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A magnetoelastic model of nonlinear behaviors of Tb-Dy-Fe alloys based on domain rotation

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Abstract. A magnetoelastic constitutive theory of nonlinear anhysteretic behaviors based on domain rotation is presented in this paper for giant magnetostrictive materials (GMMs). For a Tb-Dy-Fe polycrystal with preferred orientations, the magnetic moments of domains are constrained by the anisotropy to lie along the <111> easy magnetization directions. Thus, the fractional occupancies of only the eight <111> directions and the field direction are calculated using an inverse exponential distribution function for simplicity. The linear fitting of the potential distribution parameter with the pre-stress is obtained from the magnetization curves under different compressive stresses for a [110] oriented Tb-Dy-Fe rod, which is used to predict the magnetostriction behaviors. It shows good quantitative agreement with the experimental results under a range of constant uniaxial compressive stresses.

1. Introduction

Magnetostrictive material is able to change its physical dimensions in response to the change of magnetization state [1]. In other words, it opens the opportunity to control the strain of magnetostrictive material by magnetic field. Giant magnetostrictive material, such as Terfenol-D, has two orders of magnitude larger than Nickel. At present, Terfenol-D is the most widely used magnetostrictive material, because it can offer high room temperature positive magnetostriction of 0.1%–0.2% at a low magnetic field of a few kOe [2,3]. In general, the materials always are subjected to the coupled mechanical and magnetic fields. The magnetic properties, such as the susceptibility, coercive field, and saturation magnetostriction, are strongly dependent on the pre-stress. The magnetomechanical behaviors of the GMMs are essentially nonlinear and coupled, although linear piezomagnetic model is still employed widely in engineering applications. Obviously, accurate theoretical description or prediction of the magnetomechanical response will play an increasingly important role in devices design to improve their precision.

Many researches have paid their efforts to establish nonlinear constitutive models in the past for magnetostrictive materials, which can be simply divided into three main types. Firstly, Preisach-type models have been developed to predict the magnetostriction and shown good agreement with experimental results [4–6]. Secondly, the phenomenological constitutive models of magnetostrictive materials are

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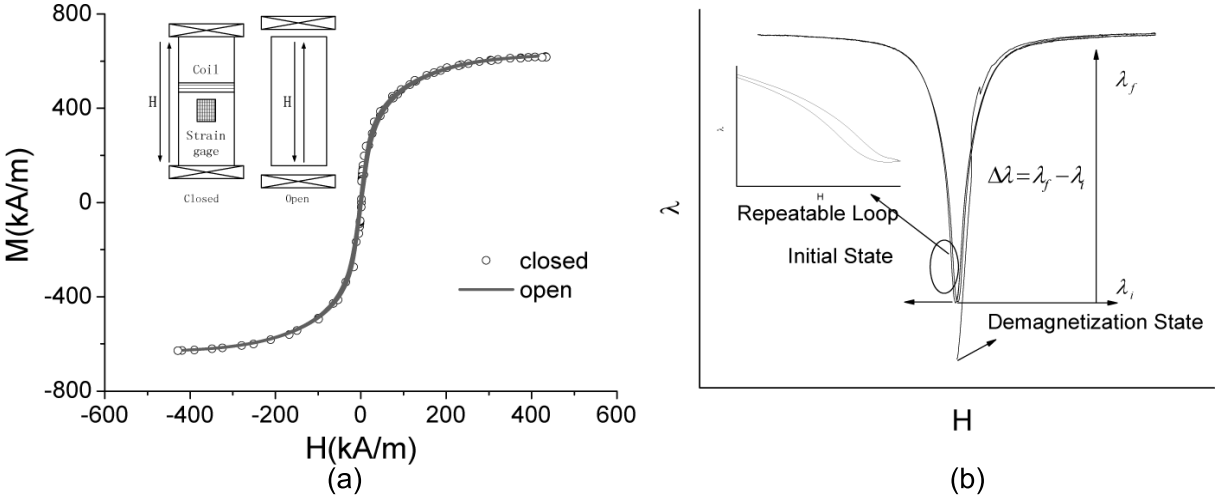

Fig. 1. The experimental results of a) closed and open magnetization and b) entire magnetostriction.

developed on the basis of thermodynamic framework [7]. In these models, the domain mechanism has not been taken into consideration so that they cannot describe the difference of magnetostriction behaviors of [110], [112] and [111] oriented crystals. Thirdly, the domain rotation or domain wall motion theory is widely used to describe the evolution of domain structures though it takes no account of internal interactions. Jiles and Thoelke generalized the anisotropic domain rotation (ADR) model without considering the hyeteresis response [8]. Based on such an assumption that the probability of the occupation of magnetization potential follows an inverse exponential distribution, the ADR theoretical description of the magnetoelastic behaviors becomes smoother [9–11]. However, they cannot quantitatively describe the magnetomechanical behaviors over the entire range of magnetization under different pre-stresses with a constant selected parameter.

In this paper, simultaneous measurements of magnetization and magnetostriction have been carried out for the <110> oriented polycrystal $Tb_{0.3}Dy_{0.7}Fe_{1.95}$ alloys. The ADR model has been modified for investigating the magnetoelastic response of GMMs under various pre-stresses. The fractional occupancies of only the eight <111> directions and the field direction are calculated using an inverse exponential distribution function for simplicity as previous researches [9–11]. The linear fitting of the potential distribution parameter with the pre-stress is obtained from the magnetization curves under different compressive stresses for a [110] oriented Tb-Dy-Fe alloy, which is used to predicted the magnetostriction behaviors for validity. The comparison between the theoretical and experimental results shows that the presented model can quantitatively describe the magnetoelastic behaviors over the entire range of magnetization under different pre-stresses

2. Experiments and results

The pseudobinary rare earth iron compound $Tb_{0.3}Dy_{0.7}Fe_{1.95}$ was prepared from high purity Tb, Dy, and Fe metals. The conventional temperature gradient zone melting technique was employed to prepare the [110] oriented polycrystalline rods. The specimens were cut from these rods with the dimensions of ϕ 10 $\times$ 40 mm. The large aspect ratio of 4:1 ensures that a large portion in middle of the sample experiences a uniform uniaxial stress. In addition, the large aspect ratio reachs 4:1, so the demagnetization

field effect is not considered. The open and closed magnetization experiments have been carried out without stress to confirm that. It is shown in Fig. 1a that the magnetic hysteresis curves almost match together for two experimental results.

The axial magnetization change was determined using an integrating device with a pickup Coil. The compressive stresses applied in this experiment were 0, 2.5, 5, 10, 25, 50 MPa. The axial strains were measured at room temperature by using a standard resistance strain gauge method, where two strain gauges were bonded on the opposite faces of the sample. The magnetostriction is half of the sum of strains. If bending occur during experiments, the addition signals which are opposite in sign and equal to each other in magnitude are produced. However, the sum of the addition signals is zero. Thus, the measurement errors due to bending effect can be avoided. It should be mentioned that a saturation magnetic field was applied to establish a same initial magnetization state for each experimental procedure. Therefore, the magnetization loops coincide with each other and can be repeated. Furthermore, the induced magnetostriction $\Delta\lambda = \lambda_f - \lambda_o$ depends on the final and initial domain orientations, which is the property of most interest in practical applications. Here, λ_o is the initial magnetostriction after the sample experiences the saturation magnetization, which is larger than that in demagnetization state. Figure 1b shows the entire magnetostriction curves and the definition of the induced magnetostriction.

3. Anisotropic domain rotation model

To deeply understand the magnetization mechanism of GMMs, a three-dimensional anisotropic domain rotation model is chosen. In general, it is used to calculate the magnetization and magnetostriction in single crystal. For the [110] oriented polycrystal, we assume here that it consists of the single crystals with the [110] directions along the rod axis. It means that the single crystal should rotate randomly around the [110] axis in Terfenol-D alloys. Otherwise, the internal interaction and domain wall motion will not be taken into account for simplicity as those in previous researches [8–11]. Therefore, the domain particles will show a Stoner-Wohlfarth type behavior.

In the present calculation, the demagnetization field effect will be neglected because it is so small in the experiment for the sample with large aspect ratio. The elastic, magnetoelastic and anisotropy energy can be combined into one simple term with the introduction of $K' = K_1 + \Delta K_1 \approx K_1 + 9/4\lambda_{100}^2(C_{11} - C_{12}) - 9/2\lambda_{111}^2 C_{44}$, where K_1 is the magnetocrystalline anisotropy constant. C_{ij} is the elastic constant. λ_{100} and λ_{111} are the magnetostriction along [100] and [111] when the magnetization is also in the same direction.

$$E_{K'} = E_K + E_{el} + E_{me} = K'(\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^2 + \alpha_3^2\alpha_1^2) \tag{1}$$

where E_K, E_{el} and E_{me} are the magnetocrystalline anisotropy, elastic and magnetoelastic energies, respectively. α_i is the direction cosine of the magnetization with respect to the crystal axes. The free energy can thus be written simply as,

$$E = E_{K'} + E_H + E_\sigma \tag{2}$$

whereE_H, and E_σ are the magnetic field and stress energies, respectively. They can be given, respectively, as,

$$E_H = -\mu_0 H M_s(\alpha_1\beta_1 + \alpha_2\beta_2 + \alpha_3\beta_3) \tag{3}$$

where μ_0 is the coefficient of permeability of free space. H denotes the magnetic field and M_s is the saturation magnetization. β_i is the direction cosine of magnetic field with respect of the crystal axes.

$$\begin{aligned} E_\sigma = &-\tfrac{3}{2}\lambda_{100}(\alpha_1^2\sigma_{11} + \alpha_2^2\sigma_{22} + \alpha_3^2\sigma_{33}) \\ &-3\lambda_{111}(\alpha_1\alpha_2\sigma_{12} + \alpha_2\alpha_3\sigma_{23} + \alpha_1\alpha_3\sigma_{13}) \end{aligned} \tag{4}$$

Where σ_{ij} denotes the component of the applied stress.

Generally, the magnetization orientation can be calculated according to the principle of energy minimum. However the simulation results are not as smooth as the experimental results observed, because it assumes that the domains along one direction produce the identical domain process in the perfect material, such as the same coercive field. Indeed, some domains will retain at that direction even a large mount of equivalent domains has a simultaneous switch due to the presence of defects. Amstrong has assumed that the probability of occupation of magnetization potential follows a parameter selected, inverse exponential potential distribution familiar from the study of statistical thermodynamics. It means the probability of the magnetization occupying one direction is highly dependent on the total free energy. To simplify the calculation, the field direction is considered as the ninth easy direction besides the eight <111> easy directions as that in previous researches [11]. The orientation distribution function is given as

$$P_i = N\exp[-E_i/\omega] \tag{5}$$

where N is a normalization factor which should be evaluated at each applied field and stress conditions. ω is the energy distribution parameter. A reduction of ω will sharpen the magnetization distribution. The subscript i refer to the domain magnetization along the ith easy direction. Thus the total magnetization M and magnetostriction λ in any crystallographic direction can be calculated by summing the projection of M and λ contributed by each domain along one of easy directions.

$$M = \sum_i P_i M_s(\alpha_{i1}\beta_1 + \alpha_{i2}\beta_2 + \alpha_{i3}\beta_3) \tag{6}$$

and

$$\begin{aligned} \lambda = \sum_i P_i\{&\tfrac{3}{2}\lambda_{100}(\alpha_{i1}^2\beta_1^2 + \alpha_{i2}^2\beta_2^2 + \alpha_{i3}^2\beta_3^2) \\ &+3\lambda_{111}(\alpha_{i1}\alpha_{i2}\beta_1\beta_2 + \alpha_{i2}\alpha_{i3}\beta_2\beta_3 + \alpha_{i3}\alpha_{i1}\beta_3\beta_1)\} \end{aligned} \tag{7}$$

where α_{ij}, β_j are the direction cosines of the domain magnetization along the ith easy direction and the measuring direction with respect of the crystal axis. The net magnetization and magnetostriction measured experimentally can be calculated as

$$\Delta M = M_f - M_o \quad \text{and} \quad \Delta\lambda = \lambda_f - \lambda_o \tag{8}$$

where subscripts o and f denote the initial and final states, respectively. The overall magnetostriction of a polycrystal can be calculated using a superposition rule

$$\lambda_p = \sum_j \lambda_j V_j \tag{9}$$

where λ_j is the magnetostriction contributed by each texture and V_j is the corresponding volume fractions.

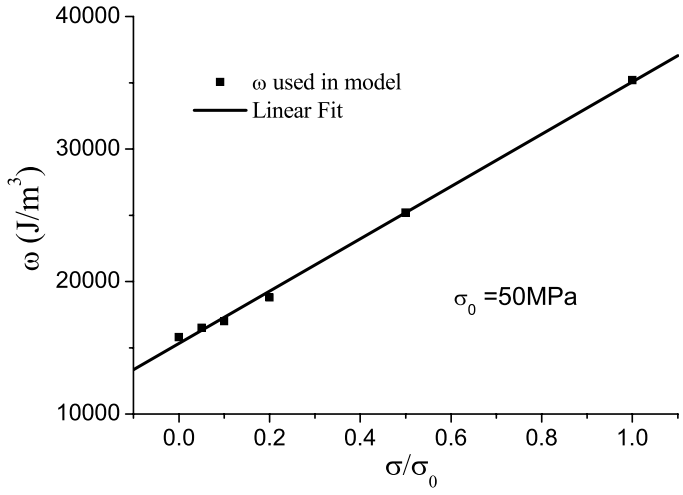

Fig. 2. The linear stress dependence of energy distribution parameter.

In the demagnetization state, magnetic domains will randomly orient to one of <111> easy axes. No magnetization and magnetostriction can be generalized along the rod axis. When a compressive stress is applied along the rod axis, the domain distribution is changed. Some magnetic domains will rotate or switch away from the [110] direction, such as from [111] to $[1\bar{1}1]$ direction. However, the materials experience a saturation magnetic field before experiments, so the initial magnetization state is not a real demagnetization state. And it is often obtained in practical application. To predict the initial state after the saturation process, the [110] oriented domains are also taken into consideration besides eight <111> easy directions. Thus, the remanent magnetostriction can be obtained compared the initial state with the demagnetization state as shown in Fig. 1b. Then the experimental process is predicted in the calculation program including the application order of pre-stress and the magnetic field.

The material parameters are taken from [2,12], where $\lambda_{100} = 90 \times 10^{-6}$, $\lambda_{111} = 1640 \times 10^{-6}$, $C_{11} = 1.41 \times 10^{11} N/m^2$, $C_{12} = 6.48 \times 10^{10} N/m^2$, $C_{12} = 6.48 \times 10^{10} N/m^2$, $C_{44} = 4.87 \times 10^{10}\ N/m^2$, $K_1 = 60000$ J/m^3. It fails to quantitatively predict the entire range of magnetoelastic response when a constant ω is adopted under different pre-stresses. Furthermore, it is difficult to accurately predict the field dependence of magnetization and magnetostriction simultaneously. Therefore, various values are used to describe the nonlinear behaviors of Terfenol-D when the applied stresses are different. Therefore, we assume that the energy distribution parameter ω is a function of prestress σ. It is interest and important to determine the value of ω in the proposed model. Several values of ω are selected to fit the magnetization curves under the application of different prestresses using the minimum variance estimate. And a linear stress dependence of energy distribution parameter ω is obtained as shown in Fig. 2. The linear fitting expression can be given as

$$\omega(\sigma) = A + B\frac{\sigma}{\sigma_0} \tag{10}$$

where $A = 15$ kJ/m^3, $B = 20$ kJ/m^3, are the fitting parameters, respectively. It can be shown in Fig. 3a that a good agreement between the theoretical simulation and the experimental results is obtained under a range of constant uniaxial compressive stresses 0, 2.5, 5, 10, 25, 50 MPa. To validate the linear relation, the magnetostriction curves under different compressive stresses are calculated with the same

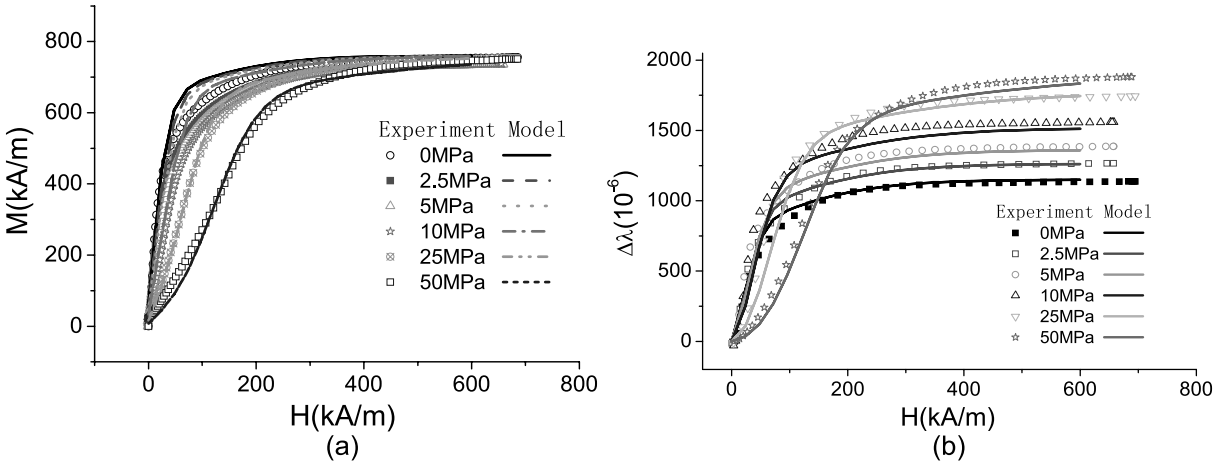

Fig. 3. The experimental results and theoretical predictions of a)magnetization and b)magnetostriction.

fitting expression as shown in Fig.3b. It is easy to conclude that the proposed model can predict well the nonlinear magnetoelastic behaviors.

It is clearly shown that the coercive field increases when stress increases. It is the result of the competition between the compressive stress and the magnetic field. At the same time, the maximum susceptibility decreases with the increase of the prestress. When the applied magnetic filed is high enough, the same saturation magnetization can be obtained even the stress is up to 50 MPa. The experimental measured value $M_s = 760$ kA/m is used in the calculation. In the magnetization process, the magnetic domains will firstly jump to one of $<111>$ family of easy directions that is closest to the field direction, and then rotate to the field direction slowly. Thus, the magnetization increases rapidly, then slowly to almost one saturation value. The amplitude of the magnetostriction is improved by the compressive stress, which proves that more non -180° domain rotations are induced. The maximum strain derivative $d\lambda/dH$ also increases firstly at a low stress, then decreases at a higher stress. All of the phenomena can be explained well by the ADR mechanism.

4. Conclusions

In the present work, magnetomechanical experiments have been carried out for the $<110>$ oriented polycrystral $Tb_{0.3}Dy_{0.7}Fe_{1.95}$ alloys. The ADR model has been modified to investigate the magnetoelastic response of GMMs by considering the volume fractions of domains along eight $<111>$ directions and the field direction. The linear fitting of the potential distribution parameter with the pre-stress was obtained from the magnetization curves under different compressive stresses, which was used to predict the magnetostriction behaviors for validity. Although the interaction of magnetic domains was not taken into consideration, it showed good quantitative agreement with the experimental results under a range of constant uniaxial compressive stresses up to 50 MPa.

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