. Figs. 4 and 5 show the simulated average BER for two cases: 1) the symmetric network with $d_{0,1} = 0.5$, $d_{0,2} = 0.5$; 2) the dissimilar network with $d_{0,1} = 0.1$, $d_{0,2} = 0.9$. It can be seen that the proposed BER expressions overlap the simulation results. Moreover, the BER curves for both the single-relay and two-relay networks demonstrate 3 dB shifts to the left, from noncoherent BFSK to coherent BFSK and from coherent BFSK to coherent BPSK, which can be expected from coherent and noncoherent communication theories.

[5]This assumption has also been made in [9]–[11] because it is very convenient to model the fading parameters for both dissimilar and symmetric cooperative diversity networks by simply choosing the positions of the relays in a straight line.

VI. Conclusions

In this letter, we have analyzed the average BER of the ML detection for uncoded DF cooperative diversity networks. Specifically, two typical cooperative diversity networks were considered: the single-relay cooperative diversity network with the direct source-destination link and the two-relay cooperative diversity network with the direct source-destination link. First, we derived the PDFs and CDFs of the sufficient statistics for the ML detection at the destination. Then we applied the accurate PL approximation and derived closed-form approximate average BER expressions with the help of the obtained PDFs and CDFs. Our BER expressions were shown to be valid for the general dissimilar DF networks adopting both coherent and noncoherent binary signallings. We also showed that our BER expressions can be considered as generalizations of the previously reported results in the literature. Throughout our analysis, only one approximation, i.e., the accurate PL approximation was made. Simulation results match excellently with the theoretical analysis, which validates our proposed BER expressions.

Acknowledgement

The authors would like to thank Dr. S. Gazor for valuable discussions on the derivation of (10).

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