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Delay-dependent stabilization for stochastic fuzzy systems with time delays☆

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Abstract

This paper is concerned with the delay-dependent stabilization problem for a class of time-delay stochastic fuzzy systems. The time delays are assumed to appear in both the state and the control input. The purpose is the design of a state-feedback fuzzy controller such that the resulting closed-loop system is asymptotically stable in the mean square. A delay-dependent condition for the solvability of this problem is obtained in terms of relaxed linear matrix inequalities (LMIs). By solving these LMIs, a desired controller can be obtained. Finally, a numerical example is given to demonstrate the effectiveness of the present results.

Keywords: Delay-dependent stabilization; Fuzzy systems; Linear matrix inequalities; Stochastic systems; Time delays

1. Introduction

The Takagi–Sugeno (T–S) fuzzy model [19] has proven to be a powerful tool for modeling complex nonlinear systems. It is known that by using this fuzzy model a nonlinear system can be described as a weighted sum of some simple linear subsystems and then can be stabilized by a model-based fuzzy control. For this reason, many issues related to the stability analysis and control synthesis of T–S fuzzy systems have been reported over the past two decades; see, for example, [1,9,12,15,21,20,30] and references therein. Recently, the stability and stabilization problems for time-delay T–S fuzzy systems have been investigated. For instance, some stability results were presented in [2,3], where stabilizing controllers were also designed by using a *linear matrix inequality* (LMI) approach. For T–S fuzzy systems with bounded uncertain delays, an algebraic inequality method was provided in [28] for the problems of stability analysis and fuzzy control design. In [31] the robust stabilization problem for discrete-time fuzzy systems with both time delays and parameter uncertainties was studied based on a basis-dependent Lyapunov function approach.

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It is worth noting that the results in [2,3,28,31] are delay-independent. The delay-dependent stability conditions for time-delay T–S fuzzy systems have also been reported [10,11,13,22], the results in which are considered less conservative than the delay-independent ones, especially in the case when delays are small.

On the other hand, the study of stochastic systems with time delays has received much attention, and various results on stability analysis [16,18,27,29], H_∞ controller design [5,6,25,26], and H_∞ filter design [24] have been reported in the literature. The problem of delay-dependent stability analysis for stochastic time-delay systems has also been studied and various criteria have been obtained; see, for example, [16,27]. The delay-dependent stabilization problem for stochastic systems with uncertainties and time-varying delays was studied in [6].

Recently, there has been a growing attention on the study of stochastic T–S fuzzy systems. It has been known that a class of nonlinear stochastic systems can be approximated by the T–S fuzzy model. Thus, we can deal with the stability analysis and controller design problems of nonlinear stochastic systems via fuzzy logic approach. The stability and stabilization problems for stochastic T–S fuzzy systems was investigated in [14], where the LMI approach was developed for state-feedback fuzzy controller design. When time delays appear in a stochastic fuzzy system, a delay-independent stability condition was given in [23]. A sliding mode fuzzy controller was designed in [8] to stabilize a stochastic T–S fuzzy system with unknown nonlinearities and constant time delays. To the authors' knowledge, so far, the problem of delay-dependent stabilization for stochastic T–S fuzzy systems with both state delays and input delays has not been addressed in the literature, which is still open and remains unsolved. This motivates the present study.

In this paper, we investigate the delay-dependent stabilization problem for stochastic fuzzy systems with state and input delays. We aim at designing a state-feedback fuzzy controller such that the resulting closed-loop system is asymptotically stable in the mean square. A delay-dependent sufficient condition for the solvability of the formulated problem is proposed in terms of relaxed LMIs [9,15]. The desired state-feedback controller is constructed by solving certain LMIs, which can be implemented by using standard numerical algorithms [7]. A numerical example is presented at last to demonstrate the effectiveness of the proposed method.

Notations: Throughout this paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, and $\mathbb{R}^{m\times n}$ denotes the set of all $m \times n$ real matrices. A real symmetric matrix $P > 0(\geqslant 0)$ denotes that P is a positive definite (or positive semi-definite) matrix, and $A > (\geqslant) B$ means $A - B > (\geqslant) 0$. I denotes an identity matrix of appropriate dimension. The superscript 'T' represents the transpose. $*$ is used as an ellipsis for terms that are induced by symmetry. Let $\tau > 0$ and $C([-\tau, 0]; \mathbb{R}^n)$ denote the family of continuous functions φ from $[-\tau, 0]$ to $\mathbb{R}^n$ with the norm $\|\varphi\| = \sup_{-\tau\leqslant\theta\leqslant 0} |\varphi(\theta)|$, where $|\cdot|$ is the Euclidean norm in $\mathbb{R}^n$. Denoted by $L^p_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$, the family of all $\mathcal{F}_0$-measurable $C([-\tau, 0]; \mathbb{R}^n)$-valued random variables $\xi = \{\xi(\theta) : -\tau \leqslant \theta < 0\}$ such that $\sup_{-\tau\leqslant\theta\leqslant 0} \mathbb{E}|\xi(\theta)|^p < \infty$, where $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure P. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. Problem formulation

Consider a T–S fuzzy stochastic model with state and input delays, in which the ith rule is formulated in the following form:

Plant rule i: **IF** $\xi_1(t)$ **is** θ_1^i **and** $\ldots$ $\xi_r(t)$ is θ_r^i, **THEN**

$$\dot{x}(t) = A_i x(t) + A_{di} x(t-\tau) + B_i u(t) + B_{di} u(t-\tau) + C_i x(t)\omega(t), \tag{1}$$

$$x(t) = \phi(t), \quad t \in [-\tau, 0], \quad i = 1, 2, \ldots, s, \tag{2}$$

where $\xi_1(t), \ldots, \xi_r(t)$ are the premise variables; $\theta_1^i, \ldots, \theta_r^i$ are the fuzzy sets; $x(t) \in \mathbb{R}^n$ is the state; $u(t) \in \mathbb{R}^m$ is the control input; $\omega(t)$ is a scalar zero mean Gaussian white noise process with unit covariance; the scalar $\tau > 0$ is a time delay that appears in both the state and the input; $\phi(t)$ is a continuous vector valued initial function. A_i, A_{di}, B_i, B_{di} and C_i are known constant matrices with appropriate dimensions. s is the number of IF–THEN rules.

As in [23], we assume that the premise variables do not depend on the control input variables $u(t)$ and the noise-input variables $\omega(t)$. The fuzzy basis functions are given by

$$h_i[\xi(t)] = \frac{\prod_{j=1}^{r} \mu_{ij}[\xi_j(t)]}{\sum_{l=1}^{s} \prod_{j=1}^{r} \mu_{lj}[\xi_j(t)]}, \quad i = 1, \ldots, s, \tag{3}$$

where $\mu_{ij}[\xi_j(t)]$ is the grade of membership of $\xi_j(t)$ in F_j^i and $\xi(t) = (\xi_1(t), \ldots, \xi_r(t))$. In what follows, we will drop the argument of $h_i[\xi(t)]$ for simplicity. By definition, the fuzzy basis functions satisfy

$$h_i \geqslant 0, \quad i = 1, \ldots, s, \quad \text{and} \quad \sum_{i=1}^{s} h_i = 1. \tag{4}$$

The defuzzified output of the delayed T–S fuzzy stochastic model (1) is given by

$$\dot{x}(t) = \sum_{i=1}^{s} h_i[A_i x(t) + A_{di} x(t-\tau) + B_i u(t) + B_{di} u(t-\tau) + C_i x(t)\omega(t)]. \tag{5}$$

Now, consider the following fuzzy control law:
Controller rule i: **IF** $\xi_1(t)$ **is** θ_1^i **and** $\ldots$ $\xi_r(t)$ is θ_r^i, **THEN**

$$u(t) = K_i x(t), \quad i = 1, 2, \ldots, s. \tag{6}$$

The overall state feedback controller is given by

$$u(t) = \sum_{i=1}^{s} h_i K_i x(t). \tag{7}$$

Under control law (7), the closed-loop system is obtained as follows:

$$\dot{x}(t) = \sum_{i=1}^{s}\sum_{j=1}^{s} h_i h_j A_{ij} x(t) + \sum_{i=1}^{s}\sum_{j=1}^{s} h_i h_{dj} A_{dij} x(t-\tau) + \sum_{i=1}^{s} h_i C_i x(t)\omega(t), \tag{8}$$

where

$$A_{ij} = A_i + B_i K_j, \quad A_{dij} = A_{di} + B_{di} K_j, \quad h_{dj} = h_j[\xi(t-\tau)].$$

Throughout the paper we shall adopt the following definition.

Definition 1 (*Wang et al. [23]*). For every $\eta \in L^2_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$, the system (8) is called to be asymptotically stable in the mean square if $\lim_{t\to\infty} \mathbb{E}|x(t;\eta)|^2 = 0$.

The purpose of this paper is to design state-feedback fuzzy controllers in the form of (7), such that, for a prescribed scalar $\bar{\tau} > 0$ and any time-delay τ satisfying $0 < \tau \leqslant \bar{\tau}$, the closed-loop system (8) is asymptotically stable in the mean square.

3. Main results

In this section, we will give some relaxed LMI-based conditions for the solvability of the delay-dependent stabilization problem for stochastic time-delay T–S fuzzy systems. We first present the following definition and lemma, which will be used in the proof of our main results.

Definition 2 (*Mao [17] and Wang et al. [23]*). (Itô's differential operator) Consider a general stochastic system

$$\dot{x}(t) = f(x(t), t) + g(x(t))\omega(t),$$

where $\omega(t)$ is an m-dimensional white noise with unit intensity, $f : \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^n$ and $g : \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^n$ satisfy the local Lipschitz condition and the linear growth condition. Let $C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+ \times \mathcal{S}; \mathbb{R}_+)$ denote the family of all nonnegative functions $Y(x, t, i)$ on $\mathbb{R}^n \times \mathbb{R}_+ \times \mathcal{S}$ that are continuously twice differentiable in x and once differentiable in t. An Itô's differential operator $\mathcal{L}$ acting on $C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+ \times \mathcal{S}; \mathbb{R}_+)$ is defined by

$$\mathcal{L}V(x,t) = V_t(x,t) + V_x(x,t) f(x,t) + \tfrac{1}{2}[g^{\mathrm{T}}(x,t) V_{xx}(x,t) g(x,t)] + V_x(x,t) g(x,t)\omega(t),$$

where $V_x = (V_{x_1}, \ldots, V_{x_n})$ and $V_{xx} = (V_{x_i} V_{x_j})_{n\times n}$.

Lemma 1 (*Cao and Lin [4]*). *Suppose that matrices $\{\mathcal{M}_i\}_{i=1}^{s} \in \mathbb{R}^{n\times m}$ and a positive-semidefinite matrix $\mathcal{P} \in \mathbb{R}^{n\times n}$ are given. If $\sum_{i=1}^{s} h_i = 1$ and $0 \leqslant h_i \leqslant 1$, then*

$$\left(\sum_{i=1}^{s} h_i\mathcal{M}_i\right)^{\mathrm{T}} \mathcal{P} \left(\sum_{i=1}^{s} h_i\mathcal{M}_i\right) \leqslant \sum_{i=1}^{s} h_i\mathcal{M}_i^{\mathrm{T}}\mathcal{P}\mathcal{M}_i.$$

Our first theorem is given as follows.

Theorem 1. *Assume that the controller gains $\{K_i\}_{i=1}^{s}$ are given. For a given scalar $\bar{\tau} > 0$, the system* (8) *is asymptotically stable in the mean square for any time-delay τ satisfying $0 < \tau \leqslant \bar{\tau}$, if there exist matrices $P > 0$, $Q > 0$, $R > 0$, $\{E_i, F_i, 1\leqslant i\leqslant s\}$, $\{\Lambda_{il}, 1\leqslant i\leqslant s, 1\leqslant l\leqslant s\}$ and $\{\Theta_{ijl}, 1\leqslant i < j\leqslant s, 1\leqslant l\leqslant s\}$ with*

$$\Lambda_{il} = \begin{bmatrix} G_{il}^{(11)} & G_{il}^{(12)} & G_{il}^{(13)} \\ * & G_{il}^{(22)} & G_{il}^{(23)} \\ * & * & G_{il}^{(33)} \end{bmatrix} \quad (1\leqslant i\leqslant s, 1\leqslant l\leqslant s)$$

and

$$\Theta_{ijl} = \begin{bmatrix} H_{ijl}^{(11)} & H_{ijl}^{(12)} & H_{ijl}^{(13)} \\ H_{ijl}^{(21)} & H_{ijl}^{(22)} & H_{ijl}^{(23)} \\ H_{ijl}^{(31)} & H_{ijl}^{(32)} & H_{ijl}^{(33)} \end{bmatrix} \quad (1\leqslant i < j\leqslant s, 1\leqslant l\leqslant s),$$

such that the following conditions are satisfied:

$$\Psi_{iil} < \Lambda_{il} \quad (1\leqslant i\leqslant s, 1\leqslant l\leqslant s), \tag{9}$$

$$\Psi_{ijl} + \Psi_{jil} < \Theta_{ijl} + \Theta_{ijl}^{\mathrm{T}} \quad (1\leqslant i < j\leqslant s, 1\leqslant l\leqslant s), \tag{10}$$

$$\begin{bmatrix} \Lambda_{1l} & \Theta_{12l} & \cdots & \Theta_{1sl} \\ * & \Lambda_{2l} & \cdots & \Theta_{2sl} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & \Lambda_{sl} \end{bmatrix} < 0 \quad (1\leqslant l\leqslant s), \tag{11}$$

where

$$\Psi_{ijl} = \begin{bmatrix} \Omega_{ij} & \Gamma_{ijl} & \bar{\tau}E_j \\ * & \Upsilon_{il} & \bar{\tau}F_l \\ * & * & -\bar{\tau}R \end{bmatrix},$$

with

$$\Omega_{ij} = A_{ij}^{\mathrm{T}}P + PA_{ij} + Q + C_i^{\mathrm{T}}PC_i + \bar{\tau}A_{ij}^{\mathrm{T}}RA_{ij} - E_j - E_j^{\mathrm{T}}, \tag{12}$$

$$\Gamma_{ijl} = PA_{dil} + \bar{\tau}A_{ij}^{\mathrm{T}}RA_{dil} + E_j - F_l^{\mathrm{T}}, \tag{13}$$

$$\Upsilon_{il} = -Q + \bar{\tau}A_{dil}^{\mathrm{T}}RA_{dil} + F_l + F_l^{\mathrm{T}}. \tag{14}$$

Proof. Denote

$$\varphi(t) = \sum_{i=1}^{s}\sum_{j=1}^{s} h_ih_jA_{ij}x(t) + \sum_{i=1}^{s}\sum_{j=1}^{s} h_ih_{dj}A_{dij}x(t-\tau)$$

and define a Lyapunov functional candidate for the system (8) as follows:

$$V(x_t,t)=x^{\mathrm{T}}(t)Px(t)+\int_{t-\tau}^{t}x^{\mathrm{T}}(\alpha)Qx(\alpha)\,\mathrm{d}\alpha+\int_{-\tau}^{0}\int_{t+\beta}^{t}\varphi^{\mathrm{T}}(\alpha)R\varphi(\alpha)\,\mathrm{d}\alpha\,\mathrm{d}\beta, \tag{15}$$

where

$$x_t=\{x(t+\sigma):-\tau\leqslant\sigma\leqslant 0\}.$$

Then, by Itô's formula (see Definition 2), we obtain the stochastic differential as

$$\frac{\mathrm{d}}{\mathrm{d}t}V(x_t,t)=\mathbb{L}V(x_t,t)+2x^{\mathrm{T}}(t)P\sum_{i=1}^{s}h_iC_ix(t)\omega(t), \tag{16}$$

where

$$\begin{aligned}\mathbb{L}V(x_t,t)=&\,2x^{\mathrm{T}}(t)P\left[\sum_{i=1}^{s}\sum_{j=1}^{s}h_ih_jA_{ij}x(t)+\sum_{i=1}^{s}\sum_{j=1}^{s}h_ih_{dj}A_{dij}x(t-\tau)\right]\\&+x^{\mathrm{T}}(t)Qx(t)-x^{\mathrm{T}}(t-\tau)Qx(t-\tau)\\&+\int_{t-\tau}^{t}[\varphi^{\mathrm{T}}(t)R\varphi(t)-\varphi^{\mathrm{T}}(\alpha)R\varphi(\alpha)]\,\mathrm{d}\alpha\\&+\left[\sum_{i=1}^{s}h_iC_ix(t)\right]^{\mathrm{T}}P\left[\sum_{i=1}^{s}h_iC_ix(t)\right]\\=&\,\frac{1}{\tau}\int_{t-\tau}^{t}\left\{2x^{\mathrm{T}}(t)P\sum_{i=1}^{s}\sum_{j=1}^{s}h_ih_jA_{ij}x(t)+2x^{\mathrm{T}}(t)P\sum_{i=1}^{s}\sum_{l=1}^{s}h_ih_{dl}A_{dil}x(t-\tau)\right.\\&+x^{\mathrm{T}}(t)Qx(t)-x^{\mathrm{T}}(t-\tau)Qx(t-\tau)+\tau\varphi^{\mathrm{T}}(t)R\varphi(t)-\tau\varphi^{\mathrm{T}}(\alpha)R\varphi(\alpha)\\&\left.+\left[\sum_{i=1}^{s}h_iC_ix(t)\right]^{\mathrm{T}}P\left[\sum_{i=1}^{s}h_iC_ix(t)\right]\right\}\mathrm{d}\alpha.\end{aligned} \tag{17}$$

By Lemma 1 and noting that $0<\tau\leqslant\bar{\tau}$, one has

$$\left[\sum_{i=1}^{s}h_iC_ix(t)\right]^{\mathrm{T}}P\left[\sum_{i=1}^{s}h_iC_ix(t)\right]\leqslant\sum_{i=1}^{s}h_ix^{\mathrm{T}}(t)C_i^{\mathrm{T}}PC_ix(t) \tag{18}$$

and

$$\begin{aligned}\tau\varphi^{\mathrm{T}}(t)R\varphi(t)\leqslant&\left[\sum_{i=1}^{s}h_i\left(\sum_{j=1}^{s}h_jA_{ij}x(t)+\sum_{j=1}^{s}h_{dj}A_{dij}x(t-\tau)\right)\right]^{\mathrm{T}}\bar{\tau}R\\&\times\left[\sum_{i=1}^{s}h_i\left(\sum_{j=1}^{s}h_jA_{ij}x(t)+\sum_{j=1}^{s}h_{dj}A_{dij}x(t-\tau)\right)\right]\end{aligned}$$

$$
\begin{aligned}
&\leqslant \sum_{i=1}^{s} h_i \left[\left(\sum_{j=1}^{s} h_j A_{ij} x(t) + \sum_{j=1}^{s} h_{dj} A_{dij} x(t-\tau) \right)^{\mathrm{T}} \bar{\tau} R \right. \\
&\quad \left. \times \left(\sum_{j=1}^{s} h_j A_{ij} x(t) + \sum_{j=1}^{s} h_{dj} A_{dij} x(t-\tau) \right) \right] \\
&= \sum_{i=1}^{s} h_i \left[\left(\sum_{j=1}^{s} h_j A_{ij} x(t) + \sum_{l=1}^{s} h_{dl} A_{dil} x(t-\tau) \right)^{\mathrm{T}} \bar{\tau} R \right. \\
&\quad \left. \times \left(\sum_{j=1}^{s} h_j A_{ij} x(t) + \sum_{l=1}^{s} h_{dl} A_{dil} x(t-\tau) \right) \right] \\
&\leqslant \sum_{i=1}^{s} \sum_{j=1}^{s} \sum_{l=1}^{s} h_i h_j h_{dl} [A_{ij} x(t) + A_{dil} x(t-\tau)]^{\mathrm{T}} \bar{\tau} R [A_{ij} x(t) + A_{dil} x(t-\tau)]. \qquad (19)
\end{aligned}
$$

It follows from (17)–(19) that

$$
\begin{aligned}
\mathbb{L}V(x_t, t) \leqslant \frac{1}{\tau} \int_{t-\tau}^{t} \sum_{i=1}^{s} \sum_{j=1}^{s} \sum_{l=1}^{s} h_i h_j h_{dl} [& x^{\mathrm{T}}(t)(A_{ij}^{\mathrm{T}} P + P A_{ij} + Q + C_i^{\mathrm{T}} P C_i + \bar{\tau} A_{ij}^{\mathrm{T}} R A_{ij}) x(t) \\
& + 2x^{\mathrm{T}}(t)(P A_{dil} + \bar{\tau} A_{ij}^{\mathrm{T}} R A_{dil}) x(t-\tau) + x^{\mathrm{T}}(t-\tau)(-Q + \bar{\tau} A_{dil}^{\mathrm{T}} R A_{dil}) x(t-\tau) \\
& - \tau \varphi^{\mathrm{T}}(\alpha) R \varphi(\alpha)] \, \mathrm{d}\alpha. \qquad (20)
\end{aligned}
$$

Note that

$$
x(t) - x(t-\tau) = \int_{t-\tau}^{t} \dot{x}(\alpha) \, \mathrm{d}\alpha = \int_{t-\tau}^{t} \varphi(\alpha) \, \mathrm{d}\alpha + \int_{t-\tau}^{t} \sum_{i=1}^{s} h_i C_i x(\alpha) \omega(\alpha) \, \mathrm{d}\alpha.
$$

Then, for any matrices $\{E_j\}_{j=1}^{s}$ and $\{F_l\}_{l=1}^{s}$, one has

$$
\begin{aligned}
& 2x^{\mathrm{T}}(t) \sum_{j=1}^{s} h_j E_j [x(t) - x(t-\tau)] \\
& \quad = 2x^{\mathrm{T}}(t) \sum_{j=1}^{s} h_j E_j \int_{t-\tau}^{t} \varphi(\alpha) \, \mathrm{d}\alpha + 2x^{\mathrm{T}}(t) \sum_{i=1}^{s} \sum_{j=1}^{s} h_i h_j E_j C_i \int_{t-\tau}^{t} x(\alpha) \omega(\alpha) \, \mathrm{d}\alpha,
\end{aligned}
$$

and

$$
\begin{aligned}
& 2x^{\mathrm{T}}(t-\tau) \sum_{l=1}^{s} h_{dl} F_l [x(t) - x(t-\tau)] \\
& \quad = 2x^{\mathrm{T}}(t-\tau) \sum_{l=1}^{s} h_{dl} F_l \int_{t-\tau}^{t} \varphi(\alpha) \, \mathrm{d}\alpha + 2x^{\mathrm{T}}(t-\tau) \sum_{i=1}^{s} \sum_{l=1}^{s} h_i h_{dl} F_l C_i \int_{t-\tau}^{t} x(\alpha) \omega(\alpha) \, \mathrm{d}\alpha.
\end{aligned}
$$

These two equations imply that

$$0=\frac{1}{\tau}\int_{t-\tau}^{t}\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{l=1}^{s}h_ih_jh_{dl}[x^{\mathrm T}(t)(-E_j-E_j^{\mathrm T})x(t)$$

$$+2x^{\mathrm T}(t)E_jx(t-\tau)+2\tau x^{\mathrm T}(t)E_j\varphi(\alpha)]\,\mathrm d\alpha$$

$$+2x^{\mathrm T}(t)\sum_{i=1}^{s}\sum_{j=1}^{s}h_ih_jE_jC_i\int_{t-\tau}^{t}x(\alpha)\omega(\alpha)\,\mathrm d\alpha,$$

and

$$0=\frac{1}{\tau}\int_{t-\tau}^{t}\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{l=1}^{s}h_ih_jh_{dl}[2x^{\mathrm T}(t)(-F_l^{\mathrm T})x(t-\tau)$$

$$+x^{\mathrm T}(t-\tau)(F_l+F_l^{\mathrm T})x(t-\tau)+2\tau x^{\mathrm T}(t-\tau)F_l\varphi(\alpha)]\,\mathrm d\alpha$$

$$+2x^{\mathrm T}(t-\tau)\sum_{i=1}^{s}\sum_{l=1}^{s}h_ih_{dl}F_lC_i\int_{t-\tau}^{t}x(\alpha)\omega(\alpha)\,\mathrm d\alpha.$$

Therefore, by recalling (16) and (20) one has

$$\frac{\mathrm d}{\mathrm dt}V(x_t,t)\leqslant\frac{1}{\tau}\int_{t-\tau}^{t}\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{l=1}^{s}h_ih_jh_{dl}\left\{\begin{bmatrix}x(t)\\x(t-\tau)\end{bmatrix}^{\mathrm T}\begin{bmatrix}\Omega_{ij}&\Gamma_{ijl}\\\Gamma_{ijl}^{\mathrm T}&\Upsilon_{il}\end{bmatrix}\begin{bmatrix}x(t)\\x(t-\tau)\end{bmatrix}\right.$$

$$\left.+2\tau x^{\mathrm T}(t)E_j\varphi(\alpha)+2\tau x^{\mathrm T}(t-\tau)F_l\varphi(\alpha)-\tau\varphi^{\mathrm T}(\alpha)R\varphi(\alpha)\right\}\mathrm d\alpha+\psi(t),\tag{21}$$

where Ω_{ij}, Γ_{ijl} and Υ_{il} are given in (12–14), respectively, and

$$\psi(t)=2x^{\mathrm T}(t)P\sum_{i=1}^{s}h_iC_ix(t)\omega(t)+2x^{\mathrm T}(t)\sum_{i=1}^{s}\sum_{j=1}^{s}h_ih_jE_jC_i\int_{t-\tau}^{t}x(\alpha)\omega(\alpha)\,\mathrm d\alpha$$

$$+2x^{\mathrm T}(t-\tau)\sum_{i=1}^{s}\sum_{l=1}^{s}h_ih_{dl}F_lC_i\int_{t-\tau}^{t}x(\alpha)\omega(\alpha)\,\mathrm d\alpha.$$

The condition $0<\tau\leqslant\bar\tau$ ensures that $-\tau^{-1}\leqslant-\bar\tau^{-1}$. Then, it can be verified that

$$\begin{aligned}&2\tau x^{\mathrm T}(t)E_j\varphi(\alpha)+2\tau x^{\mathrm T}(t-\tau)F_l\varphi(\alpha)-\tau\varphi^{\mathrm T}(\alpha)R\varphi(\alpha)\\&\quad=2x^{\mathrm T}(t)E_j(\tau\varphi(\alpha))+2x^{\mathrm T}(t-\tau)F_l(\tau\varphi(\alpha))-\tau^{-1}(\tau\varphi(\alpha))^{\mathrm T}R(\tau\varphi(\alpha))\\&\quad\leqslant2x^{\mathrm T}(t)E_j(\tau\varphi(\alpha))+2x^{\mathrm T}(t-\tau)F_l(\tau\varphi(\alpha))-\bar\tau^{-1}(\tau\varphi(\alpha))^{\mathrm T}R(\tau\varphi(\alpha))\\&\quad=2\bar\tau x^{\mathrm T}(t)E_j(\tau\bar\tau^{-1}\varphi(\alpha))+2\bar\tau x^{\mathrm T}(t-\tau)F_l(\tau\bar\tau^{-1}\varphi(\alpha))-\bar\tau(\tau\bar\tau^{-1}\varphi(\alpha))^{\mathrm T}R(\tau\bar\tau^{-1}\varphi(\alpha)).\end{aligned}$$

This, together with (21), implies that

$$\frac{\mathrm d}{\mathrm dt}V(x_t,t)\leqslant\frac{1}{\tau}\int_{t-\tau}^{t}\vartheta^{\mathrm T}(t,\alpha)\left\{\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{l=1}^{s}h_ih_jh_{dl}\Psi_{ijl}\right\}\vartheta(t,\alpha)\,\mathrm d\alpha+\psi(t),\tag{22}$$

where

$$\vartheta(t,\alpha)=[x^{\mathrm T}(t)\;\;x^{\mathrm T}(t-\tau)\;\;\tau\bar\tau^{-1}\varphi^{\mathrm T}(\alpha)]^{\mathrm T}.$$

Now, by using the relaxed technique as in [9,15], it follows from (9)–(11) that

$$\begin{aligned}\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{l=1}^{s} h_i h_j h_{dl}\Psi_{ijl} &= \sum_{l=1}^{s} h_{dl}\left(\sum_{i=1}^{s}\sum_{j=1}^{s} h_i h_j \Psi_{ijl}\right)\\ &= \sum_{l=1}^{s} h_{dl}\left[\sum_{i=1}^{s} h_i^2 \Psi_{iil} + \sum_{i=1}^{s-1}\sum_{j=i+1}^{s} h_i h_j(\Psi_{ijl}+\Psi_{jil})\right]\\ &\leqslant \sum_{l=1}^{s} h_{dl}\left[\sum_{i=1}^{s} h_i^2 \Lambda_{il} + \sum_{i=1}^{s-1}\sum_{j=i+1}^{s} h_i h_j(\Theta_{ijl}+\Theta_{ijl}^{\mathrm{T}})\right]\\ &= \sum_{l=1}^{s} h_{dl}\left\{\begin{bmatrix} h_1 I\\ h_2 I\\ \vdots\\ h_s I\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\Lambda_{1l} & \Theta_{12l} & \cdots & \Theta_{1sl}\\ * & \Lambda_{2l} & \cdots & \Theta_{2sl}\\ \vdots & \vdots & \ddots & \vdots\\ * & * & \cdots & \Lambda_{sl}\end{bmatrix}\begin{bmatrix} h_1 I\\ h_2 I\\ \vdots\\ h_s I\end{bmatrix}\right\}\\ &< 0.\end{aligned}$$

Therefore,

$$\frac{1}{\tau}\int_{t-\tau}^{t}\vartheta^{\mathrm{T}}(t,\alpha)\left(\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{l=1}^{s} h_i h_j h_{dl}\Psi_{ijl}\right)\vartheta(t,\alpha)\,\mathrm{d}\alpha < 0,$$

which guarantees that

$$\mathbb{E}\left\{\frac{\mathrm{d}}{\mathrm{d}t}V(x_t,t)\right\} < 0.$$

This means that the system (8) is asymptotically stable in the mean square. The proof is completed here. □

Now we are in a position to give the main result on the solvability of the delay-dependent stabilization problem for the investigated stochastic time-delay fuzzy systems.

Theorem 2. *Consider the stochastic fuzzy time-delay system* (5) *and give a scalar* $\bar{\tau} > 0$. *Then, there exists a state feedback controller in the form of* (7) *such that the closed-loop system* (8) *is asymptotically stable in the mean square for any time delay* τ *satisfying* $0 < \tau \leqslant \bar{\tau}$, *if there exist matrices* $X > 0$, $Y > 0$, $Z > 0$, $\{W_i, M_i, N_i, 1\leqslant i\leqslant s\}$, $\{\Sigma_{il}, 1\leqslant i\leqslant s, 1\leqslant l\leqslant s\}$ *and* $\{\Xi_{ijl}, 1\leqslant i < j\leqslant s, 1\leqslant l\leqslant s\}$ *with*

$$\Sigma_{il} = \begin{bmatrix} U_{il}^{(11)} & U_{il}^{(12)} & U_{il}^{(13)}\\ * & U_{il}^{(22)} & U_{il}^{(23)}\\ * & * & U_{il}^{(33)}\end{bmatrix}\quad (1\leqslant i\leqslant s,\ 1\leqslant l\leqslant s),$$

and

$$\Xi_{ijl} = \begin{bmatrix} V_{ijl}^{(11)} & V_{ijl}^{(12)} & V_{ijl}^{(13)}\\ V_{ijl}^{(21)} & V_{ijl}^{(22)} & V_{ijl}^{(23)}\\ V_{ijl}^{(31)} & V_{ijl}^{(32)} & V_{ijl}^{(33)}\end{bmatrix}\quad (1\leqslant i < j\leqslant s,\ 1\leqslant l\leqslant s),$$

such that the following LMIs are satisfied:

$$\begin{bmatrix} \Sigma_{1l} & \Xi_{12l} & \cdots & \Xi_{1sl} \\ * & \Sigma_{2l} & \cdots & \Xi_{2sl} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & \Sigma_{sl} \end{bmatrix} < 0 \quad (1 \leqslant l \leqslant s), \tag{23}$$

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \bar{\tau}M_i - U_{il}^{(13)} & XC_i^{\mathrm{T}} & \bar{\tau}XA_i^{\mathrm{T}} + \bar{\tau}W_i^{\mathrm{T}}B_i^{\mathrm{T}} \\ * & \Phi_{22} & \bar{\tau}N_l - U_{il}^{(23)} & 0 & \bar{\tau}XA_{di}^{\mathrm{T}} + \bar{\tau}W_l^{\mathrm{T}}B_{di}^{\mathrm{T}} \\ * & * & \bar{\tau}Z - 2\bar{\tau}X - U_{il}^{(33)} & 0 & 0 \\ * & * & * & -X & 0 \\ * & * & * & * & -\bar{\tau}Z \end{bmatrix} < 0 \quad (1 \leqslant i \leqslant s,\ 1 \leqslant l \leqslant s), \tag{24}$$

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & XC_i^{\mathrm{T}} & XC_j^{\mathrm{T}} & \bar{\tau}XA_i^{\mathrm{T}} + \bar{\tau}W_j^{\mathrm{T}}B_i^{\mathrm{T}} & \bar{\tau}XA_j^{\mathrm{T}} + \bar{\tau}W_i^{\mathrm{T}}B_j^{\mathrm{T}} \\ * & \Pi_{22} & \Pi_{23} & 0 & 0 & \bar{\tau}XA_{di}^{\mathrm{T}} + \bar{\tau}W_l^{\mathrm{T}}B_{di}^{\mathrm{T}} & \bar{\tau}XA_{dj}^{\mathrm{T}} + \bar{\tau}W_l^{\mathrm{T}}B_{dj}^{\mathrm{T}} \\ * & * & \Pi_{33} & 0 & 0 & 0 & 0 \\ * & * & * & -X & 0 & 0 & 0 \\ * & * & * & * & -X & 0 & 0 \\ * & * & * & * & * & -\bar{\tau}Z & 0 \\ * & * & * & * & * & * & -\bar{\tau}Z \end{bmatrix} < 0 \quad (1 \leqslant i < j \leqslant s,\ 1 \leqslant l \leqslant s), \tag{25}$$

where

$$\Phi_{11} = XA_i^{\mathrm{T}} + A_iX + W_i^{\mathrm{T}}B_i^{\mathrm{T}} + B_iW_i + Y - M_i - M_i^{\mathrm{T}} - U_{il}^{(11)},$$

$$\Phi_{12} = A_{di}X + B_{di}W_l + M_i - N_l^{\mathrm{T}} - U_{il}^{(12)},$$

$$\Phi_{22} = -Y + N_l + N_l^{\mathrm{T}} - U_{il}^{(22)},$$

$$\Pi_{11} = X(A_i^{\mathrm{T}} + A_j^{\mathrm{T}}) + (A_i + A_j)X + W_j^{\mathrm{T}}B_i^{\mathrm{T}} + W_i^{\mathrm{T}}B_j^{\mathrm{T}} + B_iW_j + B_jW_i$$
$$+2Y - M_i - M_j - M_i^{\mathrm{T}} - M_j^{\mathrm{T}} - V_{ijl}^{(11)} - (V_{ijl}^{(11)})^{\mathrm{T}},$$

$$\Pi_{12} = (A_{di} + A_{dj})X + (B_{di} + B_{dj})W_l + M_i + M_j - 2N_l^{\mathrm{T}} - V_{ijl}^{(12)} - (V_{ijl}^{(21)})^{\mathrm{T}},$$

$$\Pi_{13} = \bar{\tau}M_i + \bar{\tau}M_j - V_{ijl}^{(13)} - (V_{ijl}^{(31)})^{\mathrm{T}},$$

$$\Pi_{22} = -2Y + 2N_l + 2N_l^{\mathrm{T}} - V_{ijl}^{(22)} - (V_{ijl}^{(22)})^{\mathrm{T}},$$

$$\Pi_{23} = 2\bar{\tau}N_l - V_{ijl}^{(23)} - (V_{ijl}^{(32)})^{\mathrm{T}},$$

$$\Pi_{33} = 2\bar{\tau}Z - 4\bar{\tau}X - V_{ijl}^{(33)} - (V_{ijl}^{(33)})^{\mathrm{T}}.$$

In this case, the controller gains are given by

$$K_j = W_jX^{-1}, \quad 1 \leqslant j \leqslant s. \tag{26}$$

Proof. It is easy to see that

$$(Z - X)^{\mathrm{T}}Z^{-1}(Z - X) = Z - 2X + XZ^{-1}X \geqslant 0,$$

which implies that

$$-XZ^{-1}X \leqslant Z - 2X.$$

Therefore, it follows from (24) and (25) that

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \bar{\tau}M_i - U_{il}^{(13)} & XC_i^{\mathrm{T}} & \bar{\tau}XA_i^{\mathrm{T}} + \bar{\tau}W_i^{\mathrm{T}}B_i^{\mathrm{T}} \\ * & \Phi_{22} & \bar{\tau}N_l - U_{il}^{(23)} & 0 & \bar{\tau}XA_{di}^{\mathrm{T}} + \bar{\tau}W_l^{\mathrm{T}}B_{di}^{\mathrm{T}} \\ * & * & -\bar{\tau}XZ^{-1}X - U_{il}^{(33)} & 0 & 0 \\ * & * & * & -X & 0 \\ * & * & * & * & -\bar{\tau}Z \end{bmatrix} < 0 \quad (1 \leqslant i \leqslant s,\ 1 \leqslant l \leqslant s), \tag{27}$$

and

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & XC_i^{\mathrm{T}} & XC_j^{\mathrm{T}} & \bar{\tau}XA_i^{\mathrm{T}} + \bar{\tau}W_j^{\mathrm{T}}B_i^{\mathrm{T}} & \bar{\tau}XA_j^{\mathrm{T}} + \bar{\tau}W_i^{\mathrm{T}}B_j^{\mathrm{T}} \\ * & \Pi_{22} & \Pi_{23} & 0 & 0 & \bar{\tau}XA_{di}^{\mathrm{T}} + \bar{\tau}W_l^{\mathrm{T}}B_{di}^{\mathrm{T}} & \bar{\tau}XA_{dj}^{\mathrm{T}} + \bar{\tau}W_l^{\mathrm{T}}B_{dj}^{\mathrm{T}} \\ * & * & \hat{\Pi}_{33} & 0 & 0 & 0 & 0 \\ * & * & * & -X & 0 & 0 & 0 \\ * & * & * & * & -X & 0 & 0 \\ * & * & * & * & * & -\bar{\tau}Z & 0 \\ * & * & * & * & * & * & -\bar{\tau}Z \end{bmatrix} < 0 \quad (1 \leqslant i < j \leqslant s,\ 1 \leqslant l \leqslant s), \tag{28}$$

respectively, where

$$\hat{\Pi}_{33} = -2\bar{\tau}XZ^{-1}X - V_{ijl}^{(33)} - (V_{ijl}^{(33)})^{\mathrm{T}}.$$

Now, we introduce the following matrices

$$\begin{aligned} J_1 &= \mathrm{diag}\{X^{-1}, X^{-1}, X^{-1}\}, \\ J_2 &= \mathrm{diag}\{X^{-1}, X^{-1}, X^{-1}, X^{-1}, Z^{-1}\}, \\ J_3 &= \mathrm{diag}\{X^{-1}, X^{-1}, X^{-1}, X^{-1}, X^{-1}, Z^{-1},\ Z^{-1}\}. \end{aligned}$$

Set

$$\begin{cases} P = X^{-1}, \quad Q = X^{-1}YX^{-1}, \quad R = Z^{-1}, \\ E_i = X^{-1}M_iX^{-1}, \quad F_i = X^{-1}N_iX^{-1} \quad (1 \leqslant i \leqslant s) \\ \Lambda_{il} = J_1\Sigma_{il}J_1 \quad (1 \leqslant i \leqslant s, 1 \leqslant l \leqslant s), \\ \Theta_{ijl} = J_1\Xi_{ijl}J_1 \quad (1 \leqslant i < j \leqslant s,\ 1 \leqslant l \leqslant s). \end{cases}$$

Note that $W_j = K_jX$. Then, by pre-multiplying J_2 and post-multiplying J_2 to (27), and by pre-multiplying J_3 and post-multiplying J_3 to (28), respectively, we obtain

$$\begin{bmatrix} \tilde{\Phi}_{11} & \tilde{\Phi}_{12} & \bar{\tau}E_i - G_{il}^{(13)} & C_i^{\mathrm{T}}P & \bar{\tau}A_{ii}^{\mathrm{T}}R \\ * & \tilde{\Phi}_{22} & \bar{\tau}F_l - G_{il}^{(23)} & 0 & \bar{\tau}A_{dil}^{\mathrm{T}}R \\ * & * & -\bar{\tau}R - G_{il}^{(33)} & 0 & 0 \\ * & * & * & -P & 0 \\ * & * & * & * & -\bar{\tau}R \end{bmatrix} < 0 \quad (1 \leqslant i \leqslant s, 1 \leqslant l \leqslant s), \tag{29}$$

$$\begin{bmatrix} \tilde{\Pi}_{11} & \tilde{\Pi}_{12} & \tilde{\Pi}_{13} & C_i^{\mathrm{T}}P & C_j^{\mathrm{T}}P & \bar{\tau}A_{ij}^{\mathrm{T}}R & \bar{\tau}A_{ji}^{\mathrm{T}}R \\ * & \tilde{\Pi}_{22} & \tilde{\Pi}_{23} & 0 & 0 & \bar{\tau}A_{dil}^{\mathrm{T}}R & \bar{\tau}A_{djl}^{\mathrm{T}}R \\ * & * & \tilde{\Pi}_{33} & 0 & 0 & 0 & 0 \\ * & * & * & -P & 0 & 0 & 0 \\ * & * & * & * & -P & 0 & 0 \\ * & * & * & * & * & -\bar{\tau}R & 0 \\ * & * & * & * & * & * & -\bar{\tau}R \end{bmatrix} < 0 \quad (1 \leqslant i < j \leqslant s, 1 \leqslant l \leqslant s), \tag{30}$$

where

$$\tilde{\Phi}_{11} = A_{ii}^{\mathrm{T}}P + PA_{ii} + Q - E_i - E_i^{\mathrm{T}} - G_{il}^{(11)},$$

$$\tilde{\Phi}_{12} = PA_{dil} + E_i - F_l^{\mathrm{T}} - G_{il}^{(12)},$$

$$\tilde{\Phi}_{22} = -Q + F_l + F_l^{\mathrm{T}} - G_{il}^{(22)},$$

$$\tilde{\Pi}_{11} = (A_{ij}^{\mathrm{T}} + A_{ji}^{\mathrm{T}})P + P(A_{ij} + A_{ji}) + 2Q - E_i - E_j - E_i^{\mathrm{T}} - E_j^{\mathrm{T}} - H_{ijl}^{(11)} - (H_{ijl}^{(11)})^{\mathrm{T}},$$

$$\tilde{\Pi}_{12} = P(A_{dil} + A_{djl}) + E_i + E_j - 2F_l^{\mathrm{T}} - H_{ijl}^{(12)} - (H_{ijl}^{(21)})^{\mathrm{T}},$$

$$\tilde{\Pi}_{13} = \bar{\tau}E_i + \bar{\tau}E_j - H_{ijl}^{(13)} - (H_{ijl}^{(31)})^{\mathrm{T}},$$

$$\tilde{\Pi}_{22} = -2Q + 2F_l + 2F_l^{\mathrm{T}} - H_{ijl}^{(22)} - (H_{ijl}^{(22)})^{\mathrm{T}},$$

$$\tilde{\Pi}_{23} = 2\bar{\tau}F_l - H_{ijl}^{(23)} - (H_{ijl}^{(32)})^{\mathrm{T}},$$

$$\tilde{\Pi}_{33} = -2\bar{\tau}R - H_{ijl}^{(33)} - (H_{ijl}^{(33)})^{\mathrm{T}}.$$

By applying the Schur complement formula to (29) and (30), we readily get the inequalities in (9) and (10), respectively. Moreover, it follows from (23) that, for $l = 1, 2, \ldots, s$,

$$\begin{bmatrix} \Lambda_{1l} & \Theta_{12l} & \cdots & \Theta_{1sl} \\ * & \Lambda_{2l} & \cdots & \Theta_{2sl} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & \Lambda_{sl} \end{bmatrix} = \begin{bmatrix} J_1 & 0 & \cdots & 0 \\ * & J_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & J_1 \end{bmatrix} \begin{bmatrix} \Sigma_{1l} & \Xi_{12l} & \cdots & \Xi_{1sl} \\ * & \Sigma_{2l} & \cdots & \Xi_{2sl} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & \Sigma_{sl} \end{bmatrix} \begin{bmatrix} J_1 & 0 & \cdots & 0 \\ * & J_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & J_1 \end{bmatrix} < 0.$$

Therefore, we can see that all the conditions in Theorem 1 are satisfied. The proof is thus completed. □

Remark 1. Theorem 2 provides a sufficient condition for the solvability of the delay-dependent stabilization problem for stochastic T–S fuzzy systems with both state and input delays. Observe that the conditions in (23)–(25) are certain LMIs that can be readily solved using standard numerical software [7]. It is also worth noting that a desired controller can be constructed by solving the LMIs in (23)–(25). Moreover, based on Theorem 2 in this paper, one can readily obtain the delay-dependent results on the solvability of the stabilization problem for stochastic fuzzy systems without input delays (i.e., $B_{di} = 0$) and without state delays (i.e., $A_{di} = 0$), respectively.

Remark 2. In the stochastic fuzzy system (5), the input delays are the same as the state delays. For the case where these two classes of delays are not equal, the delay-dependent stabilization problem is much more complicated and still remains open. This is one of our future research topics.

4. Numerical example

Consider the following stochastic fuzzy system with state and input delays:
Plant rule 1: **IF** $x_2(t)/0.5$ **is about** 0, **THEN**

$$\dot{x}(t) = A_1x(t) + A_{d1}x(t-\tau) + B_1u(t) + B_{d1}u(t-\tau) + C_1x(t)\omega(t),$$

Plant rule 2: **IF** $x_2(t)/0.5$ **is about** π **or** $-\pi$, **THEN**

$$\dot{x}(t) = A_2x(t) + A_{d2}x(t-\tau) + B_2u(t) + B_{d2}u(t-\tau) + C_2x(t)\omega(t),$$

where

$$A_1 = \begin{bmatrix} 1-a & 0.8 \\ 0.9 & 1.1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1 & 1.2 \\ 1.5 & -1 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 1 & 0 \\ 0.5 & -1 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 1 & 0 \\ 0.5 & 1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_{d1} = \begin{bmatrix} 0.1 \\ -0.2 \end{bmatrix}, \quad B_{d2} = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}.$$

Table 1 shows the maximum allowable size of the delay τ and the corresponding controller gains for different a. For instance, when $a = 2$, the maximum allowable size of the delay τ for the solvability of the proposed stabilization problem is 0.9027. This means that, for any time delay τ satisfying $0 < \tau \leqslant 0.9027$, there exists a state-feedback fuzzy controller such that the resulting closed-loop system is asymptotically stable in the mean square. For this example, if we choose the time delay as $\tau = \bar{\tau} = 0.9027$, then by using the Matlab Control Toolbox to solve the LMIs in (23)–(25) we obtain

$$X = \begin{bmatrix} 3.8008 & -1.8670 \\ -1.8670 & 6.7355 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} -2.0109 & -16.6902 \end{bmatrix},$$

$$W_2 = \begin{bmatrix} -4.1847 & -9.7837 \end{bmatrix}.$$

By Theorem 2, we can obtain the desired state-feedback fuzzy controller as follows:

$$u(t) = \left(\begin{bmatrix} -2.0215 & -3.0383 \end{bmatrix} h_1 + \begin{bmatrix} -2.1005 & -2.0348 \end{bmatrix} h_2 \right) x(t).$$

Table 1
Numerical results for different cases

Cases	$\bar{\tau}$	Controller gain K_1	Controller gain K_2
$a = 0$	0.2862	[−12.6597 − 4.4983]	[−8.8176 − 3.4106]
$a = 0.5$	0.3838	[−9.5177 − 4.3750]	[−6.1294 − 3.0959]
$a = 1.0$	0.5279	[−6.3131 − 4.0531]	[−3.9444 − 2.6995]
$a = 1.5$	0.7471	[−3.4632 − 3.5959]	[−2.4733 − 2.2598]
$a = 2.0$	0.9027	[−2.0215 − 3.0383]	[−2.1005 − 2.0348]

5. Conclusions

The problem of delay-dependent stabilization for stochastic T–S fuzzy systems with time delays has been studied in this paper. A delay-dependent condition for the existence of a state-feedback fuzzy controller, which guarantees the asymptotic stability (in the mean square sense) of the closed-loop system, has been obtained in terms of relaxed LMIs. The numerical example provided finally has demonstrated the effectiveness of the proposed method.

References

[1] S.G. Cao, N.W. Rees, G. Feng, Analysis and design of a class of continuous time fuzzy control systems, Int. J. Control 64 (1996) 1069–1087.

[2] Y.Y. Cao, P.M. Frank, Analysis and synthesis of nonlinear time-delay systems via fuzzy control approach, IEEE Trans. Fuzzy Systems 8 (2) (2000) 200–211.

[3] Y.Y. Cao, P.M. Frank, Stability analysis and synthesis of nonlinear time-delay systems via linear Takagi-Sugeno fuzzy models, Fuzzy Sets and Systems 124 (2001) 213–229.

[4] Y.Y. Cao, Z. Lin, Robust stability analysis and fuzzy-scheduling control for nonlinear systems subject to actuator saturation, IEEE Trans. Fuzzy Systems 11 (1) (2003) 57–67.

[5] B. Chen, W. Zhang, Stochastic H_2/H_∞ control with state-dependent noise, IEEE Trans. Automat. Control 49 (1) (2004) 45–57.

[6] W.-H. Chen, Z.-H. Guan, X. Lu, Delay-dependent robust stabilization and H_∞-control of uncertain stochastic systems with time-varying delay, IMA J. Math. Control Inform. 21 (3) (2004) 345–358.

[7] P. Gahinet, A. Nemirovskii, A.J. Laub, M. Chilali, LMI Control Toolbox User's Guide, The Math. Works Inc., Natick, MA, 1995.

[8] H. Huang, D.W.C. Ho, Y. Niu, Sliding mode based fuzzy control of stochastic systems with time delay, in: Proc. 6th World Congress on Intelligent Control and Automation, Dalian, China, 2006, pp. 295–299.

[9] E. Kim, H. Lee, New approaches to relaxed quadratic stability condition of fuzzy control systems, IEEE Trans. Fuzzy Systems 8 (5) (2000) 523–534.

[10] C. Li, H. Wang, X. Liao, Delay-dependent robust stability of uncertain fuzzy systems with time-varying delays, IEE Proc. Control Theory Appl. 151 (2004) 417–421.

[11] C.H. Lien, Further results on delay-dependent robust stability of uncertain fuzzy systems with time-varying delay, Chaos Solitons Fractals 28 (2006) 422–427.

[12] C. Lin, Q.G. Wang, T.H. Lee, Improvement on observer-based H_∞ control for T–S fuzzy systems, Automatica 41 (2005) 1651–1656.

[13] C. Lin, Q.G. Wang, T.H. Lee, Delay-dependent LMI conditions for stability and stabilization of T–S fuzzy systems with bounded time delay, Fuzzy Sets Systems 157 (2006) 1229–1247.

[14] H. Liu, K. He, F. Sun, Z. Sun, Analysis and synthesis of fuzzy stochastic systems via LMI approach, in: Proc. 17th IEEE Region 10 Internat. Conf. on Computers, Communications, Control and Power Engineering, Beijing, China, 2002, pp. 1700–1703

[15] X. Liu, Q.L. Zhang, New approaches to controller designs based on fuzzy observers for T–S fuzzy systems via LMI, Automatica 39 (2003) 1571–1582.

[16] X. Mao, Robustness of exponential stability of stochastic differential delay equations, IEEE Trans. Automat. Control 41 (1996) 442–447.

[17] X. Mao, Stochastic Differential Equations and Applications, Horwood, West Sussex, UK, 1997.

[18] X. Mao, N. Koroleva, A. Rodkina, Robust stability of uncertain stochastic differential delay equations, Systems Control Lett. 35 (1998) 325–336.

[19] T. Takagi, M. Sugeno, Fuzzy identification of systems and its application to modeling and control, IEEE Trans. Systems Man Cybernet 15 (1) (1985) 116–132.

[20] K. Tanaka, T. Ikeda, H.O. Wang, Fuzzy regulators and fuzzy observers: relaxed stability conditions and LMI-based designs, IEEE Trans. Fuzzy Systems 6 (2) (1998) 250–265.

[21] K. Tanaka, M. Sugeno, Stability analysis and design of fuzzy control systems, Fuzzy Sets Systems 45 (1992) 135–156.

[22] E. Tian, C. Peng, Delay-dependent stability analysis and synthesis of uncertain T–S fuzzy systems with time-varying delay, Fuzzy Sets Systems 157 (2006) 544–559.

[23] Z. Wang, D.W.C. Ho, X. Liu, A note on the robust stability of uncertain stochastic fuzzy systems with time-delays, IEEE Trans. Systems Man Cybernet A 34 (4) (2004) 570–576.

[24] S. Xu, T. Chen, Reduced-order H_∞ filtering for stochastic systems, IEEE Trans. Signal Processing 50 (12) (2002) 2998–3007.

[25] S. Xu, T. Chen, Robust H_∞ control for uncertain stochastic systems with state delay, IEEE Trans. Automat. Control 47 (12) (2002) 2089–2094.

[26] S. Xu, T. Chen, H_∞ output feedback control for uncertain stochastic systems with time-varying delays, Automatica 40 (2004) 2091–2098.

[27] S. Xu, J. Lam, X. Mao, Y. Zou, A new LMI condition for delay-dependent robust stability of stochastic time-delay systems, Asian J. Control 7 (4) (2005) 419–423.

[28] Z. Yi, P.A. Heng, Stability of fuzzy control systems with bounded uncertain delays, IEEE Trans. Fuzzy Systems 10 (2002) 92–97.

[29] C. Yuan, X. Mao, Robust stability and controllability of stochastic differential delay equations with Markovian switching, Automatica 40 (3) (2004) 343–354.

[30] S. Zhou, G. Feng, J. Lam, S. Xu, Robust H_∞ control for discrete-time fuzzy systems via basis-dependent Lyapunov functions, Inform. Sci. 174 (3–4) (2005) 197–217.

[31] S. Zhou, T. Li, Robust stabilization for delayed discrete-time fuzzy systems via basis-dependent Lyapunov–Krasovskii function, Fuzzy Sets Systems 151 (2005) 139–153.